Anomalous magnetoresistance of germanium bicrystals at low temperatures

O. V. Zharikov

Institute of Solid State Physics, Academy of Sciences of the USSR
(Submitted 28 November 1984)
Zh. Eksp. Teor. Fiz. **88**, 2138–2147 (June 1985)

The resistance R and the magnetoresistance anisotropy of germanium bicrystals containing large-angle intercrystallite boundaries are studied as functions of temperature at $0.035 \leqslant T \leqslant 10$ K and in fields $H \leqslant 20$ kOe. The $R(T)$ and $R(H)$ dependences are found to be anomalous in all of the samples investigated. The results are analyzed in terms of the theories of weak electron localization and of electron-electron interaction in two-dimensional systems.

I. INTRODUCTION

The galvanomagnetic properties of semiconductor bicrystals (primarily germanium) have been actively studied for more than two decades.[1] The studies show that Ge samples containing large-angle intercyrstallite boundaries have a high metallic hole conductivity that is independent of temperature (T) below $\lesssim 10$ K and unaffected by the degree and type of doping.[1]

Subsequent work[2,3] using purer and better-quality crystals essentially corroborated the early findings; however, some important new results were also found. For example, the effect of the misorientation angle θ on the temperature behavior of Ge bicrystals was established in Ref. 2. In addition, Shubnikov–de Haas oscillations were observed at fields $H \sim 10^5$ Oe in the purest crystals (with the highest mobilities $\mu \sim 10^3$ cm^2/V · s at $T = 4.2$ K).[2,3] These results were used to calculate the mass and mobility of the light and heavy holes, the carrier densities, and the elastic relaxation time, thereby providing information about the energy band structure of the bicrystals which suggested that the carriers move two-dimensionally. Under certain conditions, similar results are found from measurements of the cleavage-surface conductance[4] of Ge in liquid helium; this suggests that the galvanomagnetic phenomena may be similar for surface and internal boundaries in semiconductors.

The theory of quantum coherence in disordered systems has developed rapidly in recent years. In particular, weak electron localization (WEL) and electron-electron interactions (EEI) accompanied by impurity scattering have been studied, and various novel effects have been predicted which show up most clearly in two-dimensional systems.[5–8] For example, the theory gives corrections $\sim \ln(T)$ to the residual resistance, and it predicts an anomalous magnetoresistance for weak fields in the classical range. These predictions have been confirmed and studied experimentally for a wide class of materials, including thin metal and semiconductor films,[9,10] inversion layers,[11,12] cleavage surfaces of Ge in helium,[13] and many others. Because of their properties, Ge bicrystals are good candidates for observing the effects of WEL and EEI at low temperatures and were studied in Ref. 14, where the corrections to the conductivity and anomalous mganetoresistance (AMR) were found. Similar results were obtained independently in Refs. 15 and 16 at about the same time.

In the present paper we discuss the results of further, more detailed studies of the AMR for a wide range of H and T; we also investigate the anisotropy of the magnetoresistance and the temperature dependence $G(T)$ of the conductivity in several Ge bicrystals. We then use the theory of WEL and EEI developed in Ref. 6 to analyze the experimental results, and some inadequacies of the theory are noted.

2. SAMPLES AND EXPERIMENTAL METHOD

The Ge bicrystals were grown by the Czochralski technique on double seeds, exactly as described in Ref. 1. The growth axis was initially along the [100] direction, but was then rotated toward the [001] axis (Fig. 1). Crystals with this structure have the highest boundary conductivity and have been used in most previous experiments.[1] Our samples contained a symmetric inclination boundary with misorientation angle θ equal to 16° or 20°. Portions of the bicrystal were rotated about the other crystallographic axes, but these rotations were small (less than a few degrees). The samples were grown from very pure p-type or n-type Ge, which were doped with Ga and Sb, respectively. The resistivities at room temperature were $\rho_{300\,\mathrm{K}} \approx 40\ \Omega \cdot$ cm and $43\ \Omega \cdot$ cm for the p- and n-type specimens, respectively.

After growth, the bicrystals for which $\rho_{300\,\mathrm{K}}$ differed from $\rho_{300\,\mathrm{K}}$ for the initial mixture were discarded (in a few cases, we also tested samples for which $\rho_{300\,\mathrm{K}}$ was at most 30–50% less than the initial value). We used thermal etching

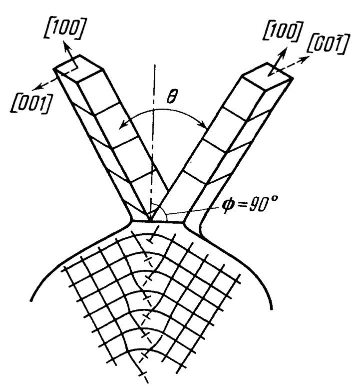

FIG. 1. Growth diagram of the germanium bicrystals (taken from Ref. 1, p. 308).

 0038-5646/85/061264-06$04.00

to expose the intercrystallite boundary during the growth process; after the formed crystal bar was cleaved into samples, we exposed the boundary by buffing and etching the samples in CP-4 solution (a mixture of fluoric, nitric, and acetic acids[17]). High-quality indium solder contacts were appplied to the crystals immediately after the chemical etching. In a few cases, gold contacts were microwelded using an MKS-2 welding device. The boundary contact was ohmic at all temperatures, and the recorded I-V characteristics were linear for $0.1\,\mu A \leqslant I \leqslant 1$ mA. The resistance ($\sim 10^{-4}\ \Omega$) of the indium contacts was negligible in all cases; the resistance of the gold contacts was some ten times greater. The resistance of the samples at $T = 4.2$ K was typically $\sim 10^3\ \Omega$, and the square resistance R_{sq} of the conducting layer of the boundary varied from 3.5 to 5 kΩ. The conductivity increases somewhat with the misorientation angle (this was analyzed in detail in Ref. 2).

We used an ultra-low-temperature H^3/He^4 cryostat built by the "Geliĭmash" All-Union Scientific-Industrial Association to measure the electrical and magnetic transport properties of the Ge bicrystals for $0.035 \leqslant T \leqslant 10$ K and $H \leqslant 20$ kOe. The temperature was measured by a "SPEER" carbon thermometer, which was inserted into the dissolution chamber of the cryostat in the immediate vicinity of the samples. The measuring current in the thermometer was equal to 0.1 μA, and the thermometer was calibrated by measuring the susceptibility of paramagnetic cerium-magnesium nitrate. The other (magnetic) thermometer was calibrated at $T = 0.6$–1.5 K in terms of the He^3 vapor pressure. The final step in calibrating both the carbon and the magnetic thermometers was to measure the superconducting-transition temperature T_c using as reference materials highly pure Cd and Ir, for which $T_c = 0.517$ and 0.133 K, respectively.

We studied the anisotropy of the magnetoresistance $R(H)$ down to 1.35 K in an ordinary helium cryostat by rotating an external magnet. We also carried out a few measurements for T down to 0.4 K by pumping off He^3 from the chamber through a cryogenic insert. The insert made it possible to rotate the sample relative to the magnetic field, which in this case was generated by a superconducting solenoid.

3. EXPERIMENTAL RESULTS

We will first describe the measured temperature dependence $R(T)$, which is shown in Fig. 2a for an n-type Ge bicrystal with $\theta = 20°$, $\rho_{300\,K} = 40\ \Omega \cdot$cm, $R_{4.2\,K} = 285\ \Omega$, $R_{sq} \approx 3.5$ kΩ; the measuring current I was 1 μA, the electric field inside the sample was $\approx 7 \cdot 10^{-3}$ V/cm, and the evolved power was $\approx 3 \cdot 10^{-10}$ W. We see that R increased steadily with $1/T$ for $T < 10$ K, and $\Delta R / R_{4.2\,K}$ was porportional to $-\ln(T)$. A plateau (saturation) was reached for $T \approx 0.3$ K, beyond which $R(T)$ remained constant. The saturation temperature (0.3 K) remained the same when I was decreased to 0.1 μA. Nevertheless, the saturation was probably caused[18] by low-frequency pickups and hum in the measuring devices (at 50 Hz and harmonics), which heated the electron subsystem.

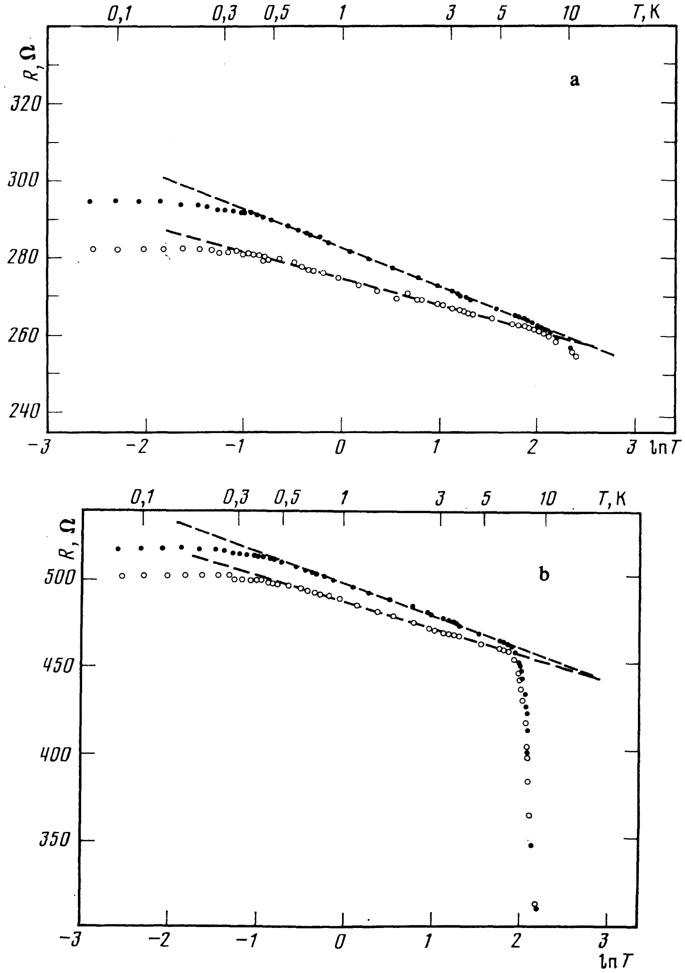

FIG. 2. Resistance of Ge bicrystals as a function of temperature: a) n-type samples; ○, $H = 0$; ●, $H = 5$ kOe; b) p-type samples; ○, $H = 0$; ●, $H = 5$ kOe.

We also measured $R(T)$ for p-type Ge bicrystals with $\theta = 20°$; the results were similar in all respects to those for n-type Ge. However, when θ was decreased to 16° the $R[\ln(T)]$ curve became $\approx 10\%$ steeper, although it continued to obey the relation $\Delta R/R \propto -\ln(T)$ (see Fig. 2b, which shows the data for a sample with $\theta = 16°$, $\rho_{300\,K} = 20\ \Omega \cdot$cm, $R_{4.2\,K} = 460\ \Omega$, $R_{sq} \approx 4650\ \Omega$). The resistance dropped abruptly when T increased above ≈ 10 K due to shunting by the bulk of the sample at higher temperatures.[1] This drop in $R(T)$ was much less pronounced for n-type bicrystals because the In contacts were blocking (they were ohmic for the p-type material). We will discuss in what follows only the behavior for $T < 10$ K. Measurements of the Hall voltage at $T = 4.2$ K revealed that holes were responsible for the conduction in the Ge bicrystals, in agreement with the findings in Ref. 1.

The relative change $\Delta R / R_{4.2\,K}$ remained proportional to $-\ln(T)$ when an external magnetic field $H = 5$ kOe was applied normal to the boundary and to the direction of the current; however, the proportionality constant increased somewhat (see Fig. 2a,b), by as much as 27–62% in some cases. Again (as for $H = 0$), the dependence $R(T)$ at $H = 5$ kOe saturated at $T \lesssim 0.3$ K.

We will now describe the results for the transverse magnetoresistance. The magnetoresistance $\Delta R(H) = R(H) - R(H=0)$ was nonzero for all of the samples studied, even for very weak fields $H \approx 10^2$ Oe; the magnitude of $\Delta R(H)$ depended on T and on whether the bicrystal was n- or p-type. For n-type crystals as grown $\Delta R(H)$ was positive for all H and increased with $1/T$. However, the dependence was more complicated for the p-type samples (Fig. 3). In weak fields ΔR could be either positive or negative, and the relative magnitude and characteristic scale of the variations $\Delta R(H)$ changed in a complicated nonmonotonic way as T decreased. There was one sample for which $\Delta R(H)$ was particularly large and negative (4% of $R_{4.2\,K}$ at $T \approx 0.65$ K and $H \approx 300$ Oe). Similar effects were also observed for p-type crystals equipped with gold contacts; in this case, the higher contact resistance degraded the signal/noise ratio considerably. We found that $R(H)$ increased as the field increased to $H \lesssim 20$ kOe in all of the sample, and saturated gradually in fields $H \approx 10^4$ Oe. The slope of the $\Delta R(H)$ curves was independent of T at high fields.

We investigated the anisotropy in $R(H)$ for $\mathbf{H} \perp \mathbf{I}$ by varying the angle α between $\mathbf{H}$ and the plane of the boundary. The results are shown in Figs. 4a and b. The change in $R(H)$ was similar for all of the samples (except at the weakest fields $H \sim 10^2$ Oe, see below); $\Delta R(H)$ was independent of H and T for $\alpha \approx 0°$, and as α increased, $R(H)$ rose steadily to a maximum at $\alpha \approx 90°$. On the other hand, the magnetoresistance for weak fields $0 \leqslant H \lesssim 10^2$ Oe depended on the properties of the mixture from which the bicrystals were grown, but in all cases $R(H)$ was independent of α for all T (see Fig. 4; note that the uppper limit of the "weak-field" region depended on T and also varied somewhat from sample to saple). We did not study $R(H)$ systematically for fields parallel to I; however, test measurements did not reveal any significant differences from the case $\mathbf{H} \parallel \mathbf{I}$. The $R(H)$ curves became independent of temperature at $T \lesssim 0.3$ K, probably (see Fig. 2) because of heating due to the noise in the electron subsystem.

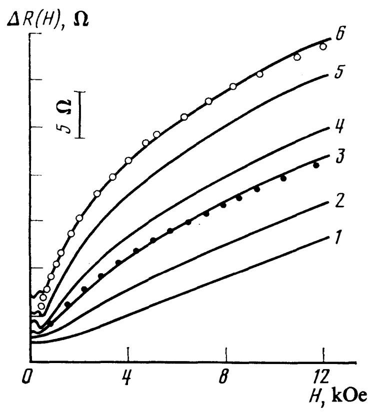

FIG. 3. Transverse magnetoresistance of a Ge bicrystal grown from a p-type mixture: 1) $T = 6.5$ K; 2) 4.0 K; 3) 3.10 K; 4) 1.85 K; 5) 0.84 K; 6) 0.32 K; ○ and ● show theoretical values calculated from (2). The curves are displaced along the vertical axis for clarity.

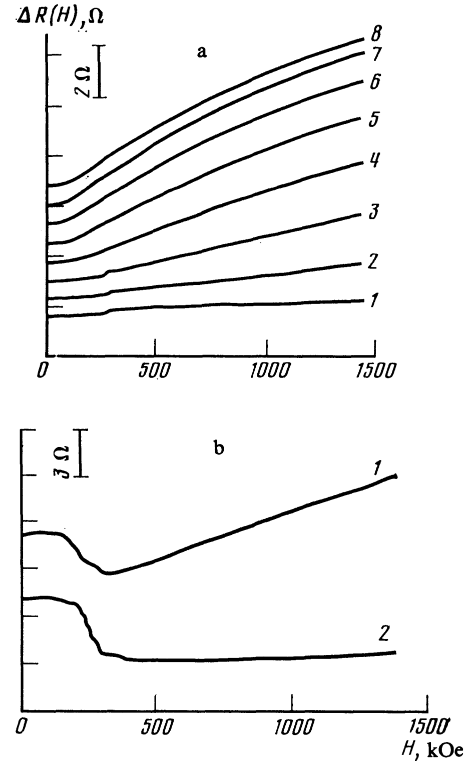

FIG. 4. Magnetoresistance of Ge bicrystals as a function of the orientation of the field ($T = 1.35$ K). a) n-type sample: 1) $\alpha \approx 0°$; 2) 10°; 3) 20°; 4) 30°; 5) 45°; 6) 65°; 7) 80°; b) for p-type samples: 1) $\alpha \approx 90°$; 2) 0°. α is the angle between the magnetic field and the plane of the boundary. For clarity, the curves are shifted along the vertical axis.

4. DISCUSSION

Our experimental results and the preliminary findings in Refs. 14–16 clearly indicate that quantum coherence phenomena (WEL,EEI) manifest themselves in the low-temperature behavior of the magnetoresistance of Ge bicrystals. However, we must answer the following two questions before we can properly apply the theories in Ref. 6:

1. To what extent can the observed behavior of the magnetoresistance be regarded as abnormal? In other words, what is the possible contribution from the classical magnetoresistance to the observed effects in the investigated field interval?

2. Is the system two- or three-dimensional?

The magnetoresistance measured in our experiments was in fact anomalous, as is easily seen by estimating the possible classical contribution. According to Ref. 19, $\Delta R/R \propto (\omega\tau)^2$, where ω is the cyclotron frequency and τ the elastic relaxation time. Since $\omega \propto eH/mc$ and $\tau \propto \mu m/e$, we have $\Delta R/R \propto (\mu H/c)^2$. If we recall that $\mu \approx 200\,\mathrm{cm^2/V\cdot s}$ for germanium,[2] we get $\Delta R/R \sim 10^{-8}$ for a magnetic field $H \sim 10^4$ Oe. This is much less than the experimentally observed magnetoresistance and is completely negligible; we thus conclude that the magnetoresistance in the Ge bicrystals was anomalous for all $H \lesssim 10^4$ Oe.

We can deduce the functional dependence $H(\alpha)$ from the experimentally measured magnetoresistance anisotropy; Fig. 5 shows the ratios H_α/H_{90} for one of the samples. We see that $H_\alpha/H_{90°}$ is proportional to $\sin\alpha$, and the same is

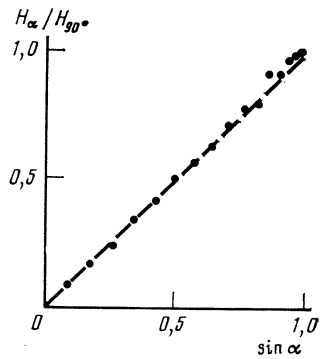

FIG. 5. The ratio H_α/H_{90} for the values in Fig. 4, as a function of sin α; H_α is the magnetic field directed at an angle α to the plane of the boundary; H_{90} is the field normal to the boundary.

true for the other bicrystals. According to Ref. 6, this attests to two-dimensional character of the observed behavior. $\Delta R(H)$ was not exactly equal to zero for fields parallel to the plane of the boundary, possibly because of errors in aligning the samples. Also, of course, the boundary in the bicrystals had microscopic roughnesses and was not a perfect plane. According to Refs. 5–8, the two-dimensional nature of the conductivity also follows from the behavior of $R(T)$ and $R(H)$ with varying T. We will discuss these questions in more detail below.

According to Ref. 6, the anomalous magnetoresistance in two-dimensional conducting systems has the general form

$$G(H)-G(0)=[C_2-C_2^{int}\beta(T)]\Delta G(H)+C_2^{int}\Delta G_{int}(H), \quad (1)$$

i.e., both weak electron localization and electron-electron (e-e) interactions contribute. Here G is the square conductivity and the constants C_2 and $C_2^{\rm int}$ take the values $\pm 1/2$ or ± 1, depending on the scattering mechanism and the shape of the conduction band; the first and third term correspond to WEL and EEI, respectively, while the second gives the Maki-Thompson corrections associated with fluctuations in the superconductivity. It is helpful to introduce the following scaling parameters: $L_\varphi = (D\tau_\varphi)^{1/2}$, where D is the diffusion coefficient, τ_φ is the phase relaxation time for the electron wave function, $L_T = (\hbar D/kT)^{1/2}$, and the magnetic length is $L_H = (\hbar c/eH)^{1/2}$ (the characteristic scale of the varying magnetic field). Here L_φ and L_T are the characteristic lengths for the WEL and EEI theories, respectively. We have $d < L_\varphi, L_T$ in the two-dimensional case, where d is the thickness of the conducting system. The theory discussed in Ref. 6 also contains the two magnetic-field scale lengths $H_{\rm WEL} = \hbar c/4eD\tau_\varphi = \hbar c/4eL_\varphi{}^2$ and $H_{\rm EEI} = \pi ckT/2eD = \tau c\hbar/2eL_T{}^2$. If $T\tau_\varphi > \hbar/4\pi k$ (as in the experiments), $H_{\rm WEL} < H_{\rm EEI}$. Physically, this implies that the field-induced localization is suppressed at lower H than is the case for the EEI processes. Since the magnetoresistance saturated at the experimental fields $\sim 10^4$ Oe (which corresponds to $L_H \approx 220$ Å) and the condition $H_{\rm WEL} < H_{\rm EEI}$ was satisfied, we conclude that the WEL suppression as H increased was primarily responsible for the anomalous behavior of the magnetoresistance. Neglecting the small second term[20] in Eq. (1), we get the final result

$$\Delta R(H)/R_{4,2\ \mathrm{K}} = C_2(e^2R_{\text{кв}}/2\pi^2\hbar)f_2(4eHL_\varphi^{1/2}/\hbar c), \quad (2)$$

where the logarithmic derivative $f_2(x) = \ln x + \psi(1/2 + 1/x)$ of the gamma-function behaves as $\sim x^2$ and as $\sim \ln(x)$ for $x \ll 1$ and $x \ll 1$, respectively.[6]

We deduced $C_2 \approx -1/2$ from the experimental magnetoresistance curves at large H. This value agrees with the theory,[6] which predicts a positive magnetoresistance for p-type Ge and a value $C_2 = -1/2$ when the complex structure of the valence band and the interaction between the light and heavy holes are allowed for. The theory is further supported by the results in Ref. 3, where the cyclotron mass m_c as deduced from observations of the Shubnikov–de Haas oscillations and it was suggested that light-and heavy-hole subbands are present in Ge bicrystals. A positive magnetoresistance was also found in Ref. 21 for heavily doped p-type Ge; moreover, the magnetoresistance became negative when the valence-band degeneracy was decreased by applying a uniaxial compression.

In comparing the experimental data with the theoretical predictions,[6] we selected the single free parameter $L_\varphi(T)$ of the WEL theory in Eq. (2) so as to obtain the best agreement. The points in Fig. 3 show values calculated from (2); clearly, they agree closely with the experimental results except for the initial weak-field region, in which $\Delta R(H)$ has a "fine structure." The values $L_\varphi(T)$ ranged from ≈ 200 to ≈ 500 Å, so that L_φ was greater than the boundary thickness $d \approx 20$ Å (Refs. 1,2). The saturation of the curves at fields $H \sim 10^4$ Oe indicates that $L_\varphi \gtrsim L_H \gtrsim l \gtrsim d$, where l is the mean free path. There is a small discrepancy between the experimental data and (2) at high fields; it probably reflects the classical component of $\Delta R(H)$ plus the contribution from EEI processes.[6]

According to (2), changes in the phase relaxation time $\tau_\phi(T)$ were responsible for the temperature dependence of the anomalous magnetoresistance. The above procedure for detemining

$$L_\varphi(T) = [D\tau_\varphi(T)]^{1/2}$$

enabled us to find the temperature dependence $\tau_\phi(T)$ under the assumption that the diffusion coefficient $D(T)$ was constant. We found that $\tau_\varphi \sim T^{-p}$ in all cases, with $p \approx 0.5$–0.7, depending on the sample. According to Ref. 22, the dependence should be $\tau_\varphi \sim T^{-1}$, which corresponds to inelastic e-e scattering in the two-dimensional case (changes in the phase of the electron wave function produced by inelastic processes, such as electron-phonon scattering or scattering by impurity ions, would require even larger values $p = +3$ and 2, respectively,[23] which are inconsistent with the experimental result $p = 0.5$–0.7). Further study is needed to explain why these discrepancies occur. For instance, it is conceivable that the temperature dependence $\tau_\varphi(T)$ represents a "superposition" of two different mechanisms, for one of which p is smaller or even zero (this may be true, e.g., for spin-orbit effects in p-type Ge).[6]

The above values of $L_\varphi(T)$ were also used to estimate the absolute values of τ_φ. If we assume that $D \sim lV_F/2 \sim 2\pi n\tau\hbar^2/2m^2 \approx 5.7$ cm²/s, we find that at $T = 3.1$ K, say $\tau_\varphi \sim 10^{-12}$ s for $L_\varphi \approx 200$ Å. We thus have $\tau_\varphi \gg \tau$, where

$\tau \sim 10^{-14}$ s (Ref. 2), and this estimate for τ_φ has the same order or magnitude as for doped p-type Ge (Ref. 21). Moreover, for $D \approx 5.7$ cm^2/s and $T = 3.1$ K we readily obtain $L_T \approx 400$ Å. Consequently, $L_\varphi < L_T$ and $H_{\rm WEL} \approx 4100$ Oe $< H_{\rm EEI} \approx 7400$ Oe, which shows that the WEL and EEI contributions can indeed be separated as assumed above.

The theory cannot satisfactorily account for the "fine structure" observed in many of the samples, nor for the fact that it is temperature-dependent but independent of the orientation of H relative to the plane of the boundary; in addition, the theory does explain why the characteristic length L_H is present. We merely note that magnetoresistance anomalies are apparently not connected with changes in the indium contacts as T drops below T_c (the critical field for indium is close to the characteristic scale of the magnetic field) because the same behavior at $T > T_c$ also for samples with gold contacts. A dip in the magnetoresistance in Ge bicrystals at weak fields was also observed in Ref. 16.

We will next discuss the $R(T)$ temperature dependence. According to Ref. 5–8, WEL and EEI processes in general both necessitate corrections to the residual resistance of two-dimensional systems:

$$\frac{\Delta R(T)}{R_{4,2\,\rm K}} = -\left\{[C_2 - \beta(T)]p + \left[1(j=0) - \frac{3}{4}F(j=1)\right] - g(T)\right\}\frac{e^2 R_{\rm sq}}{2\pi^2\hbar}\ln T. \qquad (3)$$

Here $[C_2 - \beta(T)]p$ is the contribution from the WEL effects, including the Maki-Thompson corrections[6]; the $[1(j=0) - 3/4 \cdot F(j=1)]$ are the interaction parameters of the EEI theory for the diffusion channel corresponding to total spins j equal to 0 and 1, respectively[8,24]; $g(T)$ is the interaction constant of the EEI theory for the Cooper channel.[7,8,25] According to Ref. 6, $C_2 = -1/2$ for a strong spin-orbit interaction, and this is confirmed by our experimental data on the magnetoresistance. In addition, the parameter of the EEI theory corresponding to $j = 1$ (Ref. 8) drops out, but the constant $g(T)$ remains, as was shown in Ref. 25 (these results differ from the earlier predictions in Ref. 8). We can therefore write

$$\frac{\Delta R(T)}{R_{4,2\,\rm K}} = -\left\{\left[-\frac{1}{2} - \beta(T)\right]p + 1 - g(T)\right\}\frac{e^2 R_{\rm sq}}{2\pi^2\hbar}\ln T. \qquad (4)$$

The dashed line in Fig. 2 plots the dependence (4), which is seen to accurately describe the experimental results for a wide range of T. The experimental values of the coefficients in the curly brackets ranged from 0.5 to 0.6 for the different samples.The WEL effects are preferentially suppressed in fields $H = 5$ kOe, and in this case the correction to the conductivity takes the form[5–8,25]

$$\Delta R(T)_H / R_{4,2\,\rm K} = -[1 - g(T)]\,(e^2 R_{\rm sq}/2\pi^2\hbar)\ln T. \qquad (5)$$

According to Fig. 2, the experimental dependence $\Delta R(T)_H$ was steeper for $H = 5$ kOe. Here we can calculate the value of $1 - g(T)$, which varied from 0.7 to 0.9 for our samples. We thus find that $g(T) \approx 0.1$–0.3 and $p \approx 0.5$–0.7, which agrees with the values found by deducing $\tau_\varphi(T)$ from the magnetoresistance curves. Moreover, since $\beta(T) \sim [g(T)]^2$ for small $g(T)$ (Ref. 6), we see that the above assumption regarding the smallness of $\beta(T)$ is justified. The temperature dependence $R(T)$ in Ge bicrystals for $H = 0$ is thus determined jointly by the localization and interaction effects. The parameter $g(T)$ in the EEI theory was 0.1–0.3 for the samples, which confirms one of the conclusions reached in Ref. 25, where $g(T)$ was found to be nonzero for systems with a strong spin-orbit interaction.

5. CONCLUSIONS

We have studied the magnetic transport properties and the conductivity of germanium bicrystals at low and infralow temperatures. Our results show that the anomalous temperature dependences of the resistance and magnetoresistance are accurately described by the theory[6] of weak electron localization and e-e interactions (with impurity scattering) in two-dimensional systems. We found that both localization and interaction effects determine $R(T)$, whereas the WEL theory alone accurately describes most of the features of the anomalous magnetoresistance for $H \leqslant 20$ kOe. Taken together, the results indicate that the conduction bands in Ge bicrystals are two-dimensional, as was predicted in earlier work antedating the theory in Ref. 6.

Neither the experimentally observed "fine structure" of the weak-field magnetoresistance curves nor the temperature dependence of the phase relaxation time $\tau_\varphi(T)$ can as yet be accommodated by the theory. Additional information will be provided by future experiments in which the magnetotransport properties of Ge bicrystals, e.g., are studied for crystals with various misorientation angles θ and dopings. Theoretical studies of semiconductors with an internal boundary will also be of interest and should stimulate the developement of models for the boundary conductivity and quantum coherence effects.

In closing, I thank D. E.Khmel'nitskiĭ for many fruitful discussions and for kindly supplying numerical data on the logarithmic derivative of the gamma-function, and A. M. Finkel'shteĭn for helpful discussions and valuable criticism. I am also grateful to M. E. Gershenzon, V. T. Dolgopolov, and S. I. Dorozhkin for a discussion of some of the topics treated in this work, to S.T. Boldarev and V. M. Mishachev for consultations on cryogenics and thermometry at ultralow temperatures, and to E. P. Vol'skiĭ for his interest and encouragement.

[1]H. Matare, *Electronics of Defects in Semiconductors*, Chaps. 10,11.
[2]B. M. Vul and É. I. Zavaritskaya, Zh. Eksp. Teor. Fiz. **76**, 1089 (1979) [Sov. Phys. JETP **49**, 551 (1979)].
[3]S. Ishida, L. Landwehr, and E. Bangert, Solid State Comm. **45**, 869 (1983).
[4]B. M. Vul, É. I. Zavaritskaya, and E. G. Sokol, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 517 (1979) [JETP Lett. **30**, 485 (1979)].
[5]E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979); L. P. Gor'kov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 228 (1979) [sic].

[6]B. L. Al'tshuler, A. G. Aronov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Zh. Eksp. Teor. Fiz. **81**, 768 (1981) [Sov. Phys. JETP **54**, 411 (1981)]; S. Hikami, A. I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980).
[7]B. L. Altshuler, A. L. Aronov, and P. A. Lee, Phys. Rev. Lett. **44**, 1278 (1980).
[8]B. L. Altshuler and A. G. Aronov, Solid State Comm. **46**, 429 (1983); B. L. Al'tshuler, A. A. Varlamov, and M. Yu. Reĭzer, Zh. Eksp. Teor. Fiz. **84**, 2280 (1983) [Sov. Phys. JETP **57**, 1329 (1983)].
[9]G. J. Dolan and D. D. Osheroff, Phys. Rev. Lett. **43**, 721 (1979).
[10]A. K. Savchenko, V. N. Lutskiĭ, and V. I. Sergeev, Pis'ma Zh. Eksp. Teor. Fiz. **36**, 150 (1982) [JETP Lett. **36**, 185 (1982)].
[11]D. J. Bishop, D. C. Tsui, and R. C. Dynes, Phys. Rev. Lett. **44**, 1153 (1980).
[12]O. V. Zharikov and Yu. V. Krutenyuk, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 45 (1983) [JETP Lett. **38**, 49 (1983)].
[13]B. M. Vul, É. I. Zavaritskaya, and V. N. Zavaritskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **37**, 87 (1983) [JETP Lett. **37**, 105 (1983)].
[14]O. V. Zharikov, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 108 (1983) [JETP Lett. **38**, 126 (1983)].
[15]B. M. Vul and É. I. Zavaritskaya, Pis'ma Eksp. Teor. Fiz. **37**, 571 (1983) [JETP Lett. **37**, 681 (1983)].
[16]G. Remenyi, S. Ishida, G. Landwehr, A. Briggs, and E. Bangert, in: Fifth Int. Conf. Electronic Properties of 2D-Systems, Sept. 1983, England, p. 49.
[17]A. I. Kurnosov and V.V. Yudin, Tekhnologiya Proizvodstva Poluprovodnikovykh Priborov (Semiconductor Fabrication Techniques), Vysshaya Shkola, Moscow (1974), p. 48.
[18]P. W. Anderson, E. Abrahams, and T. V. Ramakrishnan, Phys. Rev. Lett. **43**, 718 (1979).
[19]A. A. Abrikosov, Vvedenie v Teoriyu Normal'nykh Metallov (Introduction to the Theory of Normal Metals), Nauka, Moscow (1972), p. 81.
[20]M. L. Cohen, "Superconductivity in low-carrier-density systems: degenerate semiconductors," Chap. 12, p. 615; G. Gladstone, M. A. Jensen, and J. R. Schrieffer, "Superconductivity in the transition metals: theory and experiment," Chap. 13, p. 665, in: Superconductivity (ed. by R. D. Parks), Dekker, New York (1969).
[21]A. N. Ionov, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 378 (1982) [JETP Lett. **35**, 466 (1982)].
[22]B. L. Altshuler, A. G. Aronov, and D. E. Khmelnitsky, J. Phys. C **15**, 7367 (1982).
[23]M. E. Gershenzon, V. N. Gubankov, and Yu. E. Zhuravlev, Zh. Eksp. Teor. Fiz. **85**, 287 (1983) [Sov. Phys. JETP **58**, 167 (1983)].
[24]A. M. Finkel'shteĭn, Zh. Eksp. Teor. Fiz. **84**, 168 (1983); **86**, 366 (1984) [Sov. Phys. JETP **57**, 97 (1983); **59**, 212 (1984)].
[25]A. M. Finkel'stein, Z. Phys. B **56**, 189 (1984).

Translated by A. Mason