Energy gaps and the role of disorder under conditions of fractional quantization of the Hall resistance in silicon metal–oxide–semiconductor structures

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The activation gaps W in the energy spectrum of interacting two-dimensional electrons corresponding to Landau sublevel occupation factors $\nu = 1/3$, $2/3$; $4/3$; $4/5$ have been determined from the temperature dependence of the diagonal component of the magnetoresistance tensor in silicon metal–oxide–semiconductor (MOS) structures containing highly mobile two-dimensional electrons in a strong magnetic field ($H \leqslant 20$T). It is found that the magnitude of the activation gap depends on the mobility (μ_e) of the two-dimensional electrons and grows with increasing mobility. The experimental results obtained for various structures and in different magnetic fields can be well described by the relation $W \sim H^{1/2}(1\text{-const}/\mu_e)$. The values of W found and their dependence on H and μ_e are consistent with the concept of the condensation of the electron gas into a new type of quantum, incompressible, Fermi liquid.

1. INTRODUCTION

The energy spectrum of noninteracting two-dimensional ($2D$) electrons in a strong transverse magnetic field consists of a collection of Landau sublevels of a certain width, separated from one another by gaps which in silicon MOS structures correspond to the cyclotron, spin and intervalley splittings. The existence of localized states in the energy gaps of the electron spectrum is important and because of them a plateau is observed at low temperatures in the dependence of the Hall component ρ_{xy} of the magnetoresistance tensor on the concentration n_s of $2D$ electrons or of the magnetic field H, while there are broad and deep minima for the diagonal component ρ_{xx}. These features are observed when each successive Landau sublevel is completely filled, i.e. the condition $n_s = \nu eH/h$ is fulfilled, where the filling factor ν is a whole number (e is the electronic charge and h is Planck's constant). This phenomenon, called the quantum Hall effect (QHE), was discovered by von Klitzing *et al*[1] and was explained in terms of a model of noninteracting electrons.[2]

Two years after the discovery of the QHE, workers at the Bell Laboratories[3–6] found that in systems in which the $2D$ electrons have very high mobility, which is achieved in modulation doped GaAs-AlGaAs heterojunctions, a plateau and minima in the $\rho_{xy}(\nu)$ and $\rho_{xx}(\nu)$ variations are observed not only for whole numbers but also for fractional values of ν with odd denominators: $\nu = p/q$, where $p = 1, 2, 3\ldots$; $q = 3, 5, 7$. A similar behavior was also found in MOS structures in which the $2D$ electrons have exceptionally high mobility.[7–9] The features corresponding to fractional ν with odd denominators were explained by Laughlin[10,11] as a condensation of the $2D$ electron gas into a new type of quantum, strongly correlated, incompressible Fermi liquid, in the energy spectrum of which gapless excitations, for example phonons, are absent. The Laughlin theory postulates the existence of two bands of single-particle excitations, quasielectrons and quasiholes, separated by energy gaps from the ground state. To determine the size of the new gaps in the energy spectrum of the interacting $2D$ electrons, the temperature dependence of ρ_{xx} (or of the magnetoconductivity σ_{xx}) is studied in the region of fractional values of ν (Ref. 12) as is done in the case of the whole-number QHE. The activation gaps W in the region of fractional ν were determined this way in heterojunctions by various authors,[12–15] but all the results were obtained under different experimental conditions and agree badly both among themselves, and also with theory (see § 5).

We determined earlier[16] the activation energies W for $\nu = 2/3$ of the $2D$ electron system on a silicon MOS structure and it was shown that: 1) the size of W at the same H appreciably exceeds the corresponding values obtained in heterojunctions; 2) when the mobility of the $2D$ electrons is constant, the activation energy is inversely proportional to the magnetic length 1_H ($W \sim H^{1/2}$), which clearly illustrates the effect of interparticle Coulomb correlations. These results agree well with the Laughlin theory, in which $W \sim e^2/\varepsilon 1_H \sim H^{1/2}/\varepsilon$ (ε is the dielectric permittivity of the medium).

One of the chief aims of the present work was to trace how disorder affects the magnitude of the activation gaps in the region of fractional filling factors, associated with the new strongly correlated quantum state predicted by theory. As is well known, the transverse relaxation time and the resulting mobility of the charge carriers associated with it can serve as a measure of the disorder in real two-dimensional structures. In fact, transverse relaxation is determined by the finite width of the quantum states in a magnetic field.[2] Therefore, in real structures for $T \neq 0$, the threshold mobility separating localized and mobile electron states differs from the position of a single-particle quantum level. With increasing disorder the mobility thresholds of neighboring single-particle states can start to overlap because the levels begin to get wider. This causes overlap of mobility thresholds causes W to vanish, and the magnetoconductivity is no longer thermally activated. In this connection it is interesting to know how the sizes of the activation gaps depend on the mobility

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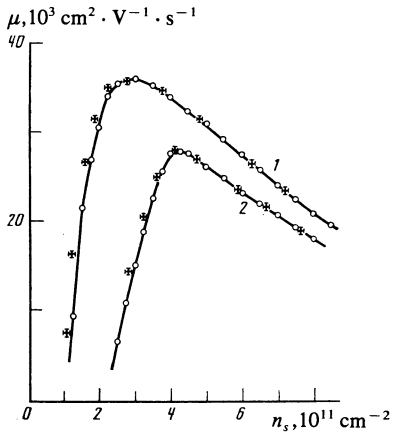

FIG. 1. The dependence of the mobility of the two-dimensional electrons on the density n_s, determined from the magnitude of the conductivity for $H = 0$ (○) and of the Hall mobility (+), for MOS structures Nos. 1 and 2 at $T = 1.5$ K.

for fractional quantum Hall resistivity, and also whether a minimum mobility exists, starting from which the activation character of the magnetoconductivity disappears as do other anomalies in the magnetotransport properties caused by the fractional QHE (FQHE). For this pupose, the temperature dependences of the transverse magnetoconductivity in the region of fractional values of the filling factor $\nu = 1/3, 2/3, 4/3, 4/5$ (§3) were studied on different MOS structures, and also the dependence of the corresponding activation energies on the magnitude of the mobility of the $2D$ electrons (§4). The experimental results on the dependence of the activation gaps on the magnitude of the magnetic field and mobility for silicon MOS structures are compared with published results on GaAs-AlGaAs heterojunctions (§5).

2. METHOD OF THE MEASUREMENTS AND THE STRUCTURES

In this work two MOS structures were studied, prepared on the (100) surface of p-type silicon with boron concentration $N_A = 7.3\times10^{14}$ cm^{-3}. The geometrical and other characteristics of the structures have been described previously.[9] The dependence of the mobility of the $2D$ electrons, μ_e^σ, determined from the value of the conductivity at $H = 0$, on n_s is shown for both structures in Fig. 1. In the same figure values of the Hall mobility μ_e^H are shown for various n_s, obtained by comparing $\rho_{xy}(H)$ and $\rho_{xx}(H)$ variations in weak magnetic fields. It can be seen that the values of μ_e^σ and μ_e^H practically coincide. The following experimental facts are evidence of the high quality of the MOS structures selected: 1) the value of the maximum mobility is high, equal to 3.6×10^4 and 2.8×10^4 cm$^2\cdot$V$^{-1}\cdot$s^{-1} at $T = 1.5$ K; 2) the concentration at which the maximum mobility is achieved is reduced, to $\sim3\times10^{11}$ cm^{-2}; 3) the mobility at the maximum of the $\mu_e(n_s)$ curve, and also of the magnetoconductivity $\sigma_{xx}^{\max}$ at the maxima of the Shubnikov oscillations increases when the temperature decreases from 4.2 to 1.5 K; 4) the magnetoconductivity $\sigma_{xx}^{\max}$ increases with magnetic field, so that for $H > 10$T, $\sigma_{xx}^{\max} \gtrsim (e^2/\pi^2\hbar)(N + 1/2)$, where N is the Landau level number. The last fact is evidence that potential fluctuations at the semiconductor-oxide interface are of short period with characteristic length < 50Å (Ref. 2). It was necessary to carry out measurements with small currents $I_0 < 100$ nA in order to observe the fractional quantum Hall effect. This is associated with the fact that the Joule power per electron is determined by the product $j^2\rho_{xx}$, and since ρ_{xx} at the minima corresponding to fractional ν is several orders of magnitude larger than for integral ν, the electron system begins to heat up at appreciably smaller measuring currents in the region of fractional ν. The electron heating was monitored by the linearity of the current-voltage characteristics. It was found that the characteristics remained linear for $I_0 < 100$ nA for all fractional ν. The measurements were carried out with 30 Hz alternating current.

3. ACTIVATED MAGNETORESISTANCE AND GAPS IN THE ENERGY SPECTRUM OF TWO-DIMENSIONAL ELECTRONS FOR $\nu = 1/3, 2/3, 4/3$, and $4/5$

The temperature dependence of the magnetoconductivity σ_{xx} (or ρ_{xx}) near a minimum for integral ν can be studied to determine the gaps in the energy spectrum of noninteracting $2D$ electrons, corresponding to cyclotron ($\hbar\omega_c$), spin ($g\mu_B H$) and intervalley (ΔE_v) splitting, i.e., when the Fermi level of the electrons is in the region of localized states half way between the corresponding levels of the energy spectrum (Fig. 2). The magnetoconductivity in this case differs from zero thanks to the thermal activation of electrons and holes from the Fermi level into an empty and completely full Landau sublevel: $\sigma_{xx} \propto \rho_{xx} \propto \exp(-W/kT)$, $W = \Delta E/2$, where W is the activation energy, ΔE is the energy gap between the corresponding Landau sublevels (Fig. 2). Fluctuations of the potential at the semiconductor-oxide interface (or the interface between two semiconductors with different forbidden gap width) lead to a broadening of the energy levels and to a change in the relation between ΔE and W. This is related to the fact that in real structures the mobility threshold, separating mobile from localized electron states, does not coincide with the position of a Landau level, but departs from it by an amount Γ. As a result, the activation gap decreases: $W = \Delta E/2 - \Gamma$. The worse the quality of the structure (and therefore the lower the mobility of the $2D$ electrons), the greater is Γ, and the measured value of W differs appreciably from $\Delta E/2$. A hopping conductivity mechanism can be observed for structures with very low mobility of the $2D$ electrons and also for very low temperatures (i.e. for $kT \ll \Gamma$), which is not described by the exponential

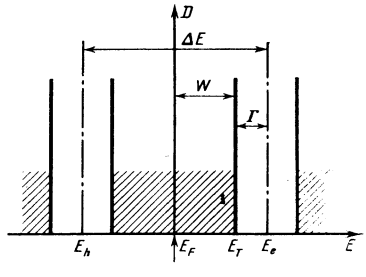

FIG. 2. Qualitative picture of the change in the density of states with energy. The shaded regions correspond to localized states; E_e, E_h, E_F, and E_T refer to the positions of the levels for, respectively, electrons, holes, the Fermi energy and the mobility threshold.

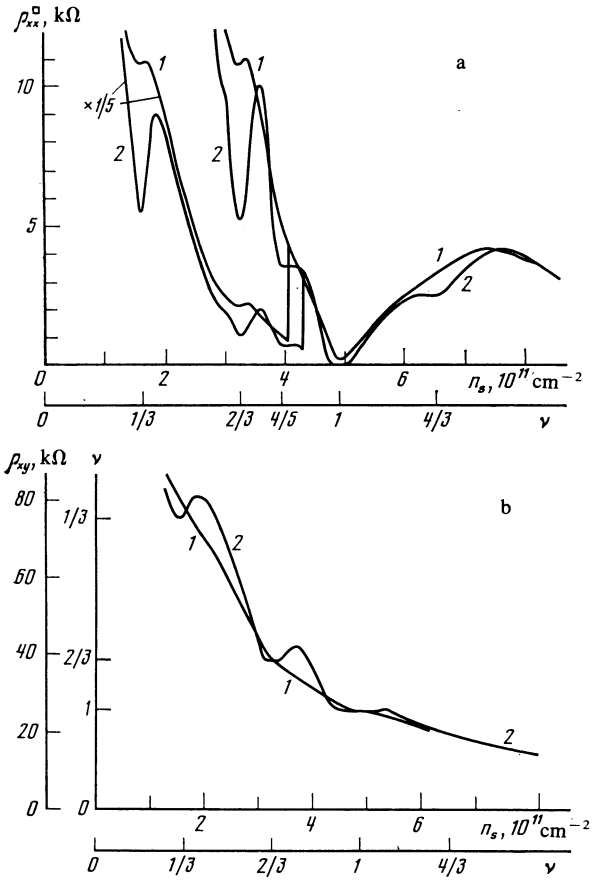

FIG. 3. a) The dependence of $\rho_{xx}^{\square}$ on the density of 2*D* electrons, obtained for MOS structure No. 1 for $H = 20$ T at two temperatures: $T = 3.0$ (curve 1) and 1.5 K (curve 2); b) the $\rho_{xy}(n_s)$ relation for MOS structure No. 1 for $H = 20$ T and two temperatures: $T = 3.0$ (curve 1) and 1.5 K (curve 2).

dependence given above.[17] However, it was found for high-qulity specimens with high 2*D* electron mobility, for $\Gamma < kT \ll \Delta E$, both for MOS structures and for heterojunctions, that the energy gaps determined from the temperature dependence of the magnetoconductivity agree well with $\hbar\omega_c$, $g\mu_B H$ and ΔE_v.[18]

The energy of the ground state of a system of interacting 2*D* electrons has minima for $\nu = p/q$ (where $p = 1,2,3.....$; $q = 3,5,7....$), corresponding to the formation of an incompressible Fermi liquid, as proposed by Laughlin. The change in density in the region of a fractional value of ν can be described in the language of the creation of excitations-quasielectrons and quasiholes with fractional charge. According to Laughlin, the quasielectron and quasihole bands are separated from the ground state by energy gaps and for $\nu = p/q$ the magnetoconductivity differs from zero thanks to thermal activation of the excitations (see Fig. 2). Consequently, for $\nu = p/q$ one should expect that $\sigma_{xx} \propto \rho_{xx} \propto \exp(-W/kT)$. The activation gaps for $\nu = 1/3$ and 2/3 were determined in this way in GaAs-AlGaAs heterojunctions by different authors,[12-15] but the results obtained agree badly both among themselves and with theory. The latter discrepancy is evidently connected with the results being obtained on different structures for different values of μ_e and H.

The 2*D* electron system in silicon MOS structures differs usefully from the electron systems in heterojunctions in permitting the density n_s over a wide range. This enables activation energies for different ν to be determined for any fixed H. The $\rho_{xx}(n_s)$ and $\rho_{xy}(n_s)$ variations for $H = 20$ *T* and various temperatures are shown in Fig. 3. Apart from the integral features, additional minima and plateaus are clearly visible on these curves, corresponding to fractional values of $\nu = 1/3$; 2/3; 4/3; 4/5. As the temperature is raised, the anomalies in the magnetotransport properties for fractional ν disappear, both for ρ_{xx} and for ρ_{xy}. The dependences of $\log \rho_{xx}^{\min}$ on inverse temperatures are shown in Fig. 4 for $\nu = 1/3$; 2/3; 4/3; 4/5 for $H = 20$ T. It can be seen that, starting from a certain temperature, the magnetoconductivity has a thermally activated character for all the fractional ν found in the experiment, and is well described by the expected relationship. The values of the activation energies at $H = 20$ T are: $W_{1/3} = (2.4 \pm 0.05)$ K, $\mu_e = 2.5 \times 10^4$ cm$^2 \cdot$ V$^{-1} \cdot$ s^{-1}, $W_{2/3} = (2.6 \pm 0.05)$ K, $\mu_e = 3.5 \times 10^4$ cm$^2 \cdot$ V$^{-1} \cdot$ s^{-1}, $W_{4/3} = (1.6 \pm 0.05)$ K, $\mu_e = 2.6 \times 10^4$ cm$^2 \cdot$ V$^{-1} \cdot$ s^{-1}, $W_{4/5} = (1.1 \pm 0.05)$ K, $\mu_e = 3.5 \times 10^4$ cm$^2 \cdot$ V$^{-1} \cdot$ s^{-1}. It can be seen that for the same μ_e and H, the values of the activation energies corresponding to fractions with denominator 3 are more than double the values obtained for fractions with denominator 5. This fact also agrees well with theoretical calculations.[11]

The maximum value of the activation energy, for example, in the region of $\nu = 2/3$ corresponds to $\nu = 0.666 \pm 0.003$ and is halved when ν departs from 2/3 by $\Delta\nu = 0.02$. The temperature dependences of ρ_{xx} for $\nu = 0.667$, 0.710 and 0.625 are shown in Fig. 5 in log (ρ_{xx}), T^{-1} coordinates, from which it can be seen that the $\rho_{xx}(T)$ variation for $\Delta\nu = \pm 0.04$ now is not activated. This is evidently associated with the fact that for $|\Delta\nu| \geqslant 0.04$, the quasielectron and quasihole bands are appreciably filled and the Fermi level reaches the corresponding mobility thresholds

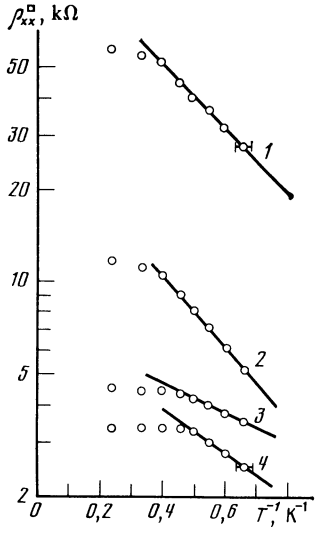

FIG. 4. The temperature variation of the diagonal component of the magnetoresistance tensor at the minima of the Shubnikov oscillation, represented in $\log \rho_{xx}^{\min}$ vs T^{-1} coordinates for different values of ν at $H = 20$ T; 1) $\nu = 1/3$, $W = 2.4$ K; 2) 2/3, 2.6 K; 3) 4/5, 1.1 K; 4) 4/3, 1.6 K.

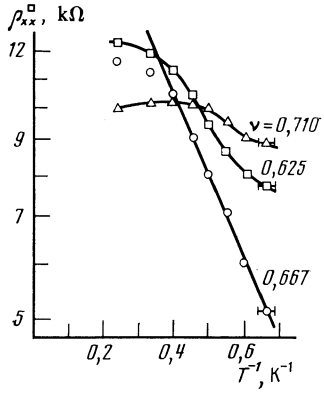

FIG. 5 The temperature dependence of $\rho_{xx}^{\square}$ for different values of the filling factor $\nu = 0.667; 0.710; 0.625$, represented in $\log \rho_{xx}$ vs T^{-1} coordinates.

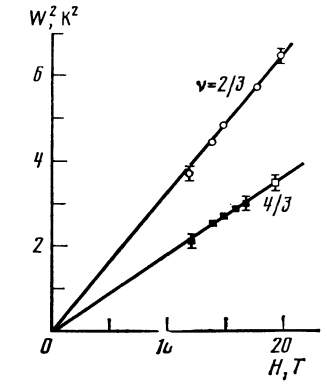

FIG. 6. The variation of the square of the activation energy W^2 with magnetic field, obtained on two MOS structures under constant-mobility conditions: ○) $\mu_e = 3.5 \pm 0.1)\ 10^4$ cm$^2 \cdot$ V$^{-1} \cdot$ s^{-1}, MOS structure No. 1; □, ■) $\mu_e = (2.7 \pm 0.1)\ 10^4$ cm$^2 \cdot$ V$^{-1} \cdot$ s^{-1}, MOS structures Nos. 1 and 2 respectively.

and lies in the band of mobile states. It can be concluded from this that only 12% of states in the quasielectron and quasihole bands are localized at $H =$ 20 T and $\mu_e = 3.5 \times 10^4$ cm$^2 \cdot$ V$^{-1} \cdot$ s^{-1}. About this fraction of localized states is obtained for $H = 15$ T, $\mu_e = 3.5 \times 10^4$ cm$^2 \cdot$ V$^{-1} \cdot$ s^{-1} for $\nu = 2/3$ and $\nu = 4/3$. When the mobility is reduced, both the value of the activation energy (at $H =$ const.) and the value of $\Delta\nu$ become smaller: for $\mu_e = 2.8 \times 10^4$ cm$^2 \cdot$ V$^{-1} \cdot$ s^{-1}, $H = 15$ T and $\nu = 4/3$, $\Delta\nu = 0.03$. These facts are evidence of the influence of disorder on the position of the mobility threshold in the energy spectrum of $2D$ electrons at non-zero temperatures.

4. THE DEPENDENCE OF ACTIVATION ENERGIES ON THE MOBILITY OF TWO-DIMENSIONAL ELECTRONS

As was mentioned in §3, the magnitude of the energy gap determined from analysis of the temperature dependence $\rho_{xx}(T)$ corresponds to the activation energy from the Fermi level to the band of mobile states, separated from localized electron states by the threshold mobility. As is known from the intergral QHE, the position of the mobility threshold relative to the corresponding Landau level depends on μ_e and H.[2,19] It is just for this reason that values of W obtained on different structures with different μ_e for the same ν and H differ: for example, for $H = 20$ T, $\nu = 4/3$ for structure No 1, $W = (2.08 \pm 0.03)$ K$(\mu_e = 31.5 \times 10^3$ cm$^2 \cdot$ V$^{-1} \cdot$ s$^{-1})$, while for structure No. 2, $W = (1.62 \pm 0.03)$ K $(\mu_e = 27 \times 10^3$ cm$^2 \cdot$ V$^{-1} \cdot$ s$^{-1})$. A comparison of the W's obtained on different MOS structures for the same ν and H showed that when μ_e is reduced, the value of the activation energy decreases regularly.

The interval on n_s within which μ_e appears constant to high accuracy can be deduced from the broach maximum which is present in the $\mu_e(n_s)$ relation (see Fig. 1): for structure No. 1: $\mu_e = (35 \pm 1) \times 10^3$ cm$^2 \cdot$ V$^{-1} \cdot$ s^{-1} for $2.1 < n_s \times 10^{-11}$ cm$^{-2} < 4$, while for structure No. 2 $\mu_e = (27 \pm 1) \times 10^3$ cm$^2 \cdot$ V$^{-1} \cdot$ s^{-1} for $3.8 < n_s \times 10^{-11}$ cm$^{-2} < 5$. This indicates that for structure No. 1 for $\nu = 2/3$, $\mu_e \approx 35 \times 10^3$ cm$^2 \cdot$ V$^{-1} \cdot$ s^{-1} for $12 < H < 25$ T, while for structure No. 2 for $\nu = 4/3$, $\mu_e \approx 27 \times 10^3$ cm$^2 \cdot$ V$^{-1} \cdot$ s^{-1} for $11 < H < 17$ T. The $W(H)$ relations for $\mu_e \approx$ const., obtained on two MOS structures for $\nu = 2/3$ and $\nu = 4/3$, are shown in Fig. 6. It can be seen that $W \sim H^{1/2}$, so that the magnetic field dependence of the activation energy W_ν, corresponding to a filling factor ν, can be represented conveniently in the form

$$W_\nu = G_\nu(\mu_e) e^2/\varepsilon l_H, \qquad (1)$$

where $G_\nu(\mu_e)$ is a dimensionless quantity which depends on the mobility of the $2D$ electrons but not on H, while $e^2/\varepsilon l_H$ is the characteristic Coulomb energy to which the value of the gap is usually normalized in the theory.[10] For structure No. 1, $G_{2/3}(\mu_e = 35 \times 10^3$ cm$^2 \cdot$ V$^{-1} \cdot$ s$^{-1}) = 0.067 \pm 0.003$, while for structure No. 2, $G_{4/3}(\mu_e = 27 \times 10^3$ cm$^2 \cdot$ V$^{-1} \cdot$ s$^{-1}) = 0.050 \pm 0.003$.

It is characteristic that the values of $G_{4/3}(\mu_e = 27 \times 10^3$ cm$^2 \cdot$ V$^{-1} \cdot$ s$^{-1})$ is determined for MOS structures No. 1 and No. 2 agree, as can be seen in Fig. 6. This means that the function $G_\nu(\mu_e)$ is universal, independent of the structure of H. To determine it we used the values of W_ν obtained on two MOS structures for different values of μ_e and H. The dependence of $G_{4/3}$ on μ_e^{-1} obtained in this way is shown in Fig. 7. It is seen that the values obtained on different MOS

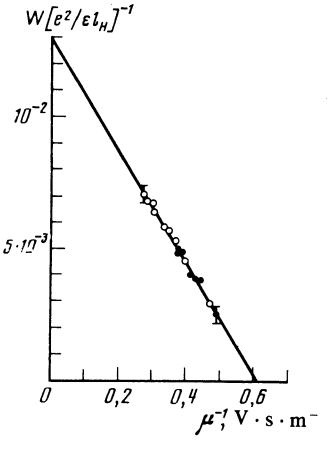

FIG. 7. The magnitude of the activation gap, normalized to the characteristic Coulomb energy, vs. the inverse of the mobility, obtained for $\nu = 4/3$ for different values of H on different MOS structures: ○ and ● correspond to MOS structures No. 1 and 2.

TABLE I.

ν	W, K	μ_e, $m^2 \cdot V^{-1} \cdot s^{-1}$	H, T	G_ν^∞	μ_ν^0, $m^2 \cdot V^{-1} \cdot s^{-1}$	Specimen	Reference
	2,4	2,5	20	0,018	1,7	Si(100)	p.w.*
	3,2	45	18,5	–	–	GaAs – AlGaAs	[12]
	2,7	110	14,6	–	–	»	[14]
1/3	–	–	–	0,028	–	Theory	[11]
	–	–	–	0,034	–	»	[21]
	–	–	–	0,055	–	»	[20]
	2,6	3,5	20	0,013	1,7	Si(100)	p.w
	0,38	45	9,6	–	–	GaAs – AlGaAs	[12]
	1,2	100	12	–	–	»	[13]
	0,5	60	9,6	–	–	»	[13]
2/3	0,9	110	7,4	–	–	»	[14]
	1,4	105	14	–	–	»	[15]
	–	–	–	0,028	–	Theory	[11]
	–	–	–	0,034	–	»	[21]
	2,2	3,5	15	0,013	1,7	Si(100)	p.w.
4/3	0,25	105	6,9	–	–	GaAs – AlGaAs	[15]
	–	–	–	0,028	–	Theory	[21]
5/3	0,09	105	5,5	–	–	GaAs – AlGaAs	[15]
	–	–	–	0,028	–	Theory	[21]
	1,1	3,5	20	0,006	2,3	Si(100)	p.w.
	–	–	–	0,007	–	Theory	[11]
4/5	–	–	–	0,0098	–	»	[21]
	–	–	–	0,012	–	»	[20]

*p.w.-present work

structures (light and dark symbols) for different μ_e and H agree well among themselves and lie on a single straight line which intersects the axes at the values G_ν^∞ and M_ν^0: G_ν^∞ corresponds to the activation energy in a perfect structure (for $\mu_e \to \infty$), μ_ν^0 is the minimum mobility of 2D electrons, below which the activation energy goes to zero and the anomalies in the magnetotransport properties for fractional values of ν are not observed. It is the value of G_ν^∞ which should be compared with theoretical calculations,[11,20,21] since they do not take account of the effect of disorder. The value of G_ν^∞ for $\nu = 1/3$, 2/3, 4/3, and 4/5 are shown in Table I, where the values of G_ν^∞ obtained theoretically by different authors are shown for comparison. Taking into account the difference between the theories, the agreement between the experimental and theoretical values of G_ν^∞ can be said to be satisfactory.

The presence of a minimal mobility μ_ν^0, essential for observing the fractional Hall effect, at first sight contradicts the rule we established earlier:[9] $\mu_e H > 36$ (for fractions with denominator 3) at $T = 1.5$ K. However, specific investigations showed that for $\mu_e H \approx 36$ for $\nu = 1/3$, 2/3, 4/3, the activation energy turns out to be 1–1.5 K, i.e., close to the minimum bath temperature in the experiment. The criterion $\mu_e H > 36$ is therefore, most likely, a special rule referring to the specific temperature $T = 1.5$ K. In the general case, it is necessary that $kT < (e^2/\varepsilon l_H) G_\nu^\infty$ hold for observation of the anomalies in the transport properties corresponding to a fractional filling factor ν.

The function $G_\mu(\mu_e)$, as can be seen from Fig. 6, is well described by the expression

$$G_\nu(\mu_e) = G_\nu^\infty (1 - \mu_\nu^0/\mu_e).$$

It must, however, be remarked that because of the narrowness of the range of values of μ_e studied, the dependence of G_ν on μ_e is also represented satisfactorily by the expression

$$G_\nu(\mu_e) = G_\nu^\infty [1 - (\mu_\nu^0/\mu_e)^\alpha],$$

if $0.4 < \alpha < 1.3$. When α is changed within this range, the values of μ_ν^0 remain practically unchanged while the values of G_ν^∞ can change by 30%. Nevertheless, the best representation is for the experimental points to lie on a straight line for $\alpha \approx 1$. It is interesting that the values of the minimum mobility obtained for different filling factors with denominator 3, coincide within the limits of experimental error, while the value of $\mu_{4/5}^0$ is appreciably larger (see Table I).

5. COMPARISON OF EXPERIMENTAL RESULTS OBTAINED WITH SILICON MOS STRUCTURES AND WITH GaAs-AlGaAs HETEROJUNCTIONS

It is interesting to compare the values of activation energies (for $\mu_e \to \infty$) found experimentally for different filling factors in silicon MOS structures with the existing theoretical calculations carried out for ideal two-dimensional systems (without taking account of disorder). Such a comparison is shown in Table I, which also shows the experimental results of different authors for a two-dimensional electron gas in variably doped GaAs-AlGaAs heterojunctions. It can be seen that the values of G_ν^∞ for silicon MOS structures are closest to the theoretical values. The departure of the experimental values from the theoretical calculations is noticeably greater in the case of heterojunctions. In both cases the experimental values of the gaps are smaller than are calculated. It is significant that the dependence of the activation energy in the form

$$W_\nu = G_\nu^\infty \left(1 - \frac{\mu_\nu^0}{\mu_e}\right) \frac{e^2}{\varepsilon l_H},$$

describes the results for GaAs-AlGaAs heterojunctions obtained by various authors[12–15] on different structures. It then appears that the value of $G_{2/3}^\infty$ in heterojunctions is 30% less than in silicon MOS structures. In our view this could arise because the potential of the interelectron interaction depend appreciably on the channel thickness:[2,22]

$$V(q)=\frac{2\pi e^2}{\varepsilon q}F\left(\frac{q}{b}\right), \tag{2}$$

where q is the wave vector, $F(q/b)$ is the form factor which for the wave function describing the electron distribution in a direction perpendicular to the layer

$$\psi(z)=(b^3/2)^{1/2}z\exp(-bz/2),$$

is equal to

$$F\left(\frac{q}{b}\right)=\frac{1}{16}\left(1+\frac{\varepsilon_{SiO_2}}{\varepsilon_{Si}}\right)\left(1+\frac{q}{b}\right)^{-3}\left[8+\frac{9q}{b}+3\left(\frac{q}{b}\right)^2\right]$$
$$+\frac{1}{2}\left(1-\frac{\varepsilon_{SiO_2}}{\varepsilon_{Si}}\right)\left(1+\frac{q}{b}\right)^{-6}. \tag{3}$$

The channel thickness is $\langle z_0\rangle + = 3/b$. The effect of channel thickness on the interelectron interaction potential is especially marked for the heterojunctions. In this case the thickness $\langle z_0\rangle$ is 60 Å and is comparable with the magnetic length for $H\gtrsim 20$ T. At the same time, the characteristic channel width on a MOS structure prepared on a silicon (100) surface is almost three times smaller and is, consequently, appreciably smaller than the magnetic length for $H\ll 20$ T, thanks to the large electron mass in a direction perpendicular to the $2D$ layer. It should be noted that the value of $F(1/bl_H)$ is noticeably different from unity: for heterojunctions at $H=15$ T, $F(1/bl_H)\approx 0.6$, while for silicon MOS structures $F(1/bl_H)\approx 0.8$, so that the existence of this form factor has to be included in theoretical calculations. Unfortunately, MacDonald and Aers,[22] who determined the energy of the ground state of the Laughlin Fermi liquid taking the form factor into account, did not calculate the expected change in the gaps in the energy spectrum on interacting $2D$ electrons with channel thickness.

As was mentioned at the beginning of this section, there is some minimal value μ_ν^0 of the mobility of $2D$ electrons in silicone MOS structures and in GaAs-AlGaAs heterojunctions, starting from which $W_\nu>0$ holds and the fractional quantum Hall effect is observed. It is surprising that while in silicon MOS structures $\mu_{2/3}^0\approx 17\times 10^3$ cm$^2\cdot$V$^{-1}\cdot$s^{-1}, in the electron channel of heterojunctions $\mu_{2/3}^0\approx 4\times 10^5$ cm$^2\cdot$V$^{-1}\cdot$s^{-1} (see also Ref. 13). It is essential to take strict account of the effect of disorder theoretically in order to discover what determines μ_ν^0. However, a simple estimate for μ_ν^0 can be obtained just from purely qualitative considerations. Laughlin[10] noted tht features corresponding to a fractional value of ν should disappear as the disorder increases when the characteristic width of a level $\Gamma\sim\hbar/\tau\equiv\hbar e/\mu_e m$ is comparable with the magnitude of the Coulomb gap corresponding to the given ν. However, one finds from the relation

$$\hbar e/\mu_e m=\text{const}\, e^2/\varepsilon l_H$$

that

$$\mu^0\infty\varepsilon l_H\hbar/em\infty H^{1/2}. \tag{4}$$

This answer contradicts the experimental result according to which μ^0 does not depend on H.

Another expression for the width of a Landau level, $\Gamma\approx\hbar\omega_c/(\mu_e H)^{1/2}$, has been obtained by different means.[23,24] Equating the level width described in this way with the characteristic Coulomb energy, we obtain

$$\mu^0=\text{const}\,(\hbar/e)^3(\varepsilon/m)^2. \tag{5}$$

Here μ^0 is independent of magnetic field in agreement with experiment and is only determined by parameters of the semiconductor. To obtain agreement between Eq. (5) and the experimental values obtained in silicon MOS structures for fractions with denominator 3, the constant has to be taken equal to $\sim(2\pi)^3$, and then

$$\mu^0\approx(h/e)^3(\varepsilon/m)^2. \tag{6}$$

Values of μ^0 estimated by using Eq. (6) for GaAs-AlGaAs heterojunctions are equal: in the case of the electron channel $\mu^0=3.8\times 10^5$ cm$^2\cdot$V$^{-1}\cdot$s^{-1}, while for the hole channel $\mu^0=1.2\times 10^4$ cm$^2\cdot$V$^{-1}\cdot$s^{-1}. These estimates agree surprisingly well with experiment.[5,13]

6. CONCLUSIONS

The main result of the present work is thus that the dependence of the sizes of the activation gaps in the region of the QHE, measured in different magnetic fields and on various structures, can be fairly well understood using Eq. (1), in which the dependences of W on magnetic field and mobility are separated. It has been shown, for different MOS structures and for various filling factors, that at a fixed value of the $2D$ electron mibility the sizes of the activation gaps grow with increasing magnetic field proportionally with the inverse of the magnetic length, which illustrates directly the role of interparticle interactions in the FQHE. Another important consequence of such a factorization is the mobility dependence of the size of the gaps, for fixed magnetic field. From this an experimental procedure for estimating the magnitude of the activation gap in the absence of disorder ($\mu_e\to\infty$) can be formulated, and an understanding and a way can be found of estimating the minimal mobility μ_ν^0, starting from which the fractional Hall effect disappears ($W_\nu\to 0$) due to the overlapping of the bands of mobile states of quasielectrons and quasiholes.

In this paper we have touched on one of the manifestations of disorder associated with fluctuations of the potential of various random defects at the semiconductor-oxide interface. It is just the scattering by random defects which determines the magnitude of the resulting electron mobility in silicon MOS structures. Also, it is known that the value of the magnetoconductivity at the maxima of the Shubnikov oscillations and its temperature dependence are sensitive to the size of the fluctuations of the random potential. In the structure investigated here the fluctuations are of short period. It remains unclear how a change in the size of the fluctuations of the random defect potential affects the magnitude of the activation gaps under FQHE conditions. We note that FQHE has so far not been observed in silicon MOS structures with high electron mobility but with long period fluctuations.

In discussing other possible experimental means for studying activation gaps under the fractional quantum Hall

effect conditions, the method of resonance absorption of electromagnetic radiation should be singled out (the spectral region of the corresponding resonance absorption corresponds to the millimeter band for silicon MOS structures). this method is also promising for studying the expected exciton effect under FQHE conditions—the Coulomb coupling of quasielectrons and quasiholes with fractional charges into excitons.

In addition, it seems to us interesting to investigate the activation energy corresponding to fractional values of ν as a function of the channel thickness of two-dimensional electrons which can be varied appreciably in silicon MOS structures using a bias voltage (a difference between the potentials of the channel and semiconducting substrate) and in heterojunctions by using photoexcitation.

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