# Superconductivity of the twinning plane of niobium and a topological phase transition in a two-dimensional superconducting system

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The superconductivity of double crystals with boundary orientations close to the twinning orientation is studied. It is found that the superconductivity associated with an isolated twinning plane arises at a higher temperature than in a bulk sample in zero magnetic field and at temperatures below the bulk critical temperature in large magnetic fields. For the samples studied, the increase in the zero-field critical temperature is 0.11 K, and the critical field exceeds the bulk critical field  $H_{c2}$  by 350 Oe at  $T \approx 3$  K. The (H,T) phase diagram of the twinning-plane superconductivity in niobium is measured in the temperature interval 2.5–10 K. A phase transition of the Berezinskiĭ-Kosterlitz-Thouless type is detected from the change in the behavior of the persistent electrical currents in the two-dimensional superconducting system along the twinning plane. The critical temperature  $T_{ci}$  of the topological phase transition is 0.016 K higher than critical temperature  $T_{c0}$  for bulk superconductivity.

## INTRODUCTION

The phase diagram of the twinning-plane superconductivity<sup>1</sup> has been studied in detail<sup>2,3</sup> for the particular case of tin, a type-I superconductor. It was established that in the presence of a magnetic field the transition to the state of localized superconductivity (LS) occurs by a first-order thermodynamic phase transition at a magnetic field  $H_d(T)$ ; at the transition from the normal to the LS state there is a region of supercooled metastable states with an absolute instability boundary  $H_M(T)$ . At the transition from the LS to the normal state the metastability is absent because there are normal regions in the interior of the sample, far from the twinning plane (i.e., at distances much greater than the coherence length  $\xi_0$ ), and nucleation of the new phase is not required.<sup>2</sup> A second characteristic feature of the phase diagram for the twinning-plane superconductivity of type-I superconductors is a lower bound on the temperature range in which localized superconductivity exists; this feature is reflected in the experiments on tin.<sup>3</sup>

Twinning-plane superconductivity has been detected in twins of tin, indium, rhenium, and thallium. In twinned samples of aluminum and lead the effect has not been detected at temperatures above the bulk critical temperature. The relevant data for the materials mentioned are given in Table I. The last row gives the data for niobium, which is the subject of this paper.

It has been suggested<sup>1,2</sup> that a twinning plane forms a new two-dimensional metal in the twinned crystal, with its own electrons and phonons; the temperature  $T_c$  of its transition to the LS state in zero field is found to be higher than the temperature  $T_{c0}$  at which the bulk superconductivity arises if the spectrum of the two-dimensional phonons of the twinning plane is "softer" than the spectrum of the three-dimensional phonons. Indeed, of the metals mentioned above, only in aluminum and lead does the twinning occur along a closepacked plane of the crystal, with the result that the upper boundary of the phonon spectrum of the twinning plane coincides with that of the bulk, and  $T_c$  is less than  $T_{c0}$ .

Niobium is an interesting object of study because, first, its crystal lattice has a body-centered cubic symmetry, a type not encountered in the metals studied previously, and, second, it is a type-II superconductor, and the thermodynamics of the transition to the LS state of its twinning plane may differ from that of the materials studied previously. We note that the (112) orientation of the twinning plane in niobium suggests that the spectrum of two-dimensional phonons should be softer than that of the three-dimensional phonons.

The influence of twinning of the superconductivity of niobium has been investigated in a number of studies (e.g., Refs. 4 and 5), but the technique used to twin the samples in those studies (by means of plastic deformation at liquid-helium temperatures) leaves some doubt as to whether the observed effects are in fact due to the twinning. The existence of other physical mechanisms, involving lattice defects, which are capable of influencing the superconductivity of niobium is suggested by the anomalously high defect mobility observed in niobium samples at temperatures considerably below the Debye temperature.<sup>6</sup>

	Lattice type	Twinning plane	Т <sub>с0</sub> , К *	$ \begin{vmatrix} \Delta T_c = T_c - T_{c0}, \\ \mathbf{K} \end{vmatrix} $
Sn In Re Tl Al Pb Nb	Tetr. Tetr. Hex. fcc fcc bcc	$(301) \\ (101) \\ (1012) \\ (1012) \\ (1112) \\ (111) \\ (111) \\ (112) $	3,72 3,41 1.70 2,38 1,18 7,20 9.3	0,04 0,01 0,006 0,003 - - 0,11

\*Rounded-off data from literature.

Experiments on niobium double crystals grown from two seeds were described in Ref. 7, but the properties of the twinned crystals were not studied in detail.

## SAMPLES AND EXPERIMENTAL TECHNIQUES

In this paper we study niobium double crystals grown from two seeds and having orientations close to the twinning orientation (the samples were taken from the series described in Ref. 7). The method of growing the double crystals is described in detail in Refs. 8 and 9. The growth technique yielded samples with an orientation of the boundary between crystallites that was close to the specified orientation; the boundary was planar, at least over regions with linear dimensions of  $\sim 10 \,\mu$ m or more, as was checked by electron-microscope studies at resolutions down to 30 Å.

It must be noted that in studies of specials kinds of boundaries, of which twinning planes are a particular case, the objects of study are usually<sup>10,11</sup> double crystals in which the boundaries deviate from an exact orientation of the lattice of coincident angles (the coherent plane) by an angle in the range  $\sim 5-6^{\circ}$ . Therefore, we can regard the double-crystal niobium samples used in the present study, which has a 3–5° angular deviation of the boundary from the special position  $\Sigma = 3$ , as twins. The position of the coherent twinning plane in niobium is described by the crystallographic indices (112) in the bcc lattice of niobium.

The double crystals studied has a resistivity ratio  $\rho_{300 \text{ K}} / \rho_{4.2 \text{ K}} \sim 200\text{--}300$ , which for niobium indicates a comparatively high purity of the initial material and a good crystal structure of the samples. Single and double crystals of the necessary size and shape were cut from a double-crystal bar 20 mm in diameter and 60 mm long on an electricaldischarge machine. The twinning plane in the double crystals lay approximately in the centers of the samples. The damaged surface layer of the cut samples was removed by chemical polishing in a mixture of nitric and hydrofluoric acids. To suppress the surface superconductivity, which could affect the results, the prepared samples were oxidized by heating to 300 °C in air for 3-5 min until a slight straw color appeared. In this way we were able to reduce the critical current for the surface superconductivity to  $\sim 3 \cdot 10^{-1}$ A. The value of the critical surface current did not depend on the direction of the measuring current and changed only slightly as the magnetic field was increased.

The first group of experiments on the superconductivity of niobium twins was done by a technique developed for studying critical currents in double crystals.<sup>7</sup> A cylindrical sample 2 mm in diameter and 15 mm long was mounted in such a way that the transport current I flowed along the cylinder—along the twinning plane in the double crystals. The external magnetic field was oriented parallel to the twinning plane and perpendicular to the current I. A goniometer head permitted measurements for magnetic-field directions not parallel to the twinning plane as well. For control experiments we prepared both single-crystal samples and samples in which the boundary between the crystals was not a twinning plane. To exclude effects due to the anisotropy<sup>7</sup> of the upper critical field  $H_{c2}$ , all the samples had the [111] crystallographic direction oriented perpendicular to the long axis of the sample (along which the current I flowed) and, accordingly, parallel to the external magnetic field H. The voltage across the potential contacts of the sample was measured to an accuracy of  $1 \cdot 10^{-9}$  V, where the criterion for the onset of resistivity was  $5 \cdot 10^{-9}$  V. For the given geometric dimensions of the samples, we were thus able to detect superconductivity if the critical superconducting current exceeded  $\sim 10^{-2}$  A. The temperature of the samples was measured by a KGG brand graduated germanium resistance thermometer.

The second group of experiments was done on the same samples by a magnetometric method, which has a number of advantages in studies of this sort. The most important advantage is that the resistance of the sample under conditions of weak pinning becomes nonzero somewhat before the superconductivity vanishes. This circumstance is especially important in studying two-dimensional or quasi-two-dimensional objects, where, because of the large demagnetizing factor, even insignificant misalignments of the directions of the plane of the sample and the magnetic field give rise to vortices which perforate the sample. The motion of these vortices is accompanied by dissipation of energy and thus gives rise to resistance in the sample. The extra diamagnetism of the sample (above that of the normal state) vanishes simultaneously with the total disappearance of the superconductivity. Such a method has been used previously to study the phase diagram of the twinning-plane superconductivity in tin.<sup>3</sup> A detailed description of the magnetometer is found in Refs. 12 and 13. The sensitivity of the magnetometer permitted detection of the presence of superconducting regions in the sample by the Meissner effect in a magnetic field  $H \sim 1$  Oe if the total volume of these regions exceeded  $\sim 10^{-8}$ -10<sup>-9</sup> cm<sup>3</sup> (this estimate was made under the assumption that the magnetic susceptibility of the bulk superconductor is  $-1/4\pi$ ). Measurements of the magnetic moment also permit estimation of the value of the electrical current if one knows the path along which it is flowing. Converted into terms of a current flowing through a circuit with the dimensions of the transverse cross section of the sample (i.e.,  $1.5 \times 1.5$  mm), the noise level of the magnetometer comes to  $5 \cdot 10^{-7}$  A. We note that magnetometric measurements of the critical current of the twinning plane are possible only in the case when the twinning plane is inclined to the field H. The measurements of the sample temperature relative to the reference point ( $T_{c0}$  of niobium) were accurate to  $\sim 0.001$  K. The design of the magnetometer permitted measurements in magnetic fields up to 200-300 Oe.

#### EXPERIMENTAL RESULTS

In the first group of experiments we measured the dependence of the critical current  $I_c$  on the magnetic field H in the temperature range ~ 2.5–8.5 K. Examples of the experimental  $I_c$  (H) curves at T = 3.67 K are shown in Fig. 1. The upper part of Fig. 1 shows the results of measurements on the twin, while the lower part shows the results for a singlecrystal control sample. The characteristic sharp drop on the measured  $I_c$  (H) curves permits determination of the upper



FIG. 1. Examples of the results of our experimental measurements of the critical current at T = 3.67 K; a) twins, b) single crystal.

critical magnetic field  $H_{c2}$  to relatively high accuracy. The results of measurements at various temperatures on singlecrystal and double-crystal niobium samples are shown in Fig. 2 by the points and line representing  $H_{c2}$ .

When the object of study was a twinned double crystal and the external magnetic field lay in the twinning plane, it turned out that the critical current in an external magnetic field  $H_{c2}$  did not fall to the value of the current for surface superconductivity but had a substantially larger value. The sample resistance measured at a small current through the



FIG. 2. Phase diagram for twinning-plane superconductivity of niobium according to measurements of the critical current: •) upper critical field  $H_{c2}$  for bulk superconductivity of niobium; O) critical field  $H_{d}^{*}$  for twinning-plane superconductivity; I) region of bulk superconductivity; II) region of normal state; III) region in which twinning-plane superconductivity is observed.

sample (but larger than the surface-superconductivity current) remained zero as the magnetic field was increased all the way to  $H_d^*$ . It must be noted that for magnetic fields between  $H_{c2}$  and  $H_d^*$  the critical current for twinning-plane superconductivity depends on the direction of the measuring current *I*, apparently because of residual structural imperfections of the twinning planes of the samples. In a magnetic field  $H_d^*$  the critical current *I*<sub>c</sub> becomes identical for opposite directions of the current *I*. The values of  $H_d^*$  measured at various temperatures for one of the niobium twins which we investigated are also shown in Fig. 2. A deviation of the direction of the external magnetic field from the twinning plane by an angle of ~ 1° completely depresses the critical current for twinning-plane superconductivity in magnetic fields greater than  $H_{c2}$ .<sup>7</sup>

The lines constructed in Fig. 2 divided the (H,T) phase plane into three regions (without allowance for the surface superconductivity): Region I, lying below the  $H_{c2}$  line, is the existence region of bulk superconductivity in niobium, region II, above the  $H_d^*$  line, is the region of the normal state, and region III, between lines  $H_{c2}$  and  $H_d^*$ , is the region in which one can observe twinning-plane superconductivity.

Thus, over the wide range of temperatures from ~2.5 to 8.5 K the (H,T) phase diagram for the superconductivity of a twin exhibits a region in which bulk superconductivity is completely destroyed but the twinning-plane superconductivity still exists. Because of the small value of the critical current we were unable to study the phase diagram for twinning-plane superconductivity by this method in the immediate vicinity of  $T_{c0}$ .

In the second group of experiments we did a magnetometric study at temperatures close to  $T_{c0}$ . Examples of the resulting M(H) curves for three different temperatures are shown in Fig. 3. The solid curves are the results of measure-



FIG. 3. Examples of experimental chart recordings of the magnetic moment of the samples versus the magnetic field at various temperatures;  $T_{co}$ is the bulk critical temperature of niobium. The solid curves are for a twin, the dashed curves for a single crystal. The inset shows in expanded scale the region in which the magnetic moment of the twin exhibits hysteresis due to the existence of macroscopic persistent currents.



FIG. 4. Phase diagram for the twinning-plane superconductivity of niobium according to measurements of the magnetic moment:  $\bigoplus$ ) upper critical field  $H_{c2}$  for bulk superconductivity of niobium;  $\bigcirc$ ) critical field  $H_d$ for the twinning-plane superconductivity;  $\square$ ) the field *h* is at which the magnetic moment of the localized superconductivity reaches a maximum;  $\triangle$ ) the field  $H_{ci}$  of the topological phase transition; I) region of bulk superconductivity; II) region of the normal state; III) region of observation of twinning-plane superconductivity in which the screening currents flow around microscopic regions; IV) region of observation of twinningplane superconductivity in which the persistent electrical currents flow along macroscopic paths. The diagram in the inset is expanded 10 times.

ments on a twin, while the dashed curves are for a single crystal at the same temperatures. We see from the curves that the twinned sample exhibits substantial diamagnetism at temperatures  $T > T_{c0}$  even in magnetic fields  $H > H_{c2}$  up to the field  $H_d$  (we arbitrarily take  $H_d$  to be the field at which the diamagnetic moment of the sample is 10 times greater than the noise level of the magnetometer); here there are none of the superheating-supercooling effects that are observed in the case of tin.<sup>2</sup>

The results of the measurements of the fields  $H_{c2}$  and  $H_d$  for niobium are plotted on an (H,T) phase diagram in Fig. 4. The lines  $H_{c2}(T)$  and  $H_d(T)$  divide the (H,T) phase plane into three regions: Region I, lying below the  $H_{c2}$  line, is the region of bulk superconductivity; region II, above the  $H_d$  line, is the region of the normal state; regions III and IV together, lying between the  $H_{c2}$  and  $H_d$  lines, are the region where twinning-plane superconductivity is observed. We emphasize that the control samples are found in the normal state at  $T > T_{c0}$  and  $H > H_{c2}$ .

At a temperature  $T = T_{c0} + 0.003$  K the M(H) curves of the niobium twins (Fig. 3) exhibit hysteresis in small fields that vanishes at  $H_{ci}$ . The values of  $H_{ci}$  measured at different temperatures are also shown on the (H,T) phase diagram (Fig. 4). The shape of the hysteresis loop of the magnetic moment of the niobium twins is typical for the field dependence of the magnetic moment of a thin superconducting disk in a magnetic field which is inclined to the plane of the disk.<sup>14</sup> In this case the hysteresis is due to the persistent supercurrents flowing in the macroscopic circuit formed by the periphery of the disk.

To verify that the supercurrent causing the hysteresis

flows along the twinning plane in a niobium twin, the sample was oriented so that the twinning plane of the samples was approximately parallel to the magnetic field H (to an accuracy of  $\sim 10^{\circ}$ ). This led to an approximately tenfold decrease in the magnetic moment of the current as a result of the decrease in the coupling coefficient between the circuit with the current and the receiving coil of the magnetometer. At the same time, no changes were observed in the amplitude of the diamagnetic moment or the position of the  $H_{ci}$  line on the phase diagram.

Thus the region on the phase diagram where the twinning-plane superconductivity is observed is divided into two parts: region III, in which the sample has an excess diamagnetic moment but there is no hysteresis, and the persistent currents causing the diamagnetism flow around only simply connected microscopic regions, and region IV, in which the diamagnetism is accompanied by hysteresis, i.e., there can be trapping of the magnetic flux of the persistent electrical currents flowing along macroscopic paths.

Similar changes in the type of magnetic-field dependence of the magnetic moment of the sample have been noted previously—in experiments on tin, for example.<sup>15</sup> However in tin the effect was observed only in samples in which a rather dense three-dimensional network of randomly distributed twinning planes has been created in a layer  $\sim 3 \cdot 10^{-4}$  cm thick. As the temperature was lowered the superconducting regions of different twinning planes overlapped and created a comparatively thick superconducting layer in which macroscopic current paths could arise. For an isolated twinning plane in tin, no electrical current was observed to flow without attenuation at macroscopic distances at any temperature.

We note that in view of our chosen criteria for determining the values of the critical fields, the  $H_d^*$ ,  $H_d$ , and  $H_{ci}$  lines constructed on the phase diagrams in Fig. 2 and 4 give a minimum estimate of the extent of the existence regions of twinning-plane superconductivity in niobium.

Another qualitative feature that distinguishes the behavior of the twinning-plane superconductivity in niobium from that observed previously in tin is the variable temperature dependence of the value h of the magnetic field at which the diamagnetic moment  $M_D$  is maximum (see Fig. 3). In the case of niobium (Fig. 4), h decreases linearly with increasing temperature, and extrapolation of h(T) to zero gives the same temperature  $T_c$  as does the extrapolation  $H_d(T)$  to zero. In the experiments on tin no such dependence of h(T) was observed: at all temperatures of observation of the effect in question, h remained constant to within the accuracy of the experiment.<sup>16</sup>

The experimental results described above permit a number of quantitative estimates. The superconducting transition temperature of niobium determined by extrapolating  $H_{c2}$  (T) in Fig. 2 to where it intersects the temperature axis is 9.4  $\pm$  0.1 K. The slope  $dH_{c2}/dT$  obtained from both the current and magnetic measurements is equal to -490 Oe/K at  $T = T_{c0}$ . The good agreement of the numerical values of the parameters characterizing the bulk superconductivity with the values from the literature<sup>17</sup> is further evidence

of the high quality of the niobium samples used in the present study. The increase in the critical temperature  $T_c$  of the transition to the LS state near the twinning plane as compared to the critical temperature of the superconducting transition in a bulk sample at zero magnetic field is  $T_c - T_{c0} = 0.11$  K. For  $dH_d/dT$  and  $dH_d^*/dT$  the values obtained are -560 Oe/K and -550 Oe/K, respectively. The twinning plane exhibits zero resistance at H = 0 at temperatures  $T < T_{ci} = T_{c0} + 0.016$  K. For the temperature dependence of the inclined (to the twinning plane) magnetic field  $H_{ci}(T)$  below which a macroscopic supercurrent can flow along the twinning plane we obtain the estimate  $dH_{ci}/dT = -270$  Oe/K. The h(T) curve has a slope dh/dT = -130 Oe/K.

The value of the diamagnetic moment  $M_D$  has been used<sup>1</sup> to obtain the estimate  $w_{\rm Sn} = 3500$  Å for the "effective" thickness of the superconducting layer near the twinning plane in tin for  $T = T_{c0}$ , under the assumption that the layer is continuous and has a magnetic susceptibility  $\chi = -1/4\pi$ . An estimate made in the same way from the results of experiments to niobium gives an effective layer thickness  $w_{\rm Nb} = 400$  Å; as in the case of tin, this value is almost exactly equal to the coherence length  $\xi_0 = 390$  Å).<sup>18</sup>

Finally, from the measurements (by both the current and magnetometric methods) of the critical current in fields somewhat higher than  $H_{c2}$ , and under the assumption that the supercurrent flows along a layer of thickness  $\sim \xi_0$ , we obtain the critical current density  $j_c \sim 10^7$  A/cm<sup>2</sup> at  $T = T_{c0}$ ; this value agrees in order of magnitude with the value of the depairing current.

#### **DISCUSSION OF RESULTS. CONCLUSIONS**

1. The experiments on niobuim have shown that it is also possible to observe twinning-plane superconductivity at temperatures and magnetic fields higher than the critical values for the single crystal in superconductors with cubic lattices. For this, however, it is apparently necessary that the twinning plane of the crystal not be a plane of close packing, as is the case in twins of aluminum and lead, which have fcc lattices.

The fact that niobium exhibits twinning-plane superconductivity at  $T > T_{c0}$  confirms the hypothesis of Refs. 1 and 2 that the phonon spectrum of a double crystal with a twinning plane develops a singular branch of two-dimensional phonons and that the critical parameters for superconductivity can increase on account of the "softness" of the two-dimensional-phonon spectrum in comparison with the three-dimensional.

2. A substantial difference in the behavior of the twinning-plane superconductivity in niobium from that studied previously in tin is the absence, to within the accuracy of the measurements, of superheating and supercooling effects at the transition from the normal state to the LS state on the  $H_d(T)$  line. The reversible behavior of the magnetic moment as a function of the magnetic field agrees with the theoretical results<sup>19,20</sup> that showed that for type-II superconductors the transition to the state with superconductivity should be a second-order phase transition. 3. As was shown by Buzdin and Khvorikov,<sup>21</sup> who used the Ginzburg-Landau theory to construct a phenomenological theory of the effect, the structure of the phase diagram depends on the value of the Ginzburg-Landau parameter  $\varkappa$ . This parameter influences the relationship between the derivatives  $dH_c/dT$  and  $dH_d/dT$ . In the case of the type-I superconductor ( $\varkappa < 1/\sqrt{2}$ ) one has

$$|dH_c/dT| > |dH_d/dT|$$
,

and, accordingly, the  $H_{c2}(T)$  and  $H_c(T)$  lines on the phase diagram intersect, and there exists a lower critical temperature  $t_c$  for the observation of localized superconductivity near the twinning plane, as has also been observed in tin.<sup>3</sup> In the case of a type-II superconductor  $(\varkappa > 1/\sqrt{2})$  the relationship between the above derivatives is the opposite, and the effect can be observed over a much wider temperature range. In the present study we have shown experimentally that the region in which niobium exhibits twinning-plane superconductivity extends at least from ~2.5 to 9.5 K.

4. A new effect not previously observed for an isolated twinning plane is the topological phase transition on the line  $H_{ci}$  separating regions II and IV on the phase diagram in Fig. 4. These regions of the phase diagram differ in the topology of the supercurrents. In region III they flow only around simply-connected microscope superconducting regions formed near the twinning plane, while the region IV they flow along the twinning plane for macroscopic distances and form a multiply-connected path. This, of course, is immediately manifested in the shape of the hysteresis loop on the curves of the magnetic moment of the sample versus the magnetic field.

The existence of such topological phase transitions in two-dimensional systems has been predicted theoretically.<sup>22,23</sup> The behavior of a two-dimensional superconductor has been considered in detail by Doniach and Huberman, who made a theoretical study of a model of a very thin superconducting film and attempted to construct the phase diagram for such a system. A noteworthy feature is the presence of two critical temperatures for the transition of a two-dimensional system to the superconducting state, denoted<sup>24</sup> by  $T_{BCS}$  and  $T_{KT}$ ;  $T_{BCS}$  corresponds to the critical temperature for the onset of superconductivity in a three-dimensional sample made of the same material as the film. At temperatures  $T < T_{BCS}$  a two-dimensional superconducting system exhibits strong diamagnetism, but the resistance between two widely separated points remains nonzero. The resistance vanishes only at temperatures below the critical temperature  $T_{KT}$  of the topological phase transition.

We note that the twinning plane of the crystals is a unique example of a physical two-dimensional system (only a single monolayer of atoms is found in a crystallographically preferred position in the crystal) and is an ideal object for experimental study of the properties of such systems. Moreover, the study of the superconductivity of the two-dimensional systems that arise in twins is, of course, strongly complicated by the existence of a proximity effect of the normal metal and by the onset of bulk superconductivity. The proximity effect of the normal metal surrounding the twinning plane should markedly lower the critical temperature  $T_{BCS}$  of the two-dimensional layer, making it approach the temperature  $T_{c0}$  of the bulk transition, so that the region in which the twinning-plane superconductivity is observed is strongly diminished. The bulk superconductivity that arises at lower temperatures screens the twinning plane from external influences. At the same time, the twinning-plane superconductivity can evidently be observed (e.g., from the appearance of "pinning" at this region)<sup>7,25</sup> even for  $T < T_{c0}$ .

Since the qualitative features of the behavior of the M(H) and M(T) curves in the region of the (H,T) phase diagram for the twinning-plane superconductivity of niobium are in total agreement with the predictions of Ref. 24, we can state that  $T_c$  is in fact the temperature  $T_{BCS}$  for the twinning plane with allowance for the proximity effect and that  $T_{ci}$  is the temperature  $T_{KT}$  of the two-dimensional system. The fact that the particular values of the temperatures  $T_{c0}$ ,  $T_{ci}$ , and  $T_c$  in niobium twins obey  $T_{c0} < T_{ci} < T_c$  has enabled us to observe the topological phase transition. In the topological phase transition in the two-dimensional super-conducting system is not observable by the techniques used. The reason for this is most likely the small difference between the temperature  $T_c$  and  $T_{c0}$ .

We do not think it would be meaningful to compare the numerical results of this study with the theoretical results of Ref. 24, since the theoretical model left out many of the essentials (discussed above) of the actual experimental situation.

5. Let us finally take up the question of the mutual agreement of the phase diagrams for the twinning-plane superconductivity in Figs. 2 and 4, which were obtained by different methods. According to the phase diagram in Fig. 2, the twinning plane exhibits zero resistance below the line  $H_d^*$ , which has a slope  $dH_d^*/dT = -550$  Oe/K, while according to the diagram in Fig. 4 the twinning-plane resistance is zero only below the line  $H_{ci}$ , with a slope  $dH_{ci}/dT = -270$  Oe/K.

The difference is due to the fact that the normal and tangential components of the external magnetic field have different effects on the twinning-plane superconductivity. Only the field component normal to the twinning plane produces vortices which penetrate the thin superconducting layer near the twinning plane and give rise to a finite resistance in comparatively weak fields. Thus, in accordance with the experimental conditions, a topological phase transition occurs on the line  $H_{ci}$  (see Fig. 4) for the case of magnetic field is exactly parallel to the twinning plane (as in the experiments for measuring the critical current) the external magnetic field does not induce vortices in the two-dimensional superconducting system, and zero resistance is observed in magnetic fields all the way up to  $H_{d}^*$ . The line

 $H_{d}^{*}(T)$  at which the zero-resistance state arises (the topological phase transition) in a magnetic field parallel to the twinning plane (this line should evidently cross the temperature axis at a point  $T_{ci}$  far from  $T_{c0}$ , as follows from the phase diagrams shown in Fig. 2 and 4) agrees within the accuracy of the experiments with the line  $H_{d}(T)$  at which the twinning-plane superconductivity is completely destroyed.

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