Numerical simulation of shock waves near comets: structural features and energy dissipation mechanisms

A. A. Galeev, A. S. Lipatov, and R. Z. Sagdeev

Institute of Cosmic Research, USSR Academy of Sciences
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The structure of a shock wave near a comet is investigated by means of numerical simulation. It is shown that the main contribution to energy dissipation at the wave front is made by ions from the comet produced in the solar wind due to the ionization of the neutral atmosphere of the comet. Due to the high cyclotron gyration velocity, some of them go out ahead of the wave front and are further accelerated in the electric field of the solar wind. However, this dissipation is insufficient and a resistive jump with a much smaller scale is established inside the shock front on a scale comparable to the Larmor radius of a cometary ion.

INTRODUCTION

The structure of the detached shock wave in front of a comet moving in a supersonic flow of solar wind plasma[1] is of interest not only in view of the recently launched probes of Halley's comet, but also because of the basic differences between the way the solar wind interacts with comets and with planets. The point is that, in the case of a planet, the solar wind encounters a localized obstruction (the magnetosphere or the ionosphere of the planet) but in the case of a comet the influence of the obstruction is felt at much greater distances. This is because the gravitational field of the comet's nucleus is negligible, so that the neutral gas which evaporates from the surface of the nucleus under the influence of solar radiation expands freely into interplanetary space. At a distance of one astronomical unit from the Sun, i.e., at the Earth's orbit, the rate of expansion is of the order of 1 km/sec and therefore the neutral gas manages to escape to millions of kilometers from the comet's nucleus before it is ionized under the influence of the solar radiation and solar wind electrons or owing to charge exchange (the characteristic times of all these processes are of the order of 10^6 sec). Under the influence of the electric field $\mathbf{E} = -[\mathbf{U}\cdot\mathbf{B}]/c$ of the solar wind plasma moving with a local hydrodynamic velocity $\mathbf{U}$ in the interplanetary magnetic field $\mathbf{B}$, the cometary ions drift together with the plasma across the magnetic field and simultaneously gyrate in a cyclotron orbit with the same velocity. In other words, the cometary ions are immediately picked up by the solar wind plasma flow, increasing its mass and thereby retarding it. Also, due to the rapid cyclotron gyration, the cometary ions make the main contribution to the plasma pressure. Moreover, in the process of deceleration of the solar wind the cyclotron gyration energy of the ions, and at the same time the plasma pressure, increase adiabatically. We note that the distribution of the cometary ions can be altered because of the development of plasma instabilities. However, the latter do not have a fundamental influence on the process described above.

The investigation of this "loading" of the solar wind by cometary ions[2] showed that the loading effect is not adequate to ensure continuous transition of the supersonic plasma flow far from the comet into a subsonic flow near it, i.e., a detached shock wave must be established in the flow of the loaded solar wind before the local Mach number, calculated using the magnetosonic velocity with allowance for the pressure of the cometary ions, falls to unity from the value $M = 5$–10 typical for the solar wind. Numerical calculations of solar wind flow past a comet[3] showed that for typical rates of evaporation of the gas from the surface of the comet's nucleus the shock wave is established at the point where the local Mach number is $M = 2$. Thus, by the time that the modified solar wind flow is just ahead of the shock front its state differs considerably from the state of the unperturbed solar wind and so we can anticipate that the jump of the plasma parameters will have a different structure at the front. It is clear that the heavy cometary ions must influence this structure.

Here it should be noted that the typical Mach numbers $M = 5$–10 for the solar wind flow at shock waves near planets significantly exceed the critical value at which hydrodynamical breaking of the shock front profile occurs[4]; this is a process which does not admit rigorous analytic description. Numerical simulation by the particle method[5] showed that in the kinetic description of the plasma the breaking corresponds to the appearance of a significant fraction of ions either reflected from the shock front or streaming out ahead of the wave front owing to the high thermal velocity of the ions behind the front.[6] At the same time, these ions acquire significant energy in the self-consistent electric field $\mathbf{E} = -[\mathbf{U}\cdot\mathbf{B}]/c$ of the plasma flow ahead of the wave front, before they again fall behind the shock front and are carried away by the plasma flow. This indicates that part of the kinetic energy of the plasma flow goes over into the cyclotron gyration energy of the ions. The irreversibility of such a transition is associated with the mixing of the phases of the cyclotron gyration of the ions in the strongly inhomogeneous magnetic field at the wave front. In accordance with the classification of anomalous dissipation mechanisms at the front of collisionless shock waves, this mechanism plays the part of an anomalous viscosity. The viscosity ν is estimated as $\nu \sim lv$, where the characteristic mean free path l is found to be of the order of the Larmor radius of the reflected ions, $l \sim r_{ci}$, and the "thermal" velocity is the velocity of their cyclotron gyration, i.e., it is of the order of the solar wind velocity.

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The characteristic thickness of the front here is of the order of the Larmor radius of the reflected ions. The calculated Mach number at the front of the shock wave near a comet is below the critical number and therefore the effect of the reflection of solar wind ions from the front must be insignificant. Nevertheless, we can anticipate that the cometary ions produced in the solar wind will play the role of reflected ions in this case and, therefore, the characteristic thickness of the front will be of the order of the Larmor radius of these ions. In this case the question remains open: Can the dissipative processes in the cometary component of the plasma by themselves ensure the required dissipation of energy at the shock front or is an even more abrupt resistive jump of the solar wind parameters necessary? We note that similar questions arise in the problem of a shock wave in space plasmas when the significant (with respect to the energy) fraction of ultrarelativistic charged particles (cosmic rays[7]) is treated; this problem is in many respects similar to the present one.

In this paper we investigate the structure of a shock wave near a comet by numerical simulation. For simplicity, we confine ourselves to the case of propagation of a shock wave strictly at right angles to the magnetic field and consider only the region immediately adjacent to the wave front. Accordingly, the velocity distribution of the cometary ions and their mass density ahead of the wave front are assumed given. The use of the adiabatic approximation makes it easy to find these parameters.[8] The ratio of the mass densities of the cometary ions and the protons is $\rho_i/\rho_p = 0.185$. In this case, the cometary ions are distributed almost uniformly with respect to the cyclotron gyration velocities in the magnetic field in the range U_1 to $1.5U_1$, where U_1 is the local hydrodynamic velocity of the plasma.

MATHEMATICAL FORMULATION OF THE PROBLEM AND NUMERICAL TECHNIQUE

To investigate the structure of a one-dimensional collisionless shock wave with "loading," we treat the combined flow of electrons, protons, and heavy ions along the z axis. The electrons are described in the hydrodynamic approximation and the protons and ions by the equation of motion of macroparticles. The magnetic field has only one component, B_y. All the hydrodynamic parameters are functions of the variable z. The basic equations have the following dimensionless form:

The equation of motion of the protons and the ions,

$$\frac{d}{dt} v_x = \frac{1}{\varepsilon M_j}(E_x - v_z B_y), \tag{1}$$

$$\frac{d}{dt} v_z = \frac{1}{\varepsilon M_j}(E_z + v_x B_y); \tag{2}$$

the generalized Ohm's law without allowance for electron inertia,

$$E_x = U_z B_y - \frac{1}{\mathrm{Re}}\frac{\partial}{\partial z} B_y, \tag{3}$$

$$E_z = -U_x B_y + \frac{\varepsilon}{M_a^2 n} j_x B_y - \frac{\varepsilon}{M_{se}^2 \gamma n}\frac{\partial}{\partial z} P_e; \tag{4}$$

the equation of state for the electron component,

$$P_e = n_e^{\gamma}; \tag{5}$$

Maxwell's induction equation:

$$\frac{\partial}{\partial t} B_y = -\frac{\partial}{\partial z}(U_z B_y) + \frac{1}{\mathrm{Re}}\frac{\partial^2}{\partial z^2} B_y, \quad j_x = -\frac{\partial B_y}{\partial z}. \tag{6}$$

Here ε is the ratio of the proton Larmor radius in the unperturbed magnetic field B_∞ to the characteristic dimension of the problem L,

$$\mathbf{U} = (\langle n_p \mathbf{v}_p\rangle + \langle n_i \mathbf{v}_i\rangle)/(\langle n_p\rangle + \langle n_i\rangle) \tag{7}$$

is the mean macroscopic plasma velocity, M_j is the particle mass divided by the proton mass, B and E are the magnetic and electric fields, $n_e \approx n_p$ and P_e are the number density and pressure of the electrons, M_a and M_s are the Alfvén and acoustic Mach numbers:

$$M_a = M_{se}(\gamma\beta_e/2)^{1/2}, \quad \beta_e = P_{e\infty}/(B_\infty^2/8\pi);$$

$\mathrm{Re} = 4\pi U_\infty L\sigma/c^2$ is the magnetic Reynolds number, U_∞ and n_∞ are the velocity and density of the protons in the unperturbed flow, γ is the effective specific heat ratio (in the calculations $\gamma = 2$), and σ is the effective conductivity.

The transformation from dimensional to dimensionless variables, which are used everywhere in this paper, has the form

$$t = Lt'/U_\infty, \quad r = Lr', \quad U = U_\infty U', \quad n = n_\infty n',$$

$$B = B_\infty B', \quad E = (U_\infty B_\infty/c)E', \quad P = P_\infty P'. \tag{8}$$

At the initial time we specify a hydrodynamic shock wave with the characteristic initial front width $\delta \neq \sim 0.5$, located in the center of the region ($z = 3$). The jumps in the parameters in the shock wave are determined by the continuity relations for the mass flow, the momentum, and the energy. For the values of the mean velocity, the density, and the magnetic field behind the shock wave we have the following expressions:

$$\frac{n_{p1}}{n_{p2}} = \frac{n_{i1}}{n_{i2}} = \frac{U_2}{U_1} = \frac{B_1}{B_2}$$

$$= \frac{1}{3}\left[1 + \frac{2/M_a^2 + 2(M_s^{-2} + {}^1/_2 n_{i1} M_i v_{Ti1}^2)}{1 + n_{i1} M_i}\right]. \tag{9}$$

The subscripts 1 and 2 correspond to the parameters ahead of and behind the shock wave, and v_{Ti} is the thermal velocity of the ions. The velocity distribution function of the protons at the initial time is Maxwellian:

$$f_p \propto \exp\left[\frac{-(v_z - U_z)^2 - v_x^2}{2v_{Tp}^2}\right]. \tag{10}$$

The velocity distribution function of the ions is specified to be uniform on the ring

$$v_{Ti}^2 < (v_z - U_z)^2 + v_x^2 < v_{Ti}^{*2} \tag{11}$$

in v_x-v_z space.

The temperature of the electrons, the protons, and the ions behind the shock wave is calculated on the assumption that the adiabatic invariant is conserved. At the left-hand end of the region ($z = 0$) an unperturbed flow of protons and ions ($n_p U_\infty$, $n_i U_\infty$) is specified. At the right-hand end ($z = z_k = 10$) a subsonic flow out of the calculated region was modeled by means of a return flow of particles with velocity $v_z > 2U_2$ into the region.

The processes on the shock wave were simulated by the particle in cell (PIC) method using a spatial grid of 500 points, with 250 000 macroprotons and 250 000 macroions. The equations of particle motion and magnetic field evolution were integrated using implicit schemes (see, for example, Ref. 9). The integration timestep τ satisfied the condition $\tau = \Delta z U_\infty^{-1}/10$ and the zone width along the z axis satisfied the condition $\Delta z \ll r_{cp2}$, where r is the Larmor radius of the protons behind the shock wave. The discrete distribution function of the protons and ions was reproduced by means of random number generators.

RESULTS AND DISCUSSION OF THE NUMERICAL SIMULATION

Figure 1 shows the profiles of the magnetic field, the density and velocity of the plasma, the Alfvén and magnetosonic Mach numbers, and the velocity distributions of the protons and cometary ions obtained by numerical simulation. In the calculations considered here the dimensionless parameters of the plasma in front of the shock wave are typical for the solar wind at the Earth's orbit:

$$M_a=8, \quad \beta_e=1, \quad \beta_p=0.5, \quad \varepsilon=0.7.$$

In order to reduce the running time of the calculation, the mass of the cometary ion was assumed to be equal to five proton masses ($M_i = 5$). At the same time, the ratio of the mass densities of the ion and proton components was kept close to the value obtained from a numerical calculation of the dynamics of loading of the solar wind by cometary ions.[6] For the case shown in Fig. 1, the ratio of the ion and proton densities was $n_{i1} M_i = 0.30$ and cyclotron gyration velocities for the cometary ions on the boundary $v_{ri} = 1, v_{ri}^* = 1.5$ were selected. With this choice the magnetosonic Mach number was $M = 1.8$.

The profiles of the basic plasma parameters shown in Fig. 1 are typical for all the calculations. They clearly show the resistive jump of the electron-proton plasma component, in front of which is the "foot" associated with the outflow of part of the cometary ions into the region preceding the wave front. A similar picture was observed in the laboratory experiments of Ref. 10. However, unlike these experiments, the outflow of ions into the region ahead of the shock front in our case occurred not because ions are reflected under the influence of the electrostatic field at the wave front and the Lorentz force, but owing to the high cyclotron gyration velocity of the cometary ions behind the wave front. We note that the heavy cometary ions hardly feel the electrostatic electric field at the front of the electron-proton jump and therefore immediately behind this shock move with almost the same velocity as in front of it. At the same time, the larger part of the ions that cross the proton jump in the negative phase of cyclotron gyration ($v_x < 0$, see Fig. 1) returns

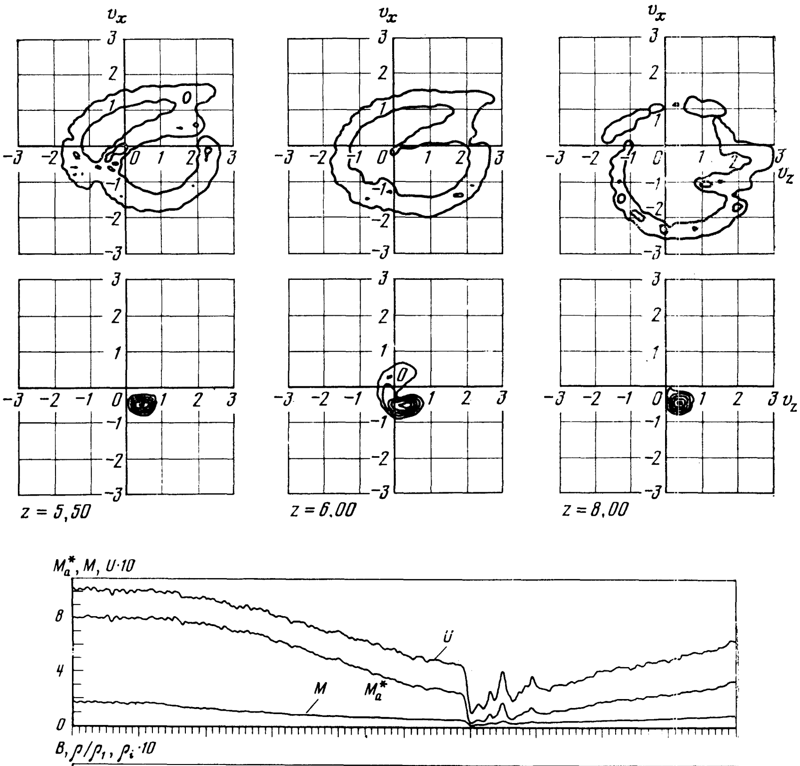

FIG. 1. Profiles of the magnetic field B, the density ρ (normalized by the perturbed value $\rho_1 = 1 + n_i M_i$), the plasma velocity U, the ion density ρ_i and the Alfvén Mach number M_a^* (calculated with allowance for the ions and normalized so that $M_a^* = M_a/\rho$) and the magnetosonic Mach number M. The velocity distribution functions for the protons and the ions.

towards it after a certain time and re-enter the region in front of the shock. Moving in the region in front of the jump, these ions are accelerated by the electric field $\mathbf{E} = -[\mathbf{U}\cdot\mathbf{B}]/c$ and after dropping behind the wave front they have an energy substantially greater than their initial energy. The contours of the velocity distribution function of the accelerated ions form a crescent-shaped structure in the plane (v_x, v_z), on the left above the ring of the basic distribution of ions in the foot region (Fig. 1). In turn, the supersonic plasma flow in front of the jump of the electron-proton plasma component is decelerated to velocities corresponding to the Alfvén Mach number $M_a \approx 3$–4 under the influence of the cometary ions which have streamed forward.

It should be noted that the "foot" in the picture of the shock wave described above is not steady, but pulsates with a characteristic time on the order of half the cyclotron gyroperiod of the cometary ions T_{ci}. To illustrate the pulsations of the "foot" Fig. 2 shows the profiles of the magnetic field, obtained at equal time intervals $\Delta t = 0.9T_{cp1}$. We see that, after its formation, the smooth foot again begins to accumulate in a narrow region and finally merges with the proton jump. Then the whole picture is again repeated. The pulsations are associated with the fact that the times for the cometary ions to stream into the region in front of the electron-proton jump and to return back to the jump add up to approximately the same total for most of these ions. In other words, they are due to the effect of the incomplete phase mixing of the outgoing ions. A further argument in favor of this conclusion is the fact that as the spread in the cyclotron gyration velocities decreases the local minima of the plasma velocity and maxima of the density in the region of the "foot" are enhanced significantly and can even give rise to the formation of a second jump there when such a velocity spread is absent.

Thus, the basic energy dissipation mechanism of the directed motion of the plasma in the shock wave described above is the acceleration in the electric field of the plasma flow $\mathbf{E} = -[\mathbf{U}\cdot\mathbf{B}]/c$ of the cometary ions which go out ahead of the proton jump. The energy of the ion cyclotron gyration increases and is ultimately transformed into heat due to the development of plasma instabilities. As the measurements on the Earth's bow shock wave show,[11] the most effective of them is the lower-hybrid instability predicted in Ref. 12 for the case of counterstreaming ions. However, this instability develops too slowly (over times of the order of tens of cometary ion gyroperiods) to appear in our calculations. Therefore, the motion of the ions can be assumed to be partially reversible. This fact appreciably influences the structure of the shock front.

In fact, in the presence of sufficient energy dissipation at the shock front the plasma parameters change monotonically from their values in the unperturbed plasma ahead of the front to values behind the front satisfying the familiar Rankine-Hugoniot relation. In this case, the electron-proton jump develops inside the shock front if the following condition is met: The velocity of the plasma behind the wave front must be less than the magnetosonic velocity in the electron-proton plasma component, i.e.,

$$\frac{U_2}{U_1} < \left[2\left(\frac{1}{M_a^2}+\frac{1}{M_s^2}\right)\frac{U_1}{U_2}\right]^{1/2}. \tag{12}$$

Here we have used the fact that the compression of the magnetic field and the proton and electron plasma components is adiabatic on the scale of the Larmor radius of the cometary ions, which characterizes the width of the shock front. Using relation (9), we rewrite this condition in the form of a bound on the ratio of the pressure of the cometary ions to the total pressure:

$$1-\frac{M^2(M_a^2+M_s^2)}{M_a^2M_s^2} \equiv K < 1-\frac{(M^2+2)^3}{27M^4}, \tag{13}$$

where

$$\frac{1}{M^2}=\frac{1}{M_a^2}+\frac{1}{M_s^2}+\frac{n_iM_iv_{Ti}^2}{(1+n_iM_i)U^2}, \qquad \frac{1}{M_s^2}=\frac{1}{M_{se}^2}+\frac{1}{M_{sp}^2}.$$

This is the parameter which is usually used to evaluate the possibility of the occurrence of a narrow plasma jump inside the shock front in a plasma in which most of the pressure comes from the cosmic rays.[13,14]

For typical parameters of the supersonic plasma flow of the solar wind at the near-comet shock front the pressure of the cometary ions dominates and condition (13) is not fulfilled. For example, for $M_a = 8$, $\beta_e = 1$, $\beta_p = 0.5$, $M = 2$ the parameter $K \approx 11/16 > K_{cr} = 0.5$. Nevertheless, the numerical calculations show that a resistive jump develops inside the front of such a shock wave. The solution of the paradox lies in the fact that in this problem the irreversibility of the motion of the cometary ions is associated only with the mixing of their cyclotron gyration phases. Therefore, on the scale of the Larmor radius of the cometary ions, their motion can be assumed to be reversible. In such a situation the plasma parameters do not at once reach the values determined by

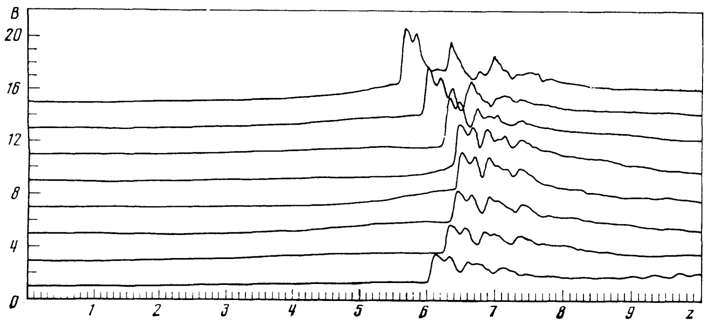

FIG. 2. Variation of the magnetic field profiles with time.

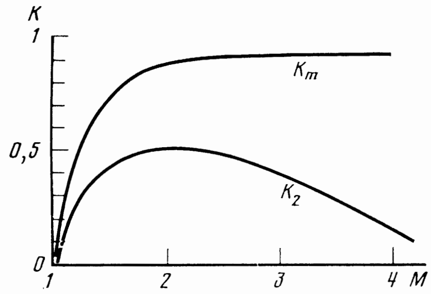

FIG. 3. Critical Mach number as a fraction of the cometary ions in the total pressure of the plasma.

the Rankire-Hugoniot relation, but execute several oscillations about them. At the same time the plasma pressure and the density overshoot their final values behind the front. One can imagine two simple mechanisms for such an overshoot. One of them is associated with the effects of the dispersion of waves in a weakly dissipative plasma.[4] The other treats the overshoot as a means of controlling the number of ions reflected from the shock front with the aim of achieving the necessary rate of energy dissipation at the front.[5] Even the model analytic calculation of self-consistent plasma flow with allowance for the reflection of ions is too approximate.[15] Therefore, in the Appendix we estimated the overshoot of the magnetic field and the plasma density for the case when the dispersion of magnetosonic waves due to the effect of the finite Larmor radius of the cometary ions determines the formation of the shock front.[16,17] The value found for the overshoot was used to determine the dependence of the critical Mach number on the fraction of cometary ions in the total pressure of the plasma (Fig. 3), which showed that for our parameters a jump of the electron-proton plasma must, in fact, have developed in the shock front.

These arguments make it possible to explain the behavior of the plasma parameters inside the shock front. Thus, the reduction of the Alfvén and total Mach numbers, the decrease of the plasma velocity, and the increase of its density in front of the jump are explained by the deceleration of the plasma flow by the heavy ions which stream out ahead. In the jump the plasma velocity falls below the local velocity of a magnetosonic wave in the electron-proton component of the plasma and, consequently, below the terminal velocity of the plasma behind the shock wave. The plasma reaches the parameters predicted by the Rankire-Hugoniot relations because behind the jump the plasma expands as it accelerates. The very fact of the expansion is associated with the partial reversibility of the energy increase and the compression of the ion component of the plasma in front of the jump. The expansion, like the compression, occurs on the scale of the Larmor radius of the cometary ions.

CONCLUSIONS

Thus, the problem of the cometary shock wave contains three entirely different scales. The first of them is characterized by the "loading" of the plasma flow of the solar wind by photoionized atoms and molecules from the cometary atmosphere, and at a heliocentric distance of about one astronomical unit it is from several hundred thousand to millions of kilometers. The second is associated with the outflow of cometary ions into the region ahead of the front because of their high cyclotron gyration velocity and, in order of magnitude, is equal to their Larmor radius. Taking into account that water is the basic component of cometary atmospheres, we find that for typical values of the interplanetary magnetic field $H = 5$ nT and solar wind velocity $U_1 \approx 400$ km/sec this scale is $\sim 10\,000$ km. Finally, inside the shock front there is a resistive jump in the density of the electron-ion plasma component with a typical scale of $(7–10)(c/\omega_{pe}) \approx 10$ km.[18] The presence of an electric current in the plasma because of the large gradient of the magnetic field at the jump gives rise to the excitation of strong ion-acoustic turbulence, which ensures energy dissipation owing to the anomalous resistance it has created to the electric current. In turn, the nonequilibrium velocity distribution of the cometary ions behind the jump acts as a source of very oblique whistler oscillations with frequencies on the order of the lower-hybrid resonance frequency. The growth rate of the excitation of these oscillations due to the admixture of heavy ions is $(m_i/m_p)^2$ times smaller than in the case of excitation by protons.[19] Accordingly, it must be anticipated that the strength of the excited oscillations will be significantly less than that measured at the Earth's bow shock.[11] In contrast, the energy of the electrons accelerated by these oscillations[20] increases by more than an order of magnitude due to the decrease of their fraction. This last is associated with the necessity of decreasing the Landau damping of these oscillations on the accelerated electrons, so that their growth rate nevertheless exceeds their damping.

Finally, the "hot" cometary ions stream far out into the region ahead of the shock front comparatively freely where the front forms a large angle with the magnetic field lines ($\geqslant 40°$). Conversely, the protons are heated very weakly by adiabatic compression in the region of the foot and when the plasma is heated by the electric current in the jump. Therefore, the main part in the process of Fermi acceleration in the shock wave[7] is played by heavy ions, and not protons, as in the case of the Earth's bow shock wave.

APPENDIX. CONDITION FOR FORMATION OF A JUMP INSIDE THE FRONT OF WEAK SHOCK WAVES IN PLASMA CONTAINING AN ADMIXTURE OF HOT HEAVY IONS

It is natural to expect that in plasma with an admixture of hot heavy ions whose pressure is a significant fraction of the total pressure of the plasma the dispersion effects associated with the finiteness of the Larmor radius of these ions must determine the thickness of the front of the weak shock waves. In Ref. 17, the dispersion of linear magnetosonic waves in a proton plasma with a small admixture of helium ions was investigated. A simple generalization of this paper to include an admixture of hot heavy ions after straightforward but involved algebraic calculations, yields the dispersion relation

$$\omega^2 = k^2 \frac{2P}{\rho}\left(1 - \frac{3}{2} k^2 R_i^2 K\right), \tag{A1}$$

where $R_i = v_{Ti}/\Omega_i$ is the Larmor radius of the ions. Here we have taken into account that $n_i M_i \ll n_p M_p$, $M_i v_{Ti}^2 \gg T_p$.

For plasma waves whose dispersion is negative as in Eq. (A1), magnetosonic compression solitons and shock waves with a trailing oscillatory tail can develop. To describe the nonlinear motions in the small-amplitude limit it is sufficient to take into account the dispersion of the waves in a linear approximation. As a result, the equations of continuity, motion, and heat balance are easily reduced to one equation for the hydrodynamic velocity:

$$4M^{-2}KR_i^2\frac{d^2U}{dx^2}=-\frac{\partial}{\partial U}W(U), \tag{A2}$$

where

$$W(U)=\frac{2(M^2+2)}{3M^2}U_1\left[\ln\frac{U_1}{U}-U_1+U\right]-(U_1-U)^2. \tag{A3}$$

Investigation of this equation[12] shows that the velocity U_m at the maximum of the overshoot of the pressure and velocity U_2 far behind the front are determined from the relations

$$\frac{\partial}{\partial U}W(U)\,|_{U=U_2}=0, \qquad W(U_m)=0. \tag{A4}$$

Substitution of the values of U_2 and U_m found here in condition (12) makes it possible to find the critical Mach numbers as functions of the ratio of the pressure of the heavy hot ions for two limiting cases: viscous shock wave without overshoot of the field and shock wave with a weakly damped oscillatory tail (curves 1 and 2 in Fig. 3). The estimate of the critical Mach number thus obtained is not at all rigorous, since it does not take into account the dissipation effects associated with the outflow of hot ions into the region ahead of the wave front.

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