Two-dimensional superfluid Fermi liquid with *p*-pairing

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The axial and planar phases of the two-dimensional superfluid Fermi liquid are discussed. The difference between the free energies of these phases is calculated with corrections for strong coupling at arbitrary temperature. The sign of this difference turns out to be temperature-independent in this approximation. Phase transitions occurring in the axial and planar phases as the temperature is increased, and the structure of ordered states, are investigated. Ordering is examined with allowance for the spin-orbit interaction. A schematic phase diagram is constructed.

1. INTRODUCTION

In weak solutions of ^{3}He in ^{4}He at low temperatures, practically all the ^{3}He is absorbed on the free surface, forming a submonomolecular layer.[1,2] Since the concentration can be varied between wide limits and the substrate (^{4}He) uniformity is perfect, this layer is an attractive object for the investigation of the properties of the two-dimensional Fermi liquid. Theoretical estimates show that the submonomolecular layer of ^{3}He on ^{4}He can undergo a transition to the superfluid state either by singlet pairing (in the case of a thin ^{4}He film[3]) or by triplet pairing.[4] The properties of the superfluid Fermi liquid with *s*-pairing are, in many ways, analogous to those of the superfluid Bose-liquid, whereas *p*-pairing leads to a whole series of new and interesting properties.[5] The shape of the phase diagram and the structure of the ordered states in the case of the two-dimensional Fermi liquid with *p*-pairing have not, however, been adequately investigated.

In the BCS approximation, the minimum of the free energy of the two-dimensional superfluid Fermi liquid with *p*-pairing is attained for two different phases simultaneously (the axial and the planar phases).[6,7] Following Brusov and Popov,[7] we shall denote them by lower-case Latin letters (respectively, *a* and *b*) in order to avoid confusion with the three-dimensional case. The relative stability of the *a* and *b* phases has not been investigated with more accurate (as compared with the BCS approximation) allowance for the interaction. The belief that the *a*-phase is stable[6] is based on a direct transfer of the Ginzburg-Landau expansion coefficients (calculated in the paramagnon approximation) from the three-dimensional to the two-dimensional case. This approach cannot be regarded as valid because the Ginzburg-Landau expansion coefficients are expressed in terms of integrals of the quasiparticle scattering amplitude and are known to be different in the three- and two-dimensional cases.

In this paper, we use the results reported by Rainer and Serene[8] to calculate the difference $\Delta\Phi_{a-b}$ between the free energies of the *a*- and *b*-phases in the higher-order (as compared with BCS approximation in the ratio of the transition temperature T_c to the Fermi energy ε_F. In the two-dimensional case, the difference $\Delta\Phi_{a-b}$ can be calculated not only near T_c but at arbitrary temperature. The sign of the difference depends on the type of interaction between the quasiparticles but, in this particular approximation, it is independent of temperature. Since, in the BCS approximation, the free energies of the *a*- and *b*-phases are equal, the inclusion of the corrections evaluated below becomes important for the shape of the phase diagram even in the case of parameter values that are satisfactory in the BCS approximation.

We shall also examine the sequences of phase transitions that occur in the *a*- and *b*-phases as the temperature is raised. In the case of the *a*-phase, the transition to the normal state occurs through three successive phase transitions, whereas in the case of the *b*-phase, there are two such transitions. The phase transitions in the *a*-phase were investigated by Stein and Cross,[9] who confined their attention to the orbital part of the order parameter.

We emphasize that, when we speak of the superfluid (ordered) state, or of symmetry breaking, we follow Berezinskiĭ[10] and have in mind the gradual reduction in correlation, and not the long-range order which is not possible in two-dimensional, continuously degenerate systems at finite temperature.[11,12]

2. STRONG COUPLING CORRECTIONS TO THE FREE ENERGY

The order parameter for the two-dimensional superfluid Fermi liquid with *p*-pairing is a complex matrix A_{pj} with three columns (subscript *p*, spin space) and two rows (subscript *j*, orbital space). For the *a*-phase, we have

$$A_{pj}=\Delta d_p(\Delta_j'+i\Delta_j''), \tag{1}$$

where d_p is a unit vector in spin (three-dimensional) space and Δ_j' and Δ_j'' are mutually perpendicular unit vectors in orbital (two-dimensional) space. The vector $\mathbf{l}=[\mathbf{\Delta}'\mathbf{\Delta}'']$ can only be parallel or antiparallel to the normal $\boldsymbol{\nu}$ to the surface. The position of the vectors Δ_j' and Δ_j'' is conveniently parametrized in terms of the phase φ by expressing the combination $\Delta_j'+i\Delta_j''$ in term of some fixed unit vectors $\Delta_j^{(1)}$ and $\Delta_j^{(2)}$ in orbital space:

$$\Delta_j'+i\Delta_j''=(\Delta_j^{(1)}+il\Delta_j^{(2)})\,e^{i\varphi}, \tag{2}$$

where $l=\mathbf{l}\cdot\boldsymbol{\nu}=\pm 1$.

The *b*-phase order parameter is

$$A_{pj}=\Delta\,(d_p^{(1)}\,\Delta_j^{(1)}+d_p^{(2)}\,\Delta_j^{(2)})\,e^{i\varphi}, \tag{3}$$

where $d_p^{(1)}$ and $d_p^{(2)}$ are mutually perpendicular unit vectors in spin space and $\Delta_j^{(1)}$ and $\Delta_j^{(2)}$ are fixed unit vectors in orbital space.

In contrast to the three-dimensional case, the gap in the spectrum is isotropic (independent of the direction of $\mathbf{k}$) in both phases under consideration. The corresponding normal Green functions

$$G_{\alpha\beta}(\mathbf{k}\omega_n)=\frac{1}{Z}\frac{i\omega_n+\xi}{\omega_n^2+\xi^2+\Delta^2}\delta_{\alpha\beta},$$

$$\xi=v_F(k-k_F),\qquad \omega_n=2\pi n(k_BT/\hbar),$$

are therefore the same for both phases provided the gap W is the same (we consider that the Green functions have the same form as in the BCS approximation; this is sufficient for the evaluation of the corrections in which we are interested[8]). The anomalous Green functions

$$F_{\alpha\beta}(\mathbf{k}\omega_n)=-\frac{1}{Z}\frac{(i\sigma^p\sigma^y)_{\alpha\beta}A_{pj}\hat{k}^j}{\omega_n^2+\xi^2+\Delta^2},\qquad \hat{k}^j=k^j/k, \tag{4}$$

then differ only by the form of the matrix A_{pj} in the numerator. In the BCS approximation, the free energy at an arbitrary temperature is the same for both phases.

Rainer and Serene[8] have investigated the corrections to the BCS approximation in the next (third) order of small quantities in T_c/ε_F (the BCS approximation itself yields the free energy in the second-order approximation in this ratio). They have shown that, to evaluate these corrections, it is sufficient to consider diagrams consisting of two vertex functions joined by four lines (of the form shown in Fig. 1). One can then assume that the Green functions have the same form as in the BCS approximation, and take the vertex functions as for the normal Fermi liquid, neglecting their dependence on the frequencies and absolute magnitudes (but not directions) of the momenta. This argument is also valid for the two-dimensional Fermi liquid.

If we wish to compare the free energies of the a- and b-phases, it is obvious that diagrams that do not contain the anomalous Green functions will yield the same contribution (because the normal Green functions are identical). Diagrams containing two anomalous Green functions each, will provide a contribution proportional to $A_{pj}A^*_{pj}$, which is also the same for the two phases.

The differences between the free energies of the a- and b-phases in this approximation are thus seen to be determined exclusively by the diagram with four anomalous Green functions (Fig. 1). It corresponds to the following contribution to the free energy:[8]

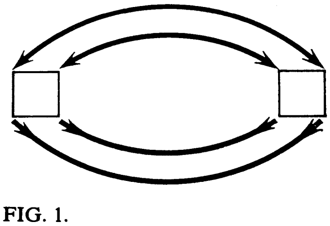

FIG. 1.

$$-\frac{1}{8}T^3\sum_{\omega_{n_1},\omega_{n_2},\omega_{n_3}}\int\frac{d^2\mathbf{k}_1}{(2\pi)^2}\frac{d^2\mathbf{k}_2}{(2\pi)^2}\frac{d^2\mathbf{k}_3}{(2\pi)^2}$$
$$\times\{\Gamma^{(4)}_{\alpha\beta\gamma\delta}(\mathbf{k}_1,\mathbf{k}_2;\mathbf{k}_3,\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3)$$
$$\times\bar{F}_{\alpha\lambda}(\mathbf{k}_1\omega_{n_1})\bar{F}_{\beta\mu}(\mathbf{k}_2\omega_{n_2})F_{\gamma\nu}(\mathbf{k}_3\omega_{n_3})$$
$$\times F_{\delta\rho}(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3,\omega_{n_1}+\omega_{n_2}-\omega_{n_3})$$
$$\times\Gamma^{(4)}_{\lambda\mu\nu\rho}(\mathbf{k}_1,\mathbf{k}_2;\mathbf{k}_3,\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3)\}. \tag{5}$$

Here and henceforth, the factor k_B is included in the definition of the temperature T.

Substituting the anomalous Green functions (4) in (5) for the a- and b-phases, summing over the spin indices, and integrating by parts with respect to the momenta, we find that, in the leading order in T_c/ε_F, the difference between the free energies is

$$\Delta\Phi_{a-b}=-\gamma\tilde{Y}(\Delta,T)\int\frac{d\psi}{2\pi}|\sin\psi|[\tilde{\Gamma}_a^2(\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2;\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2)$$
$$-\tilde{\Gamma}_a^2(\hat{\mathbf{k}}_1,-\hat{\mathbf{k}}_1;\hat{\mathbf{k}}_2,-\hat{\mathbf{k}}_2)], \tag{6}$$

where

$$\tilde{Y}(\Delta,T)$$
$$=T^3\Delta^4\sum_{n_1,n_2,n_3}\{(\omega_{n_1}^2+\Delta^2)(\omega_{n_2}^2+\Delta^2)(\omega_{n_3}^2+\Delta^2)$$
$$\times[(\omega_{n_1}+\omega_{n_2}-\omega_{n_3})^2+\Delta^2]\}^{-1/2},$$

$\gamma\sim N(0)\varepsilon_F^{-3}$, $\tilde{\Gamma}_a$ is antisymmetric in spin indices and represents the dimensionless amplitude for the scattering of quasiparticles with momenta on the Fermi surface, and ψ is the angle between $\hat{\mathbf{k}}_1$ and $\hat{\mathbf{k}}_2$.

The sign of the expression in (6) depends on $\tilde{\Gamma}_a$. When scattering in the "hole-particle" channel predominates, the a-phase is the more stable, whereas the b-phase is the more stable in the "particle-particle" channel. In the three-dimensional case, the shape of the phase diagram and the thermodynamic parameters indicate that scattering in the particle-hole channel is the more important. However, there is no reason to suppose that, for all concentrations, the interaction of quasiparticles in the submonomolecular ^{3}He layer has the same qualitative characteristics as in the three-dimensional ^{3}He. Hence, a change in the concentration (and, correspondingly, in the scattering amplitude) may result in a change in the sign of the integral in (6) and, correspondingly, in the phase transition between the a- and b-phases.

We note that the sign of $\Delta\Phi_{a-b}$ does not depend on temperature in this approximation. This means that the line representing the phase transition between the a- and b-phases (if this transition actually occurs) on the temperature vs concentration diagram should be almost parallel to the temperature axis.

In the three-dimensional case, an analogous comparison between the free energies of the A- and B-phases cannot be made as simply at an arbitrary temperature. The point is that, for the A-phase, the gap is anisotropic and the free energy is very sensitive to the precise angular dependence of the gap width. It follows that, in the case of three dimensions, comparison between the free energies of different phases is possible only near T_c, and this is usually done by having recourse to the calculation of the Ginzburg-Landau expansion coefficient.[8]

Let us now consider phase transitions occurring in the a- and b-phases as the temperature is raised. In two-dimensional, continuously-degenerate systems, orientational fluctuations of the order parameter play a dominant role. If the condition $T_c \ll \varepsilon_F$ is satisfied, the transition to the disordered state occurs at temperatures at which the Ginzburg-Landau criterion is still satisfied, and fluctuations in the magnitudes and "form" of the order parameter can be neglected. Disorder is then due to the appearance of topological singularities (domain walls of infinite length, single vortices, and so on).

3. PHASE TRANSITIONS IN THE *a*-PHASE

Let us consider, to begin with, the a-phase in the absence of spin-orbit coupling. The degeneracy space of the order parameter

$$A_{pj}=\Delta d_p(\Delta_j^{(1)}+il\Delta_j^{(2)})e^{i\varphi} \tag{7}$$

is $((S^2\times S^1)/Z_2^{(1)})\times Z_2^{(2)}$. The two-dimensional sphere S^2 is the domain of $\mathbf{d}$, and the one-dimensional sphere S^1 is the domain of φ. Factorization with respect to the group $Z_2^{(1)}$ reflects the fact that the pairs of variables $\mathbf{d}$, φ and $-\mathbf{d}$, $\varphi+\pi$ correspond to the same value of the order parameter (7). The group $Z_2^{(2)}$ is the domain of l.

In their analysis of phase transitions in the a-phase, Stein and Cross[9] consider only the orbital part (2) of the order parameter (7). In accordance with the form of (2), they conclude that, in addition to the standard Berezinskiĭ-Kosterlitz-Thouless (BKT) phase transition, due to the dissociation of the vortex molecules,[10,13,14] there is also a possible Ising-type phase transition due to the appearance of disorder in the field of the parameter l. Moreover, the order parameter also has a spin part whose fluctuations are more highly developed [because of the higher dimensionality ($d=3$) of spin space].

The expression for the gradient energy of the a-phase splits into terms involving d and φ:

$$\Phi^a_{grad}={}^1/_2K_d(\nabla_j d_p)^2+\frac{K_\varphi}{2}(\nabla_j\varphi)^2, \tag{8}$$

and contains only two independent coefficients, namely, K_d and K_φ (they are equal in the BCS approximation). It follows from Polyakov's results[15] that, as we pass to larger scales, the coefficient K_d is renormalized to zero for any arbitrarily low temperature, and K_φ remains finite at sufficiently low temperature. Hence, it follows that the $\mathbf{d}$ correlations decrease exponentially, which means that any pair correlator of the form

$$\langle A_{pj}(r_1)A_{ql}^*(r_2)\rangle, \tag{9}$$

will also fall exponentially, whereas the power-law decrease (typical of ordered phases in two-dimensional, continuously degenerate systems) will occur only in "fourth-order" correlators of the form

$$\langle A_{pj}(r_1)A_{pl}(r_1)A_{qj}^*(r_2)A_{ql}^*(r_2)\rangle. \tag{10}$$

The form of (10) shows that $e^{2i\varphi}$ and not $e^{i\varphi}$ plays the role of the order parameter, contrary to the statements made in Refs. 9 and 7. This is readily understood since φ is actually determined to within π (it can be redefined by redefining $\mathbf{d}$).

Thus, there is a breaking of symmetry in the ordered phase not under the group of two-dimensional rotations $U(1)$ but under its factorization with respect to the group Z_2, which identifies rotations differing by π. When we speak of the group of two-dimensional rotations, we have in mind the relative rotation of orbital space and phase. Of course, the group $U(1)/Z_2$ is also the group $U(1)$.

Equally important is that the simplest singularities are vortices with circulation $\pm\pi$. This leads to a fourfold increase in the universal value of the jump in the superfluid density[16] in the BKT transition

$$\Delta\rho_s=\frac{8}{\pi}\left(\frac{2m_3}{\hbar}\right)^2T_{\rm BKT} \tag{11}$$

as compared with the value indicated by Stein and Cross.[9]

We note that the ordering considered here may be observed only by confirming the presence of superfluid density. Other traditional methods (for example, NMR) are useless because of the above exponential fall in the pair correlators.

Stein and Cross[9] point that, in addition to the BKT transition in the a-phase, there is also a possible Ising-type transition due to disorder in l. We shall show that these two transitions occur independently of one another.

Consider the domain wall between regions with different l. Symmetry considerations show that the order parameter within it is

$$A_{pj}=\Delta d_p(e^{i\Psi}\Delta_j^{(1)}+e^{i(\pi/2-\Psi)}\Delta_j^{(2)})e^{i\varphi}, \tag{12}$$

where Ψ varies smoothly from 0 ($l=1$) to $\pi/2$ ($l=-1$). Since the condensation energy is independent of the common phase φ and of the direction of the vector $\mathbf{d}$, the minimum of the free energy of the domain wall is attained for $\varphi=$ cost, $\mathbf{d}=$ const. Ohmi *et al.*[17] investigated the domain wall in the a-phase, but confined their attention to a special distribution of the order parameter in which φ varies across the thickness of the wall, so that the domain wall is not stable and does not minimize the free energy, whilst a mass current flows along the wall.

The quantity Δ does not vanish anywhere within the domain wall, so that the stiffness with respect to the variable φ remains finite as well. In the Ising transition due to the disorder in l, the concentration of the domain walls (proportional to energy in the Ising model) is found to be continuous. Consequently, in this transition, there is no jump in the effective stiffness with respect to φ, so that there is no jump-induced BKT transition.

On the other hand, the vanishing of the effective stiffness to φ in the BKT transition has no effect on the free energy of the domain wall for l and, consequently, on the Ising transition temperature. It follows that there is no reason for the two transitions to occur simultaneously. Order-of-magnitude estimates ($T_{\rm BKT}\simeq\frac{1}{2}\pi K_\varphi$, $T_{Is}\sim K_\varphi$) do not answer the question as to which of them occurs at the lower temperature.

We now turn to the role of the spin-orbit coupling. In the a-phase, the dipole energy has the same form as in the three-dimensional A-phase[5]

$$\Phi^{a}_{dip} = -{}^{1}/{}_{2} g_D (\mathbf{d}\cdot\mathbf{l})^2, \tag{13}$$

where $\mathbf{l}$ may be replaced with the normal $\boldsymbol{\nu}$ to the surface. The energy (13) partially lifts the degeneracy of the order parameter. The minimum of (13) is attained for $\mathbf{d} = \pm\boldsymbol{\nu}$. Precisely the same values of the order parameter are attained at zero temperature (i.e., in the absence of fluctuations). The degeneracy space is then $S^1 \times Z_2$ and can be uniquely parametrized by specifying the phase φ and the number $l = \pm 1$.

At low but finite temperatures, we must consider not only the homogeneous principal state corresponding to the absolute minimum of energy, but also states corresponding to the local minima of energy, as well as fluctuations in the neighborhood of the principal and other minima. Inclusion of the latter leads to a power-type reduction in the correlators but, in contrast to the case where there is no spin-orbit coupling, it is the pair correlator (9) and not the fourth-order correlator that decreases in the power-type manner.

Order-parameter configurations corresponding to local energy minima include not only the domain walls for l and the traditional vortices, but also solitons, i.e., linear objects in which the vector $\mathbf{d}$ changes its direction (passing through the position corresponding to a dipole energy maximum) whilst the phase φ remains constant. In the two-dimensional superfluid Bose liquid, there are no such singularities. The soliton separates regions with opposite directions of the vector $\mathbf{d}$, which is equivalent to values of the phase φ differing by π. The thickness of the soliton is of the order of the dipole length ξ_D ($\xi_D^2 \sim K_d/g_0$), and the energy (per unit length) is of the order of K_d/ξ_D.

It is important to note that solitons are not topologically removable singularities because they separate order parameter regions that cannot be taken from one into another by a continuous transformation. A soliton can end on a vortex with a half-integer number of circulation quanta (as in the three-dimensional case[18]). When $T \ll K_\varphi$, however, these break points are coupled by a strong logarithmic interaction into small, closely-spaced pairs. This enables us to speak not of the breaking up of solitons but of small beaks within them, which are unimportant as we pass to scales exceeding the average size of a break. From the standpoint of thermodynamics, solitons behave as continuous lines similar to the domain walls in the Ising model.

Comparison of the energy of a soliton with its entropy (determined by the possibility of bending) shows that, when $T \sim K_d$, the free energy of a soliton must be zero. It is important to recall, however, that fluctuations in the three-dimensional vector $\mathbf{d}$ may lead to an essential renormalization of K_d (Ref. 15) as we pass from scales of the order of the coherence length ξ to the scales of the order of ξ_D at which this renormalization ends (we consider that $\xi_D \gg \xi$). In that case, the phase transition due to the vanishing of the free energy of the soliton occurs for $T \sim K_d/\ln(\xi_D/\xi)$ (compare with Ref. 19). It is precisely the condition $\xi_D \gg \xi$ that enables us to hope that this phase transition will occur at a sufficiently low temperature to ensure that the end point of the solitons will be bound into small pairs.

The appearance of infinite-length solitons in the phase transition considered here leads to an exponential decay of the pair correlators (9). The phase corresponding to higher temperatures involves a power-type decay of the fourth-order correlators described above. Since a change in the sign of $\mathbf{d}$ is equivalent to a change in the phase by π, we may conclude that the appearance of infinite-length solitons leads to the restitution of symmetry between states with phases differing by π, which was broken only by the presence of spin-orbit coupling. Although the absence of strict long-range order in φ prevents us from isolating Ising-type variables that assume different values on either side of the soliton, thermodynamics indicates that solitons should behave like Ising domain walls. The corresponding phase transitions should resemble phase transitions in the Ising model.

The last fact becomes more readily understood if we recall that in the Hamiltonian given by the sum of the gradient energy (8) and the dipole energy (13), there is no coupling between the variables $\mathbf{d}$ and φ. The coupling between them is purely topological because the pairs $\mathbf{d}$, φ and $-\mathbf{d}$, $\varphi + \pi$ correspond to the same value of the order parameter, and this manifests itself only in the possibility of vortex-type singularities with half-integral number of circulation quanta. If, however, we are concerned with temperatures at which vortices with circulation $\pm\pi$ are coupled to small pairs over scales exceeding the mean pair size, there is practically no coupling between $\mathbf{d}$ and φ. It is well-known that it is precisely large distances that are important for fluctuational phase transitions. The transition considered above corresponds to the disordering of the three-dimensional vector $\mathbf{d}$ in the "easy axis" type anisotropy field and, according to the universality hypothesis, it should be accompanied by the same critical behavior as the phase transition in the Ising model.

Further increase in temperature gives rise to phase transitions due to the dissociation of pairs of vortices with circulation $\pm\pi$ and the disordering of l, considered in the first part of this section. Thus, when a weak spin-orbit interaction is present, the transition from the a-phase to a completely disordered state consists of three different phase transitions (two Ising transitions and a BKT transition).

When the condition $T_c \ll \varepsilon_F$ is satisfied, the weak-coupling approximation ensures that all these transitions occur in a small neighborhood of T_c (transition temperature in the BCS approximation), since it is only for $(T_c - T)/T_c \sim T_c/\varepsilon_F$ that the constants in the expression for the gradient energy turn out to be of the order of T.

4. PHASE TRANSITIONS IN THE *b*-phase

The order parameter (3) of the b-phase can be written in the form

$$A_{pj} = \Delta(\delta_{jl} - \nu_j \nu_l) R_{pl} e^{i\varphi}, \tag{14}$$

where R_{pl} is the three-dimensional rotation matrix. Because of the presence of the projecting factor $\delta_{jl} - \nu_j \nu_l$, the subscript j can only assume two values. This projection ensures that the same value of A_{pj} can be represented in two ways in the form of (14) with different matrices R_{pl} and phases differing by π. The degeneracy space of the order parameter can be written as $(SO(3)\times S^1)/Z_2$, where $SO(3)$ is the domain of R_{pl} and S^1 is the domain of φ, whereas the factorization with respect to Z reflects the above ambiguity.

The situation in the b-phase is quite analogous to that encountered in the case of the a-phase (if we disregard the presence of the discrete degree of freedom in the a-phase). The gradient energy splits into terms involving R_{pl} and φ. In the absence of spin-orbit coupling, only the stiffness related to φ (superfluid density) does not become renormalized to zero at large distances. As in the a-phase, only the fourth-order correlator (10) decreases in the power-type manner, and $e^{2i\varphi}$ plays the role of the order parameter. The transition to the disordered state occurs through the dissociation of pairs of vortices with circulation $\pm\pi$. The jump in the superfluid density at the transition point has the same value (11) as in the a-phase.

The dipole energy in the b-phase depends on the three-dimensional rotations matrix R_{pl}. It is convenient to parametrize it in the standard fashion[5] using the vector $\mathbf{n}$ defining the direction of the axis of rotation and the angle θ through which the rotation takes place. The expression for the dipole energy then follows from the general Leggett formula[20] and takes the form

$$\Phi^b_{dip}=g_D\{{}^1/_3\cos^2\theta+{}^3/_8[(1-\cos\theta)n_\parallel{}^2+{}^4/_3\cos\theta]^2\}, \quad (15)$$

where $\mathbf{n}_\parallel=\mathbf{n}-(\mathbf{n}\cdot\boldsymbol{\nu})\boldsymbol{\nu}$ is the projection of $\mathbf{n}$ onto the plane.

The energy (15) reaches a minimum for $\cos\theta=0$, $\mathbf{n}=\pm\boldsymbol{\nu}$. All the values of R_{pl} that minimize (15) can be specified by the single matrix R_{pl} (rotation by $\pi/2$ around $\boldsymbol{\nu}$) and different values of the phase φ. The degeneracy space contracts to S^1. At low temperatures, the presence of the dipole interaction leads to a power-law reduction in the pair correlators (9).

As in the a-phase, solitons may be present in the b-phase. The vector $\mathbf{n}$ can then reverse its direction in the interior of the soliton. When $\xi_D\gg\xi$, an increase in temperature is first accompanied by the Ising phase transition due to the vanishing of the free energy of the soliton. This is a transition to the phase with the exponential reduction in pair correlators but finite superfluid density, for which there is a restoration of the spontaneously broken symmetry between the states with phases differing by π.

Thus, the transition to the disordered state in the b-phase occurs via two phase transitions. As in the case of the a-phase, the temperature of these transitions for $T_c\ll\varepsilon_F$ are only slightly lower than the transition temperature T_c in the BCS approximation.

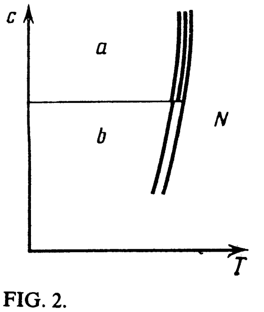

FIG. 2.

5. CONCLUSION

Our results enable us to construct a schematic phase diagram for the two-dimensional superfluid Fermi liquid with p-pairing (Fig. 2). The phase transition between the a- and b-phases is a first-order transition, whereas the other transitions are second-order transitions.

We emphasize that our prediction that the BKT transition will split into two (one of which will be the Ising transition) in the a- and b-phases of the superfluid Fermi liquid is valid only provided $\xi_D\gg\xi$, which ensures the essential renormalization of K_d. Splitting will not occur otherwise.

We must also note the favorable fact that the effective strength of the spin-orbit coupling can be reduced by a magnetic field. In the case of the a-phase (as for the three-dimensional A-phase[5]), the part of the free energy that depends on the orientation of the order parameter and is due to the magnetic field can be written in the form

$$\Phi_{mag}=\chi(\mathbf{dH})^2/2. \quad (16)$$

Comparison of (16) with (13) will show that, when $\mathbf{H}\|\boldsymbol{\nu}$, the application of the magnetic field is equivalent to the replacement of g_D with $g_D^{eff}=g_D-\chi H^2$. An increase in H is accompanied by a decrease in g_D^{eff}, which changes sign when $H_c^a=(g_D/\chi)^{1/2}$The quantity ξ_D increases correspondingly and becomes infinite. For $H>H_c^a$, the spin-orbit coupling is absent and the vector $\mathbf{d}$ lies in an anisotropy field of the "easy plane" type.[21]

The foregoing analysis leads us to the conclusion that, as H approaches H_c^a from below, the splitting of the BKT transition into two will necessarily take place. In the case of the b-phase, the anisotropic part of the magnetic energy can be written in the form $\Phi^b_{mag}=-\chi(H_pR_{pj}\nu_j)^2/2$, for $\mathbf{H}\|\boldsymbol{\nu}$, which converges to $\Phi^b_{mag}=-\chi H^2[\cos\theta-(1-\cos\theta)n_\parallel^2]^2/2$.

As the magnetic field perpendicular to the plane is increased, the height of the barrier separating the minima $\Phi^b_{\rm dip}+\Phi^b_{\rm mag}$ decreases, and vanishes together with the soliton energy and the Ising transition temperature for $H_c^b=(2g_D/19\chi)^{1/2}$.

Thus, both in the case of the a-phase and in the case of the b-phase, the splitting of the BKT transition will necessarily occur as the magnetic field is increased (at right-angles to the plane), although it is absent for $\mathbf{H}=0$ because of the unfavorable ratio of the constants.

The possible splitting of the BKT transition, the existence of the intermediate phase, and the Ising character of the transition due to the vanishing of the free energy of the soliton are more rigorously demonstrated in Ref. 22 by considering the example of a particular modification of the two-dimensional x-y model.

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