Nonexponential attenuation of electromagnetic fields in normal metals

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The structure of longitudinal electric and transverse electromagnetic fields in normal metals is analyzed. "Pulling" of the field into the metal due to a nonanalytic dielectric tensor is described. The roles played by the thermal motion of electrons, by quantum effects (the nonzero energy and momentum of the photon), and by Fermi-liquid interaction are studied. The field structure far from a plasma resonance is relatively insensitive to the nature of the electron scattering by the boundary of the metal.

§1. INTRODUCTION

The reaction of a metal to an external perturbation (an electromagnetic wave incident on its surface) is customarily described by a surface impedance (Ref. 1, for example). Study of the behavior of metals in magnetic fields has shown, however, that the electromagnetic field distribution in the interior of a metal carries important information on the conduction electrons,[2,3] and it is the field distribution which is responsible for various high-frequency size effects. Although "nonhydrodynamic" behavior of the field was observed at large distances from the surface in the classic study by Reuter and Sondheimer[4] when anomalous skin effect was present, the only discussion of a direct role of electrons carrying the electromagnetic field over a distance on the order of the mean field path l, considerably larger than the skin depth, has been in connection with various effects in a magnetic field.[2,3] Apparently the first clear assertion that there is essentially always (either with or without a magnetic field) a part of the field which is due directly to conduction electrons, and which is not seen in a macroscopic (hydrodynamic) treatment, was made in Ref. 5 (this assertion applies to both electromagnetic fields and ion displacement fields).

The role played by the nonhydrodynamic (kinetic) part of the electromagnetic field[1)] varies with the situation: In the case of a normal skin effect, it determines the structure of the surface layer, while in the case of the anomalous skin effect it determines the asymptotic behavior of the field at large distances. For this reason, this part of the field is frequently called the "pulling field." We will adopt this term here, since we are interested primarily in the collisionless limit, where l is much larger than all the other parameters with the units of length.

Pulling fields are formed by electrons which are moving normal to the surface of the metal, so there is no reason to believe that they are reflected in a specular manner by the surface.[6] We have analyzed the role played by the boundary conditions for the electron distribution for the particular case of a longitudinal electric field (§5). It is found that the transition from specular reflection to diffuse reflection usually (away from any plasma resonance) does not change the structure of the pulling field, simply changing its amplitude, by a factor of about two. This makes is possible to restrict the analysis of several comparatively subtle effects [the role played by the temperature (§§2,3), the Fermi-liquid interaction, and the complex structure of the Fermi surface (§4)] to a specular reflection of electrons by the surface. The result is to dramatically simplify the calculations.

The role played by the temperature T in many properties of metals reduces to the dependence of the mean free path on T. The structure of the pulling field is a rare exception: A rounding of the Fermi step at $T \neq 0$ is manifested at large distances from the boundary of the sample (§§2,3).

In this paper we analyze the structure of the longitudinal electric field $\mathbf{E}_\parallel(x,t) = \mathbf{E}_\parallel(x)e^{-i\omega t}$ and of the transverse electromagnetic field $\mathbf{E}_\perp(x,t) = \mathbf{E}_\perp(x)e^{-i\omega t}$ in the metal half-space $x > 0$ for the case of normal incidence at the $x = 0$ boundary of the metal, i.e., the case of a wave vector $\mathbf{k}\|\mathbf{x}$. For simplicity we assume that the x axis runs along a "good" (symmetric) direction of the crystal, so that the off-diagonal components of the electron dielectric tensor are zero.

In the case of specular reflection of electrons by the boundary of the metal, the field distribution can be determined by the Fourier method if the longitudinal field $\mathbf{E}_1(x)$ (see the discussion below) is continued as an odd function, and the transverse field $\mathbf{E}_\perp(x)$ as an even function into the $x < 0$ half-space[4,7]:

$$E_\parallel(x) = E_1(x) + E_0/\varepsilon_0, \tag{1}$$

$$E_1(x) = \frac{iE_0}{\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{\varepsilon_{xx}(\omega,k)-\varepsilon_0}{k\varepsilon_{xx}(\omega,k)}e^{ikx}dk, \tag{1'}$$

$$E_\perp(x) = \frac{cH_0}{\pi i\omega}\int_{-\infty}^{\infty}\left[1+\frac{k^2c^2}{\omega^2}-\varepsilon_{zz}(\omega,k)\right]^{-1}e^{ikx}dk, \tag{2}$$

where $\mathbf{H}\|\mathbf{y}$ is the magnetic field of the wave, $H_0 \equiv H(x=0)$, $E_0 \equiv E_\parallel(x=0)$, $\mathbf{E}_\perp\|\mathbf{z}$, $k \equiv k_x$, $\varepsilon_{\alpha\beta}(\omega,k)$ are the components of the dielectric tensor, and $\varepsilon_0 \equiv \varepsilon_{xx}(\omega,0)$.

Expressions (1) and (2) essentially describe the field structure in a boundary layer, but the thickness of this layer is macroscopic because of the long electron mean free path l. In the interior of the metal ($x \to \infty$) the longitudinal and transverse fields satisfy

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$$E_{\parallel}(+\infty)=E_0/\varepsilon_0, \quad E_{\perp}(+\infty)=0.$$

The pulling field is determined by the particular features of the components $\varepsilon_{\alpha\beta}$ as functions of k. In the semiclassical approximation,[2)] $\hbar|k|\ll p_F$, $\hbar\omega\ll\varepsilon_F$ (p_F and ε_F are the Fermi momentum and energy), we have the following expression for the components $\varepsilon_{\alpha\beta}$ of the degenerate Fermi gas of electrons in the τ approximation ($\tau = l/v_F$ is the electron relaxation time, and l is the mean free path):

$$\varepsilon_{\alpha\beta}^{(\tau)}(\omega,k)=\delta_{\alpha\beta}-\frac{8\pi e^2}{\omega(2\pi\hbar)^3}\int v_\alpha R_\tau v_\beta\frac{\partial n}{\partial\varepsilon}d^3p, \qquad (3)$$

where $n = n(\varepsilon)$ is the Fermi distribution function, and $R_\tau = (kv_x - \omega - i/\tau)^{-1}$ is the Green's function of the kinetic equation in the τ approximation.

At $T = 0$, expression (3) can be rewritten as

$$\varepsilon_{\alpha\beta}^{(\tau)}(\omega,\ k)=\delta_{\alpha\beta}+\frac{4\pi e^2}{\omega}\langle v_\alpha R_\tau v_\beta\rangle$$

$$\equiv\delta_{\alpha\beta}+\frac{8\pi e^2}{\omega(2\pi\hbar)^3}\oint_{(\varepsilon_{\mathbf{p}}=\varepsilon_F)}v_\alpha R_\tau v_\beta\frac{dS}{v}. \qquad (4)$$

The components $\varepsilon_{\alpha\beta}^{(g)}$ in (4) have singularities[5,8] in the collisionless limit ($\tau\to\infty$). These singularities stem from the multiple zeros of the expression $kv_x - \omega$. The equations

$$\Lambda(\xi,\eta)\equiv kv_x-\omega=0, \quad \partial\Lambda/\partial\xi=\partial\Lambda/\partial\eta=0, \qquad (5)$$

where ξ and η are local coordinates on the Fermi surface, determine the point $\mathbf{p}_c$ ($\xi=\xi_c,\eta=\eta_c$) on the Fermi surface and also that value ($k = k_c$) at which the singularities of $\varepsilon_{\alpha\beta}^{(g)}$ are observed. In the case of complex Fermi surfaces, Eqs. (5) may, for a single k_c, determine several points $\mathbf{p}_c$ or even a line on the Fermi surface and also several values of k_c—a spectrum of singularities. The singularity of $\varepsilon_{\alpha\beta}^{(g)}$ coincides with that of $\langle R_\infty\rangle$ if the expression $v_\alpha v_\beta$ is nonzero at $\mathbf{p}=\mathbf{p}_c$.

At a nonzero temperature the nature of the singularities in the components $\varepsilon_{\alpha\beta}^{(g)}$ changes qualitatively. The only singularity is an essential singularity at $k = 0$. If the Fermi surface is a sphere, then for $\varepsilon_{xx}^{(g)}(\omega,k)$ the point $k=0$ is an accumulation point of branch points. This nature of the singularities also determines the particular asymptotic behaviors of the fields at extremely large distances $x\gg(\varepsilon_F/T)(v_F/\omega)$. At $T\ll\varepsilon_F$, at distances

$$\frac{v_F}{\omega}\ll x\ll\frac{\varepsilon_F}{T}\frac{v_F}{\omega},$$

the intermediate asymptotic behavior of the fields remains the same as in the case $T = 0$.

In concluding this section we wish to emphasize that the diffuse scattering of electrons by the surface of a metal is examined only in §5; in the other sections of this paper we are assuming a specular reflection law.

§2. LONGITUDINAL PULLING FIELDS IN METALS WITH A SPHERICAL FERMI SURFACE (SPECULAR REFLECTION)

We begin with a study of the structure of a longitudinal electric field, assuming that the dispersion relation for the conduction electrons is quadratic and isotropic. This subsection generalizes Landau's study[7] to the case of a degenerate plasma.

1. $T = 0$. In the case of a spherical Fermi surface we see from (5) that we have $k_c = k_0\equiv\omega/v_F$, and the singularities are produced by a single point $\mathbf{p}_c$ on the Fermi surface, at which we have $p_x = p_F$, $p_y = 0$, $p_z = 0$. From (4) we have

$$\varepsilon_{xx}^{(g)}(\omega,k)\equiv\varepsilon(\sigma)\equiv1+\frac{3\beta}{\sigma^2}\left(1+\frac{1}{2\sigma}\ln\frac{1-\sigma}{1+\sigma}\right), \qquad (6)$$

where $\sigma\equiv kv_F/(\omega+i/\tau)$, $\beta\equiv\omega_0^2\,\omega^{-2}(1+i/\omega\tau)^{-1}$, and $\omega_0 = (4\pi e^2N/m^*)^{1/2}$ is the plasma frequency (m^* is the effective mass, and N is the electron density). Under these conditions we have $\varepsilon_0 = 1-\beta$.

To evaluate the integral in (1') by means of the theory of the residues, we must analytically continue the integrand into the complex plane $k = k' + ik''$. We choose the integration contour, in accordance with the sign of x ($x>0$), in the upper half-plane, $k''>0$ (Fig. 1). The branch point of the integrand, $k = k_b$, coincides with the singularity of $\varepsilon_{xx}^{(g)}$: $k_b = k_0 + i/l$. To single out the single-valued branch of the function $\varepsilon_{xx}^{(g)}(\omega,k)$, we make a cut in the k plane parallel to the imaginary axis. The field in a metal is partitioned into the sum of fields of different types: a normal, hydrodynamic field, determined by the residues [the zeros of $\varepsilon_{xx}^{(g)}(\omega,k)$] in the upper half-plane, and an anomalous field, which is written as an integral along the cut:

$$E_{\rm an}^{\parallel}(x)=\frac{iE_0}{\pi}\int_C\frac{\exp(ikx)}{k\varepsilon(\sigma)}dk, \qquad (7)$$

where C is the integration contour along the banks of the cut (the contour C is shown by the heavy line in Fig. 1); this is the pulling field.

At $x\gg k_0^{-1}$, the integral in (7) is determined primarily by the region of values $|\Delta k''|\ll k_0$, where $\Delta k'' = k''-k_b''$, so it can be rewritten as follows with an accuracy sufficient for these calculations:

$$E_{\rm an}^{\parallel}(x)=\frac{2}{3\pi}\frac{\omega^2}{\omega_0^2}\frac{E_0}{k_0}\exp(ik_0x)\int_{1/l}^{\infty}\left\{\left[\ln\left(\frac{k''-1/l}{2k_0}\right)+\frac{3\pi i}{2}\right]^{-1}-\left[\ln\left(\frac{k''-1/l}{2k_0}\right)-\frac{\pi i}{2}\right]^{-1}\right\}\exp(-k''x)\,dk''. \qquad (7')$$

We then find the asymptotic value of the longitudinal pulling field (at $x\gg k_0^{-1}$):

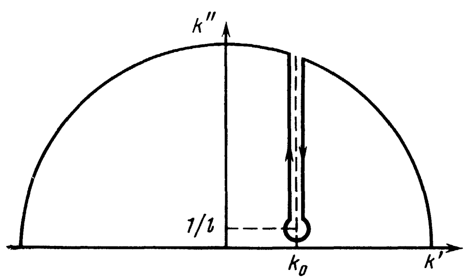

FIG. 1.

$$E^{\parallel}_{\text{an}}(x)\approx\frac{4}{3i}\frac{\omega^2}{\omega_0^2}E_0(k_0x)^{-1}\ln^{-2}(k_0x)\exp\left(ik_0x-\frac{x}{l}\right). \quad (8)$$

The inhomogeneous part of the longitudinal field is pulled into the metal by electrons over a mean free path. The wave of the pulling field propagates at the Fermi velocity, and in the collisionless limit is it damped exponentially in accordance with $(k_0x)^{-1}\ln^{-2}(k_0x)$, like a longitudinal ultrasonic wave (cf. Ref. 9).

We now consider the exponentially decaying normal part of the field. To determine the zeros of the function $\varepsilon(\sigma)$, we find its asymptotic behavior. For $|\sigma|\ll 1$ we find

$$\varepsilon(\sigma)\approx\varepsilon_0\left(1+\frac{\sigma}{\sigma_0}\right)\left(1-\frac{\sigma}{\sigma_0}\right),\quad \sigma_0=\left[\frac{5(1-\beta)}{3\beta}\right]^{1/2}, \quad (9)$$

for $|\sigma|\gg 1$ we find

$$\varepsilon(\sigma)\approx\left(1+\frac{\sigma_1}{\sigma}\right)\left(1-\frac{\sigma_1}{\sigma}\right),\quad \sigma_1=(-3\beta)^{1/2}. \quad (9')$$

It can be seen from (9) and (9') that a distinction should be made between two cases: the case far from a resonance, with $\omega\ll\omega_0$ and $|\beta|\gg 1$, and the case near a resonance, with $\omega\approx\omega_0$ and $|1-\beta|\ll 1$. At $|\beta|\gg 1$, the pole is at the point $k=k_n\approx k_0\sigma_1(1+i/\omega\tau)$, and the ordinary wave is

$$E^{\parallel}_{\text{ord}}(x)\approx E_0\exp(-3^{1/2}\omega_0x/v_F). \quad (10)$$

As expected, this wave is damped over a distance on the order of the Debye-Hückel radius for a degenerate plasma.

Near a resonance, with $|1-\beta|\ll 1$, the pole is $k_n\approx k_0\sigma_0(1+i/\omega\tau)$, and the ordinary wave is

$$E^{\parallel}_{\text{ord}}(x)\approx E_0(\beta-1)^{-1}\exp\{i[5(1-\beta)/3]^{1/2}k_0x\}. \quad (11)$$

At $\omega>\omega_0$ in the collisionless limit ($\omega\tau\to\infty$), there is an undamped wave[3)] with

$$k=k_0[{}^5/_3(\omega^2/\omega_0^2-1)]^{1/2} \quad (12)$$

in the medium.

It should be recalled, however, that in most real metals the energy of the plasma waves ($\hbar\omega_0$) is numerically close to ε_F. Accordingly [so that we do not go beyond the scope of the semiclassical approximation, (3)], we can apply results (11) and (12) under the condition $\hbar\omega_0\ll\varepsilon_F$. For metals, this condition is the same as the condition of a high density[11]: $(e^2/\hbar v_F)^{1/2}\ll 1$. For degenerate semiconductors, the condition that the resonance be semiclassical is conveniently rewritten as $e^2m^*/(\varepsilon_i\hbar^2N^{1/3})\ll 1$, from which we see that this condition can be satisfied as a result of a large ion dielectric constant ε_i and/or a small effective carrier mass m^*.

The direct application of (11) to the case of the exact resonance, $\omega=\omega_0$, leads to the result

$$E_{06}^{\parallel}(x)\approx i\omega_0\tau E_0\exp[({}^5/_6\omega_0/v_Fl)^{1/2}(i-1)x], \quad (13)$$

which obviously excludes the limit $\tau\to\infty$. In this connection we recall that we are seeking the field (1) in the linear approximation in the form of steady oscillations, but for an exact resonance this case would be meaningless for a dissipationless system. Result (13) holds for a large (but finite) value $\omega_0\tau\gg 1$ if the condition for the applicability of the linear approximation still holds for the kinetic equation (Ref. 10, for example).

2. $T\neq 0$, $l=\infty$. Mathematically, the existence of branch points of the function $\varepsilon^{(g)}_{xx}(\omega,k)$ calculated from (6) is a consequence of the existence of a limiting electron velocity at $T=0$. In this connection it is interesting to determine what changes would be caused by incorporating the thermal motion of the electrons at a nonzero temperature $T\ll\varepsilon_F$. From (3), integrating over v_y and v_z in the collisionless limit, we find

$$\varepsilon^{(r)}_{xx}(\omega,k)=1+\frac{3}{2}\frac{\omega_0^2}{\omega kv_F}\int\limits_{-\infty}^{\infty}\frac{f_T(u)u^2du}{u-(\omega+i0)/kv_F}, \quad (14)$$

where $f_T(u)\equiv\{\exp[\gamma_T(u^2-1)]+1\}^{-1}$, $\gamma_T\equiv\zeta/T$, and ζ is the chemical potential of the electrons, which is essentially the same as ε_F at $T\ll\varepsilon_F$. We have introduced the variable $u=v_x/v_F$, where $v_F=(2\zeta/m^*)^{1/2}$. Expression (14) determines two different functions upon an analytic continuation of $\varepsilon^{(g)}_{xx}(\omega,k)$ into the complex k plane: $\varepsilon_1(\omega,k)$ and $\varepsilon_2(\omega,k)$, depending on the sign of k. The two functions correspond to the possibilities of integrating around the pole $u=(\omega+i0)/kv_F$ in (14) from below ($k>0$) or from above ($k<0$) for real k:

$$\varepsilon_{1(2)}(\omega,k)=1+\frac{3}{2}\frac{\omega_0^2}{\omega kv_F}\int\limits_{\Gamma_{1(2)}}\frac{f_T(u)u^2du}{u-(\omega+i0)/kv_F}, \quad (15)$$

where the contours Γ_1 and Γ_2 circumvent the pole $u=\omega/kv_F$ from below and above, respectively. The functions $\varepsilon_1(\omega,k)$ and $\varepsilon_2(\omega,k)$ specified in this way are analytic over the entire complex k plane. The field $E_1(x)$ can be written in a form more convenient for calculations, in accordance with Ref. 7:

$$E_1(x)=\frac{iE_0}{\pi\varepsilon_0}\int\limits_{-\infty}^{\infty}\frac{\varepsilon_2(\omega,k)-\varepsilon_0}{k\varepsilon_2(\omega,k)}e^{ikx}dk+\frac{iE_0}{\pi}\int\limits_{0}^{\infty}\frac{\varepsilon_1(\omega,k)-\varepsilon_2(\omega,k)}{k\varepsilon_1(\omega,k)\varepsilon_2(\omega,k)}e^{ikx}dk. \quad (16)$$

The difference $\varepsilon_1(\omega,k)-\varepsilon_2(\omega,k)$ is given by an expression of the type in (15), but the integration is carried out over a closed contour around the pole $u=\omega/kv_F$, traversed in the clockwise direction (cf. Ref. 7):

$$\varepsilon_1(\omega,k)-\varepsilon_2(\omega,k)=3\pi i\frac{\omega_0^2}{\omega^2}\frac{k_0^3}{k^3}f_T\left(\frac{k_0}{k}\right). \quad (17)$$

The functions ε_1 and ε_2 are related by

$$\varepsilon_1(\omega,k)=\varepsilon_2(\omega,-k),\quad \varepsilon_1(\omega,k^*)=\varepsilon_2^*(\omega,k).$$

Both of the functions ε_1 and ε_2 tend toward unity at infinity and have a unique singularity $k=0$. The function $\varepsilon_1(\omega,k)$ tends toward the value $\varepsilon_0\equiv\varepsilon_1(\omega,0)$ in the limit $k\to 0$ along any path in the upper half-plane outside the rectangular sector which contains the imaginary axis and which is formed by rays emerging from the point $k=0$ and making angles of $\pm\pi/4$ with the imaginary axis. The same comments evidently apply to the function $\varepsilon_2(\omega,k)$ in the lower half-plane (the angles here are $\pm 3\pi/4$).

It can be shown that the equation $\varepsilon_2(\omega,k)=0$ has an infinite number of roots in the lower half-plane with an accumulation point $k=0$ (according to Picard's theorem regarding the value of a function near an essential singularity[12]), while in the upper half-plane it either has no roots (if $\varepsilon_0>0$) or has one root on the imaginary axis (if $\varepsilon_0<0$). Analogously, the equation $\varepsilon_1(\omega,k)=0$ has an infinite number of roots of the upper half-plane, while in the lower half-plane it either has no roots (if $\varepsilon_0>0$) or has one root on the imaginary axis (if $\varepsilon_0<0$). Displacing the integration contour in the first integral in (16) to infinity in the upper half-plane, we find that this integral either vanishes (if $\varepsilon_0>0$) or reduces to a calculation for a pole on the imaginary axis (if $\varepsilon_0<0$). In the latter case the first term in (16) falls off exponentially with distance [see (11)], and it becomes insignificant at large values of x. We denote the second term in (16) by $E^{\parallel}_{\rm an}(x)$. Substituting (17) into (16), we find

$$E^{\parallel}_{\rm an}(x)=-3E_0\frac{\omega_0^2k_0^3}{\omega^2}\int_0^\infty\frac{f_T(k_0/k)e^{ikx}}{k^4\varepsilon_1(\omega,k)\varepsilon_2(\omega,k)}dk. \qquad (18)$$

It is difficult to evaluate this integral because of the infinite number of poles with the accumulation point $k=0$. On the other hand, it is a simple matter to find an asymptotic expression describing the behavior of $E^{\parallel}_{\rm an}(x)$ at large distances $x\gg\gamma_T/k_0$ by the method of steepest descent.[7] The transition region for the function $f_T(k_0/k)$ is on the order of k_0/γ_T, so at intermediate distances, $k_0^{-1}\ll x\ll\gamma_T/k_0$, the function $f_T(k_0/k)$ can be replaced by a Fermi step, and we return to Eqs. (6)–(8). At extremely large distances, $x\gg\gamma_T/k_0$, values $k\ll k_0$ are important in the integral in (18), so that we have

$$E^{\parallel}_{\rm an}(x)\approx\frac{3E_0}{\varepsilon_0^2}\frac{\omega_0^2k_0^3}{\omega^2}\exp(\gamma_T)\int_0^\infty\exp\left(ikx-\gamma_T\frac{k_0^2}{k^2}\right)\frac{dk}{k^4}, \qquad (18')$$

and for an extremal point in the argument of the exponential function we have $|k_{ex}|=k_0^{2/3}\gamma_T^{1/3}/x^{1/3}\ll k_0$.

Calculations analogous to those in Ref. 7 lead to

$$E^{\parallel}_{\rm an}(x)\approx\frac{3^{1/2}E_0}{\varepsilon_0^2}\frac{\omega_0^2}{\omega^2}\gamma_T^{-1/2}\left(\frac{\omega x}{v_T}\right)^{1/2}\times\exp(\gamma^T)\exp\left\{-\frac{3}{4}\left(\frac{\omega x}{v_T}\right)^{2/3}(1-i\cdot3^{1/2})+\frac{2\pi i}{3}\right\}, \qquad (19)$$

where $v_T\equiv(T/m^*)^{1/2}$. At distances $x\gg\gamma_T/k_0$ the field $E^{\parallel}_{\rm an}(x)$ thus falls off in a manner characteristic of a nondegenerate plasma. Substituting $\omega_0^2=4\pi Ne^2/m^*$ into (19), and comparing the resulting expression with expression (44) of Ref. 7, we find that, in contrast with Landau's result for a Maxwellian plasma, we have an effective density $N_{\rm eff}\sim N(T/\varepsilon_F)^{3/2}$ in (19) instead of the total electron density N. This effectiveness $(T/\varepsilon_F)^{3/2}$ is significantly smaller than the usual value T/ε_F; this result is to be expected since the pulling field (19) is formed by remote thermal electrons. The "intimidating" exponential factor $\exp(\gamma_T)$ is actually "covered" by the attenuation: at $k_0x\gg\gamma_T$ we simultaneously have $\gamma_T\ll(\omega x/v_T)^{2/3}$.

At resonance ($\varepsilon_0=0,\omega=\omega_0$) we have

$$E^{\parallel}_{\rm an}(x)\approx\frac{25\cdot3^{1/2}E_0e^{\gamma_T}}{18\gamma_T^{1/2}}\left(\frac{\omega_0x}{v_T}\right)^2\times\exp\left\{-\frac{3}{4}\left(\frac{\omega_0x}{v_T}\right)^{2/3}(1-i\cdot3^{1/2})\right\}. \qquad (20)$$

By incorporating the thermal motion we can treat the resonant situation without taking dissipation into account (thermal motion is equivalent to blurring of the resonance). The behavior $E^{\parallel}_{\rm ord}(x)$ is studied by a method similar to that in the degenerate case; as a result we find (11).

At $\omega\gtrsim\omega_0$ the thermal motion of the electrons in spectrum (12) gives rise to an exponentially small attenuation:

$$k=k_0\left\{\left(\frac{5\varepsilon_0}{3}\right)^{1/2}+\frac{9\pi i}{20\varepsilon_0^2}\exp\left[-\gamma_T\left(\frac{3}{5\varepsilon_0^2}-1\right)\right]\right\}.$$

In this case we have

$$E^{\parallel}_{\rm ord}(x)\approx-\frac{E_0}{\varepsilon_0}\exp\left\{ik_0x\left(\frac{5\varepsilon_0}{3}\right)^{1/2}-\frac{9\pi}{20\varepsilon_0^2}k_0x\exp\left[-\gamma_T\left(\frac{3}{5\varepsilon_0^2}-1\right)\right]\right\}. \qquad (11')$$

We see that the field $E^{\parallel}_{\rm ord}(x)+E_0/\varepsilon_0$ initially increases in amplitude to $2E_0/\varepsilon_0$ and then tends toward the uniform value E_0/ε_0, undergoing exponentially damped oscillations around it.

§3. TRANSVERSE PULLING FIELDS IN METALS WITH A SPHERICAL FERMI SURFACE

1. $T=0$. As in the other sections below, we are interested in the long-range limit. It is customary here to distinguish between two limiting cases on the basis of the frequency:

the extremely anomalous skin effect,

$$k_0l\gg\alpha^{-1/3},\quad \alpha\gg1; \qquad (21)$$

infrared optics,

$$k_0l\gg1\gg\alpha. \qquad (22)$$

Here $\alpha=3/4(\omega_0^2/\omega^2)(v_F^2/c^2)$. For the asymptotic value of transverse pulling fields at $x\gg k_0^{-1}$ we find the following results from (2), using (4) and choosing a cut parallel to the imaginary axis in the complex plane $k=k'+ik''$ (Fig. 1):

in region (21),

$$E^{\perp}_{\rm an}(x)\approx-(v_F/c\alpha)H_0(k_0x)^{-2}\exp(ik_0x-x/l). \qquad (23)$$

in region (22),

$$E^{\perp}_{\rm an}(x)\approx-(4v_F/c)\alpha H_0(k_0x)^{-2}\exp(ik_0x-x/l). \qquad (24)$$

The asymptotic behavior in (23) was found in Ref. 4 in the region $\alpha^{-1/3}\ll k_0l\ll1$, which is contained in (21). We can also write expressions for the amplitudes of ordinary waves, which are determined by the poles of the integrands in (2). In region (21) with $1\ll k_0l\ll\alpha^{1/3}$ (the case $\alpha^{-1/3}\ll k_0l\ll1$ was studied in Ref. 4) we have

$$E^{\perp}_{\rm ord}(x)\approx\frac{2v_F}{3\pi^{1/2}c}\alpha^{-1/3}H_0\left\{\exp\left[-\frac{x}{\delta_1}(1-i\cdot3^{1/2})-i\frac{\pi}{6}\right]\right.$$

$$+\exp\left(-\frac{2x}{\delta_1}-i\frac{\pi}{2}\right)\Big\},\qquad (25)$$

while in region (22) we have

$$E^{\perp}_{\rm ord}(x)\approx\frac{3^{1/2}v_F}{2c}\alpha^{-1/2}H_0\exp\left(-\frac{x}{\delta_2}-i\frac{\pi}{2}\right),\qquad (26)$$

where the skin depth is different in the different cases:

$$\delta_1=\frac{2k_0^{-1}}{(\alpha\pi)^{1/3}}=\left(\frac{32}{3\pi}\right)^{1/3}\left(\frac{c^2}{\omega_0^2k_0}\right)^{1/3};\quad \delta_2=\frac{3^{1/2}}{2}\alpha^{-1/2}k_0^{-1}\approx\frac{c}{\omega_0}.\qquad (27)$$

Comparison of the ordinary and anomalous terms shows that in region (21) for $x\gg k_0^{-1}$ and in region (22) for $x\gg\delta_2$ we have $\text{Re}\,E^{\perp}_{\rm an}(x)\gg\text{Re}\,E^{\perp}_{\rm ord}(x)$; i.e., the quasiwave which results from electrons of the reference point on the Fermi surface plays a leading role in metals far from the boundary.

To some extent, the question of the number of roots of the dispersion relation (which determine ordinary waves) is relative. The very form of the dispersion function $D(\omega,k)=1+k^2c^2/\omega^2-\varepsilon^{(g)}_{zz}(\omega,k)$ depends on the method used to identify the single-valued branch of the multivalued function $\varepsilon^{(g)}_{zz}(\omega,k)$ (multivalued in the presence of branch points). This function can be written

$$\varepsilon^{(g)}_{zz}(\omega,k)=1+\frac{3}{4}\frac{\beta}{\sigma^3}[-2\sigma+(\sigma^2-1)\varphi(\sigma^{-1})],$$

$$\varphi(\chi)=\int_{-1}^{1}\frac{dz}{z-\chi}.$$

The function $\varphi(\chi)$ has two branch points, at $\chi_b=\pm1$. Introducing a cut from the point $\chi_b=1$ through $\chi=0$ [the corresponding cut in the k plane must not intersect the real axis—the integration path in (2)] to $\chi_b=-1$, and specifying $\arg(z-\chi)$ at any point, we determine the single-valued function $\varepsilon^{(g)}_{zz}(\omega,k)$ and its range of existence. Consequently, by specifying a cut in the k plane in calculating the anomalous terms we are simultaneously specifying the form of the function $\varepsilon^{(g)}(\omega,k)$, which in turn determines the roots of the dispersion relation $D(\omega,k)=0$, i.e., the ordinary terms.

One form of the cut is most convenient—from the points $k_b=\pm(k_0=i/l)$ parallel to the imaginary axis—in a calculation of the asymptotic values of the anomalous fields. Similarly, for this choice of the cut the integral along the cut gives the best description of the effect of the pulling of the field by conduction electrons: The exponential attenuation of the pulling field is determined exclusively by the range. Since the method for making the cut is given and does not depend on the parameters of the problem, a change in parameters (e.g., $\omega\tau$) may create a situation in which a root k_n of the dispersion relation falls at the cut. As the point k_n goes from one side of the cut to the other, the quantity $\text{Im}\,\varphi(\sigma^{-1})$ changes discontinuously by $\pm2\pi i$, so that the point k_n generally ceases to be a root of the dispersion relation. We might note that the possible appearance and the disappearance of roots singly, rather than in pairs, is due exclusively to the presence of the cut because of the branch points of the function $\varepsilon^{(g)}(\omega,k)$. In region (21), for example, as we go to lower frequencies ($\alpha^{-1/3}\ll k_0l\ll1$) the field $E^{\perp}_{\rm ord}(x)$ is described by expression (25), but it contains one term, the first. As we go to higher frequencies ($k_0l\gg\alpha^{1/3}$), it contains only the second term. As $\omega\tau$ is varied in accordance with the discussion above, the roots of the dispersion relation, moving with respect to the cut, intersect it. It should be noted that as the point k_n goes across the cut the contribution of this root to the total field in (2), which disappears in (25), appears in an anomalous term (as can be seen directly by examining the case in which k_n falls on the cut), so that the resultant (true) field in the metal is of course continuous with respect to $\omega\tau$.

In calculating the asymptotic behavior of the field at distances $x\gg k_0^{-1}$ we are dealing with exponentially small increments, so that the question of the number of poles is not very important. However, in a calculation of, say, the impedance, the contributions of the two poles and of the integral along the cut are comparable in magnitude, and to find the frequency dependence we need to study the analytic properties of the function $D(\omega,k)$ and, in particular, to study this situation in which a cut is crossed by a pole.[4)]

2. $T\neq0$, $l=\infty$. Incorporating the thermal motion of the electrons in (3) leads to the following expression for the dielectric constant:

$$\varepsilon^{(g)}_{zz}(\omega,k)=1+\frac{3}{4}\frac{\omega_0^2}{\omega kv_F}\gamma_T^{-1}\int_{-\infty}^{\infty}\frac{\ln\{1+\exp[\gamma_T(1-u^2)]\}}{u-(\omega+i0)/kv_F}du.\qquad (28)$$

Calculations from (2), analogous to those given above for the case of a longitudinal field, yield the following results for $x\gg\gamma_T/k_0$:

$$E^{\perp}_{\rm an}(x)\approx-\frac{(3\pi)^{1/2}v_F}{2^{1/2}c\alpha}H_0\frac{\exp(\gamma_T)}{\gamma_T}\left(\frac{v_T}{\omega x}\right)^{1/2}\times\exp\left\{-\frac{3}{4}\left(\frac{\omega x}{v_T}\right)^{2/3}(1-i\cdot3^{1/2})+\frac{7}{6}\pi i\right\}\qquad (29)$$

in region (21) and

$$E^{\perp}_{\rm an}(x)\approx-3^{1/2}\frac{\alpha v_F}{c}H_0\frac{\exp(\gamma_T)}{\gamma_T}\left(\frac{v_T}{\omega x}\right)^{1/2}\times\exp\left\{-\frac{3}{4}\left(\frac{\omega x}{v_T}\right)^{2/3}(1-i\cdot3^{1/2})+\frac{7}{6}\pi i\right\}\qquad (30)$$

in region (22). This "thermal" attenuation may be observable in semimetals or degenerate semiconductors, where the Fermi energy is far lower than in "good" metals.

For comparison, here is the result calculated for $E^{\perp}_{\rm an}(x)$ in a Maxwellian plasma:

$$E^{\perp}_{\rm an}(x)\approx\frac{2v_F}{3^{1/2}c\alpha}H_0\gamma_T^{1/2}\left(\frac{v_T}{\omega x}\right)^{1/2}\times\exp\left\{-\frac{3}{4}\left(\frac{\omega x}{v_T}\right)^{2/3}(1-i\cdot3^{1/2})+\frac{7}{6}\pi i\right\}.\qquad (29')$$

It can be seen from expressions (29) and (29′) that the ratio of the amplitudes of the transverse pulling fields in a degenerate plasma and a Maxwellian plasma is $\sim (T/\varepsilon_F)^{3/2}$ as in the case of a longitudinal field [cf. (19)].

§4. EFFECT OF THE LOCAL GEOMETRY OF THE PULLING FIELD, THE FERMI-LIQUID INTERACTION, AND QUANTUM EFFECTS ON THE STRUCTURE OF THE PULLING FIELD

1. Let us examine the structure of the pulling field in a metal with an arbitrary Fermi surface (we restrict the discussion to the case of a zero temperature). The wave vectors of the pulling field are found from Eqs. (5). Clearly, the law describing the attenuation of the pulling field depends on the type of singularity in $\varepsilon_{\alpha\beta}$. The nature of the singularity of the components $\varepsilon_{\alpha\beta}$ is determined to a large extent by the nature of the singularity in $\langle R_\infty \rangle$. In particular, if $v_\alpha^c v_\beta^c \neq 0$, where $\mathbf{v}^c = \mathbf{v}(\mathbf{p}_c)$, the singular part of $\varepsilon_{\alpha\beta}^{(g)}$ can be expressed in terms of the singular part of $\langle R_\infty \rangle$:

$$\mathrm{SP}\ \varepsilon_{\alpha\beta}^{(g)} = \frac{4\pi e^2}{\omega} v_\alpha{}^c v_\beta{}^c \quad \mathrm{SP} \quad \langle R_\infty \rangle.$$

The expression for $\langle R_\infty \rangle$ becomes infinite at $\Delta k \equiv k - k_c = 0$. The nature of the divergence of $\langle R_\infty \rangle$ is related to the local structure of the Fermi surface near the points $\mathbf{p}_c$. The cases encountered most frequently are[5,8] an O-type singularity ($\mathrm{Re}\langle R_\infty \rangle \propto \ln|k_c/\Delta k|$, $\mathrm{Im}\langle R_\infty \rangle$ has a discontinuity) and an X-type singularity ($\mathrm{Re}\langle R_\infty \rangle$ has a discontinuity, $\mathrm{Im}\langle R_\infty \rangle \propto \ln|k_c/\Delta k|$). In addition to these singularities in metals whose Fermi surfaces have lines of parabolic points, a singularity is intensified for selected directions of the wave vector $\mathbf{k}$: $|\langle R_\infty \rangle| \propto (k_c/\Delta k)^v$, $0 < v < 1$ (see Ref. 8, where a dumbbell-shaped Fermi surface was studied as an example).

Evaluating the integrals along the cuts drawn from the branch points $\varepsilon_{\alpha\beta}^{(g)}$: $k_b = k_c + i/l$, we can find characteristic expressions for the distribution of the pulling field in metals with arbitrary Fermi surfaces. These expressions are given in Tables I and II.

2. Up to this point we have been employing a gas model for conduction electrons. The incorporation of the Fermi-liquid interaction leads to changes in the nature of the singularity of $\varepsilon_{\alpha\beta}$. It was shown in Ref. 8 that in the collisionless limit with $|\Delta k| \ll k_c$ the components $\varepsilon_{\alpha\beta}$ of the electron liquid of a metal can be described by

$$\varepsilon_{\alpha\beta}^{(\mathrm{liq})}(\omega, k) = \varepsilon_{\alpha\beta}{}^c + A_{\alpha\beta}/\langle R_\infty \rangle, \tag{31}$$

where $\varepsilon_{\alpha\beta}^c \equiv \varepsilon_{\alpha\beta}^{(\mathrm{liq})}(\omega, k_c)$; the quantities $\varepsilon_{\alpha\beta}^c$ and $A_{\alpha\beta}$ are determined by integral equations and are finite [in order of magnitude, $\varepsilon_{\alpha\beta}^c \sim (\omega_0/\omega)^2$, $A_{\alpha\beta} \sim (\omega_0/\omega)^2 N/(\omega\varepsilon_F)$]. They depend on the Landau correlation function and also on the shape of the Fermi surface.

Incorporating the Fermi-liquid interaction eliminates the divergence of the components $\varepsilon_{\alpha\beta}$ and weakens the singularities. It is true that in some cases the singularities of the transverse components $\varepsilon_{\alpha\beta}$ may in fact be strengthened (Ref. 8). If the Fermi surface is a sphere, the Fermi-liquid renormalization does not change the nature of the singularity of the transverse part of $\varepsilon_{\alpha\beta}$ at all.

For the longitudinal part of $\varepsilon_{\alpha\beta}$ in the case of a spherical Fermi surface, we find from (31), making use of the finite value of l,

$$\varepsilon_{xx}^{(\mathrm{liq})}(\omega, k) = \varepsilon_{xx}{}^c + \frac{4}{3}\frac{\omega\varepsilon_F}{N} A_{xx} \ln^{-1}\left(\frac{k_0 + i/l - k}{k_0 + i/l + k}\right). \tag{32}$$

Let us examine the anomalous term in (1), using expression (32) instead of (6) for ε_{xx}. For the asymptotic value of the longitudinal pulling fields we then find, in place of (8),

$$E_{\mathrm{an}}(x) \approx \frac{4}{3i}\frac{\omega^2}{\omega_0{}^2} G E_0 (k_0 x)^{-1} \ln^{-2}(k_0 x) \exp\left(ik_0 x - \frac{x}{l}\right), \tag{33}$$

where

$$G = -\frac{2\omega_0{}^2 \varepsilon_F}{\omega N} \frac{A_{xx}}{(\varepsilon_{xx}{}^c)^2}. \tag{34}$$

If the Fermi-liquid interaction is ignored, the factor here is $G \sim 1$ for an arbitrary Fermi surface, and it is strictly equal to one for a spherical Fermi surface.

If the Fermi surface is a sphere, the dimensionless Landau function $F(\mathbf{p}_F, \mathbf{p}_F') \equiv F(\theta)$, where θ is the angle between $\mathbf{p}_F$ and $\mathbf{p}_F'$, is conveniently expanded in a series in Legendre polynomials:

TABLE I.

Nature of singularity of $\varepsilon_{xx}^{(g)}$	Longitudinal field $E_{\mathrm{an}}^{\parallel}(x)/(E_0\omega^2/\omega_0^2)$, $x \gg k_c^{-1}$
$\ln\lvert k_c/\Delta k\rvert$	$(k_c x)^{-1}$
$(k_c/\Delta k)^v$	$(k_c x)^{-(1+v)}$

TABLE II.

Nature of singularity of $\varepsilon_{zz}^{(g)}$	Transverse field $E_{\mathrm{an}}^{\perp}(x)/(H_0 v_F/c)$, $x \gg k_c^{-1}$, $\alpha \gg 1$	$\alpha \ll 1$
$\frac{\Delta k}{k_c}\ln\left\lvert\frac{k_c}{\Delta k}\right\rvert$	$\alpha^{-1}(k_c x)^{-2}$	$\alpha(k_c x)^{-2}$
$\ln\left\lvert\frac{k_c}{\Delta k}\right\rvert$	$\alpha^{-1}(k_c x)^{-1}\ln^{-2}(k_c x)$	$\alpha(k_c x)^{-1}$, $x \ll k_c^{-1}e^{1/\alpha}$; $\alpha^{-1}(k_c x)^{-1}\ln^{-2}(k_c x)$, $x \gg k_c^{-1}e^{1/\alpha}$
$(k_c/\Delta k)^v$	$\alpha^{-1}(k_c x)^{-(1+v)}$	$\alpha(k_c x)^{v-1}$, $x \ll k_c^{-1}\alpha^{-1/v}$; $\alpha^{-1}(k_c x)^{-(1+v)}$, $x \gg k_c^{-1}\alpha^{-1/v}$

$$F(\theta)=\sum_{n=0}^{\infty}(2n+1)F_nP_n(\cos\theta). \qquad (35)$$

The coefficients F_n fall off rapidly with n (in particular, for Na and K we have $F_0\approx-0.6$; $|F_1|$, $|F_2|,...\ll1$; Ref. 15), so we can write

$$G\approx\left(\frac{1+F_2}{1-3F_2/2}\right)^2\approx1+5F_2. \qquad (36)$$

The Fermi-liquid interaction is thus essentially unseen in the structure of the pulling field. Expression (36) was derived for the case $\omega\ll\omega_0$.

In an anisotropic metal, the Fermi-liquid interaction changes the amplitude of the pulling field, and in some cases it changes the way in which the pulling field falls off. In metals with complicated Fermi surfaces the factor G depends on the Landau function much more strongly than in an isotropic metal. The change in the amplitude and in the decay law as we go from a Fermi-gas description to a Fermi-liquid description is particularly obvious in the infrared region [see (22)]. If the transverse pulling fields are due to singularities

$$\varepsilon_{zz}^{(g)}\infty\ln|k_c/\Delta k|\quad(k_c/\Delta k)^\nu,$$

then at great distances ($x\gg k_c^{-1}$) there will be a change in the dependence of their amplitude of the parameter α, while in the intermediate region there will be a change in the decay law of the pulling field. For a logarithmic singularity we have

$$E_{\mathrm{an}}^{\perp}(x)\sim\frac{v_F}{c}\alpha\frac{\omega^2}{\omega_0^2}A_{zz}\frac{\omega\varepsilon_F}{N}H_0(k_cx)^{-1}\ln^{-2}(k_cx)\times\exp\left(ik_cx-\frac{x}{l}\right), \qquad (37)$$

while for a fractional-power singularity we have

$$E_{\mathrm{an}}^{\perp,\mathrm{liq}}(x)\sim\frac{v_F}{c}\alpha\frac{\omega^2}{\omega_0^2}A_{zz}\frac{\omega\varepsilon_F}{N}H_0(k_cx)^{-(\nu+1)}\exp\left(ik_cx-\frac{x}{l}\right), \qquad (38)$$

(cf. Table II for the case of a transverse field with $\alpha\ll1$).

If the Fermi-liquid interaction strengthens the singularity [SP $\varepsilon_{zz}^{(g)}\infty(\Delta k/k_c)\ln|k_c/\Delta k|$, while SP $\varepsilon_{zz}^{(\mathrm{liq})}\infty\ln^{-1}|k_c/\Delta k|$], the decay law

$$E_{\mathrm{an}}^{\perp,\mathrm{liq}}(x)\sim(k_cx)^{-1}\ln^{-2}(k_cx)\exp(ik_cx-x/l)$$

also differs from the decay law

$$E_{\mathrm{an}}^{\perp,\mathrm{g}}(x)\sim(k_cx)^{-2}\exp(ik_cx-x/l)$$

at $x\gg k_c^{-1}$ [cf. (23)].

3. What role do quantum effects play in the structure of the pulling field? Going over to a quantum treatment means taking into account the nonzero momentum $\hbar\mathbf{k}$ and energy $\hbar\omega$ of the photon. To avoid complicating the discussion, we will use the "gas" expression for the dielectric tensor. It can be shown (S. Shipl'kin, personal communication) that incorporating the nonzero energy and momentum of the photon does not change the structure of the singular parts of the components of the tensor $\varepsilon_{\alpha\beta}^{(\mathrm{liq})}$ (from the semiclassical expressions[8]). As we will see below, the nonzero values of $\hbar\omega$ and $\hbar\mathbf{k}$ give rise to beats in the structure of the pulling field. These beats are related to a splitting of the singularities of the components $\varepsilon_{\alpha\beta}$ in the quantum treatment. We can demonstrate the situation in the example of longitudinal pulling fields. For $\varepsilon_{xx}^{(g)}(\omega,\mathbf{k})$ we have

$$\varepsilon_{xx}^{(g)}(\omega,\mathbf{k})=1-\frac{4\pi e^2}{k^2}\int\frac{n(\varepsilon_{\mathbf{p}})-n(\varepsilon_{\mathbf{p}+\hbar\mathbf{k}})}{\varepsilon_{\mathbf{p}}+\hbar\omega-\varepsilon_{\mathbf{p}+\hbar\mathbf{k}}+i\hbar/\tau}\frac{2d^3p}{(2\pi\hbar)^3}. \qquad (39)$$

For clarity we consider an isotropic electron dispersion relation; we find

$$\varepsilon_{xx}^{(g)}(\omega,k)=1+\frac{3\omega_0^2}{2\omega^2}\left\{1-\frac{2\varepsilon_F}{\hbar\omega}\left[\frac{\Delta k_+}{k_0}\ln\left(\frac{i/l-\Delta k_+}{2k_0}\right)-\frac{\Delta k_-}{k_0}\ln\left(\frac{i/l-\Delta k_-}{2k_0}\right)\right]\right\},\quad k\approx k_0. \qquad (40)$$

Here $\Delta k_\pm=k-k_0^\pm$, where $k_0^\pm=k_0(1\pm\hbar\omega/4\varepsilon_F)$.

We see that in the upper k half-plane, two branch points[5)] of $\varepsilon_{xx}^{(g)}$ appear when $|k|\ll p_F/\hbar$: $k_{b_1}=k_0^++i/l$ and $k_{b_2}=k_0^-+i/l$. The integration contour used in this case [see (1')] is shown in Fig. 2. The splitting of the singularities must be taken into account under the condition $|\Delta k|\ll k_0\hbar\omega/\varepsilon_F$, i.e., in the region $x\gg k_0^{-1}\varepsilon_F/\hbar\omega$. As a result of the addition of two pulling fields with approximately equal wave vectors we find "quantum" beats in the pulling field:

$$E_{\mathrm{an}}^{\parallel}(x)\approx\frac{16}{3i}\frac{\omega^2}{\omega_0^2}E_0\frac{\varepsilon_F}{\hbar\omega}\ln^{-2}\left(\frac{\varepsilon_F}{\hbar\omega}\right)\sin\left(\frac{\hbar\omega}{4\varepsilon_F}k_0x\right)\times(k_0x)^{-2}\exp\left(ik_0x-\frac{x}{l}\right),\quad x\gg k_0^{-1}\varepsilon_F/\hbar\omega. \qquad (41)$$

We see that at $k_0^{-1}\ll x\ll k_0^{-1}\varepsilon_F/\hbar\omega$ we can use the semiclassical approximation, (8), while at $x\gg k_0^{-1}\varepsilon_F/\hbar\omega$ we must use the quantum treatment, and it does in fact give rise to beats. We wish to emphasize that the splitting of the singularities and the beats caused by this splitting stem from the nonzero value of the ratio $\hbar\omega/\varepsilon_F$.

For an arbitrary Fermi surface, the splitting of the singularities of $\varepsilon_{\alpha\beta}$ of other types can be taken into account in a similar way.

For a Fermi surface of complex shape we need to take

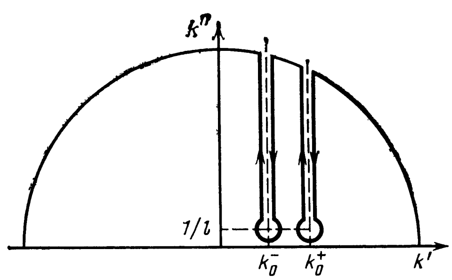

FIG. 2.

the quantum approach even in the limit $\hbar\omega/\varepsilon_F \to 0$ if the band $v_x = 0$ on the Fermi surface has a self-intersection point or if a loop is generated there (or disappears there).[8] As was shown in Ref. 8, this situation is possible if the wave vector is directed nearly along the tangent to a parabolic point on the Fermi surface. We denote the wave vector in this case by $\mathbf{k}_{cc}$, emphasizing that not only the length of the wave vector but also its direction is critical. In this case, the value found for k_{cc} from (5) tends toward infinity, and to analyze the singularities (e.g., in the longitudinal part of $\varepsilon_{\alpha\beta}$) we should use not (4) but expression (39), from which we easily find $k_{cc} \sim k_c(\varepsilon_F/\hbar\omega)^{1/2} \sim (p_F/\hbar)(\hbar\omega/\varepsilon_F)^{1/2}$. The pulling field[6)] decays in the region $k_{cc}^{-1} \ll l \ll k_{cc}^{-1}\varepsilon_F/\hbar\omega$ in accordance with

$$E^{\parallel}_{\text{an}}(x) \sim \frac{\omega^2}{\omega_0^2}\left(\frac{\varepsilon_F}{\hbar\omega}\right)^{1/4} E_0(k_{cc}x)^{-1}\exp\left(ik_{cc}x - \frac{x}{l}\right), \quad (42)$$

while at $x \gg k_{cc}^{-1}\varepsilon_F/\hbar\omega \sim k_c^{-1}(\varepsilon_F/\hbar\omega)^{1/2}$ we again find beats [cf. (41)]: In the critical direction, quantum beats begin closer to the surface of the metal than in the case of an arbitrary direction.

We emphasize that there may be cases in which the critical direction is one of the "good" directions in the crystal. In copper, for example, the direction of the critical wave vector k_{cc} is the [100] direction.[16]

Quantum effects are particularly marked at $x \ll l$. In a metal in an rf electromagnetic field there is a "nonremovable" electron mean free path l_{eff} (Ref. 17; this mean free path stems from electron-electron collisions), given in order of magnitude at $T \ll \hbar\omega$ by

$$l_{\text{eff}} \sim \frac{\hbar}{p_F}\left(\frac{\varepsilon_F}{\hbar\omega}\right)^2 W^{-1} \sim k_c^{-1}\frac{\varepsilon_F}{\hbar\omega}W^{-1}. \quad (43)$$

The dimensionless factor W depends on the exact value of the probability for a transition during a collision. It is clear from (41) and (43) that the quantum beats of a pulling field with a wave vector k_c should be clearly observed at $W \ll 1$. The quantum asymptotic behavior due to the critical direction (a pulling field with a wave vector k_{cc}) may also be observed at $W \sim 1$ (since in this case we have $l_{\text{eff}} \gg k_{cc}^{-1}\varepsilon_F/\hbar\omega$).

§5. PULLING FIELD DURING THE DIFFUSE REFLECTION OF ELECTRONS FROM THE SURFACE OF A METAL

To determine the role played by the countary conditions, we examine the structure of longitudinal pulling fields for the case of a purely diffuse reflection. For simplicity we assume that the Fermi surface is spherical. We ignore the Fermi-liquid interaction, and we set $T = 0$. It follows directly from Maxwell's equations that

$$E^{\parallel}(x) - \frac{4\pi}{i\omega}j(x) = 0, \quad (44)$$

where the field in vacuum is $E^{\parallel}(x<0) = E_0$, and the conduction current

$$j(x) = e\langle v_x\Psi\rangle = \frac{m^{*2}e}{2\pi^2\hbar^3}\int_{-v_F}^{v_F} v_x\Psi(x, v_x)\,dv_x$$

can be found by solving the kinetic equation for the electron distribution function. This equation is given in the τ approximation by

$$v\frac{\partial\Psi}{\partial x} + \left(\frac{1}{\tau} - i\omega\right)\Psi = ev_xE^{\parallel}(x), \quad \Psi = -\delta n\Big/\left(\frac{\partial n}{\partial\varepsilon}\right), \quad (45)$$

where δn is the deviation of the distribution function from the equilibrium Fermi function. The boundary conditions on (45) in the limiting case of purely diffuse scattering of electrons by the surface are chosen in the usual form:

$$\Psi_+(x=0) = eA, \quad (46)$$

where the constant A satisfies the condition that a conduction current does not flow across the metal-vacuum interface.[18] The subscript + specifies the value of the function with $v_x > 0$. Using (45) and (46), we find from (44) the following integral equation for $E^{\parallel}(x)$:

$$E^{\parallel}(x) - E_0 = \int_0^\infty dy E^{\parallel}(y)K(|x-y|) + AK(x),$$
$$K(x) = \frac{4\pi e^2}{i\omega}\left\langle v_x \exp\left(\frac{i\omega - 1/\tau}{v_x}x\right)\right\rangle_+. \quad (47)$$

Here $\langle\ldots\rangle_+$ means an integration over that part of the Fermi surface on which the condition $v_x > 0$ holds [cf. (4)]. Transforming to dimensionless variables $X \equiv k_0 x$, $u \equiv v_x/v_F$, and using

$$\varepsilon_0 \equiv 1 - \beta = 1 - 2\int_0^\infty K(x)\,dx,$$

we find the equation

$$\frac{\varepsilon_0}{E_0}\int_0^\infty E_1(Y)L(|X-Y|)\,dY - \frac{\varepsilon_0}{\beta E_0}E_1(X)$$
$$= \int_X^\infty L(Y)\,dY + BL(X), \quad (48)$$
$$L(X) \equiv 3\frac{(\omega\tau)^{-1} - i}{2}\int_0^1 u\exp\left(\frac{i - (\omega\tau)^{-1}}{u}X\right)du$$

for the component of field (1) which decays at $+\infty$. The constant $B \equiv \beta\omega A/E_0 v_F$ must be determined from the boundary condition $E_1(x=0) = E_0(\varepsilon_0 - 1)/\varepsilon_0$, which is equivalent to the condition for no flow of a conduction current.

Equation (48) can be solved by the Wiener-Hopf method by means of Fourier transforms. For this purpose, the function $L(X)$ is continued as an even function, and the first term on the right side of (48) as an odd function, onto the negative semiaxis $X < 0$. We introduce the field

$$E_1^+(X) = \theta(X)E_1(X), \quad E_1^-(X) = \theta(-X)E_1(X),$$

where $\theta(X) = 0$ at $X < 0$ and $\theta(X) = 1$ at $X > 0$.

Following the Wiener-Hopf method (Ref. 19, for example), we factorize the dielectric constant $\varepsilon(\sigma)$, the Fourier transform of the kernel of Eq. (48): $\varepsilon(\sigma) = r_k r_{-k}$, where the function

$$r_k = \exp\left\{\frac{1}{2\pi i}\int_{-\infty+i0}^{\infty+i0} \ln\varepsilon\left(\frac{\varkappa k_0^{-1}}{1+i/\omega\tau}\right)\frac{d\varkappa}{\varkappa-k}\right\} \quad (49)$$

is analytic and has no zeros at $k'' < 1/l$. We recall that $\sigma = kv_F/(\omega + i/\tau)$ [see (6)], and we note that in the limits $|k| \to 0$ and $|k| \to \infty$ the factorization is given directly by expressions (9) and (9'). Using (49), we find from Eq. (48), extended to $X < 0$, the following equation:

$$\left[1 - i\frac{k}{k_0}B + i\frac{k\varepsilon_0}{E_0}E_1^+(k)\right]r_k$$

$$= (r_{-k})^{-1}\left[1 - i\frac{k}{k_0}B - \beta - i\frac{k\varepsilon_0}{E_0}E_1^-(k)\right], \quad (50)$$

where the $E_1^{\pm}(k)$ are Fourier transforms of the functions $E_1^{\pm}(X)$.

Since we have $E_1^+(X = \infty) = 0$ and, as can be seen from the continuation of Eq. (48), $E_1^-(X = -\infty) = 0$, the half-planes in which the left and right sides of (50) are analytic intersect in a band around the real k axis. Consequently, both sides of (50) are equal to an entire function, which we must choose here in the form $r_0 - ikB/k_0$ (see §1.7 in Ref. 19), so we can find $E_1^+(k)$ [and, if necessary, $E_1^-(k)$ also]. The constant B, as we have already stated, is found from the boundary condition

$$E_1(+0) = \lim_{k\to\infty}[ikE_1^+(k)] = -\beta E_0/\varepsilon_0$$

and from the asymptotic form of r_k [see (9')]. We finally find

$$E_1^+(k) = \frac{E_0}{ik\varepsilon_0}\left\{\frac{r_0}{r_k}\left(1 + k\frac{1-r_0}{k_1}\right) - 1 - k\frac{r_0 - r_0^2}{k_1}\right\}, \quad (51)$$

where $r_0 \equiv r_{k=0} = (1-\beta)^{1/2}$, $k_1 = \sigma_1 k_0(1 + i/\omega\tau)$, and σ_1 is given in (9'). Expression (51) gives us the exact solution of the problem in the k representation.

Since all the terms in braces in (51) except the first give us zero during an integration along the banks of the cut, the expression for the pulling field can be written in the form

$$E_{\text{an}}^{\parallel}(x) = \frac{iE_0}{2\pi}\int_C \frac{e^{ikx}}{k\varepsilon(\sigma)}\frac{r_{-k}}{r_0}\left(1 + k\frac{r_0 - 1}{k_1}\right)dk, \quad (52)$$

by multiplying and dividing (51) by r_{-k}. The contour C is shown (by the heavy line) in Fig. 1. At large x, this integral is dominated by the vicinity of the branch point $k_b = k_0 + i/l$ (or $\sigma_b = 1$); at this point, r_{-k} is analytic by definition. We thus have from (52)

$$E_{\text{an}}^{\parallel}(x) = \frac{E_0}{2\pi i}\frac{r_{-k_b}}{r_0}\left(1 + \frac{r_0 - 1}{\sigma_1}\right)\int_C \frac{e^{ikx}}{k\varepsilon(\sigma)}dk. \quad (52')$$

Comparing (52') and (7), we reach the conclusion that the asymptotic expressions for $E_{\text{an}}^{\parallel}(x)$ (at $x \gg k_0^{-1}$) for the specular and purely diffusion scattering of electrons by the surface differ only by a constant factor

$$Q = \frac{r_{-k_b}}{2r_0}\left(1 + \frac{r_0 - 1}{\sigma_1}\right).$$

The quantity r_{-k_b} can be calculated by using an integral factorization form (49), which gives us, after some manipulation,

$$Q = \frac{1+\sigma_n}{2(1-\beta)^{1/2}}\left(1 - \frac{(1-\beta)^{1/2} - 1}{(-3\beta)^{1/2}}\right)$$

$$\times \exp\left\{-\int_{-\infty}^{+\infty}\frac{\ln(1+\sigma(w))}{w^2+\pi^2}dw\right\}, \quad (53)$$

where $\sigma(w)$ is the function which is the inverse of

$$w(\sigma) = \frac{2\sigma^3}{3\beta} + 2\sigma + \ln\left(\frac{\sigma-1}{\sigma+1}\right), \quad \sigma > 1.$$

The parameter σ_n in the extreme cases in terms of β coincides with either σ_0 (if $|\beta| \sim 1$) or σ_1 ($|\beta| \gg 1$):

$$\sigma_n \approx \begin{cases} [5(1-\beta)/3\beta]^{1/2}, & |1-\beta| \ll 1 \\ (-3\beta)^{1/2}, & |\beta| \gg 1 \end{cases}. \quad (54)$$

Evaluating the integral in (53) approximately, and making use of (54), we find

$$Q \approx \begin{cases} 0{,}215(1 + i\sqrt{3})(1-\beta)^{-1/4}, & |1-\beta| \ll 1 \\ 0{,}47, & |\beta| \gg 1 \end{cases}. \quad (55)$$

We see that in the most interesting case—far from a resonance ($|\beta| \gg 1$, $\omega \ll \omega_0$)—not only the structure of the pulling field but also the wave amplitude is relatively insensitive to the nature of the scattering of electrons by the surface. Near a resonance ($\omega \approx \omega_0$) the boundary conditions is seen to be important; the amplitude of the pulling wave increases substantially.

CONCLUSION

1. It is meaningful to distinguish a pulling field only at distances $x \gg k_0^{-1}$, i.e., at distances large in comparison with the path traversed by a Fermi electron over the field period. Analysis of the quantum and temperature effects introduces some new length scales: at $x \gg k_0^{-1}\varepsilon_F/\hbar\omega$, "quantum" beats of the pulling-field waves arise because the branch points of $\varepsilon_{\alpha\beta}$ move away from each other; at $x \gg k_0^{-1}\varepsilon_F/T$, the pulling field falls off in the manner characteristic of a nondegenerate plasma. Just which of the two types of asymptotic behavior will be seen first depends on the relation between $\hbar\omega$ and T. The x axis is thus partitioned into intervals in each of which certain groups of conduction electrons or others are manifested. In anisotropic metals, this partitioning depends strongly on the direction of the normal to the surface with respect to the crystallographic axes.

2. In a study of the structure of a high-frequency transverse field one should distinguish between two frequency regions: the region of the extremely anomalous skin effect, (21), and the region of infrared optics, (22). In region (21),

with $\delta_1 \ll k_0^{-1}$, the hierarchy of length scales is the same as for a longitudinal field. The pulling fields determine the "actual" asymptotic behavior at large distances from the surface of the metal. In region (22), and under the condition $k_0^{-1} \ll x \ll \delta_2$, the amplitude of the anomalous wave exceeds the amplitude of an ordinary wave. The pulling field determines the actual asymptotic behavior.

3. If the band $v_x = 0$ at the Fermi surface has a self-intersection point, or if a loop begins (or disappears) there, macroscopic pulling fields can propagate with a wave vector $k_{cc} \gg k_0$.

4. In isotropic metals, the Fermi-liquid interaction is essentially not manifested in the structure of the pulling field. In anisotropic metals, this interaction can both change the amplitude of the pulling field and also change its decay law [see (37) and (38)].

5. Since the results of the calculation of the longitudinal pulling fields far from a resonance differ by only a numerical factor in the limiting cases of purely diffuse and specular reflection of electrons by the boundary, we can assume quite confidently that this result will also hold in the intermediate case (with a specular coefficient $0 < q < 1$). This circumstance justifies a study of the qualitative features of the pulling field in the case simplest for calculations, that with specular reflection of electrons.

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[1)]In this paper we are discussing only electromagnetic fields.

[2)]We are considering macroscopic pulling fields here, for which the condition $|k_c|a \ll 1$ holds (k_c is the wave vector of the pulling field, and $a \sim \hbar / p_F$ is an interatomic distance).

[3)]The dispersion relation for this wave, $\omega^2 = \omega_0^2 + 3k^2 v_F^2/5, \omega \gtrsim \omega_0$, naturally agrees with the dispersion relation for a longitudinal plasma wave.[10]

[4)]This point is particularly important in a study of the impedance and of other characteristics as functions of the magnitude of the magnetic field normal to the surface of a metal, whose effect is seen in a replacement of $\omega\tau$ by $(\omega \pm \omega_c)\tau$, where ω_c is the cyclotron frequency.[13]

[5)]A finite mean free path l smoothes over the singularities in $\varepsilon_{xx}^{(g)}$, and we can speak of a splitting of these singularities only in the case $k_+ - k_- \gg 1/l$.

[6)]The wave vector $\mathbf{k}_{cc}$ corresponds to a macroscopic pulling field because of the condition $|k_{cc} a| \ll 1$. The factor of $\hbar\omega / \varepsilon_F$ in the expression for k_{cc} and in (42) does not arise from the Fermi function. It is written in this form for convenience in the calculations:

The quantities m_c^* and v_c are the effective mass and velocity of the electrons at the critical point.

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