Mechanism of triggered emission in the magnetospheric plasma

R. Z. Sagdeev, V. D. Shapiro, and V. I. Shevchenko

Institute of Space Research, USSR Academy of Sciences
(Submitted 4 January 1985)
Zh. Eksp. Teor. Fiz. **89**, 22–33 (July 1985)

Triggered radiation, a monochromatic response of the magnetospheric plasma to a whistler propagating in it, is studied. The response is produced by the modulation of the distribution of particles in Cherenkov resonance with the main wave. Phase mixing is absent because the particle velocity equals the group velocity of the response. It is shown that the experimentally observed amplitude and frequency characteristics of the response are in satisfactory agreement with the theory presented.

INTRODUCTION

It is well known that the magnetospheric plasma is a fairly good waveguide for electromagnetic waves with frequencies below the electron cyclotron frequency, the so-called whistlers, which propagate effectively along the geomagnetic field. In many experiments (see, for example, Refs. 1 and 2, a powerful terrestrial transmitter produced a whistler in the magnetosphere, and an electromagnetic signal was detected at the magnetically conjugate point. Besides the waveguide properties of the magnetosphere, various nonlinear effects produced by the interaction between the whistler and the magnetosphere plasma have been investigated. Some of these effects, such as the amplitude and frequency modulation of the electromagnetic signal, can be explained fairly readily by the appropriate nonlinear theory of the interaction of a wave with resonant particles (see Ref. 3), but the interpretation of some other experimental results has proved to be much more difficult. This applies particularly to the triggered emission phenomenon—the monochromatic response of the magnetosphere to a whistler propagating in it. It is generally observed together with the triggering wave at the magnetically conjugate point and it has a frequency that is modulated in time. Although this interesting phenomenon, discovered in 1964 by Helliwell and his collaborators,[1] was subsequently the subject of detailed experimental investigation (see the reviews Refs. 4, 5 and the literature quoted in them), there is still no quantitative theory of it. Typical spectra of oscillations detected at the conjugate point are shown in Fig. 1. It can be clearly seen that besides the original signal, whose frequency spectrum is broadened by nonlinear effects by an amount $\Delta\omega \sim 10^{-2}\omega_0$, a monochromatic signal with monotonically increasing (1) or s-shaped (2) frequency characteristic is also observed after a delay time of 0.5–1 sec, which is comparable to the time of propagation of a wave between the conjugate points. This is the triggered emission; the maximal deviation of its frequency from that of the original signal is 10–20%.

Sudan and Ott[6] proposed the following scheme for generation of the triggered radiation. It is based on the triggered emission arising as a result of modulation of the distribution function of the resonance particles by the triggering wave. For a whistler propagating along the magnetic field (circularly polarized wave with the direction of polarization coincident with the direction of gyration of the electrons), resonance with electrons is possible only through the normal Doppler effect: $\omega_0 - k_0 v_z = \omega_H$, where ω_0 and $k_0(z)$ are the frequency and wave number of the wave, v_z is the velocity of the electrons along the magnetic field, and $\omega_H(z) = eH_0/mc$ is the cyclotron frequency. Since $\omega_0 < \omega_H$ for a whistler, $v_z < 0$, i.e., the resonance particles propagate in the opposite direction to the wave. When such particles move in a nonuniform magnetic field, bunching occurs, analogous to the well-known mechanism of klystron bunching (the particles that come into resonance later have a higher velocity and catch up with the ones in front of them). The blob which develops as a result of the bunching produces radiation, whose frequency increases as the bunch moves to a stronger magnetic field. A theory of such radiation was constructed in Refs. 6 and 7, but although the main kinematic characteristics of the radiation spectrum reflect the characteristic features of the triggered emission, the radiation amplitude was found to be inadequate for satisfactory explanation of this phenomenon.

In the present paper, we propose a different scheme for generation of triggered emission, applicable when the whistler is "oblique," i.e., propagates at an angle to the magnetic field. In this case, modulation of the electron distribution function by the triggering wave is also possible at Cherenkov resonance,

$$\omega_0 = k_{0z}(z_1) v_z, \tag{1}$$

in which case the resonance particles move in the same direction as the wave.

The modulation of the distribution function induces an electric field which can be represented as a superposition of

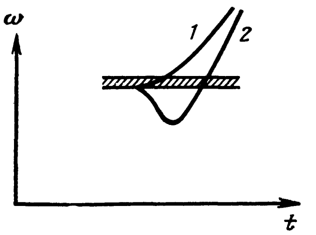

FIG. 1. Time dependence of the frequency of signals detected at the conjugate point; the triggering signal is shown by the hatching, the triggered emission by curves 1 and 2.

 0038-5646/85/070012-07$04.00

van Kampen harmonics with different frequencies ω and wave numbers ω/v_z. The fundamental harmonic is the one whose wave number ω/v_z is equal to the characteristic wave number of the whistler, determined from the dispersion relation

$$k_z(\omega,\ z)=\omega/v_z. \tag{2}$$

With increasing separation of the point of formation z of the response from the point of modulation z_1, the macroscopic response field is rapidly damped by phase mixing of the van Kampen waves corresponding to different v_z. However, such mixing does not occur for the group of resonant particles, which remain in resonance with the van Kampen wave as its frequency is shifted, this, as readily seen from (2), consistent with the condition

$$\partial k_z/\partial\omega=1/v_z. \tag{3}$$

The finite width of this resonance is due to the growth rate of a whistler propagating in the radiation belt: $\Delta v_z \sim \gamma/k_z$, $\gamma = \mathrm{Im}\omega$. However, as usual in such plasma resonances, the small number of particles in resonance, $\Delta n_{\rm res} \propto \gamma$, is compensated by the large resonance contribution, $\propto 1/\gamma$, and the total response field does not depend on γ.

The calculations made below for typical parameters of the magnetosphere plasma show that the intensity E_R of the electric field of the response is comparable to the intensity E_0 of the triggering wave, whereas in the case of the klystron mechanism $E_R \sim E_0 n^*/n_{\rm res}$, where $n^* = n_{\rm res}\ (\mathrm{k_0L})^{-1/2}$ is the density of the bunching particles, L is the length of the magnetic field line, and $n_{\rm res}$ is the total density of the resonance particles (see Ref. 7).

If it is assumed that the triggered emission, like the triggering wave, propagates at a small angle to the magnetic field, $k_\perp \ll k_z$, then to determine the dependence $k_z(\omega)$ one can use the dispersion relation corresponding to longitudinal propagation of the whistler mode:

$$\omega=\omega_H c^2k_z^2(\omega_p^2+c^2k_z^2)^{-1}.$$

Then from conditions (2) and (3) we find that the frequency of the response must be equal to the half-harmonic of the local cyclotron frequency at the point of formation of the response,

$$\omega={}^1/_2\omega_H(z), \tag{4}$$

in agreement with the well-known experimental condition for the existence of triggered emission (see, for example, Ref. 5).

The mechanism considered here is a modification of the linear "echo" effect in an inhomogeneous plasma (see Ref. 8), the absence of phase mixing being ensured by the resonance condition (3). The possibility of using an approximation linear in the amplitude of the triggering wave to calculate the modulation of the distribution function simplifies the problem considerably. Let us consider the applicability of this approximation.

For the existence of triggered emission, the particles must not come out of resonance with the wave too slowly, in order to avoid mixing as a result of phase oscillations of the resonance particles, which causes an ergodic distribution function to be established and the phase modulation to disappear. The corresponding condition entails a sufficiently rapid variation of the geomagnetic field, so that the length over which the particles come out of resonance,

$$l_1\sim\Omega_{tr}|d\omega_H/dz|^{-1}$$

is appreciably less than the phase mixing length $l_2 \sim v_2/\Omega_{\rm tr}$, i.e.,

$$\Omega_{tr}^2<\left|v_z\frac{d\omega_H}{dz}\right|. \tag{5}$$

In this condition, $\Omega_{\rm tr} = (\Omega_H\omega_0^2/\omega_H)^{1/2}v_\perp/v_z$ is the frequency of the phase oscillations of the trapped particles in Cherenkov resonance with the oblique whistler; this frequency determines the "nonlinear" width of the resonance: $k_z\Delta v_z \approx \Omega_{\rm tr}$; here, $\Omega_H = eH_w/mc$ is the electron cyclotron frequency in the magnetic field of the triggering wave. Substituting in condition (5) in the neighborhood of the equatorial plane $|d\omega_H/dz| \sim \omega_H\delta z/L^2$, where δz is the dimension of the region in which the triggered emission is generated, $\delta z \sim L\,|\delta\omega|/\omega_0$, in which $\delta\omega = \omega - \omega_0$ is the offset of the triggered emission frequency, we write (5) in the form

$$\frac{H_w}{H_0}<\left\langle\frac{v_z}{v_\perp}\right\rangle^2_{res}\frac{|\delta\omega|}{\omega_0}\frac{1}{k_0L}\frac{\omega_H}{\omega_0}. \tag{6}$$

For typical magnetosphere conditions, the threshold value of the magnetic field of the wave determined by the condition (6) is $H_w^{\rm cr} \approx 10^{-7}$–$3\times10^{-8}$ Oe. The condition (6) is the condition for the existence of triggered emission and, at the same time, the condition of applicability of the linear approximation. In the opposite limiting case $H_w > H_w^{\rm cr}$, the nonlinear "rearrangement" of the distribution function and "spilling" of the resonance particles must be important.

CALCULATION OF THE RESPONSE FIELD

We consider an electromagnetic wave propagating at an angle to an external magnetic field $\mathbf{H}_0 = H_0(z)\mathbf{e}_z$ whose intensity varies over distances appreciably greater than the wavelength. We shall assume that the original wave has a fairly sharp leading edge, so that at each point of the field line when

$$t<\int\limits_{s_0}^{z} d\zeta\left(\frac{d\omega_0}{dk_{0z}}\right)^{-1}\equiv t_0(z)$$

we have $E = 0$ and $f=f_0(v_\perp, v_z)$, where f_0 is the equilibrium distribution function. For $t>t_0(z)$,

$$E=\left(E_0\exp\left(i\int k_{0z}d\zeta-i\omega_0t\right)+E_R(t,z)\right)\exp(i\mathbf{k}_\perp\mathbf{r}_\perp),$$

where E_0 is the amplitude of the triggering wave, E_R is the response field, and the correction to the equilibrium distribution function is found from the equation

$$\frac{\partial f_1}{\partial t}+ik_\perp v_\perp\cos\theta f_1+v_z\frac{\partial f_1}{\partial z}+\omega_H\frac{\partial f_1}{\partial\theta}$$
$$=\frac{e}{m}\left\{\frac{\partial f_0}{\partial v_\perp}\left[\cos\theta\left(E_{xR}-\frac{v_z}{c}H_{yR}\right)+\sin\theta\left(E_{yR}+\frac{v_z}{c}H_{xR}\right)\right]\right.$$
$$+\frac{v_\perp}{c}\frac{\partial f_0}{\partial v_z}(H_{yR}\cos\theta-H_{xR}\sin\theta)$$
$$\left.+\left[\left(1-\frac{k_{0z}v_z}{\omega_0}\right)\frac{\partial f_0}{\partial v_\perp}+\frac{k_{0z}v_\perp}{\omega_0}\frac{\partial f_0}{\partial v_z}\right](E_{0x}\cos\theta+E_{0y}\sin\theta)\right\}.$$

In this equation, θ is the azimuthal angle in the velocity space. In writing down the equation we have used the fact that in the whistler mode $E_{0z}=0$. To solve this equation, we perform a Laplace transformation with respect to the time,

$$f_p=\int_{t_0(z)}^{\infty}\exp(-pt)f_1(t)\,dt,$$

and a Fourier transformation with respect to the angle θ. Solving the resulting differential equation with respect to z using the obvious boundary condition $f_p(z\to S_0)=0$, we obtain the expression

$$\begin{aligned}f_p=&\frac{e}{2m}\int_{S_0}^{z}\frac{dz'}{v_z}\sum_{n,n'}\sum_{\pm}J_{n\mp1}\left(\frac{k_\perp v_\perp}{\omega_H}\right)\\&\times J_{n'}\left(\frac{k_\perp v_\perp}{\omega_H}\right)\exp[i(n-n')\theta]\\&\times\exp\left\{-\int_{z'}^{z}\frac{p+in\omega_H}{v_z}d\zeta+i\int_{S_0}^{z'}k_{0z}\,d\zeta+i\mathbf{k}_\perp\mathbf{r}_\perp-(p+i\omega_0)t_0(z')\right\}\\&\times\left\{\left[\left(E_{px}+\frac{v_z}{p}\frac{dE_{px}}{dz}\right)\mp i\left(E_{py}+\frac{v_z}{p}\frac{dE_{py}}{dz}\right)\right]\frac{\partial f_0}{\partial v_\perp}\right.\\&-\frac{v_\perp}{p}\frac{\partial f_0}{\partial v_z}\left(\frac{dE_{px}}{dz}\mp i\frac{dE_{py}}{dz}\right)+\frac{E_{0x}\mp iE_{0y}}{p+i\omega_0}\left[\left(1\right.\right.\\&\left.\left.\left.-\frac{k_{0z}v_z}{\omega_0}\right)\frac{\partial f_0}{\partial v_\perp}+\frac{k_{0z}v_\perp}{\omega_0}\frac{\partial f_0}{\partial v_z}\right]\right\}.\end{aligned}\tag{7}$$

Here, E_p is the Laplace transform of the electric field of the response. In calculating the magnetic components of the response field, we have used relations that follow from Maxwell's equations:

$$H_{px}=\frac{c}{p}\frac{dE_{py}}{dz},\quad H_{py}=-\frac{c}{p}\frac{dE_{px}}{dz}.\tag{8}$$

Using these relations, from Maxwell's equations curl $\mathbf{H}=(4\pi/c)\mathbf{j}$ (as usual, we ignore the displacement current in the whistler mode, since $\omega\ll kc$) we readily obtain the following system of equations for $E_p^{\pm}=E_{px}\pm iE_{py}$:

$$\begin{aligned}&\frac{d^2E_p^-}{dz^2}+\frac{k_\perp^2}{2}(E_p^+-E_p^-)=-\frac{4\pi ep}{c^2}\int v_\perp e^{-i\theta}f_p d\mathbf{v},\\&\frac{d^2E_p^+}{dz^2}+\frac{k_\perp^2}{2}(E_p^--E_p^+)=-\frac{4\pi ep}{c^2}\int v_\perp e^{i\theta}f_p\,d\mathbf{v}.\end{aligned}\tag{9}$$

Calculating by means of the distribution function (7) the electron current on the right-hand side of Eqs. (9), we write these equations in the form

$$\begin{aligned}&\frac{d^2E_p^-}{dz^2}+\left(\hat g_--\frac{k_\perp^2}{2}\right)E_p^-+\left(\hat h_-+\frac{k_\perp^2}{2}\right)E_p^+\\&=E_0^-\int d\mathbf{v}\sum_n J_{n-1}^2\left(\frac{k_\perp v_\perp}{\omega_H}\right)S_n(p,\mathbf{v}),\\&\frac{d^2E_p^+}{dz^2}+\left(\hat g_+-\frac{k_\perp^2}{2}\right)E_p^++\left(\hat h_++\frac{k_\perp^2}{2}\right)E_p^-\\&=E_0^-\int d\mathbf{v}\sum_n J_{n-1}\left(\frac{k_\perp v_\perp}{\omega_H}\right)J_{n+1}\left(\frac{k_\perp v_\perp}{\omega_H}\right)S_n(p,\mathbf{v}).\end{aligned}\tag{10}$$

In Eqs. (10) we have used the fact that, because $k_\perp^2\ll k_{0z}^2$, $E_0^+\ll E_0^-$, and we have used the notation

$$\begin{aligned}S_n(p,\mathbf{v})=&\frac{(2\pi)^{1/2}e^2}{mc^2}\exp\left(\frac{5\pi i}{4}\right)\frac{pv_\perp}{p+i\omega_0}\exp\\&\times\left\{i\int_{S_0}^{z_1}k_{0z}\,d\zeta+i\mathbf{k}_\perp\mathbf{r}_\perp-\int_{z_1}^{z}\frac{p+in\omega_H}{v_z}d\zeta\right.\\&\left.-(p+i\omega_0)t_0(z_1)\right\}R_n^{-1/2}L(f_0);\end{aligned}$$

$$\begin{aligned}\hat g_\mp E_p=&\frac{2\pi e^2}{mc^2}p\int d\mathbf{v}\,v_\perp\int_{S_0}^{z}\frac{dz'}{v_z}\sum_n J_{n\mp1}^2\left(\frac{k_\perp v_\perp}{\omega_H}\right)\\&\times\exp\left[-i\mathbf{k}_\perp\mathbf{r}_1\int_{z'}^{z}\frac{p+in\omega_H}{v_z}d\zeta\right]\\&\times\exp\left[-\int_{z'}^{z}\frac{p+in\omega_H}{v_z}d\zeta+i\mathbf{k}_\perp\mathbf{r}_1\right]\\&\times\left[\frac{\partial f_0}{\partial v_\perp}\left(1+\frac{v_z}{p}\frac{d}{dz'}\right)-\frac{\partial f_0}{\partial v_z}\frac{v_\perp}{p}\frac{d}{dz'}\right]E_p(z');\end{aligned}$$

$$\begin{aligned}\hat h_\mp E_p=&\frac{2\pi e^2}{mc^2}p\int d\mathbf{v}\,v_\perp\int_{S_0}^{z}\frac{dz'}{v_z}\sum_n J_{n-1}\left(\frac{k_\perp v_\perp}{\omega_H}\right)J_{n+1}\left(\frac{k_\perp v_\perp}{\omega_H}\right)\\&\times\left[\frac{\partial f_0}{\partial v_\perp}\left(1+\frac{v_z}{p}\frac{d}{dz'}\right)-\frac{\partial f_0}{\partial v_z}\frac{v_\perp}{p}\frac{d}{dz'}\right]E_p(z'),\end{aligned}$$

$$L(f_0)=\left(1-\frac{k_{0z}v_z}{\omega_0}\right)\frac{\partial f_0}{\partial v_\perp}+\frac{k_{0z}v_\perp}{\omega_0}\frac{\partial f_0}{\partial v_z},$$

$$R_n=v_z\left(n\frac{d\omega_H}{dz_1}+v_z\frac{dk_{0z}}{dz_1}\right).\tag{11}$$

The right-hand sides of Eqs. (10) are proportional to the current of the resonant particles which arises as a result of the modulation of the distribution function and creates the response in the form of the whistler propagating in the magnetosphere. In calculating the integral over z' in this current, we used the method of steepest descent; the point z_1 is the point of stationary phase, determined from the equation

$$v_z^{-1}(n\omega_H(z_1)-ip)+k_{0z}(z_1)-(d\omega_0/dk_{0z})^{-1}(\omega_0-ip)=0.\tag{11a}$$

The expressions for the operators $\hat g_\pm$ and $\hat h_\pm$ can be transformed by separating the contributions from the nonresonant (plasma) and resonant electrons. For the frist group, the conditions $k_zv_z/|p|\ll1$ and $k_zv_z/\omega_H\ll1$ holding, the interval of integration satisfies $|z-z'|\ll k_z^{-1}$, and accordingly $E_p(z')\approx E_p(z)$. For the second group, the integrals are calculated by the method of steepest descent in the neighborhood of the stationary point, whose coordinate is determined from Eq. (11a) with the substitution $k_{0z}\to k_{pz}$. As a result, the integral operators $\hat g_\pm$ and $\hat h_\pm$ are transformed into algebraic expressions $g_\pm$ and $h_\pm$, and it is found that $h_+=h_-$. The results of the calculations can be represented in the form

$$g_{\pm}E_p=\frac{2\pi e^2}{mc^2}p\int d\mathbf{v}\, v_\perp \sum_n J^2_{n\pm1}\left(\frac{k_\perp v_\perp}{\omega_H}\right)\frac{\partial f_0}{\partial v_\perp}$$
$$\times \exp(i\mathbf{k}_\perp\mathbf{r}_\perp)\frac{E_p(z)}{p+in\omega_H}$$
$$+\frac{(2\pi)^{3/2}e^2}{mc^2}\exp(\pi i/4)\,p\int d\mathbf{v}\, v_\perp\sum_n J^2_{n\pm1}\left(\frac{k_\perp v_\perp}{\omega_H}\right)L(f_0)R_n^{-1/2}$$
$$\times E_p(z_1)\exp\left[-\int_{z_1}^{z}\frac{p+in\omega_H}{v_z}d\zeta+i\mathbf{k}_\perp\mathbf{r}_\perp\right],$$
$$hE_p=\frac{2\pi e^2}{mc^2}p\int d\mathbf{v}\, v_\perp\sum_n J_{n+1}\left(\frac{k_\perp v_\perp}{\omega_H}\right)J_{n-1}\left(\frac{k_\perp v_\perp}{\omega_H}\right)\frac{\partial f_0}{\partial v_\perp}$$
$$\times \exp(i\mathbf{k}_\perp\mathbf{r}_\perp)\frac{E_p(z)}{p+in\omega_H}$$
$$+\frac{(2\pi)^{3/2}e^2}{mc^2}\exp\left(\frac{\pi i}{4}\right)p\int d\mathbf{v}\, v_\perp J_{n+1}\left(\frac{k_\perp v_\perp}{\omega_H}\right)$$
$$\times J_{n-1}\left(\frac{k_\perp v_\perp}{\omega_H}\right)E_p(z_1)$$
$$\times L(f_0)R_n^{-1/2}\exp\left[-\int_{z_1}^{z}\frac{p+in\omega_H}{v_z}d\zeta+i\mathbf{k}_\perp\mathbf{r}_\perp\right]. \qquad (12)$$

In the terms that determine the current of the resonant particles we have substituted

$$E_p(z)\sim\exp\left(i\int^{z}k_{pz}d\zeta\right),$$

where k_{pz} is the longitudinal component of the wave vector of the response, calculated from the dispersion relation of the whistler mode:

$$D_p=(k_{pz}^2+k_\perp^2/2-g_-)(k_{pz}^2+k_\perp^2/2-g_+)-(k_\perp^2/2+h)^2=0. \qquad (13)$$

Ignoring the g and h the contribution of the resonant particles and going to the limit of zero Larmor radius $k_\perp v_\perp/\omega_H\to 0$ (at the same time $h\to 0$), we obtain from (13) the whistler dispersion relation in the hydrodynamic limit:

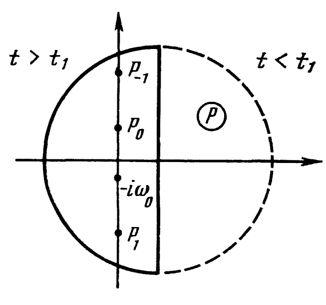

FIG. 2. Contour of integration with respect to p in Eq. (15); $t_1=t_0(z_1)+\int_{z_1}^{z}\frac{d\zeta}{v_z}$.

$$k_{pz}^2(k_\perp^2+k_{pz}^2)+\frac{2p^2}{p^2+\omega_H^2}\frac{\omega_p^2\left(\frac{1}{2}k_\perp^2+k_z^2\right)}{c^2}$$
$$+\omega_p^4/c^4\frac{p^2}{p^2+\omega_H^2}=0. \qquad (14)$$

In the case $\omega_p \gg kc$ (purely electromagnetic oscillations) and $|p|\ll\omega_H$ we obtain from this the standard equation

$$p^2=-\omega_H^2k_{pz}^2(k_\perp^2+k_{pz}^2)c^4/\omega_p^4.$$

Solving the system of equations (10) and making the inverse Laplace transformation, we obtain the following expression for electric field of the response:

$$E_R(t,\mathbf{r})=\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}e^{pt}E_p^-(\mathbf{r})\,dp=\sqrt{2\pi}\frac{e^2E_0}{mc^2}e^{3\pi i/4}\int_{\sigma-i\infty}^{\sigma+i\infty}dp\,e^{pt}$$
$$\times\frac{p}{p+i\omega_0}\int d\mathbf{v}\,v_\perp\sum_n\left\{\left[\frac{(n\omega_H-ip)^2}{v_z^2}+g_+\right]J^2_{n-1}\left(\frac{k_\perp v_\perp}{\omega_H}\right)\right.$$
$$\left.+J_{n-1}\left(\frac{k_\perp v_\perp}{\omega_H}\right)J_{n+1}\left(\frac{k_\perp v_\perp}{\omega_H}\right)h\right\}$$
$$\times\ L(f_0)R_n^{-1/2}\left\{D_p\left[k_{pz}=\frac{ip-n\omega_H(z)}{v_z}\right]\right\}^{-1}$$
$$\times\exp\left[i\int_{S_0}^{z_1}k_{0z}d\zeta+i\mathbf{k}_\perp\mathbf{r}_\perp-\int_{z_1}^{z}\frac{p+in\omega_H}{v_z}d\zeta-(p+i\omega_0)t_0(z_1)\right]. \qquad (15)$$

The integration contour is shown in Fig. 2; $\sigma>0$. The singularities of the integrand are poles on the imaginary axis at the points

$$p=-i\omega_0;\quad p_n^{\pm}=i(\mp k_{pz}v_z-n\omega_H),\quad D_p(p_n)=0.$$

It is obvious that when

$$t<t_0(z_1)+\int_{z_1}^{z}\frac{d\zeta}{v_z}$$

the contour integration with respect to p in (15) can be closed in the right half-plane, where the integrand has no singularities, and, therefore, $E_R=0$. For given velocity v_z of the resonant particles, a nonzero response arises only when

$$t>t_0(z_1)+\int_{z_1}^{z}\frac{d\zeta}{v_z},$$

i.e., at times greater than the time required for the triggering wave to move to the point z_1, where the distribution function of the resonant particles is modulated, plus the time for these particles to move from the point z_1 to the point z at which the response is determined. Omitting in E_R the contribution from the pole $p=-i\omega_0$, which corresponds to the response at the triggering-wave frequency, we obtain the following expression for the triggered emission part of the response, whose frequency must depend on the point z:

$$E_R(t,\mathbf{r})=\sqrt{2\pi^3}\frac{e^2E_0}{mc^2}e^{5\pi i/4}\int dv\, v_\perp \sum_n J_{n-1}^2\left(\frac{k_\perp v_\perp}{\omega_H}\right)\frac{\omega^*}{\omega^*-\omega_0}$$

$$\times R_n^{-1/2}L(f_0)\left[\left(\frac{\omega^*-n\omega_H}{v_z}\right)^2+\frac{\omega_p^2}{c^2}\frac{\omega^*}{\omega_H+\omega^*}\right]$$

$$\times\left\{\left(\frac{\omega_p^2}{c^2}\frac{\omega^{*2}}{\omega_H^2-\omega^{*2}}-\frac{k_\perp^2}{2}\right)^2+\frac{\omega_p^2\omega^{*2}(\omega_p^2/c^2+k_\perp^2)}{c^2(\omega_H^2-\omega^{*2})}\right\}^{-1/2}$$

$$\times\exp\left[i\int_{s_0}^{z_1}k_{0z}d\zeta+i\mathbf{k}_\perp\mathbf{r}_\perp-i\omega^*t+i\int_{z_1}^{z}\frac{\omega^*-n\omega_H}{v_z}d\zeta-i(\omega_0-\omega^*)t_0(z_1)\right]$$

$$\times\left\{\left[2k_z\left(\frac{1}{v_z}-\frac{dk_z}{d\omega}\right)\right]^{-1}_{k_z=(\omega^*-n\omega_H)/v_z}-\left[2k_z\left(\frac{1}{v_z}+\frac{dk_z}{d\omega}\right)\right]^{-1}_{k_z=(n\omega_H-\omega^*)/v_z}\right\}. \quad (16)$$

Here, $\omega^*=ip_n$, where p_n are the poles of E_p, i.e., the points for which

$$D_p\left(k_{pz}=\frac{\omega^*-n\omega_H}{v_z}\right)=0;$$

in calculating (16), we have used the hydrodynamic limit for D_p determined by Eq. (14).

In the integral over v_z, all that is important is the residues at the points

$$v_z=d\omega^*/dk_z,\quad \omega^*-k_zv_z=n\omega_H; \quad (17)$$

$$v_z=-d\omega^*/dk_z,\quad \omega^*+k_zv_z=n\omega_H. \quad (17a)$$

The integral in the principal value sense for $|z-z'|\gg v_z/\omega_H$ becomes zero fairly rapidly as a result of phase mixing in the integrand. The width of the resonance at the point of the residue is, as usual, $\sim\gamma/k_z$, and the change in the phase of the field for the integration over v_z in the neighborhood of the resonance can be ignored provided

$$\gamma|z-z_1|/v_z\ll 1.$$

Here, γ is the growth rate with which the amplitude of the triggering wave changes. For typical parameters of magnetospheric experiments with whistlers, this condition is equivalent to the requirement that the width $\delta z=|z-z_1|$ of the resonance region be small compared with the length of the field line, i.e., it is certainly satisfied.

Simple analysis shows that the resonance conditions (17) can be satisfied only for $n=0$ (Cherenkov resonance). For $n=1, 2\ldots$ (cyclotron resonance in the normal Doppler effect) the signs of k_z and v_z in (17) are opposite, and the resonance condition $v_z=d\omega^*/dk_z$ cannot be satisfied. For $n=-1, -2, -3\ldots$ (anomalous Doppler effect) the simultaneous fulfillment of the resonance conditions (17) is also impossible, since from Eqs. (14) for the whistler mode, we are considering $d\omega^*/dk_z<2\omega^*/k_z$ and $\omega^*<\omega_H$. For $n=0$, the resonant conditions (17) imply that the group and phase velocities of the whistler are equal, which for $k_\perp/k_z\ll 1$ means that the response frequency must be equal to half the local cyclotron frequency. Retaining only the contribution of the Cherenkov resonance in (16) and integrating over v_z, we obtain for the electric field of the response the expression

$$E_R(t,\mathbf{r})=\left(\frac{\pi}{2}\right)^{3/2}\frac{\omega_p^2}{n_0c^2}E_0e^{3\pi i/4}\times\int d\mathbf{v}_\perp v_\perp\left(\frac{dk_0}{dz_1}\right)^{-1/2}J_1^2\left(\frac{k_\perp v_\perp}{\omega_H}\right)\frac{v_z^{*2}}{\omega^*-\omega_0}L(f_0)$$

$$\times\exp\left[-i\omega^*t+i\mathbf{k}_\perp\mathbf{r}_\perp+i\int_{s_0}^{z_1}k_{0z}d\zeta+\frac{i\omega^*}{v_z^*}(z-z_1)+i(\omega^*-\omega_0)t_0(z_1)\right]. \quad (18)$$

The response initiates in the magnetosphere a packet of whistler waves, which can be represented in the form

$$E(t,z)=\int d\varkappa E_\varkappa\exp\left[i\left(\int_{s_0}^{z}k_z(\varkappa,\zeta)d\zeta-\omega_\varkappa t\right)\right], \quad (19)$$

where $k_z(\varkappa,z)$ is the longitudinal wave number of an individual harmonic of the packet, $\varkappa$ is its value in the equatorial plane $z=z_0$, and the frequency $\omega_\varkappa$ of the harmonic is determined by the dispersion relation (14) of the whistler mode with the substitution $ip\to\omega_\varkappa$, $k_z\to\varkappa$, $z=z_0$. For the amplitude of the packet initiated by the response field we then have the expression

$$E_\varkappa=\frac{1}{2\pi}\int dz'\,E_R[t_R(z'),z']\exp\left[-i\int_{s_0}^{z'}k_z(\varkappa,\zeta)d\zeta+i\omega_\varkappa t_R(z')\right]$$

$$t_R(z)=t_0(z_1)+(z-z_1)/v_z, \quad (20)$$

where $t_R(z)$ is the time of formation of the response at the point z.

Substituting E_R from Eq. (18) and integrating with respect to z' by the method of steepest descent, we arrive at the expression

$$E(t,\mathbf{r})=\frac{\pi i}{4}\frac{\omega_p^2}{n_0c^2}E_0\int d\omega_\varkappa\left|\frac{d\omega_\varkappa}{d\varkappa}\right|^{-1}\times\int dv_\perp v_\perp J_1^2\left(\frac{k_\perp v_\perp}{\omega_H}\right)L(f_0)v_z^{*2}\frac{1}{\omega_\varkappa-\omega_0}$$

$$\times\left\{\frac{dk_0}{dz_1}\left(\frac{dk_z}{dz_2}+2\frac{k_z}{v_z^*}\frac{dv_z^*}{dz_2}\right)\right\}^{-1/2}\exp\left[-i\omega_\varkappa t+i\mathbf{k}_\perp\mathbf{r}_\perp+i\int_{s_0}^{z_1}k_{0z}d\zeta+\frac{i\omega_\varkappa}{v_z^*}(z_2-z_1)+i\int_{z_2}^{z}k_z(\varkappa,\zeta)d\zeta+i(\omega_\varkappa-\omega_0)t_0(z_1)\right]. \quad (21)$$

The coordinate z_2 of the point of stationary phase is determined by the equation

$$k_z(z_2,\varkappa)-\frac{\omega_\varkappa}{v_z^*(z_2)}+\frac{\omega_\varkappa}{v_z^{*2}}\frac{dv_z^*}{dz_2}(z_2-z_1)=0. \qquad (22)$$

In the neighborhood of the equatorial plane, the final term in this equation is small by a factor $(z_2-z_1)^2/L^2$. Ignoring it, we find that the equation for k_z is identical to the resonance condition (17) for $n=0$ and $z=z_2$, and therefore

$$\omega_\varkappa=\omega^*(z_2)={}^1/_2\omega_H(z_2). \qquad (23)$$

The integral over $\omega_\varkappa$ in Eq. (21) can also be readily calculated by the method of steepest descent, the condition of stationary phase making it possible to separate in the packet (21) the harmonic that is the main one at the point of observation z at the time t ($\omega_\varkappa=\omega_{\rm tr}$). This condition has the form

$$t=\int_{s_0}^{z_1}d\zeta\left(\frac{d\omega_0}{dk_{0z}}\right)^{-1}+\int_{z_1}^{z_2}\frac{d\zeta}{v_z^*}+\int_{z_2}^{z}d\zeta\left(\frac{d\omega_\varkappa}{d\varkappa}\right)^{-1}. \qquad (24)$$

Its meaning is rather obvious; the time of observation is the sum of the time required by the triggering wave to propagate to the point z_1, where resonant particles with velocity v_z^* undergo modulation without phase mixing, the time for these particles to move to the point z_2, the point at which the response is formed, and, finally, the time required for the fundamental harmonic of the packet initiated by the response to propagate to the point of observation z.

Using the method of steepest descent to calculate the integral over $\omega_\varkappa$ in (21), we write down the final expression for the electric field at the point of observation:

$$E(t,\mathbf{r})=\sqrt{2\pi^5}\,\frac{e^2E_0}{mc^2}\,e^{3\pi i/4}\int d\mathbf{v}_\perp\,v_\perp(\omega_{tr}-\omega_0)^{-1}J_1^2\left(\frac{k_\perp v_\perp}{\omega_H}\right)$$

$$\times v_z^*L(f_0)$$

$$\times\left[\int_{z_2}^{z}\frac{d^2k_z^{tr}}{d\omega^{*2}}d\zeta\,\frac{dk_{0z}}{dz_1}\left(\frac{dk_z^{tr}}{dz_2}+2\frac{k_z^{tr}}{v_z^*}\frac{dv_z^*}{dz_2}\right)\right]^{-1/2}$$

$$\times\exp\Big\{-i\omega_{tr}t+i\mathbf{k}_\perp\mathbf{r}_\perp+i\int_{s_0}^{z_1}k_{0z}d\zeta$$

$$+i\frac{\omega_{tr}}{v_z^*}(z_2-z_1)+i\int_{z_2}^{z}k_z^{tr}(\zeta)\,d\zeta+i(\omega_{tr}-\omega_0)\,t_0(z_1)\Big\}; \qquad (25)$$

$k_z^{\rm tr}(\zeta)=k_z(\varkappa,\zeta)$ provided $\omega_\varkappa=\omega_{\rm tr}$. This signal is received at the point of observation as triggered emission radiation with frequency that is modulated in time. The change in time of the coordinates of the modulation point z_1 of the distribution function and the point of formation z_2 of the response, and the change in the velocity v_z^* of the resonance particles and the triggered emission frequency $\omega_{\rm tr}$ are determined from the system of equations consisting of (24) plus Eqs. (11a), (17), and (22). If we restrict ourselves to Cherenkov resonance ($n=0$) and omit all small terms, these last equations can be rewritten in the form

$$k_{0z}(z_1,\omega_0)=\omega_0/v_z^*,\qquad k_z(z_2,\omega_{tr})=\omega_{tr}/v_z^*,$$

$$v_z^*=d\omega_{tr}/dk_z(z_2). \qquad (26)$$

The last of these is equivalent to $\omega_{\rm tr}=\frac{1}{2}\omega_H(z_2)$.

The system of Eqs. (24) and (26) is the system we are seeking; it characterizes the kinematic properties of the triggered emission, i.e., the properties that determine the change in the signal frequency with time. Omitting here a detailed analysis of these equations, we restrict ourselves to a qualitative treatment, from which the basic possibility of obtaining triggered emission with either monotonically increasing or s-shaped frequency characteristic follows. Substituting in Eqs. (26) the expansions valid in the neighborhood of the equatorial plane $z=z_0$, where $\partial k_z/\partial z_0=0$, $\partial^2k_z/\partial z_0^2<0$:

$$k_{0z}(\omega_0,z_1)=k_z(\omega,z_0)+\frac{\partial k_z}{\partial\omega}(\omega-\omega_0)+{}^1/_2\frac{\partial^2k_z}{\partial\omega^2}(\omega-\omega_0)^2$$

$$+{}^1/_2\frac{\partial^2k_z}{\partial z_0^2}(z_1-z_0)^2,$$

$$k_z(\omega,z_2)=k_z(\omega,z_0)+{}^1/_2\frac{\partial^2k_z}{\partial z_0^2}(z_2-z_0)^2,$$

we can write down the following equation relating the deviation from the equatorial plane of the point of modulation, $\delta z_1=z_1-z_0$, and the point of formation of the response, $\delta z_2=z_2-z_0$:

$$\delta z_2^2-\delta z_1^2=\frac{\partial^2k_z}{\partial\omega^2}(z_0,\omega)\left[\frac{\partial^2k_z}{\partial z_0^2}(z_0,\omega)\right]^{-1}(\omega-\omega_0)^2. \qquad (27)$$

If $\omega>\frac{1}{4}\omega_H(z_0)$,

$$\partial^2k_z/\partial\omega^2=-v_g^{-3}dv_g/dk_z>0.$$

Here, $v_g=d\omega/dk_z$ is the group velocity of the whistler waves. It then follows from (27) that $|dz_1|>|\delta z_2|$.

At the beginning of the triggered emission pulse ($t=t_0$), the triggered emission frequency is equal to the frequency of the triggering wave. The points z_1 and z_2 are found from the intersection of the field line with the line $\omega_0=\frac{1}{2}\omega_H(z)$, and one of the two following possibilities (see Fig. 3) can occur:

1. At $t=t_0$, the points z_1 and z_2 coincide. At $t>t_0$, the points z_1 and z_2 sink below the line $\omega_0=\frac{1}{2}\omega_H(z)$, as shown in Fig. 3a, and the frequency of the triggered emission increases monotonically with the time.

2. At $t=t_0$, the points z_1 and z_2 are on opposite sides of the equatorial plane. With increasing time, the points z_1 and

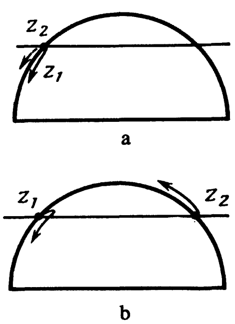

FIG. 3. Displacement along the field line of the resonance points z_1 and z_2 for monotonically (with the time) increasing (3a) and s-shaped (3b) frequency characteristics of the triggered emission. The horizontal line corresponds to $\frac{1}{2}\omega_H(z)=\omega_0$.

z_2 rise above the line $\omega_0 = \frac{1}{2}\omega_H(z)$ (see Fig. 3b) and the triggered emission frequency $\omega_{\rm tr}$ is less than ω_0. However, when the point z_2 passes through the equatorial plane the point z_1 begins to sink along the field line, as follows from Eq. (27). In this case a growth of the triggered emission frequency with time begins; at a certain time t_1, the two points z_1 and z_2 coincide, and $\omega_{\rm tr} = \omega_0$. It is readily seen that $t_1 > t_0$, since the group velocity of the whistler is minimal at $z = z_0$. Subsequently ($t > t_1$), the points z_1 and z_2 sink below the line $\omega_0 = \frac{1}{2}\omega_H(z)$, and the triggered emission frequency is higher than ω_0 (s-shaped frequency characteristic of the triggered emission). Both possibilities are observed experimentally.

We substitute in (25) the distribution function of the resonance electrons in the form of a two-temperature Maxwellian distribution,

$$f_0(v_\perp, v_z) = n_{res}\frac{m}{2\pi T_\perp}\left(\frac{m}{2\pi T_\parallel}\right)^{1/2}\exp\left(-\frac{mv_\perp^2}{2T_\perp} - \frac{mv_z^2}{2T_\parallel}\right),$$

integrate over $v_\perp$, and use the estimates

$$\frac{dk_z}{dz} \sim \frac{k\delta z}{L^2} \qquad \int\limits_{z_2}^{z} \frac{d^2k_z}{d\omega^2}\,d\zeta \sim \frac{k_z L}{\omega^2},$$

where L is the length of the field line, and $\delta z = z_2 - z_1$ is the width of the region in the neighborhood of the equatorial plane in which the response is formed. It follows from Eqs. (27) that $\delta z/L \approx |\omega_{\rm tr} - \omega_0|/\omega_0$. We then obtain an approximate expression for the amplitude of the triggered emission wave at the point of observation:

$$|E_{tr}| \approx E_0 \frac{\omega_p^2}{k^2c^2}\frac{n_{res}}{n_0}(k_0L)^{1/2}$$
$$\times \frac{\omega_0^2}{(\omega_{tr}-\omega_0)^2}\frac{k_\perp^2 T_\perp}{m\omega_H^2}(mv_z^{*2}/T_\parallel)^{1/2}\exp\left(-\frac{mv_z^{*2}}{2T_\parallel}\right). \quad (28)$$

It follows from (28) that for the real values of the parameters of a magnetospheric experiment,

$$n_{res}/n_0 \sim 10^{-4}, \ |\omega_{tr}-\omega_0|/\omega_0 \sim 10^{-1},$$
$$k_0L \sim 10^5, \ k_\perp^2 T_\perp/m\omega_H^2 \sim 1/3, \ v_z^* \sim (T_\parallel/m)^{1/2},$$

we have the estimate $E_{\rm tr} \approx E_0$. It is important that the triggered emission amplitude $E_{\rm tr}$ in (28) be proportional to E_0 and $(\omega_{\rm tr} - \omega_0)^{-2}$. The first of these dependences may be associated with the delay in the time of arrival of the triggered emission with the arrival of the triggering wave (delay by the time of amplification of the original wave due to instability in the radiation belt). It follows from the second dependence that for detectable triggered emission to occur its frequency must be close to the frequency of the triggering wave.

We thank O. A. Molchanov for discussing the results.

[1]R. A. Helliwell, J. Katzufrakis, M. Trimpi, and N. J. Brice, J. Geophys. Res. **69**, 2391 (1964).
[2]Ya. I. Likhter, O. A. Molchanov, and V. M. Chmyrev, Pis'ma Zh. Eksp. Teor. Fiz. **14**, 475 (1971) [JETP Lett. **14**, 325 (1971)].
[3]N. I. Bud'ko, V. I. Karpman, and O. A. Pokhotelov, Pis'ma Zh. Eksp. Teor. Fiz. **14**, 469 (1971) [JETP Lett. **14**, 320 (1971)].
[4]R. A. Helliwell, J. Geophys. Res. **72**, 4773 (1967).
[5]O. A. Molchanov, Doctoral Dissertation [in Russian], IZMIR AN SSSR (1981).
[6]R. N. Sudan and E. Ott, J. Geophys. Res. **76**, 4463 (1971).
[7]V. D. Shapiro and V. I. Shevchenko, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 673 (1976) [JETP Lett. **16**, 619 (1976)].
[8]H. L. Berk, C. W. Horton, D. E. Baldwin, M. N. Rosenbluth, and R. N. Sudan, Phys. Fluids **11**, 367 (1968).

Translated by Julian B. Barbour