Formation, dynamics, and explosion of a phonon hot spot

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The excitation and propagation of nonequilibrium phonons during interband optical absorption in a semiconductor are analyzed. The energy and length of the pump pulse at which a Planck distribution is established, and at which the deviation of the phonons from equilibrium can be described in terms of a nonequilibrium phonon temperature, are determined. The dynamic equations of this temperature are found in the regime of a nonlocal thermal conductivity; these are the laws which describe the time evolution of the characteristic temperature and the depth of the heating. When this depth becomes comparable to the size of the light spot on the surface of the crystal, the Planck distribution is disrupted in an explosive manner, and the nonlocal thermal conductivity converts into a quasidiffusion.

INTRODUCTION

The concept of a hot spot, introduced by Hensel and Dynes,[1] is now used widely[2–6] to refer to that situation in the region in which the phonon subsystem is being excited in which the excitation is strong, and the nonequilibrium phonons propagate in a nonballistic manner through the crystal. Although it is clear that the phonons in a hot spot have a relatively high frequency, so that scattering (by static defects and each other) are important for them, we do not have a clear picture of the state of the phonon system in the excitation region. It is this state which determines which propagation regime prevails instead of the ballistic regime. The point is that under strong excitation, in contrast to weak excitation, the propagation regime is determined not only by the relative sizes of the various phonon ranges and the propagation length but also by the excitation conditions: the size of the excitation region, the duration of the excitation, and the energy deposited. By "weak" excitation we mean that nonequilibrium phonons are generated with frequencies ω^* on the order of thermal-phonon frequencies, ω_{T_B}, and the energy density (ε^*) of the nonequilibrium phonons is small in comparison with that of the thermal phonons, ε_{T_B}. If at least one of these conditions is violated, i.e., if either $\omega^* \gtrsim \omega_{T_B}$ or $\varepsilon^* \gtrsim \varepsilon_{T_B}$, the excitation is "strong."

In the case of weak excitation, the occupation numbers of the nonequilibrium phonons, $n^*(\omega)$, are small in comparison with the equilibrium values, $n_{T_B}(\omega)$; i.e., there is almost no deformation of the equilibrium Planck distribution. In the case of strong excitation, there is a pronounced distortion of the equilibrium distribution, whether because of the appearance of a large number of nonequilibrium phonons in the ω_{T_B} region or because of the appearance of a comparatively small number of nonequilibrium phonons in the region $\omega^* \gtrsim \omega_{T_B}$, where there are exponentially few equilibrium phonons.

The hot spot is ordinarily understood as a heated region of the crystal, where the nonequilibrium phonons can be described by a phonon temperature $T > T_B$. We wish to emphasize in this connection that the temperature T can be established only as a result of phonon-phonon collisions; the scale time for the attainment of the temperature T is τ_T—the scale time of the anharmonic processes at $\omega = \omega_T$. Over this time, the phonons with ω_T are displaced a distance l_T in space (in a diffusive or ballistic manner). Accordingly, we clearly can speak in terms of a temperature T only when the minimum dimension of the heated region satisfies $L \gtrsim l_T$. As we will see below, at liquid-helium temperatures T_B is by no means always satisfied, so that the excitation of phonons will not always be equivalent to a heating. We will use the term "hot spot" only for the case in which a phonon temperature T is in fact established.

Our purpose in the present paper is to describe the state of the phonon system and its dynamics in the case of optical excitation, in which phonons are produced during the thermalization of hot electrons which are scattered high into the band (see Refs. 2, 5, 7, and 8, for example). A circumstance specific to this type of excitation is that the phonons which are produced are always high-frequency phonons: Their frequencies ω^* are on the order of the Debye frequency ω_D. At liquid-helium temperatures T_B, the excitation is always strong, since even at the weakest pumps used the occupation numbers of the nonequilibrium phonons are much higher than the equilibrium numbers at these frequencies.

1. STATEMENT OF THE PROBLEM AND BASIC ASSUMPTIONS

We assume that phonons with frequencies $\omega^* \gtrsim \omega_{T_B}$ are produced uniformly in a surface layer of depth d in a crystal of area A; the phonons have occupation numbers $n^* \ll 1$ and are distributed over the interval $\Delta\omega^* \sim \omega^*$.

The energy E which is deposited in the phonon system is deposited at a constant rate over the time θ. The area A is actually determined by the focusing of the light beam; the thickness d is the optical absorption depth plus the depth to which carriers diffuse as they cool, emitting optical or intervalley phonons The pump duration θ is the length of the light pulse plus the carrier cooling time. It is sometimes conven-

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ient to replace the energy E by $P = E/A$ or $W = P/\theta = E/A\theta$, which are respectively the energy and power per 1 cm^2 of surface area. Some typical values are $A = (30\ \mu\text{m})^2$ to (1 mm)^2, $d = 1$ to $10\,\mu$m, $\theta = 10$ ps to 100 ns, and $E < 10\,\mu$J. We ignore the phonons of the thermal background; i.e., we set $T_B = 0$.

The scattering of the phonons is determined by the scale time for elastic scattering by defects, $\tau'(\omega)$, and the lifetime with respect to spontaneous decay, $\tau(\omega)$. The crystal is assumed to be "defective"; i.e., it is assumed that for all pertinent frequencies the conditions $\tau(\omega) \gg \tau'(\omega)$ and $s\tau'(\omega) \ll L$ hold, where s is the average sound velocity, and L is the minimum dimension of the region occupied by nonequilibrium phonons. The first of these conditions means that a phonon is scattered elastically many times over its lifetime, and the second means that the motion of the phonon is a diffusive motion, with a diffusion coefficient $D(\omega) = s\tau'(\omega)/3$. The diffusion length over the lifetime is $l(\omega) = [D(\omega)\tau(\omega)]^{1/2}$.

The time τ_T and the length l_T mentioned in the Introduction are defined by $\tau_T = \tau(\omega_T)$, $l_T = l(\omega_T)$, where $\hbar\omega_T = \alpha T$, and the numerical factor a relates the temperature to the characteristic energy of a thermal phonon. If we identify the latter energy with the position of the maximum of the spectral energy density, we would have $\alpha \approx 2.82$ (even for order-of-magnitude estimates it is important to take the factor a into account because of the strong dependence of τ and τ' on ω).

At $\omega \ll \omega_D$ the scattering times are power-law functions of ω and can be written conveniently as

$$1/\tau'(\omega) = \eta\omega_D(\omega/\omega_D)^4, \tag{1}$$

$$1/\tau(\omega) = \gamma\omega_D(\omega/\omega_D)^5, \tag{2}$$

where the parameter η describes the defectiveness of the crystal, while the parameter γ describes the rate of anharmonic processes. The diffusion length is

$$l(\omega) = 3^{-1/2}(6\pi^2)^{-1/3}(\eta\gamma)^{-1/2}a_0'(\omega/\omega_D)^{-9/2}, \tag{3}$$

where $(a_0')^3$ is the volume per atom; the numerical coefficient is equal to 0.15.

We also introduce the energy density of the phonons which have occupation numbers $n \sim 1$, which have an average frequency ω, and which are distributed over the frequency interval $\Delta\omega \sim \omega$:

$$\varepsilon(\omega) = b\hbar\omega\rho(\omega)\omega, \quad b = \pi^4/15a^4 = 0.\ 10\ldots, \tag{4}$$

where $\rho(\omega)$ is the state density, and the numerical factor b is chosen to satisfy the following condition at $\omega_T \ll \omega_D$:

$$\varepsilon(\omega_T) = (\pi^2/10\hbar^3)(T^4/s^3) = \varepsilon_T = \varepsilon_0(T/T_D)^4. \tag{5}$$

Here ε_T is the energy density of the (three-mode) Planck distribution and $T_D = \hbar\omega_D$ is the Debye temperature.

We assume that the duration and size of the excitation region are "macroscopic" with respect to the phonons which are initially excited:

$$\theta \gg \tau(\omega^*), \quad d \gg l(\omega^*). \tag{6}$$

Since the size of the light spot, $A^{1/2}$, is usually greater than the thickness of the excitation layer, d, which is in turn greater than the initial diffusion length $l(\omega^*)$, the propagation of the phonons in a plane geometry. Later, when the depth (L) to which the phonons penetrate into the crystal becomes comparable to $A^{1/2}$, there is a transition to propagation in a spherical geometry.

2. FORMATION OF A HOT SPOT

In this section we determine the particular pump parameters for which a nonequilibrium phonon temperature (i.e., a Planck phonon distribution) is established. We will use the picture of phonon generations.[9,10]

As long as the occupation numbers are small, the evolution of the phonon distribution function can be described by the equation

$$[\partial/\partial t - D(\omega)\nabla^2]n(\omega, \mathbf{r}, t) = -n(\omega, \mathbf{r}, t)/\tau(\omega) + \hat{B}n(\omega, \mathbf{r}, t), \tag{7}$$

$$\hat{B}n(\omega) = \int_\omega^\infty d\omega'\rho(\omega')n(\omega')P(\omega' \to \omega).$$

The integral term $\hat{B}n$ describes the appearance of phonons ω during the decay of phonons of higher frequencies.[11] If, during the solution of Eq. (7), it turns out that the characteristic occupation numbers n increase over time and reach values $n \sim 1$, the meaning is that a phonon temperature is established.[10]

We define the characteristic frequencies ω_d and ω_θ by the conditions

$$d = l(\omega_d), \quad \theta = \tau(\omega_\theta). \tag{8}$$

It follows from (6) that $\omega_d, \omega_\theta \ll \omega^*$. We also introduce $\theta_0 = \tau(\omega_d)$, the characteristic pulse length. Let us examine the meaning of these quantities. The frequency ω_d delimits the generations with $\omega \gg \omega_d$, which do not escape from layer d, and the generations with $\omega \ll \omega_d$, which do. Phonons begin to escape from layer d at times $t \sim \theta_0$. The frequency ω_θ delimits generations with $\omega \gg \omega_\theta$, which live during the pumping conditions, from generations with $\omega \ll \omega_\theta$, which live after the pumping has ended. Here $\omega_d/\omega_\theta = (\theta/\theta_0)^{1/5}$. If $\theta \ll \theta_0$, the pumping is brief; i.e., it ends before the phonons begin to escape from the excitation layer. If $\theta \gg \theta_0$, the pumping is instead prolonged; i.e., the phonons begin to escape from the layer before the pumping ends.

We first consider the case of prolonged pumping ($\theta \gg \theta_0$). At times $t \ll \theta_0$, at which mostly phonons with $\omega \gg \omega_d$ exist, it can be assumed that there are no phonons outside layer d, while inside this layer the distribution of n does not depend on the coordinates. Equation (7) is then satisfied by the self-similar solution

$$n(\omega, t) = C\omega^\sigma f_\sigma(\eta), \quad \eta = t/\tau(\omega) \tag{9}$$

with any σ. The function f_σ satisfies the equation

$$\frac{\partial}{\partial\eta}f_\sigma(\eta) = -f_\sigma(\eta) + \int_0^1 \frac{dx}{x}x^{-5-\sigma}h(x)f_\sigma\left(\frac{\eta}{x^5}\right),$$

where

$$h(x)=P(1\to x)\left[\int_0^1 dx\, x^3 P(1\to x)\right]^{-1}.$$

The energy density of the phonons described by distribution (9) at the time t is

$$\varepsilon(t)=\int_0^\infty d\omega\rho(\omega)\hbar\omega n(\omega,t)\propto t^{-(\sigma+4)/5}. \tag{10}$$

Since the total energy of the phonons increases in proportion to t at these times $t<\theta$, we should choose $\sigma=-9$. It is convenient to normalize the function f_σ by the condition

$$\frac{1}{5}\int_0^\infty \frac{d\eta}{\eta}\eta^{(\sigma+4)/5}f_\sigma(\eta)=1.$$

Distribution (9) can then be written in the form

$$n(\omega,\ t)=[W\tau(\omega)/d\varepsilon(\omega)]bf_{-9}(\eta). \tag{11}$$

The function f_σ contains no large or small parameters, so that characteristic values are $\eta\sim 1$, and at such values of η we have $f_\sigma\sim 1$. The time dependence of the average frequency is then determined by the condition $\eta\sim 1$, i.e.,

$$\tau(\omega)\sim t, \tag{12}$$

and the occupation numbers for the average frequencies are

$$n\sim W\tau(\omega)/d\varepsilon(\omega). \tag{13}$$

If we express ω in terms of t on the basis of (12) and substitute the result into (13), we find the time dependence of the characteristic occupation numbers to be

$$n\sim(W/W_0)\,(t/\theta_0)^{9/5}, \tag{14}$$

where

$$W_0=P_0/\theta_0,\quad P_0\equiv d\varepsilon(\omega_d). \tag{15}$$

If $W\gg W_0$, the decay will come to a halt at some critical generation at the time $t\ll\theta_0$, at which the occupation numbers reach values $n\sim 1$, and a temperature $\overline{T}$ corresponding to the frequency of the critical generation, $\omega_{\overline{T}}$, will be established in the layer d. This frequency is determined from (13) by setting $n=1$; this procedure gives us an equation for $\overline{T}$:

$$d\varepsilon(\omega_{\overline{T}})=W\tau(\omega_{\overline{T}}). \tag{16}$$

It can be seen from (12), (13), and (16) that the temperature is established over a time $\bar{t}\sim\tau(\omega_{\overline{T}})$.

If $W\ll W_0$, the occupation numbers are small up to $t\sim\theta_0$, and the decay continued even after the phonons have escaped from layer d. At times $t\gtrsim\theta_0$ we must take a diffusion term into account in (7). As long as the depth to which the phonons penetrate into the crystal remains smaller than $A^{1/2}$, the propagation geometry is a plane geometry, and (7) is satisfied by the selfsimilar solution

$$n(\omega,\ z,\ t)=C\omega^\sigma f_\sigma(\xi,\ \eta),\quad \xi=z/l(\omega),\quad \eta=t/\tau(\omega), \tag{17}$$

where the function f_σ satisfies the equation

$$\left[\frac{\partial}{\partial\eta}-\frac{\partial^2}{\partial\xi^2}\right]f_\sigma(\xi,\eta)=-f_\sigma(\xi,\eta)+\int_0^1\frac{dx}{x}x^{-5-\sigma}h(x)f_\sigma\left(\frac{\xi}{x^{9/2}},\frac{\eta}{x^5}\right), \tag{18}$$

which contains no large or small parameters.

The energy of the phonons per 1 cm^2 of surface for distribution (17) is

$$\int_0^\infty dz\int_0^\infty d\omega\rho(\omega)\hbar\omega n(\omega,z,t)\propto t^{-(\sigma-1/2)/5}. \tag{19}$$

At times $t<\theta$ this energy increases in proportion to t, so that we need to choose $\sigma=-9/2$. Normalizing the function f_σ by the condition

$$\frac{1}{5}\int_0^\infty d\xi\int_0^\infty\frac{d\eta}{\eta}\eta^{(\sigma-1/2)/5}f_\sigma(\xi,\eta)=1, \tag{20}$$

we can write distribution (17) as

$$n(\omega,\ z,\ t)=[W\tau(\omega)/l(\omega)\varepsilon(\omega)]bf_{-9/2}(\xi,\ \eta). \tag{21}$$

Proceeding as in (14), we find the characteristic occupation numbers

$$n\sim(W/W_0)\,(t/\theta_0)^{9/10}\sim(P/P_0)\,(\theta/\theta_0)^{-1/10}(t/\theta)^{9/10}. \tag{22}$$

At $W\ll W_0$, the occupation numbers increase even after the phonons have escaped from layer d, as long as the pump lasts, although this increase is slower than before the phonons escape from layer d.

It can be seen from (22) that if

$$P/P_0\gg(\theta/\theta_0)^{1/10} \tag{23}$$

then the temperature $\overline{T}$ determined by the equation

$$l(\omega_{\overline{T}})\varepsilon(\omega_{\overline{T}})=W\tau(\omega_{\overline{T}}) \tag{24}$$

will be established even before the end of the pumping. This temperature is established by the time $\bar{t}\sim\tau(\omega_{\overline{T}})$ in a layer of thickness $\bar{z}\sim l(\omega_{\overline{T}})$.

When the inequality opposite (23) holds, the decay continues even after the pump pulse. At $t>\theta$, the energy in (19) must be constant, so that we have $\sigma=1/2$. Using normalization (20), we find the distribution

$$n(\omega,\ z,\ t)=[P/l(\omega)\varepsilon(\omega)]bf_{1/2}(\xi,\ \eta), \tag{25}$$

which is established at $t\gg\theta$. We now have

$$n\sim(P/P_0)\,(t/\theta_0)^{-1/10}, \tag{26}$$

so that the occupation numbers decrease. In other words, if the pumping is prolonged ($\theta\gg\theta_0$), the temperature can be established only before the end of the pumping.

The analysis of brief pumping ($\theta\ll\theta_0$) is analogous. At $t<\theta$, expression (11) holds, but (14) is conveniently rewritten as

$$n\sim(P/P_0)\,(\theta/\theta_0)^{4/5}(t/\theta)^{9/5}.$$

We thus see that if

$$P/P_0\gg(\theta/\theta_0)^{-4/5}$$

the temperature determined by Eq. (16) will be established in layer d. If, instead, the opposite inequality holds, the decay will continue even after the pulse.

At $t>\theta$, the energy in (10) remains constant, so that we must choose $\sigma=-4$ in solution (9). This value gives us the

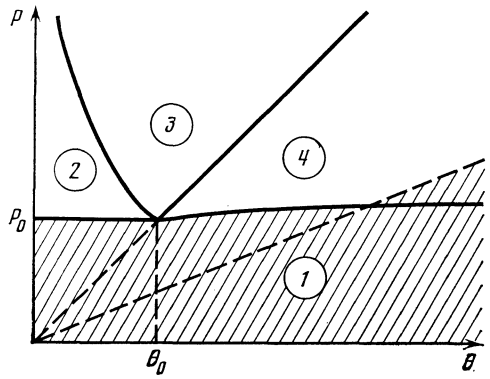

FIG. 1. Conditions for the formation of a hot spot. Heavy solid lines—Boundaries between regions described by (30); heavy dashed lines— $P/P_0 = (d/A^{1/2})(\theta/\theta_0)$; unhatched region—the temperature reaches a steady state.

distribution found in Ref. 11:

$$n(\omega, t) = [P/d\varepsilon(\omega)]\, b f_{-4}(\eta), \tag{27}$$

which is established at $t \gg \theta$. For this distribution we have

$$n \sim (P/P_0)\,(t/\theta_0)^{4/5}. \tag{28}$$

As before the end of the pumping, the occupation numbers increase, but more slowly. We see from (28) that under the condition $P \gg P_0$ the temperature $\bar{T}$ determined by the equation

$$d\varepsilon(\omega_{\bar{T}}) = P \tag{29}$$

is established in layer d. If the condition $P \ll P_0$ holds instead, the decay will continue even after escape from the layer, when the distribution takes the form in (25). The occupation numbers now decrease, according to (26), and the temperature does not reach a steady state.

Although the analysis above is slightly tedious in terms of the abundance of different cases, it is extremely instructive, since it shows how different the situations in the excitation region can be, depending on the parameters of the pump. This analysis provides an exhaustive answer to the question of which pump parameters lead to the formation of a hot spot, and it tells us the thickness and temperature of the spot. To put the answer to these questions in their most graphic form, we specify the pump to be a point in the (P, θ) plane. We partition this plane as shown in Fig. 1. The equations of the boundaries between the different regions are

$$\begin{aligned} &1/2: \quad P/P_0 = 1, \\ &2/3: \quad P/P_0 = (\theta/\theta_0)^{-4/5}, \\ &3/4: \quad P/P_0 = \theta/\theta_0, \\ &4/1: \quad P/P_0 = (\theta/\theta_0)^{1/10}. \end{aligned} \tag{30}$$

In region 1 (weak pumping) a hot spot does not form. In region 3 (high P) the temperature $\bar{T}$ determined by Eq. (16) is established in layer d even before the end of the pump pulse and before a significant number of phonons has escaped from this layer. We thus have

$$\bar{T} = T_0 (W/W_0)^{1/9}, \quad aT_0 \equiv \hbar\omega_d. \tag{31}$$

The time required for the attainment of this temperature is

$$\bar{t} = \theta_0 (W/W_0)^{-5/9}. \tag{32}$$

In region 4 (intermediate values of P; prolonged pumping) the temperature reaches its steady-state value before the end of the pump pulse, but after phonons have escaped from layer d. The initial temperature is found from (24):

$$\bar{T} = T_0 (W/W_0)^{2/9}. \tag{33}$$

The temperature relaxation time is

$$\bar{t} = \tau(\omega_{\bar{T}}) = \theta_0 (W/W_0)^{-10/9};$$

the thickness of the layer occupied by the phonons by the time at which the temperature reaches its steady-state value is

$$\bar{z} = l(\omega_{\bar{T}}) = d(W/W_0)^{-1}. \tag{34}$$

In region 2 (intermediate values of P; brief pumping), the temperature reaches its steady-state value after the pump pulse but before the phonons have escaped from layer d. The initial temperature is found from (29) to be

$$\bar{T} = T_0 (P/P_0)^{1/4}; \tag{35}$$

the temperature relaxation time is

$$\bar{t} = \tau(\omega_{\bar{T}}) = \theta_0 (P/P_0)^{-5/4}. \tag{36}$$

We assumed above that the hot spot forms in a plane geometry. This is clearly a valid assumption in regions 2 and 3, where the spot is formed inside layer d. Only for regions 1 and 4 need we consider the transition to a spherical geometry.

In the case of a spherical geometry the propagation of Eq. (7) also satisfies a self-similar solution of the form in (17), but now we have $\xi = r/l(\omega)$, and f_σ satisfies Eq. (18) in which the two-dimensional Laplacian is replaced by a spherical Laplacian. In a spherical geometry the total energy of the phonons of self-similar distribution (17) is

$$4\pi \int_0^\infty dr r^2 \varepsilon(r, t) \propto t^{-(\sigma - 19/2)/5}. \tag{37}$$

We choose the normalization

$$\frac{1}{5} \int_0^\infty d\xi\, \xi^2 \int_0^\infty \frac{d\eta}{\eta}\, \eta^{(\sigma - 19/2)/5} f_\sigma(\xi, \eta) = 1$$

for the spherical geometry. Before the end of the pump pulse we have, choosing $\sigma = 9/2$,

$$n(\omega, r, t) = [WA\tau(\omega)/l^3(\omega)\varepsilon(\omega)]\,(b/4\pi) f_{9/2}(\xi, \eta). \tag{38}$$

After the pump pulse, at $t \gg \theta$, the solution found in Ref. 11, with the constant energy (37), i.e., with $\sigma = 19/2$, is established:

$$n(\omega, r, t) = [E/l^3(\omega)\varepsilon(\omega)]\,(b/4\pi) f_{19/2}(\xi, \eta). \tag{39}$$

Using (12), we find $n \propto t^{-9/10}$ and $n \propto t^{-19/10}$ for (38) and (39), respectively; i.e., for propagation in spherical geometry the occupation numbers always decrease. This result means that if a hot spot does not manage to form in region 1 in the plane geometry it will never form.

We should thus examine the transition to spherical geometry only in region 4, where the temperature reaches a steady state at the depth in (34). The temperature does in fact reach a steady state if $\bar{z} \ll A^{1/2}$, i.e., if

$$W/W_0 \gg d/A^{1/2}. \tag{40}$$

This condition is usually satisfied in region 4. If it is not, however (as it may not be if the pump pulse is a long, sharply focused pulse), then the occupation numbers will still be small by the time of the transition to the spherical geometry, and subsequently they can only decrease. Accordingly, a hot spot does not form if condition (40) is violated.

3. DYNAMICS OF A NONEQUILIBRIUM TEMPERATURE IN PLANE GEOMETRY

Before the phonon temperature reaches a steady state, the dynamics of the deviation of the phonons from equilibrium is described by one of the self-similar solutions (11), (21), (25), (27) in plane geometry or (38) and (39) in spherical geometry. If the excitation parameters are such that a nonequilibrium temperature $\bar{T}$ is established at sometime $\bar{t}$ in the layer $\bar{z}$, the subsequent development of the deviation of the phonons from equilibrium will be described by a heat-conduction equation. This equation is nonlinear by virtue of the relation $\bar{T} \gg T_B$. Furthermore, as was shown in Ref. 12, the thermal conductivity is a nonlocal property in the case of Rayleigh scattering of phonons.

There are several regimes of a nonlocal thermal conductivity in which energy is transported by phonons of various frequencies and polarizations.[13] We will restrict the present discussion to the simplest regime, corresponding to the most defective crystals, in which energy is transported by subthermal TA phonons. This regime prevails in the region

$$l_T \ll L \ll \delta_T^{5/6} l_T. \tag{41}$$

Here T is a characteristic temperature. L is the minimum size of the heated region, and $\delta_T = \tau_T/\tau_T'$.

The state of the hot spot can be described in a semiquantitative way by a point in the (T, L) plane (Fig. 2), and its dynamics can be described by motion along trajectories which begin at the point $(\bar{T}, \bar{z})$ at the time $\bar{t}$. The line $L = A^{1/2}$ in this plane separates regions in which the hot spot expands in plane geometry ($L = z \ll A^{1/2}$) and in spherical geometry ($L = r \gg A^{1/2}$).

We begin by finding the trajectories of the system in the case $z \gg d$, in which a hot spot has been "torn away" from the excitation region. For this purpose we use the self-similar solutions of the nonlinear equations for a nonlocal thermal conductivity[14]:

$$\left[\frac{\partial}{\partial t} - D(\omega)\nabla^2\right] n = -\frac{1}{\tau(T,\omega)}\left[n - \frac{T}{\hbar\omega}\right]$$
$$\partial\varepsilon/\partial t + \operatorname{div}\mathbf{w} = 0, \quad \mathbf{w} = \int_0^\infty d\omega\, \rho(\omega)\hbar\omega[-D(\omega)\nabla n]. \tag{42}$$

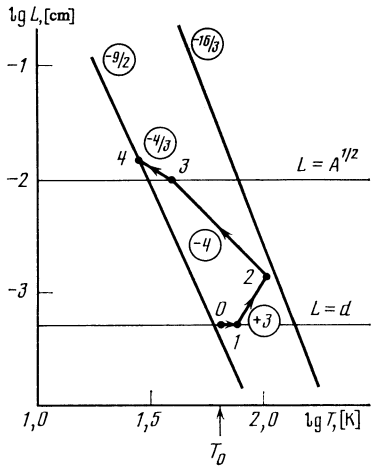

FIG. 2. Dynamics of a hot spot in Ge. Region (41) lies between the lines with slopes of $-9/2$ and $-16/3$; a temperature does not exist to the left of this region. Line with slope of $+3$—W-trajectory (53); lines with slopes of -4 and $-4/3$—P-trajectories (52) and (63); horizontal region between points 0 and 1—trajectory (56).

Here $n(\omega, \mathbf{r}, t)$ is the distribution function of the low-frequency phonons which are carrying the heat flux $w(\mathbf{r}, t)$, and $\hat{\tau}$ is the scale time for absorption of a low-frequency phonon ω by a Planck distribution with a temperature T. According to Ref. 12, the flux $\mathbf{w}$ is established rapidly in comparison with the duration of the overall process, so that we can discard the time derivative $\partial/\partial t$ from the left side of the first of Eqs. (42).

To construct self-similar solutions of system (42) in plane geometry, we choose some temperature T^+, which we leave arbitrary for the moment, and we determine the dimensionless variables

$$\omega' = \hbar\omega/T^+, \quad z' = z/l_{T^+}, \quad t' = t/\tau_{T^+}.$$

We seek self-similar solutions in the form

$$n(\omega, z, t) = (t')^{-\lambda} g(\zeta, \eta), \quad T(z,t) = T^+ (t')^{-\mu} f(\zeta) \tag{43}$$

with the self-similar variables

$$\eta = \omega'(t')^{\alpha}, \quad \zeta = z'(t')^{-\beta}. \tag{44}$$

A solution in the form in (43) "passes through" Eqs. (42) if

$$\alpha + \lambda - \mu = 0, \quad 5\alpha - 2\beta + 4\mu = 0, \quad 4\mu + 1 - \lambda - 2\beta = 0. \tag{45}$$

For the functions f and g in this case we find a system of equations which can be written as follows for plane geometry:

$$\partial^2 g/\partial\zeta^2 = \eta^5 f^4 g - \eta^5 f^5, \tag{46a}$$

$$\mu f^4 + \beta f^3 \zeta \frac{\partial f}{\partial \zeta} = \frac{15}{4\pi^4}\int_0^\infty \frac{d\eta}{\eta}\frac{d^2 g}{d\zeta^2}. \tag{46b}$$

System (46) is invariant under a scale transformation of the functions and the variables,

$$g \to C_g g, \quad f \to C_f f, \quad \zeta \to C_\zeta \zeta, \quad \eta \to C_\eta \eta,$$

If the scaling factors satisfy the three relations

$$C_g/C_\zeta^2 = C_\eta^5 C_f^4 C_g = C_\eta^4 C_f^5, \quad C_f^4 = C_g/C_\zeta^2.$$

The first two of these relations follow from (46a), and the third from (46b). The one scaling factor which remains free can be determined by the normalization condition

$$\int_0^\infty d\zeta f^4(\zeta)=1. \tag{47}$$

The energy of the phonons per 1 cm^2 of surface area at the time t is therefore

$$\int_0^\infty dz\varepsilon_T=\varepsilon_{T^+}l_{T^+}(t/\tau_{T^+})^{\beta-4\mu}. \tag{48}$$

If we are interested in the solution before the end of the pump pulse, at $t<\theta$, the integral in (48) must be equal to Wt. This result means that we must assume $\beta-4\mu=1$ and determine the temperature T^+ from the condition $\varepsilon_{T^+}l_{T^+}=W\tau_{T^+}$. In this case we find the following results for the exponents, using (45)

$$\alpha=^2/_7,\quad \beta=^3/_7,\quad \lambda=-^3/_7,\quad \mu=-^1/_7.$$

Since we are interested in the solution after the end of the pump pulse, which is established at $t\gg\theta$, we should set the integral in (48) equal to P. Doing so, we find $\beta-4\mu=0$. We then find the exponents[14]

$$\alpha=^4/_{21},\quad \beta=^{20}/_{21},\quad \lambda=^1/_{21},\quad \mu=^5/_{21},$$

and the condition for determining T^+ is

$$\varepsilon_{T^+}l_{T^+}=P.$$

In both cases, the functions f and g satisfy system of equations (46) and normalization condition (47), which contains no physical parameters (large or small). The characteristic values of the arguments are therefore $\zeta\sim1$ and $\eta\sim1$, and for such arguments we have $g\sim1$ and $f\sim1$. It is thus clear that the motion of the heating front is described by the law

$$z\sim l_{T^+}(t/\tau_{T^+})^\beta, \tag{49}$$

and the characteristic temperature behind the front is

$$T\sim T^+(t/\tau_{T^+})^{-\mu}. \tag{50}$$

The last two expressions describe the motion of the image point in the (T, L) plane along the trajectory

$$(z/l_{T^+})=(T/T^+)^{-\beta/\mu}. \tag{51}$$

Before the end of the pump pulse, at $t<\theta$, the system moves along a trajectory on which the energy flux (W) across 1 cm^2 of the surface of the crystal is constant (this is a "W-trajectory"). At the end of the pump pulse, at $t=\theta$, the system goes onto a trajectory on which the energy (P) per 1 cm^2 of surface area of the crystal is constant (a "P-trajectory"). The system moves along this trajectory at $t>\theta$.

For a W-trajectory we have $-\beta/\mu=3>0$, so that the temperature increases over time with increasing z, and the image point moves upward and to the right along a trajectory $z\propto T^3$. For a P-trajectory we have $-\beta/\mu=-4<0$, so that the temperature decreases, and the point moves along the trajectory $z\propto T^{-4}$ upward and to the left. It is thus clear that the temperature reaches its maximum by the end of the pump pulse.

It is convenient to rewrite the equation of the trajectory in such a manner that it does not contain T^+. For a P-trajectory we then find

$$P=z\varepsilon_T, \tag{52}$$

and for a W-trajectory we find

$$W=(\varepsilon_T l_T/\tau_T)(z/l_T)^{-9/5}\equiv w_{T,z}, \tag{53}$$

where $w_{T,z}$ is the heat flux which arises[12,15] because of ∇T in a spot with temperature T and size z. In terms of dimensionless quantities, Eq. (52) is

$$P/P_0=(z/d)(T/T_0)^4, \tag{54}$$

and Eq. (53) is

$$W/W_0=(z/d)^{-3/5}(T/T_0)^{9/5}. \tag{55}$$

Let us examine the trajectory of the image point under conditions such that the hot spot still coincides with the excitation region, i.e., under conditions such that the heat flux $w_{T,z}$ due to ∇T is small in comparison with the pumping W. The image point obviously moves along the trajectory

$$z=d, \tag{56}$$

on which the temperature increases in accordance with

$$d\varepsilon_T/dt=W/d. \tag{57}$$

The final point of trajectory (56) is determined by the condition

$$w_{T,z}|_{z=d}=W \tag{58}$$

if this point is reached before the end of the pump pulse; alternatively, it is determined by the condition

$$\varepsilon_T=W\theta/d=P/d, \tag{59}$$

which follows from (57), if the pumping ends earlier.

We see from (49) and (50) that the rate at which the system moves along the trajectory decreases as time elapses. Consequently, the time required to reach a given point on the trajectory is determined by the last part of the trajectory—in order of magnitude, simply by the final point. This time can be stimated from the continuity equation for energy in system (42):

$$t\sim z(\varepsilon_T/w_{T,z}). \tag{60}$$

Substituting in $w_{T,z}$ from (53), we find

$$t\sim\tau_T(z/l_T)^{9/5}=\theta_0(z/d)^{9/5}(T/T_0)^{11/5}. \tag{61}$$

The same result can be found by eliminating T^+ from (49) and (50). In the plane-geometry region the time required to reach a given point in the (T, L) plane is thus independent of the trajectory. This time, (61), is, according to Ref. 12, the "thickness-doubling time" for a heated layer of thickness z and temperature T.

4. SPHERICAL GEOMETRY. DESTRUCTION OF THE HOT SPOT

Sooner or later, the trajectories traced out in the plane geometry and studied in Section 3 come to either a boundary

of the region in which the hot spot exists, $L = l_T$, or a boundary of the transition to the spherical geometry, $L = A^{1/2}$. In the first case, the hot spot is destroyed, and the subsequent propagation of the phonons occurs in a quasidiffusion regime. In the second case the hot spot survives but expands in spherical geometry along a W-trajectory,

$$WA \sim r^2 w_{T,r}, \tag{62}$$

or along a P-trajectory,

$$r^3 \varepsilon_T \sim E. \tag{63}$$

We wish to find self-similar solutions corresponding to these trajectories. An attempt to find them in the form in (43), with the variables (44), in spherical geometry leads to $\beta < 0$; this result corresponds to a decrease in the radius of the hot spot with increasing time, which is physically meaningless. We must accordingly choose the self-similar variables in the form

$$\eta = \omega'(t_0' - t)^{\alpha}, \quad \zeta = r'(t_0' - t)^{-\beta}. \tag{64}$$

Here t_0 is a parameter which is undetermined at the moment, and $r' = r/l_{T^+}$. We seek a solution in the form

$$n(\omega, r, t) = (t_0' - t')^{-\lambda} g(\zeta, \eta), \quad T(r, t) = T^+ (t_0' - t')^{-\mu} f(\zeta). \tag{65}$$

Substituting this solution into Eq. (42), we find the same conditions—conditions (45)—on the exponents as for the plane geometry. We define the normalization of the functions f by the condition

$$4\pi \int_0^{\infty} d\zeta \zeta^2 f^4(\zeta) = 1.$$

The total energy of the phonons at time t is then

$$4\pi \int_0^{\infty} dr r^2 \varepsilon_T = \varepsilon_{T^+} l_{T^+}^3 [(t_0 - t)/\tau_{T^+}]^{3\beta - 4\mu},$$

$$t_0 = t_0' \tau_{T^+}.$$

Restricting the discussion to the case $t > \theta$, we should choose $3\beta - 4\mu = 0$; doing so, we find

$$\alpha = 4, \quad \beta = -20, \quad \lambda = -19, \quad \mu = -15,$$

and we can find T^+ from the condition

$$\varepsilon_{T^+} l_{T^+}^3 = E. \tag{66}$$

The characteristic radii and temperatures of the spot corresponding to solution (65) are, in terms of dimensional quantities,

$$r \sim l_{T^+} \left(\frac{t_0 - t}{\tau_{T^+}} \right)^{-20}, \quad T \sim T^+ \left(\frac{t_0 - t}{\tau_{T^+}} \right)^{15}. \tag{67}$$

In the limit $t \to t_0$ the spot radius goes to infinity ($r \to \infty$), while its temperature vanishes ($T \to 0$). This result means that in spherical geometry the expansion of the spot is "explosive": The energy "goes off to infinity" in a finite time t_0.

Although the transition from plane geometry to spherical geometry actually occurs gradually, we will assume for simplicity that this transition occurs abruptly, i.e., that at the time of the transition the hot spot already has a spherical shape with a radius $\hat{r} \sim A^{1/2}$ and a temperature $\hat{T}$, determined from the condition

$$\varepsilon_{\hat{T}} = E/A^{3/2}. \tag{68}$$

We place the time of the transition at $t = 0$ in (65). It is obvious that solution (65) cannot exactly satisfy arbitrary initial conditions, but there is the possibility that we might be able to satisfy these conditions, at least in order of magnitude, by requiring that we find values on the order of $\hat{r}$ and $\hat{T}$ from (67) at $t = 0$. This procedure yields

$$t_0 \sim \tau_{T^+} (\hat{T}/T^+)^{1/15}.$$

Using (66) and (68), we easily find

$$(\hat{T}/T^+) = (l_{\hat{T}}/\hat{r})^{-6/19}. \tag{69}$$

Since the initial conditions lie in the region in which a hot spot exists, i.e., since $\hat{r} \gg l_{\hat{T}}$, it follows from (69) that we have $\hat{T} \gg T^+$ and $t_0 \gg \tau_{T^+}$. Using (69), we can rewrite t_0 as

$$t_0 \sim \tau_{\hat{T}} (\hat{r}/l_{\hat{T}})^{8/5}. \tag{70}$$

According to (61), t_0 is on the order of magnitude of the time at which the system arrives at the point of the transition to the spherical geometry, and it is simultaneously the time required for a doubling of the initial radius of the hot spot, $\hat{r}$. Since the initial conditions can be satisfied only in order of magnitude, solution (65) has an exact meaning only at $r \gg \hat{r}$, at which a spot "is torn away" from the initial conditions. It is easy to verify that we would then have $t_0 - t \ll t_0$. The explosive self-similar solution thus has an exact meaning only in a small time interval near t_0, and a non-self-similar state occupies nearly the entire interval of the time t. The time of the explosion, t_0, is determined by the initial conditions; this result is an important distinction between the spherical geometry and the plane geometry. In the plane geometry the initial dimensions of the spot "are forgotten," and the duration of the process is determined by its final stage. In the spherical geometry, in contrast, the duration of the process is determined specifically by the initial dimensions.

Furthermore, it should be kept in mind that the equations for nonlocal thermal conductivity, (42), are meaningful only if the phonons which carry the heat flux are subthermal: $\hbar\omega \ll T$. Since $\alpha > 0$, we have $\omega \to \infty$ as $t \to t_0$. Solution (65) therefore loses its meaning near t_0. According to this solution, we sould have $\hbar\omega \sim T$ at $(t_0 - t) \sim \tau_{T^+}$. Accordingly, solution (65) finally has an exact meaning at

$$\tau_{T^+} \ll t_0 - t \ll t_0. \tag{71}$$

At the time $t = t_0 - \tau_{T^+}$, at which the self-similar solution becomes meaningless, the spot temperature is on the order of T^+, and its radius is on the order of l_{T^+}; i.e., the system is at the boundary of the region in which a hot spot exists. This is an exceedingly important circumstance: It shows that ordinary nonlinear local thermal conductivity does not occur in the case of Rayleigh scattering in a crystal with a sufficient defect density.

Because of the large exponents in (67), region (71) is exceedingly narrow, and solution (65) has no practical importance. However, a study of this solution allows us to justify the following important conclusion: After the thickness

TABLE I.

	a_0', Å	s, 10^5 cm/s	β	T_D, K	$\omega_D/2\pi$, THz	ε_0, 10^4 J/cm^3	γ, 10^{-4}	η, 10^{-4}
Si	2,7	5,8	0,6	640	13,4	2,5	0,37	9,3
Ge	2,8	3,5	0,6	370	7,8	1,3	0,36	27
GaAs	2,8	3,3	0,6	350	7,2	1,3	0,46	4,5

of the hot spot has become equal to its radius, over a time on the order of that in (70), the spot will be destroyed over another time interval of the same order of magnitude.

5. DISCUSSION OF RESULTS

Near all the experiments of this type have been carried out in Ge, Si, and GaAs. Table I lists the properties of these materials. The property η refers to isotopic scattering; γ is found from the scale time for the decay of an LA phonon calculated in Ref. 16 by taking an average over branches: $\gamma = \gamma_{LA}(1 + 2\beta^{-3})^{-1}$, where $\beta = s_{TA}/s_{LA}$.

The most legitimate choice of initial phonons would be phonons found after the first decay of optical or intervalley phonons. We assume that their frequency is $\omega^* = \omega_D/2$. For such phonons in Ge we have $\tau(\omega^*) = 20$ ns and $l(\omega^*) = 3\,\mu$m, so that conditions (6) on d and θ hold, although not very well. Let us estimate the initial occupation numbers n^*. Condition (6) on θ means that the phonons ω^* are produced during steady-state pumping, so that their number per 1 cm^3 is $W\tau(\omega^*)/d\hbar\omega^*$. To find n^*, we need to divide this number by the phase volume occupied by the phonons, $bp(\omega^*)\omega^*$, so that we find $n^* \sim W\tau(\omega^*)/d\varepsilon(\omega^*)$. Taking $d = 5\,\mu$m, we then find $n^* \sim 10^{-1}$ even for a very high pump level, $W = 10^6$ W/cm^2. Nevertheless, at the lowest pump level, $W = 1$ W/cm^2, we have $n^* \sim 10^{-7}$; at $T_B = 4$ K, the equilibrium values are $n_{T_B}(\omega^*) \sim 10^{-21}$ and thus much smaller than n^*.

Choosing $d = 5\,\mu$m, we find the characteristic parameter values listed in Table II. These values determine the formation of the propagation regime. It can be seen from (8) and (3) that we have $\omega_d \propto d^{-2/9}$, and from (15) and (5) we find $P_0 \propto d^{1/9}$; i.e., this power depends very weakly on d. On the other hand, the boundary between regions 1 and 4 in Fig. 1 is essentially horizontal, so that the condition for the establishment of a nonequilibrium temperature is $P \gtrsim 100\ \mu$J/mm^2, regardless of the pump duration, in pure semiconductors. With $\theta = 100$ ns, this value would correspond to $W \gtrsim 1$ kW/mm^2. These are quite high pump levels, which have by no means been attained in all experiments.

At lower pump levels, at which the excitation parameters fall in region 1 in Fig. 1, the phonons propagate by a quasidiffusion. As can be seen from (21), (25), (38), and (39), the most characteristic feature of this propagation regime is that the shape of the signal is independent of the energy deposition E. According to the experiments of Refs. 7 and 17, this is indeed the situation[1)] in GaAs with $W = 0.02$–2 W/mm^2 and $\theta = 50$ ns, corresponding to $P = 0.001$–$0.1\ \mu$J/mm^2. The shape of the signal begins to depend on E at $P \sim 1\ \mu$J/mm^2. This value is two orders of magnitude lower than the estimates of P_0, but how strong the dependence was at this value of P was not stated in Refs. 7 and 17. It must also be kept in mind that our estimate of P_0 may prove to be extremely crude, since all of the equations contain unknown numerical factors. An independence of the rate of arrival of the phonon signal in GaAs has also been observed in a study of phonons from a Joule-heated heterostructure.[23]

As an example we consider the formation and dynamics of a hot spot in Ge pumped with $E = 1\ \mu$J, $\theta = 75$ ns, $A = (0.1\text{ mm})^2$, and $d = 5\ \mu$m. The experiments of Ref. 5 on Ge used approximately this pumping. Since $P = 150\ \mu$J/mm^2 and $W = 2$ kW/mm^2, the pump parameters fall in region 3 in Fig. 1. It follows from (31) and (32) that a temperature $\overline{T} = 62$ K is established in $\bar{t} = 25$ ns in layer d. The hot spot then moves along trajectory (56) (Fig. 2). Condition (58) yields a temperature $T_1 = 73$ K, which is reached at the time $t_1 = 46$ ns, according to (57). Condition (59) gives us $T_1' = 81$ K, so that this point is not reached. At point 1 the spot moves to a W-trajectory (53), along which it moves until the end of the pumping at point 2; by the time $t_2 = \theta = 75$ ns it reaches a temperature $T_2 = 103$ K and a depth $z_2 = 14\ \mu$m. At point 2 the spot moves to a P-trajectory (52), along which it moves to point 3, which is reached, according to (61), at the time $t_3 = 1.4\ \mu$s. Here $T_3 = 38$ K. At point 3 the spot becomes spherical and then moves along P-trajectory (63) to point 4, where it is destroyed according to (66) at a temperature $T_4 = T^+ = 38$ K, having a radius $r_3 = l_{T^+} = 140\ \mu$m. The time of the motion between points 3 and 4 is on the order of t_3.

These numbers should not be taken too literally, for several reasons. First, all the equations which we have used are estimates containing unknown numerical factors. Second, power laws (1), (2), and (5), on which these equations are based, may have to be abandoned. Finally, scattering processes other than scattering by isotopes may become important, as they do in Ge at $T \gtrsim 70$ K, as can be seen by comparing the thermal conductivities[24] of natural-abundance and

TABLE II.

	$\omega_d/2\pi$, THz	T_0, K	P_0, μJ/mm^2	θ_0, ns	W_0, kW/mm^2
Si	6,7	115	130	10	13
Ge	3,5	60	45	31	1,4
GaAs	3,8	65	77	12	6,4

isotopically enriched Ge.

After the destruction of the hot spot, the propagation occurs in a quasidiffusion regime. Accordingly, if the distance between the pumping region and the detector, R, is much larger than the radius (l_{T^+}) of the hot spot at the time of its destruction, the shape of the signal should be independent of the pump energy E. Typical values of R are from 1 mm to 1 cm. The largest values of l_{T^+} are reached at the highest values of E, as can be seen from (66), (3), and (5); taking $E = 10$ μJ, we find $l_{T^+} = 0.6$ mm. The condition $R \gg l_{T^+}$ will thus usually hold; experimentally, however, one can observe[2] a strong E dependence of the signal shape.

To resolve this discrepancy we should recall that the quasidiffusion propagation sets in only after several decays of the "initial" phonons ω^* (Ref. 11). During the destruction of a hot spot, the initial phonons have a frequency ω_{T^+}, while the phonons reaching the detector have a frequency ω_R, determined from the condition $l(\omega_R) = R$ (Refs. 9 and 10). Using (3), we find $\omega_R/\omega_{T^+} = (R/l_{T^+})^{-2/9}$, from which we then find $\omega_R/\omega_{T^+} \sim 0.5$ at $R = 1$ cm and $l_{T^+} = 0.6$ mm. This result means that only a single decay occurs during the propagation time. The quasidiffusion regime actually does not form; this conclusion explains the observed E dependence of the signal shape.

If a hot spot does not form at all, the "initial" phonons are acoustic phonons with $\omega^* \approx \omega_D/2$, which arise in the decay of optical phonons.[7] We now find $\omega_R/\omega^* \approx 0.13$, which corresponds to the three decays. In this situation there is naturally the hope that a quasidiffusion regime will set in and that the shape of the signal will not depend on E.

We note in conclusion that some of the results derived here apply more generally than to the situation discussed above. For example, the method used to determine which propagation regime sets in (and, in particular, to determine whether a hot spot forms) can be used in cases more general than those in which phonons are produced during the thermalization of electron-hole pairs. An example is the production of phonons with $\omega \sim \omega_D$ in the radiationless relaxation of excited Eu centers in CaF_2 (Ref. 25) or in the many-photon absorption of the IR beam from a CO_2 laser.[26] Furthermore, the arguments regarding the trajectory of the hot spot after it is torn away from the pumping region can also be applied to other phonon excitation methods (the methods using a film injector, for example) if there is a solid basis for expecting the formation of a hot spot.

[1)]Ulbrich *et al.*[7,17] offered a completely different interpretation: They suggest that the signal results from short-wave phonons in ballistic propagation (and they cite the experiments of Ref. 18 as support for their suggestion). However, estimates and numerical calculations show[19,20] that a ballistic propagation of such phonons would be impossible because of scattering by isotopes. This conclusion is also supported by experimental results which show that there is no phonon focusing.[21] See the discussion in Ref. 22 in this regard.

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