Electron tunneling through layers with statistically rough surfaces

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The tunnel-current amplitude, the range of angles of incidence, and the yield of tunneling electrons are related theoretically to the interface roughness in a metal-insulator-metal system. The connection between these quantities and the main parameters of the roughnesses is determined. The most probable surface fluctuation distortions that carry the bulk of the current are found.

1. INTRODUCTION

Tunneling experiments provide a method for studying the band structure of semicondutors. The conductor-insulator interface is usually assumed to be mirror-smooth, so that the tangential component of the electron momentum is conserved during tunneling. In this cases, only electrons nearly normally incident on the interface can tunnel through a low-transparency barrier.[1] The amplitude of the tunnel current should thus reflect to some degree the structure of the conductor Fermi surface in the direction normal to the interface.

In practice, however, interfaces are never perfectly smooth, so that the tangential momentum component is not conserved and electrons incident at large oblique angles can also tunnel. The tunnel current will thus depend not on the structure of the electron spectrum at the Fermi surface normal to the interface, but on the characteristics of this structure integrated over a certain range of angles. In this paper we will investigate how interface roughness alters the tunnel current.

2. CALCULATION OF THE TUNNEL CURRENT

Figure 1 shows a model of the tunnel junction (I, III are the metal layers, II is the insulator). We write ε_{1F} and ε_{2F} for the Fermi energy of the electrons in layers I and III, respectively, and U for the height of the potential barrier in the insulator. The interfaces I–II and II–III are assumed to be rough (we will call them the first and second boundaries, respectively). The positions of the boundaries are described by the random functions $\xi_1(\mathbf{r}_1)$ and $\xi_2(\mathbf{r}_2)$, respectively, with expectation values $\langle\xi_1\rangle = \langle\xi_2\rangle = 0$ and variances $\langle\xi_1^2\rangle = a_1^2$, $\langle\xi_2^2\rangle = a_2^2$; b_1 and b_2 are the characteristic lengths of the surface irregularities (parallel to the interface).

We assume that the surface roughnesses are large-scale, i.e., $a_i, b_i \gg \varkappa^{-1}$ $(i = 1,2)$, where m_2 is the electron mass and $\varkappa^{-1} = [2m_2(U-\varepsilon_{1F})]^{-1/2}$ is the decay length of the wave function in the insulator[1)]; we also assume that the barrier is not too transparent. The latter condition will be satisfied if the interfaces are flat and $\varkappa d \gg 1$. Clearly, this condition will be modified somewhat if the boundaries are rough and most of the current passes through the thinnest regions of the insulator. Indeed, we will show below that the appropriate condition in this case is $\varkappa[d-2\varkappa(a_1^2+a_2^2)] \gg 1$. Moreover, since we are interested in how the roughness affects the tunnel current, we will neglect resonant electron tunneling along the impurities and electron scattering by phonons and other defects in the insulator.[2,3] The insulator will thus be taken to be homogeneous. Moreover, we will simplify the calculations by assuming that the potential energy of the electrons in the insulator is constant, and that the effective electron mass is isotropic. At the end of this paper we will discuss what happens when an electric field is applied to the insulator.

We will derive an expression for the dimension of the regions between which effective tunneling takes place; in some cases, these dimensions may be large compared to atomic. We will therefore consider the equations for the envelopes rather than for the Bloch functions themselves. This is legitimate, because local wave-function changes at the interface, which can be due either to loss of lattice periodicity or to local surface defects, alter in fact only the coefficient multiplying the exponential in the expression for the tunnel current. The main results in this paper will thus be valid up to the choice of the pre-exponential factors.

Let an electron wave

$$\psi_1 = \exp\{i\mathbf{k}_{1r}\mathbf{r} + ik_{1z}z\}$$

be incident on the first boundary from metal *I*. Green's theorem

$$\psi_{2-3}(\mathbf{r}_2,\xi_2) = \frac{1}{2\pi}\mathcal{T}_2(\mathbf{k}_3,\mathbf{r}_2)\int \psi_{1-2}(\mathbf{r}_1,\xi_1)\,\frac{\partial G_2}{\partial n_1}\,dS_1 \qquad (1)$$

then determines the wave function at a point $\mathbf{r}_2$ on the boundary II–III; here $\psi_{1-2}(\mathbf{r}_1,\xi_1) = \mathcal{T}_1(\mathbf{k}_1,\mathbf{r}_1)\exp\{i\mathbf{k}_{1r}\cdot\mathbf{r} + ik_{1z}\xi_1\}$ is the wave function at the first boundary, and $\mathcal{T}_1(\mathbf{k}_1,\mathbf{r}_1)$ and $\mathcal{T}_2(\mathbf{k}_3,\mathbf{r}_2)$ are the transmission coefficients for the first and second interface, respectively; G_2 is given by

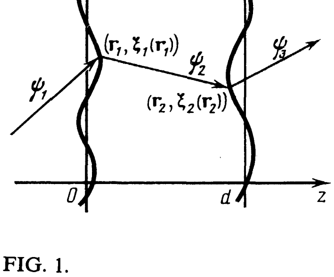

FIG. 1.

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$$G_2=(\exp(-\varkappa R_2))/R_2,\ R_2 =[(d-\xi_1(\mathbf{r}_1)+\xi_2(\mathbf{r}_2))^2+(\mathbf{r}_1-\mathbf{r}_2)^2]^{1/2}.$$

Since we are interested only in the exponential behavior, we will not discuss the form of the coefficients $\mathscr{T}_1$ and $\mathscr{T}_2$. However, we will show that most of the tunneling occurs from surface depressions, where the local inclination angle of the surface is small. We will therefore take $\mathscr{T}_1, \mathscr{T}_2$ to be given by the corresponding expessions for tunneling through mirror-smooth interfaces.

The wave function in region III is given by

$$\psi_3(\mathbf{R}_3)=\frac{1}{2\pi}\int \psi_{2-3}(\mathbf{r}_2,\xi_2)\frac{\partial G_3}{\partial n_2}dS_2, \tag{2}$$

which is of the same for as (1); here $G_3 = (\exp i_3\mathbf{k}\cdot\mathbf{R})/R_3$ and $R_3 = [(z-\xi_2)^2-(\mathbf{r}-\mathbf{r}_2)^2]^{1/2}$. If we substitute (1) into (2), expand R_2 and R_3 [assuming $\varkappa d$ and $k_3R_0 \gg 1$, respectively], and write $R_0 \equiv [(z-d)^2+r^2]^{1/2}$, we get

$$\psi_3(\mathbf{k}_1,\mathbf{k}_3)=\mathscr{T}(\mathbf{k}_1,\mathbf{k}_3)\int \exp\{-\varkappa d+(ik_{1z}+\varkappa)\xi_1-(ik_{3z}+\varkappa)\xi_2 -(\mathbf{r}_1-\mathbf{r}_2)^2/2\rho_0^2+i\mathbf{k}_{1r}\mathbf{r}_1+i\mathbf{k}_{2r}\mathbf{r}_2+i\mathbf{k}_3\mathbf{R}_0\}\,d(\mathbf{r}_1,\mathbf{r}_2), \tag{3}$$

where

$$\mathscr{T}(\mathbf{k}_1,\mathbf{k}_3)=-\frac{i\varkappa k_z}{(2\pi)^2 dR_0}\mathscr{T}_1\mathscr{T}_2,\qquad \rho_0^2=\frac{d}{\varkappa}.$$

In the analysis of the expressions for the tunnel-current density through the junction we consider for definiteness the case $k_BT \ll eV \ll (U_0-\varepsilon_{1F})/\varkappa d$, where V is the voltage applied to the junction. In this case, only electrons whose energy lies in a layer eV near the Fermi surface will contribute to the tunnel current.[4] The partial density of the tunnel current of electrons that enter and leave the barrier with momenta $\mathbf{k}_1$ and $\mathbf{k}_3$ can be expressed by using (3):

$$j_{\mathbf{k}_1,\mathbf{k}_3}=\frac{2}{(2\pi)^3}e^2V\left(\frac{\partial\varepsilon_1}{\partial k_{1z}}\frac{\partial\varepsilon_3}{\partial k_{3z}}\right)^{-1} (\mathbf{k}_3\nabla_k\varepsilon_3)\,|\nabla\varepsilon_3|\,k_{3F}^{-3}\frac{R_0^2}{S}\langle|\psi_3(\mathbf{k}_{1r},\mathbf{k}_{3r})|^2\rangle. \tag{4}$$

Here S is the area of the junction, ε_1 and ε_2 are the electron energies in regions I and III. The total current density is obviously

$$j=\int j_{\mathbf{k}_1,\mathbf{k}_3}\,d^2k_{1r}\,d^2k_{3r}. \tag{5}$$

We must know the mean value of the squared modulus of the wave function in (3) to evaluate the partial current (4). If we assume that the roughness distributions of the different surfaces are Gaussian and statistically independent, we have

$$\langle\exp\{(\varkappa+ik_{1z})\xi_1(\mathbf{r}_1)-(\varkappa+ik_{3z})\xi_2(\mathbf{r}_2)+(\varkappa-ik_{1z})\times\xi_1(\mathbf{r}_1')-(\varkappa-ik_{3z})\xi_2(\mathbf{r}_2')\}\rangle=\exp\{(\varkappa^2-k_{1z}^2)a_1^2 +(\varkappa^2-k_{3z}^2)a_2^2+(\varkappa^2+k_{1z}^2)K_1(\mathbf{r}_1-\mathbf{r}_1')+(\varkappa^2+k_{3z}^2)K_2(\mathbf{r}_2-\mathbf{r}_2')\}, \tag{6}$$

where $\langle\ldots\rangle$ denotes averaging over a surface, and the roughness correlation function $K(\mathbf{r})$ vanishes outside a region on the order of b. (Examples in which similar mean values are calculated can be found, e.g., in Refs. 5.) If

$$\frac{b_1^2}{2(\varkappa^2+k_{1z}^2)a_1^2}\exp(\varkappa^2+k_{1z}^2)a_1^2 \gg \rho_0^2$$

together with the analogous condition on the second boundary (which can be easily satisfied to the extent that $(\varkappa^2+k_{1z}^2)a_1^2 \gg 1$, $(\varkappa^2+k_{3z}^2)a_2^2 \gg 1$ and mean that the electrons can tunnel to any point on a surface from a deposition in another), we can expand the functions $K_i(\mathbf{r}_i-\mathbf{r}_i')$ as powers of $(\mathbf{r}_i-\mathbf{r}_i')^2/2b_i^2$ and keep only the first term; the result is

$$\langle|\psi_3(\mathbf{k}_1,\mathbf{k}_3)|^2\rangle=|\mathscr{T}|^2\int\exp\Big\{-2\varkappa d+2\varkappa^2(a_1^2+a_2^2) -\frac{(\mathbf{r}_1-\mathbf{r}_1')^2}{2\rho_1^2}-\frac{(\mathbf{r}_2-\mathbf{r}_2')^2}{2\rho_2^2} -\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{2\rho_0^2}-\frac{(\mathbf{r}_1'-\mathbf{r}_2')^2}{2\rho_0^2} +i\mathbf{k}_{1r}(\mathbf{r}_1-\mathbf{r}_1')-i\mathbf{k}_{3r}(\mathbf{r}_2-\mathbf{r}_2')\Big\}\,d(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_1',\mathbf{r}_2'),$$

where $\rho_1=[b_1^2/(\varkappa^2+k_{1z}^2)a_1^2]^{1/2}$ and $\rho_2=[b_2^2/(\varkappa^2+k_{3z}^2)a_2^2]^{1/2}$ are the dimensions of the characteristic depressions in the first and second surfaces, respectively. Evaluating (7), we find that

$$\langle|\psi_3(\mathbf{k}_1,\mathbf{k}_3)|^2\rangle=(2\pi)^3S\rho_1^2\rho_2^2\rho_0^2[1+(\rho_1^2+\rho_2^2)/2\rho_0^2]^{-1}|\mathscr{T}|^2 \times\exp\Big\{-2\varkappa(d-\varkappa(a_1^2+a_2^2))-\frac{(\mathbf{k}_{1r}-\mathbf{k}_{3r})^2\rho_1^2\rho_2^2}{2(\rho_1^2+\rho_2^2)} -\frac{(\mathbf{k}_{1r}\rho_1^2+\mathbf{k}_{2r}\rho_2^2)^2}{2(\rho_1^2+\rho_2^2)[1+(\rho_1^2+\rho_2^2)/2\rho_0^2]}\Big\}. \tag{8}$$

If the depressions are very gently sloping, namely, i.e., if $\rho_1^2,\rho_2^2 \gg \rho_0^2$, or $b_2/a_2 \gg \varkappa d(\varkappa^2+k_z^2)/\varkappa^2 \gg 1$ for each boundary, the factor

$$\frac{\rho_1^2\rho_2^2}{\rho_1^2+\rho_2^2}\exp\Big\{-\frac{(\mathbf{k}_{1r}-\mathbf{k}_{3r})^2\rho_1^2\rho_2^2}{2(\rho_1^2+\rho_2^2)}\Big\}$$

in (8) yields $\delta(\mathbf{k}_{1r}-\mathbf{k}_{3r})$, meaning that the tunneling preserves the tangential component of the momentum. We thus get the result

$$\langle|\psi_3(\mathbf{k}_1,\mathbf{k}_3)|^2\rangle\propto\delta(\mathbf{k}_{1r}-\mathbf{k}_{3r})\exp\{-2\varkappa(d-\varkappa(a_1^2+a_2^2)) -k_r^2\rho_0^2/4\}. \tag{9}$$

In this case the tunneling across a rough interface can be treated as tunneling between different planar surface regions (the so-called mosaic model of the junction)[6]; the only effect of the roughness on the tunnel current is to produce an extra factor $\exp[2\varkappa^2(a_1^2+a_2^2)]$ which reflects the fact that tunneling occurs where the barrier is thinnest.

For most tunnel junctions, however, $\varkappa d \gg 1$ and the inequality $b^2/a^2 \ll \varkappa d/(\varkappa^2+k_z^2)/\varkappa^2$ is more likely to hold on each boundary, i.e., $\rho_1^2,\rho_2^2 \ll \rho_0^2$. Expression (8) then becomes

$$\langle|\psi_3(\mathbf{k}_1,\mathbf{k}_3)|^2\rangle=(2\pi)^3S|\mathscr{T}|^2\rho_1^2\rho_2^2\rho_0^2 \times\exp\Big\{-2\varkappa(d-\varkappa(a_1^2+a_2^2))-{}^1/_2k_{1r}^2\rho_1^2-\frac{1}{2}k_{3r}^2\rho_2^2\Big\} \tag{10}$$

and the partial current (4) is

$$j_{\mathbf{k}_1,\mathbf{k}_3}=\frac{2}{(2\pi)^4}\frac{e^2V}{\rho_0^2}\left(\frac{\partial\varepsilon_1}{\partial k_{1z}}\frac{\partial\varepsilon_3}{\partial k_{3z}}\right)^{-1} (k_{3F}\nabla_k\varepsilon_3)\,|\nabla_k\varepsilon_3|\,\frac{k_{3z}^2}{k_{3F}^2}\,|\mathscr{T}_1|^2|\mathscr{T}_2|^2 \times\rho_1^2\rho_2^2\exp\Big\{-2\varkappa(d-\varkappa(a_1^2+a_2^2))-\frac{1}{2}k_{1r}^2\rho_1^2-\frac{1}{2}k_{3r}^2\rho_2^2\Big\}. \tag{11}$$

We thus see that the tangential electron momenta in the initial and final states are completely unrelated because of the scattering. Substitution of the expressions for $\mathscr{T}_1$ and $\mathscr{T}_2$ restores the symmetry $j_{\mathbf{k}_1,\mathbf{k}_3} = j_{\mathbf{k}_3,\mathbf{k}_1}$.

To analyze expression (11) we obtain next the form of the most probable surface depressions from which tunneling can occur, and analyze the distinguishing features of the tunneling from the fluctuations.

3. OPTIMUM SURFACE FLUCTUATIONS FOR TUNNELING

Since (11) shows that the roughnesses of the two interfaces enter symmetrically and independently, we can simplify things somewhat by assuming that only the I–II interface is rough. To this end, we assume that $k_z \ll \varkappa$. The tunnel current across a surface fluctuation $\xi(\mathbf{r})$ is given by

$$I_k = \alpha \left| \int \exp\{\varkappa\xi(\mathbf{r}) + i\mathbf{k}_r\mathbf{r}\} d^2\mathbf{r} \right|^2, \tag{12}$$

where the factor α is independent of $\mathbf{r}$. The distribution function of the current is expressed in terms of a functional of the distribution of the fluctuations, which we assume is Gaussian.

$$g(I_k) \sim \int \exp\left(\xi \frac{K^{-1}}{2} \xi\right)$$

$$\delta\left\{ I_k - \alpha \left| \int \exp[\varkappa\xi(\mathbf{r}) + i\mathbf{k}_r\mathbf{r}] d^2 r \right|^2 \right\} D\xi \tag{13}$$

Here

$$(\xi K^{-1}\xi) = \int \xi(\mathbf{r}) K^{-1}(\mathbf{r}-\mathbf{r}') \xi(\mathbf{r}') d\mathbf{r}\, d\mathbf{r}',$$

where the operator K^{-1} is the inverse of K. The functional integral can be evaluated by the method of steepest descent.[2] The integrand has a maximum when

$$\xi(\mathbf{r}) = 2\varkappa\alpha\beta\left\{ \int \exp[\varkappa\xi(\mathbf{r}') - i\mathbf{k}_r\mathbf{r}'] d\mathbf{r}' \int K(\mathbf{r}-\mathbf{r}'') \times \exp[\varkappa\xi(\mathbf{r}'') + i\mathbf{k}_r\mathbf{r}''] d\mathbf{r}'' + \text{c.c.} \right\}. \tag{14}$$

$$K(\mathbf{r}-\mathbf{r}'') = a^2 \exp(\mathbf{r}-\mathbf{r}'')^2/2b^2.$$

Here β is a Lagrange multiplier and $K(\mathbf{r}-\mathbf{r}'')$ is the roughness correlation function. Direct substitution shows that the function

$$\xi(\mathbf{r}) = A_k \exp\left[-\frac{r^2}{b^2}\left(1 - \frac{1}{\varkappa A_k}\right)\right] \tag{15}$$

satisfies (14) for the case $\varkappa A_k \gg 1$ of interest here (A_k is the amplitude of the surface fluctuation). Indeed, if we substitute (15) into (14) and expand $\xi(\mathbf{r})$ in the argument of the exponential in powers of $r^2 b^{-2} \cdot (1 - 1/\varkappa A_k)$, which is legitimate to the extent that $\varkappa A_k \gg 1$, we find that (14) is satisfied if

$$A_k = 2(2\pi)^2 \beta\varkappa a^2 b^4 \exp(2\varkappa A_k - k^2 b^2), \tag{16}$$

which thus determines the optimum fluctuation amplitude. We then find from (12), (13), (15), and (16) that

$$A_k \approx \frac{1}{2\varkappa} \ln \frac{I_k}{\alpha b^4} + \frac{k^2 b^2}{2\varkappa \ln I_k/\alpha b^4}, \tag{17}$$

$$g(I_k) \sim \exp\left\{ -\left(\ln\frac{I_k}{\alpha b^4} + \frac{k^2 b^2}{\ln I_k/\alpha b^4}\right)^2 \Big/ 8\varkappa^2 a^2 - \ln\frac{I_k}{\alpha b^4} \right\}. \tag{18}$$

The average current from state $\mathbf{k}$ is equal to

$$\bar{I}_k = \int g(I_k) I_k\, dI_k.$$

The integrand has a maximum at $\ln(I_k)/\alpha b^4 \approx 4\varkappa^2 a^2$, from which we find that the average current is

$$\bar{I}_k \propto \alpha \exp\{2\varkappa^2 a^2 - k^2 b^2/2\varkappa^2 a^2\} \tag{19}$$

(which of course agrees with (11) under the above assumptions). This current is due primarily to fluctuations of the form

$$\xi_k(r) \approx \left(2\varkappa a^2 + \frac{k^2 b^2}{4\varkappa^3 a^2}\right) \exp\left(-\frac{r^2}{b^2}\right). \tag{20}$$

In order to obtain the mean current (19), the junction area over which the averaging is performed must be sufficiently large. Indeed, although the current density for tunneling through fluctuations of the form (20) is approximately $\exp(4\varkappa^2 a^2)$ times greater than for tunneling through a flat section of the surface, the probability such a fluctuation is exponentially small:

$$g(\xi_{\text{opt}}) \sim \exp\left[-\xi_{\text{opt}} \frac{K^{-1}}{2} \xi_{\text{opt}} \right] \sim \exp(-2\varkappa^2 a^2). \tag{21}$$

The mean current is thus proportional to $\exp(2\varkappa^2 a^2)$; when it is averaged, however, the area S of the junction must satisfy the condition

$$S \gg b^2 \exp 2\varkappa^2 a^2 \tag{22}$$

since the dimension of the fluctuations in the tangential direction is $\sim b$. Condition (22) severely restricts the properties of the fluctuations and must be satisfied if (19) is averaged. Failure to average can lead to exponential fluctuations in the tunnel current amplitudes from one measurement to another for an ensemble of junctions of equal area.

4. TUNNELING FROM OPTIMUM SURFACE FLUCTUATIONS

We have seen that surface fluctuations of the form $\xi_{\text{opt}}(r) = \varkappa a^2 \exp(-r^2/2b^2)$ account for most of the tunnel current. To identify the features of tunneling from rough surface, its depressions deserve therefore special study (Fig. 2). To understand the meaning of the results of Sec. 3 we return to Eq. (1) for the wave function. We will assume as in the previous section that $\xi_2(\mathbf{r}_2) = 0$. We then have the explicit expression

$$\psi_{2-3}(\mathbf{k}_1, \mathbf{r}_2) = \frac{1}{2\pi} \mathscr{T}_1 \mathscr{T}_2 \rho_0^{-2} \exp\{-\varkappa(d - \varkappa a^2)\}$$

$$\times \int \exp\{-(\varkappa + ik_{1z})\varkappa a^2 r_1^2/2b^2 - (\mathbf{r}_1 - \mathbf{r}_2)^2/2\rho_0^2 + i\mathbf{k}_{1r}\mathbf{r}_1\} d\mathbf{r}_1 \tag{23}$$

for the wave function on the second boundary, assuming a wave incident on the depression in question from the metal layer I. Here we take $\rho_0^2 = (d - \varkappa a^2)/\varkappa$, i.e., we lift the constraint $\varkappa a^2 \ll d$. If $\rho_0^2 \ll b^2/\varkappa^2 a^2$ and $b^2/k_{1z}\varkappa a^2$, we can neglect the first term in the exponential inside the integral for the region $r_1 < b/\varkappa a$. In this case we obtain the wave function corresponding to tunneling from a flat section of surface of diameter $b/\varkappa a$.

In the opposite limit $\rho_0^2 \gg b^2/(\varkappa^2 + k_{1z}^2)a^2 \equiv \rho_1^2$, evaluation of the integral in (23) leads to

$$|\psi_{2-3}(\mathbf{k}_1, \mathbf{r}_2)|^2 = |\mathscr{T}_1|^2 |\mathscr{T}_2|^2 \rho_1^4 \rho_0^{-4} \exp\{-2\varkappa(d - \varkappa a^2)\}$$

$$\times \exp\}-(\mathbf{r}_2 - \mathbf{k}_{1r}\rho_1^2 k_{1z}/\varkappa)^2/\rho_0^2 - k_{1r}^2\rho_1^2\}.$$

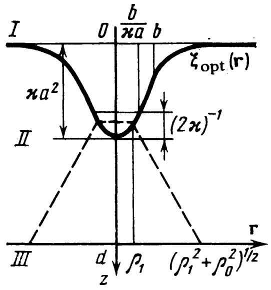

FIG. 2.

Equation (24) shows that electrons with $k_{1r} \lesssim (\varkappa^2 + k_{1z}^2)^{1/2} a/b$ participate in the tunneling, i.e., when $\rho_1 \ll \rho_0$ tunneling can occur for incidence angles much larger than for a flat surface. The explanation is made apparent by examining Eq. (23). Indeed, the integrand in (23) is effectively nonzero only in a finite region ΔS of the boundary I–II which depends on the decay of the tunneling exponential and the choice of the point $\mathbf{r}_2$ on the second boundary. Since tunneling does not change the phase of the wave function, the tunneling electrons reaching $\mathbf{r}_2$ have the same phase as initially (on the first interface). Thus the wave functions interface destructively if the phase changes rapidly within the surface ΔS. Therefore, only the electrons incident that can tunnel are those so that the phase of the wave function changes by less than 2π. If the interface I–II is planar, the size of ΔS is comparable to ρ_0 and electron tunneling occurs for incidence angles such that $k_{1r} \lesssim \rho_0^{-1}$. If the interface is rough, the region ΔS for which the tunneling exponential is effectively nonzero will be smaller, so that tunneling can occur over a wider range of angles. If $\varkappa \gg k_{1F}$, this range coincides with the diffraction angle for waves incident normally on an aperture of the corresponding diameter ($\rho \sim b/\varkappa a$, so that $k_{1r} \lesssim \varkappa a/b$). If $\varkappa \ll k_{1F}$, a depression of height $1/2\varkappa$ will contain many wavelengths; in this case, electrons incident normal to ΔS at a distance of $1/2\varkappa$ from the bottom of the depression will tunnel when $k_{1r} \lesssim k_z a/b$. In general, both types of tunneling occur if $k_{1r} \lesssim (\varkappa^2 + k_{1z}^2)^{1/2} a/b = \rho_1^{-1}$; this condition leads to the constraint

$$\operatorname{tg}\theta \leqslant \operatorname{tg}\theta_{max} = \frac{a}{b}\left(\frac{\varkappa^2 + k_{1F}^2}{k_{1F}^2 - \varkappa^2 a^2/b^2}\right)^{1/2} \tag{25}$$

on the angle of incidence θ and determines the range of angles in Eqs. (10), (11).

If $\varkappa a/k_{1F} b \geqslant 1$ holds in (25) then electrons tunnel at arbitrary angles of incidence on the interface. However, these angles may be limited by shadow effects which we have ignored. Conditions for the shadow effects to be negligible can be derived easily from the explicit form of the optimum fluctuations. Since the latter are $\sim \varkappa a^2$ deep and $\sim b$ wide, shadowing can limit the partial tunnel current only for electrons with $\theta > \tan^{-1}(b/\varkappa a^2)$. Shadowing clearly will not limit the current if $\tan(\theta_{\max}) < b/\varkappa a^2$, which is equivalent to $a^2/b^2 \lesssim k_{1F}/\varkappa a(\varkappa^2 + k_{1F}^2)^{1/2} \ll 1$ if $k_{1F} a \gg 1$ and to $a^2/b^2 \lesssim k_{1F}^2/\varkappa^2 \ll 1$ if $k_{1F} a \ll 1$ (recall that we are interested in the case when $\varkappa a \gg 1$). The latter inequalities can hold only if $a/b \ll 1$, i.e., for shallow surface irregularities. If none of the above conditions is satisfied, then shadowing effects must be considered. However, if the electron mean free path l is $< \varkappa a^2$ (which is very probable, since $\varkappa a \gg 1$), the above inequalities can be satisfied more easily and shadowing does not limit the tunneling angles even for fairly large ratios a/b. In this case, an electron incident at a depression may be scattered and tunnel at a large angle θ, thus contributing to the corresponding partial current.

We note that an increase in the angular range of tunneling does not in general imply an increase in the tunnel current amplitude, as is easily seen by substituting the wave function (24) into (4), assuming an isotropic Fermi surface, and evaluating the integral (5). This is because there is a proportional decrease in the current density from each partial component, as may be understood by comparing the mechanism of electron tunneling across a small surface element $\Delta S \sim \rho_1^2$ with the transmission of an electron wave through an aperture of the same diameter. In the latter case, the current is independent of the aperture dimension ρ_1 (we are assuming that $k_{1F}\rho_1 \gg 1$). If we then use (1) to calculate the electron wave function at a distance d from the aperture, we find that the pre-exponential factor has the same form as in (23). However, the diffraction angle is equal to $\tilde{\rho}_2 \sim \lambda_2 d/\rho_1 \sim \rho_0^2/\rho_1$. We have already seen that the diameter of the corresponding region during tunneling is equal to ρ_0. The amplitudes of the partial tunnel currents thus decrease because the region in which the tunneling exponential is effectively nonzero shrinks by a factor $\rho_0/\rho_1 \gg 1$.

5. DISCUSSION

Not surprisingly, the features of electron tunneling from optimum surface fluctuations also persist in Eq. (11) for the partial currents. The roughness of the interface affects both the current amplitude and the range of incident angles $\Delta\theta$ contributing to the current. The increase in the current by the factor $\exp[2\varkappa^2(a_1^2 + a_2^2)]$ reflects the fact that even though the tunneling area ΔS is smaller for rough surfaces, the surface depressions are deeper. We have shown that the range $\Delta\theta$ is limited by the size of the region ΔS of effective tunneling, and $k_{1r} \lesssim \rho_1^{-1}$, $k_{3r} \lesssim \rho_2^{-1}$. That is, the optimum fluctuations determine ΔS. The roughness distributions of the two boundaries are also independent, because the exponential function plays the same role in selecting ΔS on each interface (recall that the roughness functions ξ_1, ξ_2 for the two interfaces are assumed to be statistically independent).

In order to calculate the total tunnel current amplitude, we must evaluate the integral (5) with $j_{\mathbf{k}_1,\mathbf{k}_3}$ given by (11). Although the explicit form of the partial current depends on the transmission coefficients $\mathscr{T}_1(\mathbf{k}_1)$ and $\mathscr{T}_2(\mathbf{k}_3)$, they will always be greater at normal incidence than for oblique incident angles. Examination of (5) with (11) then shows that if the Fermi surface of the metal layers are isotropic, increasing $\Delta\theta$ will not increase the amplitude of the total current, just as we showed previously for the current due to optimum fluctuations. However, the tunnel current could increase if the Fermi surfaces of the electrons in the metals are not isotropic. For instance, suppose that the effective electron mass in one of the metals is greater parallel to the interface than normal to it; electron tunneling at oblique angles may then increase the current considerably. A similar increase could occur if the minimum of the conduction band of one of the metals does not lie at the center of the Brillouin zone.

Finally, we note that similar results can be derived without assuming that the potential barrier in the insulator is of constant height; we need only assume that U is nearly constant over the depth of an optimum fluctuation. It is then clear that the parameters characterizing the two interfaces will contain the characteristic decay lengths $\varkappa_1^{-1}$, $\varkappa_2^{-1}$, whose "average" determines the diameter ρ_0 of the Fresnel zone. For instance, if the electric field at the barrier is constant then $\rho_0^2 \approx 2d/(\varkappa_1 + \varkappa_2)$.

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[1)] We set $\hbar = 1$.

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