Quantum corrections to thermoelectric power of a nondegenerate electron gas

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The quantum corrections to the thermoelectric coefficient of a semiconductor are investigated. The case is considered of sufficiently high temperatures and relatively low densities of electrons subject to Boltzmann statistics. The electron scattering by acoustic phonons is quasielastic in this situation and quantum interference can result from such scattering alone in the absence of structural disorder. The dependence of the quantum corrections on the magnetic field is investigated for the following cases: (1) the interference is caused by quasielastic scattering of the electrons by acoustic phonons, and is disrupted by the low inelasticity of the scattering; (2) the interference is due to the relatively frequent collisions between the electrons and the lattice defects, and is disrupted only by collisions with phonons.

We investigate here the quantum corrections (i.e., the corrections for diagrams of the fan type) to the thermoelectric coefficient of a semiconductor. We consider the case of sufficiently high temperatures and relatively low electron densities, when the electrons obey Boltzmann statistics. (The case of Fermi statistics was considered by us earlier.[1,2]

We regard an investigation of the role of quantum corrections in this situation as particularly interesting since it is possible to manage in experiment the electron density so that the electron–electron interaction plays no role. As shown by Al'tshuler and Aronov,[3] single-particle interference corrections and corrections for electron-electron interaction coexist in the case of a degenerate electron gas, and the two types of corrections must be separated in experiment. For Boltzmann statistics, however, conditions can be created such that the interference effects are obtained in "pure" form.

Another circumstance whose importance comes into play precisely in the case of relatively high temperatures is that the semiconductor need not necessarily have structural disorder to require quantum corrections. Collisions between electrons and acoustic phonons are known to be quasielastic, whereby the electron momentum is changed by each collision by a value of the order of the momentum itself, while the energy changes by only a small fraction of the initial value. The lifetime of the electronic state relative to the phase relaxation is therefore large enough and interference is possible. If the semiconductor is pure enough, this interference-formation mechanism should predominate at relatively high temperatures. In the opposite case, the impurity mechanism is in operation.

Further, with increasing temperature, the relative role of the contribution made to the thermoelectric coefficient η by the electron drag by the phonons is decreased. This is important from the viewpoint of the possibilities of comparing our theory with experimental data, since the relative value of the "diffusion" part of the thermopower, the correction to which is the one calculated here, is also increased.

Finally, the relative value of this correction turns out to be much larger for nondegenerate electrons than for degenerate ones. The correction is therefore easier to observe in the former case.

The thermoelectric coefficient η is defined with the aid of the following relation for the current density j:

$$\mathbf{j}=\sigma\mathbf{E}-\eta\nabla T \tag{1}$$

(**E** is the electric field and T is the temperature in energy units.) In the lowest nonvanishing approximation in the parameter $\hbar/\varepsilon\tau$ (ε is the electron energy and τ is its lifetime in the state with the specified momentum), which we assume to be small, we have[1,2]

$$\Delta_c\eta=\frac{2e}{\pi\hbar T}\int_0^\infty d\varepsilon\,\frac{\partial n_0}{\partial\varepsilon}\,(\varepsilon-\mu)D(\varepsilon)\mathfrak{C}(\varepsilon). \tag{2}$$

Here $\Delta_c\eta$ is the interference correction to the coefficient η, e is the electron charge, n_0 is the electron equilibrium distribution function, μ is the chemical potential, $D(\varepsilon)$ is the diffusion coefficient of electrons having an energy ε, and $\mathfrak{C}(\varepsilon)$ is the sum of the fan diagrams (see the papers by Gor'kov, Larkin and Khmel'nitskii[4] as well as by Abrahams and Ramakrishnan[5]). This sum is usually represented in the form

$$\mathfrak{C}(\varepsilon)=\int(dq)\,[D(\varepsilon)q^2+1/\tau_c]^{-1}. \tag{3}$$

Here τ_c is the characteristic time of destruction of the single-particle interference or, in other words, the time of phase relaxation of the single-particle wave function that describes the state of an electron in a field of randomly arranged impurities; $(dq)=d^u q/(2\pi)^u$, where $u=2$ or 3.

Two situations are usually investigated in experiment. In the first, the sample is a plate or a film with thickness d much larger than the electron de Broglie wavelength $\hbar/p$. The electrons are characterized then by a three-dimensional momentum **p** and a three-dimensional density of the electronic state $2\nu(\varepsilon)$ (ν denotes everywhere the single-particle density of states without allowance for the spin dyad). At the same time, the relation between d and the length $L_c=(D\tau_c)^{1/2}$ can be arbitrary. At $d\gg L_c$ the integration with respect to **q** is three-dimensional ($u=3$); if, however, $d\ll L_c$, the integration with respect to **q** is effectively two-dimensional ($u=2$).

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In the second experimental situation one studies the carriers of an inversion or of an enriched surface layer of thickness $d \ll \hbar/p$. In this case the carriers can be regarded as two-dimensional; they are characterized by a two-dimensional density of states 2ν, and $u = 2$. We shall refer to the first and second situation as cases I and II, respectively.

We have previously[2] derived Eq. (3) with allowance for its dependence on ε. We considered in that reference a degenerate electron gas and it sufficed to investigate the $\mathfrak{C}(\varepsilon)$ dependence near the Fermi level. Here we must know the ε-dependence in an entire interval, of order T, of the values of ε; this dependence is obtained in the same manner as in Ref. 2. We reckon the energy ε here from the lower edge of the allowed band. Accordingly, the lower limit in (2) is $\varepsilon = 0$. Expression (2) can be regarded[1)] as a quantum correction to the Cuttler-Mott formula[6] for the coefficient η, a formula valid for elastic scattering of the electrons, viz.,

$$\eta=-2\varepsilon\int_0^\infty d\varepsilon\,\nu(\varepsilon)\,\frac{\varepsilon-\mu}{T}D(\varepsilon)\,\frac{\partial n_0}{\partial\varepsilon}. \tag{4}$$

Accordingly, expression (2) for $\Delta_c\eta$ can be written as

$$\Delta_c\eta=-2e\int_0^\infty d\varepsilon\,\nu(\varepsilon)\,\frac{\varepsilon-\mu}{T}\Delta_cD(\varepsilon)\,\frac{\partial n_0}{\partial\varepsilon}, \tag{5}$$

where the quantum increment to the classical diffusion coefficient is

$$\Delta_cD(\varepsilon)=-\frac{1}{\pi\hbar\nu(\varepsilon)}D(\varepsilon)\mathfrak{C}(\varepsilon). \tag{6}$$

The expression for the quantum correction to the conductivity, calculated by the Matsubara technique (cf. Ref. 4), can be represented in the same form

$$\Delta_c\sigma=\frac{2e^2}{\pi\hbar}\int_0^\infty d\varepsilon D(\varepsilon)\mathfrak{C}(\varepsilon)\,\frac{\partial n_0}{\partial\varepsilon}. \tag{7}$$

Thus, in the lowest approximation in the parameter $T/|\mu| \ll 1$ we obtain the relation

$$\Delta_c\eta=-(\mu/eT)\Delta_c\sigma. \tag{8}$$

A similar relation connects also the classical quantities η and σ.

The possible ensuing deduction is quite remarkable. The calculations yield a strongly differing behavior of $\Delta_c\sigma$ and $\Delta_c\eta$ in the case of degenerate and nondegenerate electrons. In the former case $\Delta_c\sigma/\sigma$ exceeds $\Delta_c\eta/\eta$. In the latter case these quantities are equal, so that their contributions to the differential thermopower $\alpha = \eta/\sigma$ cancel each other. Even in the next approximation in the parameter $T/|\mu|$, however, the corresponding contribution to α turns out to differ from zero and can be observed in experiment, say by investigating the dependence of α on the magnetic field H in weak fields (see below). From our viewpoint it would be of interest to measure directly the thermoelectric current to verify relation (8).

The time τ_c in (3) can be governed by various mechanisms.

The first is scattering by magnetic impurities, combined with spin-orbit scattering. This mechanism was studied for out problem by Hakami, Larkin, and Nagaoka[7] and by Lee.[8] It can play the principal role in sufficiently "dirty" semiconductors. In pure semiconductors, transitions with spin flip can occur even in quasielastic scattering by acoustic phonon, owing to the spin-orbit interaction.

At relatively high temperatures the value of $\mathfrak{C}(\varepsilon)$ is apparently determined mainly by another mechanism-inelastic scattering electrons by phonons. In this case the cooperon is likewise usually represented in the form (3), and the phase relaxation time is τ_φ. One must bear in mind here a situation wherein two scattering mechanisms "act" simultaneously. The perdominant one is pure elastic, and the other, inelastic, is relatively weak against the background of the first. If the second mechanism is substantially inelastic, the time τ_φ coincides with the departure time τ_{in} with respect to inelastic processes.

If, however, the second mechanism is quasielastic then, as shown in Ref. 9, the explicit form of the cooperon $\mathfrak{C}(\varepsilon)$ depends substantially on the ratio of the transferred energy $\hbar\overline{\omega}$ to the departure time τ_{in} relative to inelastic processes. If the relation

$$\overline{\omega}\tau_{in}\gg 1 \tag{9}$$

is satisfied, the cooperon can be represented in the form (3), as first indicated in Ref. 10, and $\tau_\varphi = \tau_{in}$. If the inequality (9) is reversed, however, we have

$$\tau_\varphi^{-1}\approx\tau_{in}^{-1}(\overline{\omega}\tau_{in})^{2/3}. \tag{10}$$

This order-of-magnitude estimate was obtained in Ref. 11.

In our earlier paper[9] we analyzed in detail the case when there is only one quasi-elastic scattering mechanism, in other words, when the energy transfer is much lower than the electron energy. In the case of scattering by acoustic phonons, such a situation is realized at[2)]

$$T\gg mw^2. \tag{11}$$

Here m is the electron effective mass (the electron spectrum is assumed for simplicity to be isotropic and quadratic) and w is the speed of sound. The characteristic energy transfer $\hbar\overline{\omega}$ is in this situation of the order of

$$\hbar\overline{\omega}\approx pw\approx T(mw^2/T)^{1/2}\ll T. \tag{12}$$

For one quasielastic scattering mechanism the result also depends substantially on the ratio of $\overline{\omega}$ to $\tau_{in.} \equiv \tau$. If the inequality (9) is reversed, each scattering act causes a small change of the phase of the electron wave function, so that the loss of phase memory is the result of many such acts. In place of (3), the correct expression for the cooperon is then

$$\mathfrak{C}(\varepsilon)=\int(dq)\int_0^\infty dt\exp[-D(\varepsilon)q^2t-t^3/\tau_\varphi^3], \tag{13}$$

where τ_φ is determined by the estimate (10).

The condition that must be satisfied for quantum interference to be possible is

$$\tau<\tau_\varphi. \tag{14}$$

Combination of this inequality with (10) leads to the condition $\overline{\omega}\tau \ll 1$. So that in the case of one quasielastic relaxation mechanism the correct expression for $\mathfrak{C}(\varepsilon)$ in all cases of interest is (13). We note that their difference notwithstanding, Eqs. (3) and (13) yield practically the same result in the absence of a magnetic field: at $u = 2$ we have $\mathfrak{C} \propto \ln(\tau_\varphi/\tau)$. In the presence of a magnetic field, as we shall show, the expressions that follow for $\mathfrak{C}$ from (3) and (13) differ more significantly.

We present now the result for the thermoelectric coefficient η in the absence of a magnetic field. The result is simplest when the cooperon is given by Eq. (3). We then obtain at $d \ll L_c$

$$\Delta_c\eta = \frac{e}{2\pi^2\hbar} e^{\mu/T}\left(\frac{\mu}{T} - 1\right)\ln\frac{\tau_c(T)}{\tau}. \tag{15}$$

This expression can be rewritten in terms of the electron density n in the form

$$\Delta_c\eta = -\frac{en}{(2\pi)^2\bar{\nu}\hbar T}\left(\ln\frac{2\bar{\nu}T}{n} + 1\right)\ln\frac{\tau_c(T)}{\tau}, \tag{16}$$

where

$$\bar{\nu} = \int_0^\infty d\left(\frac{\varepsilon}{T}\right) e^{-\varepsilon/T}\nu(\varepsilon).$$

$\bar{\nu} = 2^{-1}\pi^{1/2}\nu(T)$ in case I and $\bar{\nu} = \nu = m/2\pi\hbar^2$ in case II.

This result remains in force when the cooperon is given by (13). The fact that the ratio τ/τ_φ depends according to (10) on the energy ε is not reflected in the result obtained in the principal approximation in the parameter $\ln(\tau_\varphi/\tau) \gg 1$.

It is useful to compare (15) with the value of $\Delta_c\sigma$ calculated by the Matsubara technique and given at the accuracy assumed by

$$\Delta_c\sigma = -\frac{e^2}{2\pi^2\hbar}\ln\left[\frac{\tau_c(T)}{\tau}\right]e^{\mu/T}. \tag{17}$$

Hence, since $\exp(\mu/T) \ll 1$,

$$\Delta_c\eta = \frac{1}{e}\left(1 - \frac{\mu}{T}\right)\Delta_c\sigma = \frac{1}{e}\left[\ln\left(\frac{2\bar{\nu}T}{n}\right) + 1\right]\Delta_c\sigma. \tag{18}$$

We examine now the change of the expression for the thermopower in a magnetic field. The cooperon takes the simplest form if expressed in the form (3) at $H = 0$. In this case, as shown by Al'tshuler *et al.*,[12] allowance for a weak magnetic field (which does not bend the trajectory significantly over distances on the order of the mean free path) leads in the two-dimensional case to replacement of (3) by

$$\mathfrak{C}(\varepsilon) = \frac{4|e|H}{2\pi c\hbar}\sum_{n=0}^{m}\left[4D\frac{|e|H}{\hbar c}\left(n + \frac{1}{2}\right) + \frac{1}{\tau_c}\right]^{-1}, \tag{19}$$

where m is a large integer of the order of $c\hbar/2|e|Hl^2$. It is convenient to represent (19) in integral form

$$\mathfrak{C}(\varepsilon) = \frac{\gamma}{2\pi D(\varepsilon)}\int_0^\infty dx\,\frac{e^{-ax/2}\,\mathrm{sh}(ax/2)}{\mathrm{sh}(\gamma x/2)}\exp\left[-\frac{\tau x}{\tau_c}\right], \tag{20}$$

where $a \approx 1(m + 1 \equiv a a_H{}^2/4D\tau)$, and $\gamma = 4D\tau|e|H/\hbar c$. Equation (20) is sensible in fields such that $\gamma \ll 1$. In principal order in the parameter $\ln(\tau_c/\tau)$ we can therefore replace the lower limit of the integral in (20) by a^{-1} and put $\exp(-ax/2)\sinh(ax/2) \approx \frac{1}{2}$. We then arrive at

$$\mathfrak{C}(\varepsilon, H) = \frac{1}{2\pi D(\varepsilon)}\Phi_1(\varepsilon, H), \tag{21}$$

$$\Phi_k(\varepsilon, H) = \int_{\tau/a\tau_c}^\infty \frac{(H/H_c)\,e^{-x^k}}{2\,\mathrm{sh}(Hx/2H_c)}\,dx, \quad H_c(\varepsilon) = \hbar c/4|e|D(\varepsilon)\tau_c. \tag{22}$$

At $H \ll H_c$ the function $\Phi_k(\varepsilon, H)$ goes over into $\ln(\tau_c/\tau)$, and at $H \gg H_c$ we have $\Phi_k \propto \ln(H_c\tau_c/H\tau)$. At $H \gtrsim H_{cr} \sim c\hbar/2|e|l^2$ the value of Φ_k becomes of the same order as the unaccounted-for diagrams. We note that the contribution of the latter in the entire magnetic-field region $H \ll H_{cr}$ can be excluded by investigating the difference $\Phi_k(\varepsilon, H) - \Phi_k(\varepsilon, 0)$.

We use the same method of taking the magnetic field into account also if the cooperon is defined by (13). This can be justified by using the quasiclassical asymptotic of the Green's functions in a magnetic field, followed by a calculation similar to that in Ref. 9. It is shown in the Appendix that the result differs from (22) in that $k = 1$ is replaced by $k = 3$ and in that $\tau_c \to \tau_\varphi$. This difference leads to a different dependence on the magnetic field. In particular, the coefficient of $(H/H_c)^2$ in the expansion of Φ_1 in powers of H/H_c is equal to $-1/24$ if H/H_c is small; the corresponding coefficient in the expansion of the function Φ_3 is $-\Gamma(2/3)/72$, i.e., smaller by a factor 2.2.

The quantum corrections to the thermopower and to the conductivity are determined by substituting the expressions for $\mathfrak{C}(\varepsilon, H)$ in (2) and (7). It must be recognized here that H_c depends, generally speaking, on the electron energy ε. Accordingly,

$$\Delta_c\sigma = -\frac{e^2}{2\pi^2\hbar}e^{\mu/T}\int_0^\infty d\left(\frac{\varepsilon}{T}\right)e^{-\varepsilon/T}\Phi_k(\varepsilon, H); \tag{23}$$

$$\Delta_c\eta = -\frac{e}{2\pi^2\hbar}e^{\mu/T}\int_0^\infty d\left(\frac{\varepsilon}{T}\right)e^{-\varepsilon/T}\frac{(\varepsilon-\mu)}{T}\Phi_k(\varepsilon, H). \tag{24}$$

The temperature dependences of these corrections are determined by the actual mechanisms of the elastic and inelastic scattering.

For sufficiently thick samples ($d \gg L_c$), the expression for $\Delta_c\eta$ cannot be represented in a form as simple as (16). The point is that the integral with respect to q is determined in this case by the upper limit. For the difference $\eta(H) - \eta(0)$, however, an equation such as (24) is valid, with $\Phi_k(\varepsilon, H)$ replaced by the function

$$\Psi_k(\varepsilon, H) = \left(\frac{3}{4\pi}\right)^{1/2}\frac{1}{l(\varepsilon)}\int_0^\infty \frac{dx}{x^{1/2}}\left[\frac{H/H_c}{2\,\mathrm{sh}(Hx/2H_c)} - \frac{1}{x}\right] \times \exp[-x^k], \tag{25}$$

where $l(\varepsilon)$ is the mean free path of an electron of energy ε.

We estimate now those temperature and material-parameter regions in which the phenomena discussed can be observed. We begin with the case when there is one quasielastic scattering mechanism, namely interaction with acoustic phonons. The energies significant in a nondegenerate electron gas are $\varepsilon \approx T$, so that the perturbation-theory parameter for the interaction with phonons is $\hbar/T\tau$, this is the quantity that must be small. On the other hand, as already noted, the electronic-states interference due to interaction with phonons is significant only at $\bar{\omega}\tau \ll 1$. In the upshot, the limits of applicability of the theory at $d \gg \hbar(2mT)^{-1/2}$ are determined by the chain of inequalities

$$(T_c m w^2)^{1/2} \ll T \ll T_c, \tag{26}$$

where the characteristic temperature T_c satisfies the condition $T_c \tau(T_c) = \hbar$. Here $(T_c m w^2)^{1/2} \approx (E_a/\Lambda)^2 \Theta$, where E_a is the atom energy, Λ the deformation potential, and Θ the Debye temperature. We note that the first inequality of (26) is much more stringent than the quasielasticity condition (11).

We write down also a condition that allows us to confine ourselves to one "fan" diagram. It is equivalent to the requirement that the ratio of the quantum correction to the classical value of the kinetic coefficient be small, and takes the form

$$\left(\frac{T}{T_c}\right)^{1/2} \ln \frac{T}{(m w^2 T_c)^{1/2}} \ll 1. \tag{27}$$

It is therefore obvious that the quantum correction increases with temperature.

It must be borne in mind, however, that as the temperature is raised a larger role is assumed by scattering from optical phonons, and this scattering must also be taken into account. At sufficiently low temperatures (optical-phonon frequency $\hbar\omega_\phi \gtrsim T$) this scattering is inelastic and leads only to phase relaxation. At higher temperatures the scattering becomes quasielastic. In this case the results are similar to those given above: the condition (26) is satisfied automatically, since $T\tau_{\rm opt} \propto T^{1/2}$. In the case when the interference is due to one scattering mechanism (e.g., as is frequently in experiment, scattering by impurities), and the phase relaxation is due to another (say, scattering by phonons), the condition $\bar{\omega}\tau_{\rm ph} \gg 1$ is not necessary. As noted above, at $\bar{\omega}\tau_{\rm ph} \gg 1$ the time τ_φ is of the order of $\tau_{\rm ph}$ and all that is required is that $\tau_{\rm ph}$ be much larger than the resultant departure time τ.

We discuss now the possibility of observing quantum effects in a magnetic field. The point is that the quantum corrections can compete with the classical corrections because the electron trajectories are bent in a magnetic field. In weak fields the classical effects are of the order of $(H/H\mu)^2$, where $H_\mu = c/\mu$, $\mu = eD/T$ is the electron mobility. For the quantum corrections, which are of the order of $\hbar/pl$ compared with the known classical kinetic coefficients, to be discernible against the background of this relation, it is necessary to satisfy the condition

$$H_c/H_\mu = \hbar/4T\tau_\varphi \lesssim (\hbar/pl)^{1/2}. \tag{28}$$

APPENDIX

We have calculated in Ref. 9 a sum of the "fan" diagrams (Fig. 1) in quasielastic scattering of electrons by phonons, without allowance for the magnetic field. In this case expression (13) could be represented in the form

$$\mathfrak{C}(\varepsilon) = \tau \int (dq) \mathscr{F}(t=0, q, \varepsilon), \tag{A.1}$$

and $\mathscr{F}(t)$ was defined as

$$\mathscr{F}(t) = \cos(\omega_{2p} t) + (1-\Delta) \int_{-\infty}^{t} \frac{dt'}{\tau} e^{-(t-t')/\tau} \mathscr{F}(t') \Psi(2t-t'), \tag{A.2}$$

where

$$\Delta = Dq^2\tau - i\Omega\tau, \quad \Psi(t) = \langle \cos(\omega_{\mathbf{p}-\mathbf{p}'} t) \rangle$$

(the angle brackets denote averaging over all the directions of the directions of the vector $\mathbf{p}'$).

In the spatially inhomogeneous case the equation that determines the sum of the fan diagrams and shown in the figure can be written in the form

$$\begin{aligned} &C(\rho\mathbf{R}, \rho_1\mathbf{R}_1, \omega) \\ &= I(\rho)\delta(\mathbf{R}-\mathbf{R}_1)\delta(\rho-\rho_1) - g^2 \int \frac{d\omega'}{2\pi i} (d\rho')(dR') \\ &\times C(\rho'\mathbf{R}', \rho_1\mathbf{R}_1, \omega') I(\rho, \omega-\omega') \\ &\times G^R(\mathbf{R}+\rho/2, \mathbf{R}'+\rho'/2, \varepsilon+\Omega+\omega-\omega') \\ &\times G^A(R-\rho/2, R'-\rho'/2, \varepsilon+\omega')[N(\omega)-N(\omega'-\omega)], \end{aligned} \tag{A.3}$$

where

$$I(\rho, \omega) = \mathscr{D}^R(\rho, \omega) - \mathscr{D}^A(\rho, \omega),$$

and the Wigner coordinates are defined as follows:

$$\rho = \mathbf{r}_1 - \mathbf{r}_2, \quad \rho_1 = \mathbf{r}_3 - \mathbf{r}_4, \quad \rho' = \mathbf{r}_1' - \mathbf{r}_2';$$

$$\mathbf{R} = {}^1/_2(\mathbf{r}_1+\mathbf{r}_2), \quad \mathbf{R}_1 = {}^1/_2(\mathbf{r}_3+\mathbf{r}_4), \quad \mathbf{R}' = {}^1/_2(\mathbf{r}_1'+\mathbf{r}_2').$$

The sum of the fan diagrams can then be written in analogy with (A.1)

$$\mathfrak{C}(\varepsilon) = \tau \int (d\rho)(d\rho_1)(dR) \mathscr{F}(t=0, R\rho, R\rho_1, \varepsilon). \tag{A.4}$$

The Fourier component of the function $\mathscr{F}(t)$ is defined using the relation $F(\omega) \equiv iC(\omega)/\omega g^2$. $G(\mathscr{D})$ in Eqs. (A.3) and (A.4) denotes the electron (phonon) Green's functions, and g is the electron-phonon interaction constant. Assuming the magnetic field to be weak enough, we have

$$G(\mathbf{r}, \mathbf{r}_1) = G(\mathbf{r}-\mathbf{r}_1) \exp\left\{\frac{ie}{c} \int_{\mathbf{r}}^{\mathbf{r}_1} \mathbf{A} d\mathbf{l}\right\} \equiv G(\mathbf{r}-\mathbf{r}_1) e^{i\varphi(\mathbf{r}, \mathbf{r}_1)}.$$

Taking the Fourier transform with respect to the variables ρ:

$$G(\mathbf{p}\mathbf{R}, \mathbf{p}_1\mathbf{R}_1) = \int (d\rho)(d\rho_1) G(\rho\mathbf{R}, \rho_1\mathbf{R}_1) e^{-i\mathbf{p}\rho - i\mathbf{p}_1\rho_1},$$

we obtain from (A.3)

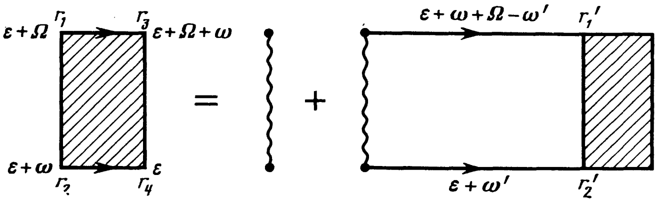

FIG. 1.

$$F(\mathbf{p}R, \mathbf{p}_1 R_1, \omega)$$

$$= I_1(\omega, \mathbf{p}-\mathbf{p}_1)\,\delta(\mathbf{R}-\mathbf{R}_1) - Tg^2 \int (dp')\,(dq)\,(dR')\frac{d\omega'}{2\pi i}$$

$$\times\, G^R\left(\varepsilon+\Omega+\omega-\omega', \mathbf{p}' + \frac{\mathbf{q}}{2}\right)$$

$$\times\, G^A\left(\varepsilon+\omega', -\mathbf{p}' + \frac{\mathbf{q}}{2}\right) F(\mathbf{p}'R', \mathbf{p}_1 R_1, \omega')$$

$$\times \exp[2i\varphi(\mathbf{R}, \mathbf{R}') + i\mathbf{q}(\mathbf{R}-\mathbf{R}')]\,I_1(\omega'-\omega, \mathbf{p}'-\mathbf{r}). \qquad \text{(A.5)}$$

Here $I_1 = iI/\omega g^2$. It is convenient next to integrate with respect to $d\varepsilon_{p'}$ and introduce

$$P(\mathbf{R}, \mathbf{R}', n_{\mathbf{p}'}) = \int (dq)\,\frac{\exp[i\mathbf{q}(\mathbf{R}-\mathbf{R}') + 2i\varphi(\mathbf{R}, \mathbf{R}')]}{1 - i\Omega\tau - iqv\tau(\mathbf{n}_\mathbf{q}, \mathbf{n}_{\mathbf{p}'})},$$

$$\mathbf{n}_\mathbf{p} = \frac{\mathbf{p}}{p}.$$

Just as in the case of elastic impurities,[12] the function $\langle P\rangle$ can be represented in the form

$$\langle P(\mathbf{R}, \mathbf{R}', n_{\mathbf{p}'})\rangle = \sum_n \lambda_n \psi_n(\mathbf{R})\,\psi_n(\mathbf{R}'),$$

$$\langle \mathscr{F}(\mathbf{p}'\mathbf{R}', \mathbf{p}_1\mathbf{R}_1, t)\rangle = \sum_n C_n(t)\,\psi_n(\mathbf{R}')\,\psi_n(\mathbf{R}), \qquad \text{(A.6)}$$

where $\psi_n(R)$ is the solution of the Schrödinger equation for a particle with mass $2D\tau$ and energy $E_n = 1 + i\Omega\tau - \lambda_n$ in a constant magnetic field. Transformations similar to those used in the problem without the magnetic field verify that the coefficient functions $C_n(t)$ satisfy Eq. (A.2) with $\Delta \to \Delta_n = (n + \frac{1}{2})\gamma - i\Omega\tau$. As shown earlier,[9] its solution is

$$C_n(t) = \int_0^\infty dx \exp\left[-\Delta_n x - \left(\frac{\tau}{\tau_\varphi}\right)^3 x^3\right]. \qquad \text{(A.7)}$$

This leads to Eq. (22) with $k = 3$.

1) This assertion is due to A. G. Aronov.

2) The inequalities (11) and (12) pertain to case I. In case II the quasielasticity condition (11) takes the form $T \gg \hbar w/d$, and $\hbar\overline{\omega} \approx \hbar w/d \ll T$.

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