Dynamic equations of metals

A. F. Andreev and D. I. Pushkarov[1)]

Institute of Physical Problems, Academy of Sciences of the USSR; Joint Institute for Nuclear Research
(Submitted 21 June 1985)
Zh. Eksp. Teor. **89**, 1883–1889 (November 1985)

A system of exact nonlinear dynamic equations for metals is derived. It consists of equations from the theory of elasticity, a kinetic equation for the conduction electrons, and Maxwell's equations. These equations are coupled to each other. The current density in a metal is calculated for an arbitrary time-dependent deformation.

In many problems in the theory of metals it is necessary to use a complete system of dynamic equations, consisting of equations from the theory of elasticity a Boltzmann kinetic equation for the conduction electrons, and Maxwell's equations for the electromagnetic field; these equations are coupled with each other. These equations have been derived by many investigators, most comprehensively and systematically by Kontorovich (Ref. 1 and the bibliography there). All of this work, however, has dealt with linearized equations. In addition to the obvious deficiency of linearized equations—they cannot be applied to nonlinear problems, e.g., that of the interaction of oscillation modes—there are some fundamental questions in this case which can be resolved systematically only by an exact nonlinear approach.

The most important question is a description of the dynamics of electrons in a time-varying deformed crystal lattice. In an exact description, all the physical quantities characterizing an electron, as in an undeformed crystal, are periodic functions of the quasimomentum, but the periods are functions of the coordinates and the time. The boundaries of the Brillouin zone in this case depend not only on the deformation at a given instant but also on the velocity of the lattice. If we expand all quantities in powers of the vectors lattice displacement, an apparent aperiodicity arises in the expressions for physical quantities (the energy, etc.); the aperiodicity is completely analogous to the secular terms in the theory of nonlinear oscillations. The kinetic equation becomes incompatible with the condition of periodicity of the distribution function. Kontorovich[1] overcame this difficulty by somewhat artificial methods: by transforming to a noninertial comoving coordinate system and by essentially postulating the form of the noninertial terms in the electron Hamiltonian. Nevertheless, we must not fail to point out that the final linearized equations found by Kontorovich are correct.

In the present paper we derive a complete system of exact nonlinear dynamic equations for metals. The only condition which must be satisfied for the applicability of these equations is that the properties of the lattice, the electron distribution function, and the electromagnetic field must be slowly varying functions of the coordinate and the time at the scale of interatomic distance and atomic times, respectively.

1. ENERGY AND HAMILTONIAN OF AN ELECTRON

Our problem includes a formulation of a nonlinear theory of elasticity. The formulation which is most convenient for our purposes is as follows (cf. Ref. 2): We denote by $\mathbf{a}_\alpha(\mathbf{r},t)$ ($\alpha = 1,2,3$) the primitive translation vectors of the crystal lattice at a given point $\mathbf{r}$ at a given time t. We introduce local reciprocal-lattice vectors $\mathbf{a}^\alpha(\mathbf{r},t)$, which satisfy the relations

$$\mathbf{a}_\alpha\mathbf{a}^\beta=\delta_\alpha{}^\beta, \quad a_{\alpha i}a_k{}^\alpha=\delta_{ik}. \tag{1}$$

We also introduce an invariant metric "tensor" $g_{\alpha\beta}=\mathbf{a}_\alpha\mathbf{a}_\beta$ and its inverse, by virtue of (1), the tensor $g^{\alpha\beta}$, which is equal by definition to $\mathbf{a}^\alpha\mathbf{a}^\beta$.

We denote by N^α the integer coordinates of the lattice ions, expressed in units of the corresponding periodicity vectors a_α, so that the differential coordinates $d\mathbf{r}$, which is a physical infinitesimal (i.e., large in comparison with the lattice constant but small in comparison with the distance over which the properties of the lattice vary substantially), at a given time can be written

$$d\mathbf{r}=\mathbf{a}_\alpha dN^\alpha. \tag{2}$$

With $dN^\alpha=0$, the time dependence of the coordinates $\mathbf{r}$ is evidently determined by the lattice velocity $\mathbf{v}(\mathbf{r},t)$: $d\mathbf{r}=\mathbf{v}dt$. In general, we can therefore replace (2) by

$$d\mathbf{r}=\mathbf{a}_\alpha dN^\alpha+\mathbf{v}dt. \tag{3}$$

We wish to stress that the quantities $N^\alpha=N^\alpha(\mathbf{r},t)$ are single-valued functions of the coordinates and the time only if we are dealing with a deformation of the crystal in the absence of dislocations. It is this case with which we will be concerned below.

Using (1), we find from (3)

$$dN^\alpha=\mathbf{a}^\alpha d\mathbf{r}-\mathbf{a}^\alpha\mathbf{v}dt,$$

so that

$$\mathbf{a}^\alpha=\nabla N^\alpha, \quad \mathbf{v}=-\mathbf{a}_\alpha\dot{N}^\alpha. \tag{4}$$

The three functions $N^\alpha(\mathbf{r},t)$ thus completely determine the configuration and velocity of the lattice. The use of these functions (in place of the three components of the displacement vector) as unknown functions is more convenient in

 0038-5646/85/111087-04$04.00

formulating a nonlinear theory of elasticity.

The following expressions for the time derivatives of the translations vectors are consequences of (1) and (4):

$$\dot{\mathbf{a}}_\alpha = -(\mathbf{v}\nabla)\mathbf{a}_\alpha + (\mathbf{a}_\alpha\nabla)\mathbf{v},$$
$$\dot{a}_i^\alpha = -(\mathbf{v}\nabla)a_i^\alpha - a_k^\alpha \partial v_k/\partial x_i. \tag{5}$$

The lattice density is

$$\rho_l = Mg^{-1/2}, \tag{6}$$

where M is the mass of the ions in a unit cell, and $g = \det g_{\alpha\beta}$. Using the identity $dg = -gg_{\alpha\beta}dg^{\alpha\beta}$, we easily see that a continuity equation follows automatically from (4) and (5):

$$\dot{\rho}_l + \operatorname{div}\rho_l\mathbf{v} = 0. \tag{7}$$

Before we determine the Hamitonian H which appears in the Boltzmann equation and the energy $\tilde{\varepsilon}$ of an electron, we note the following. In this theory which we are constructing we are assuming that the quantities a_α and $\mathbf{v}$ vary slowly in space and time, and we are expanding in derivatives of these quantities. In the expressions for H and $\tilde{\varepsilon}$ it is then sufficient to retain only terms of lowest order. In this approximation, H and $\tilde{\varepsilon}$ are the same as their values in a lattice with constant but otherwise arbitrary vectors $\mathbf{a}_\alpha$ and $\mathbf{v}$, i.e., in a crystal which is uniformly deformed (periodic) and in uniform motion.

We first assume $\mathbf{v} = 0$. The wave function of an electron which belongs to a definite energy band and which is executing a semiclassical motion must be treated[3] as a function of the discrete coordinates N^α:

$$\psi(N^\alpha, t) \propto \exp\{(i/\hbar)S_0(N^\alpha, t)\},$$

where S_0 is the classical action. In the immobile periodic lattice with which we are concerned at this point, the Hamiltonian is known to be equal to the energy $\varepsilon = \varepsilon(k_\alpha, g^{\alpha\beta})$, which is a periodic function of the components of the invariant quasimoment k_α with periods $2\pi\hbar$ and which depends on the invariant characteristics of the unit cell, determined by the quantities $g^{\alpha\beta}$. In this case the derivatives of the action are

$$(\partial S_0/\partial t)_{N^\alpha} = -\varepsilon(k_\alpha, g^{\alpha\beta}), \qquad (\partial S_0/\partial N^\alpha)_t = k_\alpha. \tag{8}$$

At $\mathbf{v} \neq 0$ the action S can be found by means of the well-known laws[4] by which the one-electron wave functions transform under Galilean transformations. We write

$$S = S_0 + m\mathbf{v}\mathbf{r} - mv^2t/2, \tag{9}$$

where m is the mass of a free electron.

Differentiating S, we find the quasimomentum

$$\mathbf{p} = (\partial S/\partial \mathbf{r})_t = k_\alpha \nabla N^\alpha + m\mathbf{v} = \mathbf{a}^\alpha k_\alpha + m\mathbf{v},$$

and thus $k_\alpha = \mathbf{a}_\alpha(\mathbf{p} - m\mathbf{v})$ and the Hamiltonian

$$H(\mathbf{p}, \mathbf{r}, t) = -(\partial S/\partial t)_{\mathbf{r}} = -k_\alpha \dot{N}^\alpha + \varepsilon + mv^2/2 = \varepsilon + \mathbf{p}\mathbf{v} - mv^2/2. \tag{10}$$

Here $\varepsilon = \varepsilon(\mathbf{a}_\alpha(\mathbf{p} - m\mathbf{v}), g^{\alpha\beta})$ is a periodic function of the quasimomentum $\mathbf{p}$ with periods $2\pi\hbar\mathbf{a}^\alpha$ determined by the local values of the reciprocal-lattice vectors.

The electron energy $\tilde{\varepsilon}$ is given directly by the Galilean transformation equation

$$\tilde{\varepsilon} = mv^2/2 + \mathbf{v}\mathbf{p}_0 + \varepsilon,$$

where $\mathbf{p}_0$ is the average value of the electron momentum in the system with $\mathbf{v} = 0$. This value is equal to the product of the mass of the electron and its velocity, which is equal to the derivative of the Hamiltonian with respect to the quasimomentum. As a result we have $\mathbf{p}_0 = m\partial\varepsilon/\partial\mathbf{p}$ and thus

$$\tilde{\varepsilon} = \varepsilon + m\mathbf{v}\partial\varepsilon/\partial\mathbf{p} + mv^2/2. \tag{11}$$

The energy $\tilde{\varepsilon}$ is a periodic function of the quasimomentum $\mathbf{p}$.

The Hamiltonian (10) is not periodic. It is this Hamiltonian which makes the kinetic equation compatible with the condition of a periodic distribution function. We write a kinetic equation for the electron distribution function $f(\mathbf{p}, \mathbf{r}, t)$, taking into account the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$:

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{r}}\frac{\partial H}{\partial \mathbf{p}} - \frac{\partial f}{\partial \mathbf{p}}\frac{\partial H}{\partial \mathbf{r}} - \frac{\partial f}{\partial \mathbf{p}}\left\{e\mathbf{E} + \frac{e}{c}\left[\frac{\partial H}{\partial \mathbf{p}}\mathbf{B}\right]\right\} = If, \tag{12}$$

where I is a collision operator.

The condition for periodicity of the distribution function

$$f(\mathbf{p}, \mathbf{r}, t) = f(\mathbf{p} + 2\pi\hbar\mathbf{a}^\alpha(\mathbf{r}, t), \mathbf{r}, t)$$

is compatible with Eq. (12), since the function on the right side of the last expression satisfies the same equation as $f(\mathbf{p}, \mathbf{r}, t)$, i.e., Eq. (12), as is easily verified.

Aperiodic quantities can be eliminated from consideration if we replace $\mathbf{p}$ by the quantities k_α as the arguments of f. The function $f(k_\alpha, \mathbf{r}, t)$ satisfies the equation

$$\frac{\partial f}{\partial t} + (\mathbf{v}\nabla)f + \mathbf{a}_\alpha\nabla f\frac{\partial \varepsilon}{\partial k_\alpha} - \frac{\partial f}{\partial k_\alpha}\left\{\mathbf{a}_\alpha\frac{\partial \varepsilon}{\partial \mathbf{r}} + \mathbf{a}_\alpha e\mathbf{E}' + \frac{e}{c}\frac{\partial \varepsilon}{\partial k_\beta}\mathbf{B}'[\mathbf{a}_\alpha\mathbf{a}_\beta]\right\} = If, \tag{13}$$

where

$$\mathbf{B}' = \mathbf{B} - \frac{mc}{e}\operatorname{rot}\mathbf{v}, \qquad e\mathbf{E}' = e\mathbf{E} + m\dot{\mathbf{v}} + m\nabla\frac{v^2}{2} + \frac{e}{c}[\mathbf{v}\mathbf{B}'],$$

and all quantities are differentiated with respect to the coordinates and the time at constant k_α. All the quantities which appear in Eq. (13) are periodic in k_α.

To expand the equations in powers of the strain, we should set $N^\alpha = N_0^\alpha - u^\alpha$, where $N_0^\alpha = \mathbf{a}_0^\alpha\mathbf{r}$, and the $\mathbf{a}_0^\alpha$ are the reciprocal-lattice vectors of the undeformed crystal. The quantity $\mathbf{u} = \mathbf{a}_{0\alpha}u^\alpha$ is then the displacement vector of the ordinary theory of elasticity. Differentiating N^α with respect to the coordinates and the time, we find the deviations, linear in $\mathbf{u}$, of all quantities from their values in the undeformed crystal:

$$\delta\mathbf{a}^\alpha = -\delta\mathbf{a}_\alpha = -a_{0k}^\alpha\partial u_k/\partial\mathbf{r}, \qquad \mathbf{v} = \dot{\mathbf{u}},$$
$$\delta g_{\alpha\beta} = -\delta g^{\alpha\beta} = 2a_{0i}^\alpha a_{0k}^\beta u_{ik},$$

where u_{ik} is the strain tensor. Linearizing (10), we find

$$H = \varepsilon_0(\mathbf{p}) + \left(\lambda_{ik}(\mathbf{p}) + p_i\frac{\partial \varepsilon_0}{\partial p_k}\right)\frac{\partial u_k}{\partial x_i} + \dot{\mathbf{u}}\left(\mathbf{p} - m\frac{\partial \varepsilon_0}{\partial \mathbf{p}}\right). \tag{14}$$

Here $\varepsilon_0(\mathbf{p})$ and $\lambda_{ik}(\mathbf{p})$ are, respectively, the values of the function $\varepsilon(\mathbf{a}_{0\alpha}\mathbf{p}, g^{\alpha\beta})$ and of its derivatives with respect to u_{ik} at $u_{ik} = 0$. Expression (14) was derived by Landau and then by Kontorovich.[1]

2. DERIVATION OF THE DYNAMIC EQUATIONS OF THE LATTICE FROM CONSERVATION LAWS

The energy density and momentum density of a metal are given by the following expressions in the quasineutral approximation, in which we should ignore the displacement current in Maxwell's equations:

$$\mathscr{E}=\frac{\rho_l v^2}{2}+\mathscr{E}_l(g^{\alpha\beta})+\langle\tilde{\varepsilon}f\rangle+\frac{B^2}{8\pi}, \tag{15}$$

$$\mathbf{P}=\rho_l\mathbf{v}+m\left\langle\frac{\partial H}{\partial\mathbf{p}}f\right\rangle=\rho\mathbf{v}+m\left\langle\frac{\partial\varepsilon}{\partial\mathbf{p}}f\right\rangle, \tag{16}$$

where ρ_l, H, and $\tilde{\varepsilon}$ are given by (6), (10), and (11); $\mathscr{E}_l(g^{\alpha\beta})$ is the elastic energy of the immobile lattice; and

$$\langle F\rangle\equiv\int 2\frac{d^3p}{(2\pi\hbar)^3}F.$$

Here the integration is over the Brillouin zone which corresponds to the local vectors $\mathbf{a}^\alpha$. The boundaries of the zone may be regarded equally well as displaced by $m\mathbf{v}$ or as not displaced, since all the quantities to be integrated in (15) and (16) are periodic in $\mathbf{p}$.

The dynamic equations of the lattice are embodied in the momentum conservation equation

$$\dot{P}_i+\partial\Pi_{ik}/\partial x_k=0, \tag{17}$$

where Π_{ik} is the symmetric momentum flux tensor. As usual, the latter is determined by the condition that conservation of the total energy, i.e., an equation of the form

$$\dot{\mathscr{E}}+\operatorname{div}\mathbf{Q}=0 \tag{18}$$

with some $\mathbf{Q}$, follows automatically from Eq. (17), kinetic equation (12), and Maxwell's equations. To actually find this condition we must differentiate (15) with respect to the time and express the time derivatives of all quantities in terms of spatial derivatives.

There is an important circumstance to be kept in mind here. The integration over $\mathbf{p}$ is over a region which depends on the coordinates and the time. The integration operations $\langle\ldots\rangle$ thus generally does not commute with differentiation with respect to $\mathbf{r}$ and t. The commutators are defined by certain integrals of the distribution function along the boundary of the Brillouin zone. Similar integrals arise when we eliminate $\dot{f}$ with the help of Eq. (12) after integrating by parts (over $\mathbf{p}$) in the terms which are proportional to the product of $\partial f/\partial\mathbf{p}$ and the aperiodic factor $\partial H/\partial\mathbf{r}$. Each of these integrals vanishes for metals having a closed, purely electron Fermi surface, since in this case there is a rather large region near the boundaries of the Brillouin zone in which the condition $f=0$ holds. If there are open or hole surfaces, the situation is different, but in general all the surface integrals will cancel each other out. The simplest way to see this is to replace the function $f(\mathbf{p}, r, t)$ and Eq. (12) by Eq. (13) for the function $f(k_\alpha, \mathbf{r}, t)$ with universal periods, to carry out all the necessary differentiations and integrations by parts, and to then go back to the original representation in the expressions for the fluxes $\mathbf{Q}$ and Π_{ik}. When the calculations are carried out in this order, surface integrals do not appear at all. Correct expressions for $\mathbf{Q}$ and Π_{ik} in the general case can be found by working in the $\mathbf{p}$ representation and by bearing in mind the simplest case, that of closed electron Fermi surfaces.

We differentiate the energy in (15) with respect to the time, using identity (7) and Maxwell's equations

$$\operatorname{div}\mathbf{B}=0,\quad \operatorname{rot}\mathbf{B}=\frac{4\pi}{c}\mathbf{j},\quad \operatorname{rot}\mathbf{E}=-\frac{1}{c}\dot{\mathbf{B}}, \tag{19}$$

where $\mathbf{j}$ is the current density; by virtue of the condition of electrical neutrality, the current density is

$$\mathbf{j}=-e\langle f\partial\varepsilon/\partial\mathbf{p}\rangle. \tag{20}$$

We find

$$\dot{\mathscr{E}}=\rho_l\mathbf{v}\dot{\mathbf{v}}-\frac{v^2}{2}\operatorname{div}\rho_l\mathbf{v}+\dot{\mathscr{E}}_l+m\mathbf{v}\dot{\mathbf{v}}\langle f\rangle+m\mathbf{v}\left\langle\frac{\partial\dot{\varepsilon}}{\partial\mathbf{p}}f\right\rangle$$
$$+m\dot{\mathbf{v}}\left\langle\frac{\partial\varepsilon}{\partial\mathbf{p}}f\right\rangle+\langle\dot{\varepsilon}f\rangle+\langle\tilde{\varepsilon}\dot{f}\rangle-\operatorname{div}\mathbf{S}-\mathbf{E}\mathbf{j}, \tag{21}$$

where $\mathbf{S}$ is the Poynting vector. Since $\varepsilon=\varepsilon(k_\alpha, g^{\alpha\beta})$, where $k_\alpha=\mathbf{a}_\alpha(\mathbf{p}-m\mathbf{v})$, we have the following equation with $\mathbf{p}=\text{const}$:

$$d\varepsilon=(\partial\varepsilon/\partial k_\alpha)\{(\mathbf{p}-m\mathbf{v})d\mathbf{a}_\alpha-m\mathbf{a}_\alpha d\mathbf{v}\}+2\lambda_{\alpha\beta}\mathbf{a}^\alpha d\mathbf{a}^\beta, \tag{22}$$

where

$$\lambda_{\alpha\beta}(\mathbf{p})=(\partial\varepsilon/\partial g^{\alpha\beta})_{k_\alpha}.$$

Using identity (22), we find

$$\dot{\varepsilon}=-(\mathbf{v}\nabla)\varepsilon-m\frac{\partial\varepsilon}{\partial\mathbf{p}}\dot{\mathbf{v}}$$
$$+\frac{\partial v_i}{\partial x_k}\left(p_i\frac{\partial\varepsilon}{\partial p_k}-mv_i\frac{\partial\varepsilon}{\partial p_k}-mv_k\frac{\partial\varepsilon}{\partial p_i}-2\lambda_{\alpha\beta}a_i^\alpha a_k^\beta\right). \tag{23}$$

We can transform the time derivative of the elastic energy of the lattice analogously:

$$\dot{\mathscr{E}}_i=-(\mathbf{v}\nabla)\mathscr{E}_l-2\sigma_{\alpha\beta}a_i^\alpha a_k^\beta\partial v_i/\partial x_k,$$

where $\sigma_{\alpha\beta}=\partial\mathscr{E}_l/\partial g^{\alpha\beta}$.

Substitution into (21) reveals that the derivative $\dot{\mathbf{v}}$ appears in the expression for $\dot{\mathscr{E}}$ with the same coefficient as in the expression for $\mathbf{v}\dot{\mathbf{P}}$, which can be found by differentiating (16). It is thus a simple matter to eliminate $\dot{\mathbf{v}}$. When we then express $\dot{\mathbf{P}}$ and $\dot{f}$ in terms of spatial derivatives with the help of Eqs. (17) and (12), respectively, we find, after some straightforward manipulations,

$$\dot{\mathscr{E}}+\operatorname{div}\left\{\mathbf{v}\mathscr{E}_l+\left\langle\varepsilon\frac{\partial H}{\partial\mathbf{p}}f\right\rangle-\frac{v^2}{2}\mathbf{P}+v_k(\Pi_{ik}+t_{ik})+\mathbf{S}\right\}$$
$$=\frac{\partial v_i}{\partial x_k}\{\Pi_{ik}+t_{ik}-\rho v_i v_k+\frac{m}{e}(v_i j_k+v_k j_i)-2\sigma_{\alpha\beta}a_i^\alpha a_k^\beta+\mathscr{E}_l\delta_{ik}$$
$$-2\langle\lambda_{\alpha\beta}f\rangle a_i^\alpha a_k^\beta\}+\langle\varepsilon I f\rangle, \tag{24}$$

where $t_{ik}=(1/4\pi)(B_iB_k-B^2\delta_{ik}/2)$ is the Maxwell tensor of the magnetic field.

If the operator I represents elastic collisions of electrons with lattice defects or with each other, we would have $\langle\varepsilon If\rangle=0$. When electron-phonon collisions occur, the electron energy is not conserved, and phonons must be taken into consideration. All the equations above, including kinetic equation (12), actually apply to any quasiparticles in crystals, in particular, to phonons. To take phonons into account, it is sufficient to substitute the sum of the corresponding integrals of the electron and phonon distribution

functions into the equations containing integrals of the electron distribution function. It must be kept in mind, of course, that for phonons we have $m = e = 0$. The last term in (24) in this case is equal to the sum

$$\langle \varepsilon I f\rangle + \langle \varepsilon_{ph} I_{ph} f_{ph}\rangle,$$

which vanishes by virtue of the conservations of the total energy of the quasiparticles. Comparing (24) and (18), we can determine the unknown fluxes:

$$\mathbf{Q}=\mathbf{v}\mathscr{E}_l+\left\langle \varepsilon \frac{\partial H}{\partial \mathbf{p}} f\right\rangle - \frac{v^2}{2}\mathbf{P}+v_k(\Pi_{ik}+t_{ik})+\mathbf{S},$$

$$\Pi_{ik}=-t_{ik}+\rho v_i v_k+2\sigma_{\alpha\beta}a_i^{\alpha}a_k^{\beta}-\mathscr{E}_l\delta_{ik}$$

$$+2\langle \lambda_{\alpha\beta} f\rangle a_i^{\alpha}a_k^{\beta}-\frac{m}{e}(v_i j_k+v_k j_i). \qquad (25)$$

The complete system of dynamic equations of the metal consists of kinetic equation (12), momentum conservation equation (17), Maxwell's equations (19), and the quasineutrality condition. The latter is written in the form $(Z/M)\rho_l = \langle f\rangle$, where Z is the total charge of the ions of a unit cell. Transforming to an integral over the invariant quasimomentum k_α, we find $\langle f\rangle = g^{-1/2}\langle f\rangle_k$, where

$$\langle F\rangle_k=\int 2\frac{d^3k}{(2\pi\hbar)^3}F.$$

Substituting in expression (6) for the lattice density, we can rewrite the quasineutrality condition in the more convenient form

$$\langle f\rangle_k=Z=\text{const}. \qquad (26)$$

3. CURRENT DENSITY IN A DEFORMED METAL

When the electron mean free path l is vanishingly small, the solution of the kinetic equation is a local equilibrium function $f=f_0(\varepsilon-\mu(\mathbf{r},t))$, where $\mu(\mathbf{r},t)$ is the chemical potential, and f_0 is the Fermi function (for simplicity we assume $T\equiv 0$). The dependence of μ on the coordinates and the time is determined by the quasineutrality condition (26).

By virtue of the inversion symmetry of a uniformly deformed lattice, current density (20) vanishes in a state with $f=f_0$; vanishing along with it is the last term in expression (25) for Π_{ik}. The next-to-last term can be transformed in the following way. We evaluate the derivative of the electron energy density with respect to $g^{\alpha\beta}$:

$$\mathscr{E}_{el}=\langle \varepsilon f_0\rangle = g^{-1/2}\langle \varepsilon f_0\rangle_k.$$

We have

$$\frac{\partial \mathscr{E}_{el}}{\partial g^{\alpha\beta}}=(\mathscr{E}_{el}g^{1/2})\frac{\partial g^{-1/2}}{\partial g^{\alpha\beta}}+g^{-1/2}\left\{\langle \lambda_{\alpha\beta}f_0\rangle_k+\left\langle \varepsilon\frac{\partial f_0}{\partial \varepsilon}\frac{\partial(\varepsilon-\mu)}{\partial g^{\alpha\beta}}\right\rangle_k\right\}.$$

The last term in braces vanishes by virtue of the δ-function nature of the derivative $\partial f_0/\partial\varepsilon$ and condition (26). As a result we find

$$\partial\mathscr{E}_{el}/\partial g^{\alpha\beta}=\mathscr{E}_{el}g_{\alpha\beta}/2+\langle \lambda_{\alpha\beta}f_0\rangle.$$

We see that the next-to-last term in the expression for Π_{ik} causes a renormalization of the energy, $\mathscr{E}_l\to\mathscr{E}_l+\mathscr{E}_{el}$, in this case, and Eq. (17) converts (as it should) into the usual equation of the nonlinear theory of elasticity.

To calculate the current density we need to go to the next higher order in the approximation in l and to take into account the deviation δf of the distribution function from the locally equilibrium value f_0. It is convenient to calculate δf from kinetic equation (13). Substituting $f=f_0+\delta f$ into this equation, and retaining δf only in the collision integral and in the term with the field $\mathbf{B}'$, we find

$$-\mathbf{a}_\alpha\frac{\partial f_0}{\partial k_\alpha}(e\mathbf{E}'+\nabla\mu)=I\delta f+\frac{\partial\delta f}{\partial k_\alpha}\frac{\partial\varepsilon}{\partial k_\beta}\frac{e}{c}\mathbf{B}'[\mathbf{a}_\alpha\mathbf{a}_\beta]. \qquad (27)$$

We have omitted from the left side of this equation terms which are even under the inversion $k_\alpha\to -k_\alpha$. These terms would be important in a calculation of the electron viscosity, but they do not contribute to the expression for the current. It follows immediately from the form of Eq. (27) that the current density is

$$\mathbf{j}=\hat{\sigma}\left(\mathbf{B}-\frac{mc}{e}\operatorname{rot}\mathbf{v}\right)\left\{\mathbf{E}+\frac{1}{c}[\mathbf{vB}]+\frac{1}{e}\nabla\mu+\frac{m}{e}\frac{d\mathbf{v}}{dt}\right\},$$

where $d\mathbf{v}/dt=\dot{\mathbf{v}}+(\mathbf{v}\nabla)\mathbf{v}$, and $\hat{\sigma}(\mathbf{B})$ is the conductivity tensor of a uniformly deformed immobile metal. The current density is Galilean-invariant, as it should be in a neutral system.

We wish to thank Ya. B. Zel'dovich, M. I. Kaganov, L. P. Pitaevskiĭ, and Yu. V. Sharvin for a useful discussion of this study.

[1)] Institute of Solid State Physics, Bulgarian Academy of Sciences, Sofia, Bulgaria.

[1]V. M. Kontorovich, Usp. Fiz. Nauk **142**, 265 (1984) [Sov. Phys. Usp. **27**, 134 (1984)]; Zh. Eksp. Teor. Fiz. **45**, 1638 (1963) [Sov. Phys. JETP **18**, 1125 (1964)]; Zh. Eksp. Teor. Fiz. **59**, 2116 (1970) [Sov. Phys. JETP **32**, 1146 (1971)].
[2]A. F. Andreev and M. Yu. Kagan, Zh. Eksp. Teor. Fiz. **86**, 546 (1984) [Sov. Phys. JETP **59**, 318 (1984)].
[3]I. M. Lifshitz, M. Ya. Azbel', and M. I. Kaganov, Élektronnaya teoriya metallov (Electronic Theory of Metals), Nauka, Moscow, 1971, p. 13.
[4]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika, Nauka, Moscow, 1974 (Quantum Mechanics, Pergamon Press, Oxford) (Problem in §17).

Translated by Dave Parsons