Gunn effect in narrow-band conductors

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An exact theory for the Gunn effect in narrow-band conductors at high temperatures is derived through a solution of the Fokker-Planck equation for the kinetics of the heavy particles in a strong electric field. Explicit analytic solutions are found for the structure of the moving domains, and their stability is studied. The effect of thermal diffusion on the velocity of the Gunn domains is studied. The dependence of this velocity on the shape of the domains and the boundary conditions is studied. The possibility of experimentally observing the Gunn effect in various types of conductors with narrow allowed bands is discussed.

1. INTRODUCTION

Several compounds and systems with narrow allowed bands have recently been studied in connection with the search for conductors of new types. Among them there have been many quasi-1D organic conductors and semiconductors[1] and some molecular crystals and polymers[2,3] in which the width of the allowed electron bands is $M \lesssim 0.01$–0.1 eV. In some of these compounds, e.g., anthracene,[2] tetracene, and tetrabenzofulvalene,[3] a substantial nonlinearity of the voltage-current characteristics $j(E)$ has been observed in strong electric fields $E \sim 10^4$–10^5 V/cm. The current reaches saturation, and the differential conductivity $\sigma_D = dj/dE$ vanishes. The characteristic field (E_0) at which the pronounced nonlinearity sets in depends strongly on the temperature T, decreasing significantly as the temperature is lowered from 290 to 140 K (Ref. 2). This behavior suggests that σ_D may go negative in stronger fields or at lower temperatures, so that a domain instability and the Gunn effect may occur.[4–8]

The simplest mechanism to explain the saturation and subsequent decrease of $j(E)$ is the finite nature of the motion of the electrons in the narrow bands in a strong field. This effect leads to the asymptotic decrease $j(E) \propto E^{-1}$ as $E \to \infty$ which was studied by Keldysh[9] and Bychkov and Dykhne.[10] The functional dependence $j(E)$ for such systems has been calculated[11] explicitly for arbitrary E. Those calculations used the Fokker-Planck equation[12] describing the kinetics of heavy particles which are scattered inelastically by phonons. This equation had been used earlier by Bychkov and Dykhne[10] to calculate the decreasing asymptotic behavior of $j(E)$ in strong fields. In the case of an additive of the electron spectrum,

$$\varepsilon(\mathbf{p}) = \sum_{\alpha} \varepsilon_{\alpha}(p_{\alpha}), \quad \alpha = x, y, z \tag{1}$$

variables can be separated[11] in the three-dimensional Fokker-Planck equation,[12] and this equation reduces to three one-dimensional equations of the type

$$\frac{\partial f}{\partial t} + v\frac{df}{\partial X} + eE\frac{\partial f}{\partial p} = \frac{\partial}{\partial p}\left(Af + B\frac{\partial f}{\partial p}\right),$$

$$A = B\frac{v}{T}, \quad v = \frac{\partial \varepsilon}{\partial p}, \tag{2}$$

where T is the temperature of the phonon heat reservoir.

It is assumed here that B does not depend on p. This is true in, for example, systems with a band whose width M is smaller than the characteristic phonon frequencies ω_0. For such systems, two-phonon Compton processes become the primary mechanism for inelastic scattering.[11] The electron spectrum is additive [see (1)] in the strong-coupling approximation, in which case we have

$$\varepsilon(p) = -M\cos(pa). \tag{3}$$

This approximation usually gives a good description of the electron dispersion relation for narrow-band organic conductors[1] and molecular crystals.[2,3] Equation (1) also describes the kinetics of electrons in quasi-1D organic conductors[1] and molecular crystals.[2,3] Equation (1) also describes the kinetics of electrons in quasi-1D organic conductors and polymers and in 1D superlattice,[13] where, despite the 3D electron spectrum, it gives a good description of the kinetics of the carriers in strong, uniform, time-dependent electric fields.[14] This success apparently indicates that the distribution function is a quasiequilibrium function of the transverse momentum components.

The region over which Eq. (2) applies to quasi-1D organic conductors and semiconductors[1] is quite broad. The reason is that the primary mechanisms for the inelastic scattering of electrons in such systems are their interactions with high-frequency intramolecular vibrations, which have a small dispersion,[15,16] $\Delta \sim 0.001$ eV $\ll \omega_0 \sim 0.1$ eV, and with low-frequency acoustic phonons with a low Debye frequency, $\omega_D \sim 0.001$ eV (Ref. 17). Because of the small quantities Δ, $\omega_D \ll M \sim 0.01$–0.1 eV, the scattering of the electrons is quasielastic, since in the case of high-frequency phonons this scattering can occur only as an effective Compton scattering with a small energy transfer $\sim\Delta$. In the quasi-1D case, a small energy transfer in forward scattering corresponds to a small momentum transfer, so that the Fokker-Planck approximation can be used.[12] By virtue of the quasielastic nature of the scattering here backscattering is equivalent to an impurity scattering whose incorporation leads to no more than an effective renormalization of certain parameters of the system, as we will show below. The slight change in energy which occurs during backscattering can also be incorporated in the Fokker-Planck approximation, in the form of corresponding differential terms.

The Fokker-Planck approximation should also hold

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well in 3D molecular crystals such as anthracene, since the bandwidth in these substances, estimated from the maximum electron velocity,[2,3] $v \sim 10^2$–10^4 cm/s, corresponds to $M \sim 10^{-3}$–10^{-4} eV $\ll \omega_D$, $\omega_0 \sim 0.01$ eV, so that again there would be an effective Compton scattering of phonons by an electron with a small momentum transfer at $T < \omega_D, \omega_0$ (Ref. 11).

An explicit expression for the voltage-current characteristic $j(E)$, with its characteristic peak, can easily be found directly from the solution of Eq. (2) in a static and uniform field E. In particular, at high temperatures, $T \gg M$, with dispersion law (3), we have[11]

$$j(E) = j_0 \frac{\beta}{2} \frac{\varepsilon}{1+\varepsilon^2}, \quad \varepsilon = \frac{E}{E_0},$$

$$E_0 = \frac{B}{e}, \quad j_0 = M\rho_0, \quad \beta = \frac{M}{T}, \tag{4}$$

where ρ_0/e is the average carrier density in the system, the lattice constant is $a = 1$, and the quantity B does not depend on p. Expression (4) gives a good description of the experimental data on the voltage-current characteristics in anthracene[2] at $E < E_0$ and provides a correct estimate at $E_0 \sim 10^5$ V/cm with reasonable values of the mean free path with respect to inelastic scattering, $l = M/B \sim 10$ Å. The substantial increase in l with decreasing temperature makes it possible to explain the significant decrease in E_0 in the interval[2] $140 < T < 290$ K, raising the hope that it will be possible to experimentally observe a descending region of the voltage-current characteristic and the Gunn effect at lower temperatures or stronger fields.

In the present paper we derive an exact theory for the Gunn effect in narrow-band conductors at temperature $T \gg M$. We show that all the characteristics of the system, including the explicit analytic solutions for the structure of moving domains, can be found directly from kinetic equation (2). The exact solutions which are found make it possible, in particular, to carry out a comparative analysis of the various phenomenological approaches which are ordinarily taken in the theory for the Gunn effect[4–8] and to test the validity of their underlying physical assumptions.

For example, it turns out that in this system, despite its pronounced deviation from equilibrium, the Einstein relation between the mobility $\mu(E)$ and the diffusion coefficient holds in lowest order in $\beta \ll 1$: $D(E) = T\mu(E)$, where T is the temperature of the phonon heat reservoir. This circumstance indicates that the phenomenological approach taken by Knight and Peterson,[4] which is based on the assumption of the Einstein relation, has certain advantages over other approximations,[5–8] which usually ignore any functional dependence $D(E)$ completely. It should be noted that the Einstein relation holds only in lowest order in β in the case under consideration in the present paper, so that in its most general form the relationship between D and μ is quite complicated. This point indicates that there are severe restrictions on the range of applicability of any phenomenological approach, including the Knight-Peterson approximation, in the theory of the Gunn effect.

The model which we discuss here can also resolve some questions which have not been studied in the phenomenological theory,[4–8] e.g., the effect of thermal diffusion on the domain velocity, which makes this velocity effectively a function of the shape of the domains.

We must emphasize that attempts to directly observe the Gunn effect experimentally in narrow-band conductors may run into definite difficulties, since even in organic compounds with a very nonlinear voltage-current characteristic[2,3] $j(E)$ it is not yet possible to go beyond the saturation region; stronger fields will apparently be required to observe the descending region of this characteristic. This is not an insurmountable problem, however, and it does not rule out the possibility that suitable systems will be found in the near future among the large number of organic compounds which are presently available[1–3] and whose behavior in strong electric fields has not been studied adequately.

The conditions are most favorable for the observation of the Gunn effect in semiconductors such as[1] $Cs_2(TCNQ)_3$ and (TEA) $(TCNQ)_2$ and also in corresponding molecular crystals and polymers.[2,3] Because of the low carrier density, $n \sim 10^{13}$–10^{15} cm^{-3}, the conductivity is $\sigma \sim 10^{-2}$–10^{-5} S/cm (Ref. 1), so that the maximum current density $j_1 \sim \sigma E_0$ which arises in the system is 0.1–1 A/cm^2 at the typical values $M \sim 0.01$ eV and $l \sim 10$–10^2 Å, at which we have $E_0 = M/el \sim 10^4$–10^5 V/cm. The low carrier density leads to a large Debye length $r_D \sim 10^2$–10^3 Å, which determines the characteristic width of the Gunn domains[4–8] and the range of applicability of the gradient expansions used in the present paper.

It is extremely difficult to observe the Gunn effect in compounds which are good conductors, with $n \sim 10^{18}$–10^{19} cm^{-3} and $\sigma \sim 10^2$–10^3 S/cm (Ref. 1), because of the small Debye length, $r_D \sim 10$ Å, and the high current density, $j_1 \sim 10^5$ A/cm^2, in strong fields $E \sim 10^3$ V/cm. This comment also applies to narrow-band metals such as U_2Zn_{17} and UBe_{13} (Ref. 18), where we find $M \sim 0.001$ eV, so that a significant nonlinearity of the voltage-current characteristic could also appear at fields as low as $E \sim 10^3$ V/cm. Furthermore, this effect may be strongly suppressed by the wider bands which may occur in these compounds along with the narrow f bands. In these systems, however, certain other types of domain instabilities, e.g., the thermoelectric instability, which occurs in normal metals and which is described by the same phenomenological methods[19] as are used to describe the Gunn effect, may be observed in relatively weak fields $E \sim 10$ V/cm.

In addition to the systems mentioned above, very narrow bands with $M \sim 10^{-4}$–10^{-5} eV correspond to the quantum tunneling of light ions[20–22] and the motion of defects in quantum crystals.[23] Narrow bands with $M \sim 0.01$ eV can also be produced artificially in semiconductors with a superlattice.[13,14] Such systems have recently attracted heightened interest,[24–26] as have the Gunn effect and other types of domain instabilities in solids.[19,26–28] A curious point is that the voltage-current characteristics of superlattice systems can be described quite well by expression (4) (Ref. 14).

2. BASIC EQUATIONS OF THE GUNN EFFECT

To derive the basic equations of the Gunn effect, we use Eq. (2) and a cosinusoidal dispersion law (3). Taking Fourier

transforms in the electron momentum p in that equations, we find the following system of equations for the Fourier compounds of the distribution function, f_m:

$$\left(\frac{\partial}{\partial t}+imeE+m^2B\right) f_m$$

$$=\frac{1}{2i}\left(-M\frac{\partial}{\partial X}+im\beta B\right)(f_{m-1}-f_{m+1}). \quad (5)$$

We see from the general structure of this equation that under the conditions $\beta\ll 1$ and $X\gg l=M/B$ it can be expanded in a series in these parameters. Doing so, we find a closed system of equations for the three lowest harmonics, f_0, f_1, and f_{-1}, which determine the charge density ρ, the current density j, and the average kinetic energy of electrons, W, respectively:

$$\rho=ef_0, \quad j=\frac{Me}{2i}(f_{-1}-f_1), \quad W=-\frac{M}{2}(f_{-1}+f_1). \quad (6)$$

In this notation, the system of equations takes the following form, where we are including terms of up to third order in β and l/X inclusively and also the Poisson equation:

$$\frac{\partial E}{\partial X}=\frac{4\pi}{\varkappa}(\rho-\rho_0), \quad \frac{\partial\rho}{\partial t}+\frac{\partial j}{\partial X}=0, \quad (7)$$

$$Ej-\left(B+\frac{\partial}{\partial t}\right)W-\frac{M}{2e}\beta B\rho=\frac{M}{e}\frac{\partial j_2}{\partial X}+\beta BW_2, \quad (8)$$

$$\left(\frac{\partial}{\partial t}+B\right)j+e^2EW+\frac{M^2}{2}\frac{\partial\rho}{\partial X}=Me\frac{\partial W_2}{\partial X}+\beta Bj_2, \quad (9)$$

where $\varkappa$ is the static dielectric constant,

$$j_2=\frac{Me}{4}\chi_0(4B\chi_2-2eE\chi_1), \quad W_2=\frac{M}{4}\chi_0(4B\chi_1+2eE\chi_2), \quad (10)$$

and the quantities χ_i are defined by

$$\chi_0=(16B^2+4e^2E^2)^{-1},$$

$$\chi_1=\frac{1}{e}\frac{\partial j}{\partial X}-\frac{2\beta B}{M}W, \quad \chi_2=2\beta\frac{Bj}{Me}-\frac{\partial W}{\partial X}. \quad (11)$$

Equations (8) and (9) describe the relaxation of the electron energy, with allowance for Joule heating, and the relaxation of the electric current, with allowance for diffusion terms, respectively.

From Eqs. (7) we find, in particular, the known relationship between j and the given external current J:

$$j=J-\frac{\varkappa}{4\pi}\frac{\partial E}{\partial t}. \quad (11a)$$

The condition $X\gg l$ clearly holds if the carrier density is not too high for Gunn domains with a scale dimension[4-8] $X\sim r_D=(\varkappa T/4\pi e\rho_0)^{1/2}$. When this condition holds, we might note, the scale times $t\sim X/v\gg l/v\sim B^{-1}$ are also quite long. Accordingly, in the lowest order in these parameters we can omit from Eqs. (8) and (9) the terms containing time derivatives and also the small terms on the right side.

As a result, these equations simplify considerably, becoming

$$BW=Ej-\frac{M}{2e}\beta B\rho, \quad Bj=-\frac{M^2}{2}\frac{\partial\rho}{\partial X}-e^2EW. \quad (12)$$

In particular, in the case of a uniform, static external field E we find expression (4) for $j(E)$ from these equations.

In terms of the dimensionless variables

$$\tau=t/\tau_0, \quad \tau_0=\varkappa E_0/4\pi\rho_0V(E_0), \quad \varepsilon=E/E_0, \quad (13)$$

$$x=X/l_0, \quad l_0=V(E_0)\tau_0=\beta r_D^2/l, \quad V(E_0)=\beta M/4 \quad (14)$$

we find the the dimensionless quantities

$$\tilde{j}=j/\rho_0V(E_0), \quad f=J/\rho_0V(E_0), \quad \tilde{\rho}=\rho/\rho_0, \quad w=4\pi W/\varkappa E_0^2 \quad (15)$$

and given in lowest order in β and X/l by the following equations:

$$\tilde{j}=f-\varepsilon_\tau, \quad \tilde{\rho}=\varepsilon_x+1, \quad b=(2r_D/l)^2, \quad (16)$$

$$bw-\varepsilon\tilde{j}=-2\tilde{\rho}, \quad b\tilde{j}+\varepsilon wb^2=-8\tilde{\rho}_x/\beta^2. \quad (17)$$

The subscripts x and τ mean differentiation. Eliminating the variables $\tilde{j}$, $\tilde{\rho}$, and w from these equations, we find the following equation for the electric field $\varepsilon(x,\tau)$:

$$-d(\varepsilon)\varepsilon_{xx}+v(\varepsilon)(\varepsilon_x+1)+\varepsilon_\tau=f, \quad \varepsilon_x\geqslant -1, \quad (18)$$

where

$$v(\varepsilon)=2\varepsilon/(1+\varepsilon^2), \quad d(\varepsilon)=d_0/(1+\varepsilon^2), \quad d_0=2(l/\beta r_D)^2. \quad (19)$$

The constant d_0, which determines the diffusion coefficient $d(\varepsilon)$ in (19), is proportional to the square ratio of the two independent small parameters $l/r_D\ll 1$ and $\beta\ll 1$. In the most general case, this constant can thus be much larger than unity, it can be much smaller than unity, or it can have an intermediate value $d_0\sim 1$.

Equation (18) has the form of a nonlinear diffusion equation and a simple physical meaning, corresponding to the usual balance between the field current and the diffusion current; the displacement current is also involved here. In lowest order in the small gradients, however, the thermal diffusion of the type ε_x^2 should also appear in this equation; in the case at hand, these terms are small, on the order of the parameter β. As we will see below, retaining these terms leads to some important effects, in particular, a dependence of the velocity of the Gunn domains on their shape.

The general structure of Eq. (18) with arbitrary functions $d(\varepsilon)$ and $v(\varepsilon)$ has been studied in the phenomenological theory in several papers.[4-8] Because of the substantial latitude in the choice of these functions and the relationship between them in this case, several different approaches[4-8] have been taken to analyze these effects.

Noteworthy among these approaches is the phenomenological theory of Knight and Peterson,[4] in which an Einstein relation is assumed to hold between $\mu(\varepsilon)=v(\varepsilon)/\varepsilon$ and $d(\varepsilon)$, and it is also assumed that the function $v(\varepsilon)$ is N-shaped. In the case of the present paper, the situation is similar, differing only in an absence of the increase in $v(\varepsilon)$ in very strong fields. As a result, there are some qualitative differences, but we can still make use of several of the general results of Ref. 4, and we can show that the Knight-Peterson approach has advantages over the other approximations.[5-8]

3. STRUCTURE OF MOVING DOMAINS

We are interested in traveling-wave solutions $\varepsilon(x,\tau)=\varepsilon(x-c\tau)=\varepsilon(\xi)$ of Eq. (18) with a periodic ξ depen-

dence corresponding to the condition of overall electrical neutrality of the system.[6] Substituting this solution into (18), we find the following ordinary differential equation for $\varepsilon(\xi)$ for a constant external current f:

$$-d(\varepsilon)\varepsilon_{\xi\xi}+(v(\varepsilon)-c)\varepsilon_{\xi}=f-v(\varepsilon), \quad \varepsilon_{\xi}\geqslant -1. \tag{20}$$

This equation can be rewritten for an arbitrary domain velocity c as[4]

$$-(\varepsilon_{\xi}+1)d(\varepsilon)\frac{d}{d\varepsilon}\bar{C}=\varepsilon_{\xi}(f-c), \tag{21}$$

where

$$-\bar{C}=\ln(\varepsilon_{\xi}+1)-\varepsilon_{\xi}+\int d\varepsilon\,(v(\varepsilon)-c)\,d^{-1}(\varepsilon). \tag{22}$$

It follows from (21) and (22) that Eq. (18) has closed integral curves and periodic solutions $\varepsilon(\xi)$ only with $c=f$, so that the domain velocity is determined exclusively by the external current. Taking into account the explicit functions $d(\varepsilon)$ and $v(\varepsilon)$ in (19), we can put the general expression for the integral of motion in (22) in the following form:

$$d_0[\varepsilon_{\xi}-\ln(\varepsilon_{\xi}+1)]=C+\varepsilon^2-\varepsilon f(1+{}^1/_3\varepsilon^2)\equiv\varphi(\varepsilon), \quad C=\bar{C}d_0. \tag{23}$$

If periodic solutions are to exist, the equation $\varepsilon_{\xi}=0$, which is equivalent to the condition $\varphi(\varepsilon)=0$, must have three real roots $\varepsilon_1\geqslant\varepsilon_2\geqslant\varepsilon_3$, so that the constant C must satisfy the condition

$$C_-\leqslant C\leqslant C_+, \quad C_{\pm}=1-\frac{2}{3f^2}[1\mp(1-f^2)^{3/2}]. \tag{24}$$

Here the quantities ε_1 and ε_2 determine the maximum and minimum values of the electric field in a domain.

In the general case $d_0\sim 1$, we would need to solve transcendental equation (23) for ε_{ξ} in order to calculate $\varepsilon(\xi)$. Figure 1 shows a plot of the corresponding function $\varepsilon(\xi)$ for the values $d_0=1, f=1/2$, and $C=-1$. We note that the shape of the domain is very asymmetric in this case; we also note that the maximum field in the domain is quite high, and with $C<0$ it goes off to infinity in accordance with $\varepsilon_1\approx 3/f$ in the limit $f\to 0$. This situation does not arise in the case of an ordinary N-shaped voltage-current characteristic,[4-8] in which case we have $f\geqslant f_0$, and f_0 is the local minimum of $v(\varepsilon)$.

The asymmetry of the shape of the domains depends strongly on d_0 and disappears entirely at $d_0\gg 1$, as obviously follows from Eq. (20), where terms of even parity in ξ are predominant at large values of d_0. In the case $d_0\ll 1$, the asymmetry becomes very pronounced, and the domain structure converts into a sawtooth curve with characteristic values $\varepsilon_{\xi}\gg 1$ and $\varepsilon_{\xi}\approx -1$:

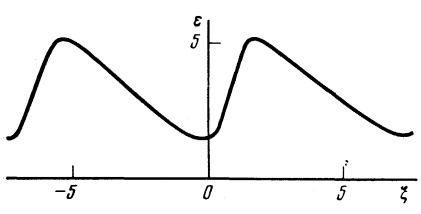

FIG. 1. Field distribution in a domain for the case $d_0=1$, $f=1/2$, $C=-1$.

$$\varepsilon_{\xi}\approx\varphi(\varepsilon)/d_0 \text{ for } \varepsilon_{\xi}>0$$

$$\text{and } \varepsilon_{\xi}+1\approx\exp(-\varphi(\varepsilon)/d_0) \text{ for } \varepsilon_{\xi}<0. \tag{25}$$

The period (Ω_0) of the domain structure is $\varepsilon_1-\varepsilon_2$ in this case.

If $d_0\gg 1$, the left side of transcendental equation (23) can be expanded in the small quantity $\varepsilon_{\xi}\ll 1$, and as a result we find

$$\xi(\varepsilon)=2\left(\frac{d_0}{\varepsilon_1-\varepsilon_3}\right)^{1/2}F(\varphi,k),$$

$$k^2=\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1-\varepsilon_3}, \quad k^2\sin^2\varphi=\frac{\varepsilon-\varepsilon_2}{\varepsilon-\varepsilon_3}, \tag{26}$$

where $F(\varphi,k)$ is an elliptic integral of the first kind. In this case the function $\varepsilon(\xi)$ can be expressed in terms of the elliptic sine[29] $\mathrm{sn}(\xi',k)$:

$$\varepsilon(\xi)=dn^{-2}(\xi',k)\,[\varepsilon_2-\varepsilon_3k^2\,\mathrm{sn}^2(\xi',k)],$$

$$dn^2(\xi',k)=1-k^2\,\mathrm{sn}^2(\xi',k), \quad \xi'=\xi\left(\frac{\varepsilon_1-\varepsilon_3}{4d_0}\right)^{1/2}. \tag{27}$$

Figure 2 shows a corresponding curve of ε versus $\tilde{\xi}=\xi/d_0^{1/2}$ for $f=1/2$ and $C=-1$. The period (Ω_0) of the superstructure is determined in this case by Eq. (26) with $\varphi=\pi/2$:

$$\Omega_0=2\left(\frac{d_0}{\varepsilon_1-\varepsilon_3}\right)^{1/2}K(k), \tag{28}$$

where K is the complete elliptic integral of the first kind. In ordinary units, we would have $\Omega_0\sim r_D$ at $d_0\gtrsim 1$ and $\Omega_0\sim l_0\sim(l/\beta r_D^2)^{-1}\gg r_D$ at $d_0\ll 1$.

In the case $f\approx 1$, the domain structure is smoothed over, and we have $\varepsilon_2\approx\varepsilon_2\approx 1$, and $\Omega_0\propto(1-f)^{-1/2}$. Analogously, this structure disappears as $C\to C_-$ for arbitrary f, with $\varepsilon_1\approx\varepsilon_2\approx\varepsilon_0$, where

$$\varepsilon_0=(1+(1-f^2)^{1/2})/f. \tag{29}$$

In this case, expressions (26)–(28) become valid for arbitrary $d_0\gtrsim 1$ and simplify considerably[29]:

$$\varepsilon(\xi)=\varepsilon_2+(\varepsilon_1-\varepsilon_2)\sin^2\xi'. \tag{30}$$

In a similar way we find the functional dependence $\varepsilon(\xi)$ as $C\to C_+$ with $\varepsilon_2\approx\varepsilon_3$ (Ref. 29):

$$\varepsilon(\xi)=\varepsilon_3+(\varepsilon_2-\varepsilon_3)/(1+(1-k^2)\,\mathrm{ch}\,2\xi'). \tag{31}$$

In this case, Ω_0 tends toward infinity logarithmically.

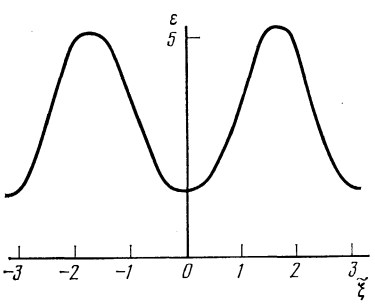

FIG. 2. Field distribution in a domain in the case $f=1/2$, $C=-1$, $d_0\gg 1$.

The structure of the moving domains with $d_0 \gg 1$ is analogous to that of the ordinary soliton solutions of the Korteweg-de Vries equation.[12,30] This comment applies, however, only to traveling-wave solutions. It does not mean that the original equation, (18), is integrable by the method of the inverse scattering problem.[30] Furthermore, analysis of Eq. (18) with the help of the general criteria for the integrability of nonlinear equations suggested by Sokolov and Shabat[31] proves that it is not integrable by the method of the inverse scattering problem or by other methods of nonlinear wave theory.[30]

4. STABILITY OF THE SOLUTIONS

Knight and Peterson[4] have derived a general theory for the stability of the solutions of Eq. (18) for arbitrary functions $d(\varepsilon)$ and $v(\varepsilon)$. They proved that the domain structures are unstable in the case of a given external current f, while they are stable in the case of a given voltage with a negative static impedance $Z(0) = d\bar{\varepsilon}/df$ (which describes the behavior of the current through a sample with a domain as a function of the average field $\bar{\varepsilon}$).

Making use the explicit functional dependence $\varepsilon(\xi)$ at $d_0 \gg 1$, in (26)–(31), we can put these general results in a more specific form and estimate the instability growth rate λ_0 in the regime of a given current. We can also calculate $Z(0)$ and determine the region in which domain structure exist. The quantity λ_0 characterizes the region of frequency dispersion, $Z(\omega)$, at small[4-8] $\omega \gtrsim \lambda_0/\tau_0$ and determines the extent to which weak signals are amplified at low frequencies imposed on the strong static field.

To evaluate λ_0 we need to linearize Eq. (18) near the zeroth-order solution $\varepsilon_0(\xi)$, determined at $d_0 \gg 1$ by Eqs. (26)–(31), and analyze a small increment $\varepsilon_1(x, \tau) = e^{\lambda\tau}\varepsilon_1(\xi)$. As a result we find an eigenvalue equation for the instability growth rate λ:

$$\left[-\frac{d^2}{d\tilde{\xi}^2} + \lambda(1+\varepsilon_0^2(\tilde{\xi})) + 2(1-f\varepsilon_0(\tilde{\xi}))\right]\varepsilon_1(\tilde{\xi}) = 0, \quad \tilde{\xi} = \xi/d_0^{1/2}. \tag{32}$$

This equation is equivalent to a Schrödinger equation with an effective potential

$$U(\tilde{\xi}) = \lambda\varepsilon_0^2(\tilde{\xi}) + 2(1-f\varepsilon_0(\tilde{\xi})) \tag{33}$$

which depends on λ.

We are interested in the maximum positive value of λ_0, so we should seek a ground state in potential (33) with a negative energy ($-\lambda_0$). That such a state exists follows from the circumstance that the eigenvalue $\lambda = 0$ in (32) corresponds to the solution $\varepsilon_1(\xi) = \varepsilon_{0\xi}$, which has zeros at the points $\varphi(\varepsilon) = 0$ [see (23)]. It follows from the fact that $U(\tilde{\xi})$ goes negative at at least certain values of $\tilde{\xi}$ that the inequality $\lambda_0 < f^2/2$ holds; from this inequality, even at $f < 1/2$, we find $\lambda_0 \lesssim 0.1$. This inequality also implies that the instability growth rate is numerically small. At small values of f, the quantity λ_0 approaches zero rapidly, and the domain structure could apparently be observed even at a fixed external current. Curiously, in the case of the ordinary mechanism for the Gunn effect an instability of this sort has been observed directly in experiments in III–V semiconductors.[6-8]

The quantity λ_0 vanishes as $f \to 1$, since here we have $\varepsilon_0(\xi) \approx 1$, and the absolute minimum $U(\tilde{\xi}) = 0$ in (33) is reached only with $\lambda = 0$. While remaining a small quantity, λ_0 depends quite strongly on the constant C in (23) and on the current f. The explicit functional dependence $\lambda_0(f)$ can be found easily in the case $C \to C_{\pm}$, in which $\varepsilon_0(\tilde{\xi})$ depends weakly on $\tilde{\xi}$, the potential $U(\tilde{\xi})$ becomes semiclassical, and λ_0 is determined by the position of its bottom at $\varepsilon_0(\xi) \approx \varepsilon_0$ in (29). In the limit $C \to C_{\pm}$ we thus find

$$\lambda_0(f) = f^2 - 1 + (1-f^2)^{1/2}. \tag{34}$$

At $f \ll 1$ we have $\lambda_0(f) \approx f^2/2$; as $f \to 1$ we have $\lambda_0 \approx (2(1-f))^{1/2}$; and the maximum value $\lambda_0 = 1/4$ is reached at $f = \sqrt{3/2}$. The small value of λ_0 at $f \ll 1$ and as $f \to 1$ indicates a substantial dispersion of the impedance $Z(\omega)$ at low frequencies $\omega \ll \tau_0^{-1}$.

The periodic structure of the functional dependence $\varepsilon_0(\xi)$ gives rise to an entire narrow band of values of λ, so that the results above refer to the maximum value of λ_0, which corresponds to the top of the band.

The criterion for the stability of the domain solutions of Eq. (18) in the case of a given external voltage reduces to the condition[4-8] $Z(0) = d\bar{\varepsilon}/df < 0$. This condition follows in an obvious way from the general structure of Eq. (18), since as $\varepsilon(\xi)$ is varied with unfixed f in (32) there is a shift of λ by an amount $Z^{-1}(0) = \delta f/\delta\varepsilon$, which can give rise to negative values of the growth rate, which would mean that the domain solutions are stable.

To calculate $Z(0)$ at $d_0 \gg 1$ we use the explicit expression for $\varepsilon(\xi)$ in (27). Averaging ε over the period Ω_0 in (28), and using some known relations,[29] we find

$$\bar{\varepsilon} = \varepsilon_3 + (\varepsilon_1 - \varepsilon_3)E(k)/K(k), \tag{35}$$

where $E(k)$ is the complete elliptic integral of the second kind. Figure 3 shows a curve of the voltage-current characteristic $f(\bar{\varepsilon})$ corresponding to (35) for the case $C = -1$. At $C < 0$, the voltage-current characteristic of a sample with a domain begins at the point $\bar{\varepsilon} = \bar{\varepsilon}_c$, $f = f_c$, determined from the conditions $C_-(f_c) = C$ and $\bar{\varepsilon}_c = (1 + (1-f_c^2)^{1/2})/f_c$; it then decreases to zero as $\bar{\varepsilon} \to \infty$ in accordance with a law determined by the asymptotic relation $\bar{\varepsilon}(f) \approx -6/f\ln f$ as $f \to 0$. In the region $\bar{\varepsilon} > \bar{\varepsilon}_c$, we have $Z(0) < 0$, and the domain structure is stable. At the point $\bar{\varepsilon} = \bar{\varepsilon}_c$ there is an abrupt change in f, corresponding to a metastability of the lower branch of the voltage-current characteristic at $\bar{\varepsilon} < \bar{\varepsilon}_c$, which is typical of Gunn systems of all types.[6-8]

In the case $0 < C < 1/3$, at large values of $\bar{\varepsilon}$ there is also a critical value $\bar{\varepsilon}_{c1}$, determined from the conditions $C = C_+(f_{c1})$, $\bar{\varepsilon}_{c1} = (1 + (1-f_{c1}^2)^{1/2})/f_{c1}$. Figure 4 shows a

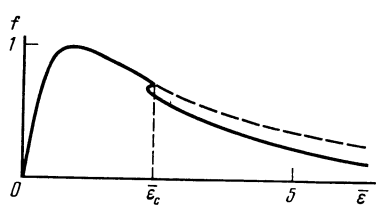

FIG. 3. Voltage-current characteristic of a sample with a domain at $C = -1$ in the case $d_0 \gg 1$.

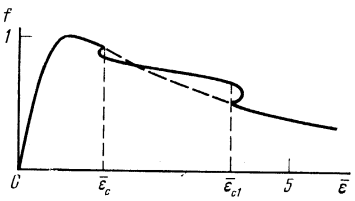

FIG. 4. Voltage-current characteristic of a sample with a domain for $C = 0.05$ in the case $d_0 \gg 1$.

plot of $f(\bar{\varepsilon})$ for $C = 0.05$. As $C \to 0$ we have $\bar{\varepsilon}_{c1} \to \infty$; as $C \to 1/3$ we have $\bar{\varepsilon}_{c1} \to \bar{\varepsilon}_c$, and the region in which domain structures exists contracts to zero. The change in the current at the point $\bar{\varepsilon}_{c1}$ also occurs abruptly; the current vanishes as $f \to 1$ in accordance with $(1-f)^2$.

With a large number of domains the condition $d\bar{\varepsilon}/df < 0$ corresponds to stability with respect to periodic perturbations with a period[6-8] Ω_0. The system may be unstable with respect to distortions of the periodic structure; this instability is described by the same growth rate[8] λ_0 as in the case of a fixed value of f. This instability accordingly develops relatively slowly, at least at low currents, $f \lesssim 0.1$, at which we have $\lambda_0 \lesssim 10^{-3}$ according to (34), and a rather large number of domains, on the order of λ_0^{-1}, can be observed in the system.

5. EFFECTS OF THERMAL DIFFUSION

As was shown above, Eq. (18), which we derived from our original equations, (7)–(11), in lowest order in β and l/r_D, has simple solutions in the form of periodic waves which are traveling at a velocity $c = f$ which does not depend on their shape and which is determined by the constant C in (23). The general structure of Eq. (18) corresponds to an ordinary gradient expansion for the current j, so that along with the purely diffusive term $d(\varepsilon)\varepsilon_{xx}$ this equation might contain terms of the type $\Gamma(\varepsilon)\varepsilon_x^2$, which have a "gradient smallness" on the same order of magnitude, and terms which are analogous to thermal diffusion,[12] caused by a dependence of d on the coordinates. As is clear from Eqs. (20) and (21), such terms may violate the condition for the existence of closed integral curves and periodic solutions $\varepsilon(\xi)$ for Eq. (20), having a significant effect on the nature of the domain instability.[6] This circumstance was originally pointed out by Anisimov *et al.*[5] in one of the phenomenological approaches possible.

In the case at hand the thermal-diffusion terms are small, on the order of $\beta \ll 1$, and in the lowest order in this parameter they do not affect the general structure of Eq. (18) or the nature of the domain solutions. These terms do, however, become important in the following orders in β, causing the velocity c to become a weak function of the domain structure.

To study this dependence we use our original equations (8) and (9), retaining in them small terms of the next higher order, which are described by the terms on the right side. In the calculation of the small quantities j_2 and W_2 we must use Eqs. (12), which were derived in the lowest order in β and l/r_D. Finally, in terms of the dimensionless variables (13)–(15), Eqs. (8) and (9) become

$$w_\tau + bw - \varepsilon\tilde{j} + 2\tilde{\rho} = -\frac{1}{8}\left(\frac{1}{b}\tilde{j}_{2x} + \beta^2 w_2\right), \tag{36}$$

$$\tilde{j}_\tau + b\tilde{j} + \varepsilon w b^2 + \frac{8}{\beta^2}\rho_x = \frac{1}{2}\left(w_{2x} + \frac{\beta^2}{4} b\tilde{j}_2\right), \tag{37}$$

where

$$\left(1+\frac{\varepsilon^2}{4}\right) w_2 = \frac{1}{2}\left(\tilde{j}_x - \frac{4}{d_0} bw\right) + 2\varepsilon\left(\tilde{j} - \frac{d_0}{4} bw_x\right), \tag{38}$$

$$\left(1+\frac{\varepsilon^2}{4}\right)\tilde{j}_2 = \tilde{j} - \frac{d_0}{4} bw_x - \frac{\varepsilon}{4}\left(\tilde{j}_x - \frac{4}{d_0} bw\right). \tag{39}$$

We are interested in a traveling-wave solution of Eqs. (36) and (37). Accordingly, substituting $\varepsilon(x, \tau) = \varepsilon(\xi)$ into these equations, and eliminating $\rho, \tilde{j}$, and w with the help of (16), we find

$$f - c = -d(\varepsilon)\varepsilon_{\xi\xi\xi} + (v(\varepsilon) - c)(\varepsilon_\xi + 1) - \Gamma(\varepsilon)\varepsilon_\xi(\varepsilon_\xi + 1) - \Gamma_1(\varepsilon)\varepsilon_\xi^2(\varepsilon_\xi + 1) - \Gamma_2(\varepsilon)\varepsilon_{\xi\xi\xi}(\varepsilon_\xi + 1) - \Gamma_3(\varepsilon)\varepsilon_{\xi\xi\xi\xi}, \tag{40}$$

where

$$\Gamma(\varepsilon) = \frac{\beta^2}{16}\frac{4}{(1+\varepsilon^2)(4+\varepsilon^2)}\left\{\varepsilon d_0\left(2 + \varepsilon f + \frac{1}{2}\varepsilon^2\right) + \frac{1}{2}\varepsilon(1-\varepsilon f) + f\left(4 - \frac{7}{2}d_0\right) + \frac{4\varepsilon}{4+\varepsilon^2}\left[\varepsilon f\left(\frac{17}{8}d_0 - 2\right) + \frac{1}{8}\varepsilon^2(\varepsilon f - 2) + 4 + \frac{1}{2}fd_0\right]\right\}, \tag{41}$$

$$\Gamma_1(\varepsilon) = \frac{\beta^2}{16}\frac{16 f d_0^2}{(1+\varepsilon^2)(4+\varepsilon^2)^2}\left(1 - \frac{1}{32}\varepsilon^2\right), \tag{42}$$

$$\Gamma_2(\varepsilon) = \frac{\beta^2}{16}\frac{4d_0}{(1+\varepsilon^2)(4+\varepsilon^2)}\left\{\frac{\varepsilon f}{16}(35d_0 + 1) - 2d_0 - \frac{1}{8}\frac{\varepsilon^2}{(4+\varepsilon^2)}[d_0(\varepsilon f - 2) + \varepsilon f]\right\}, \tag{43}$$

$$\Gamma_3(\varepsilon) = \frac{\beta^2}{16}\frac{4\varepsilon d_0}{(1+\varepsilon^2)(4+\varepsilon^2)}\left\{\frac{17}{16}d_0(\varepsilon f - 2) + \frac{1}{16}\varepsilon f\right\}. \tag{44}$$

Equations (41)–(44) were derived in lowest order in β, so we have set $c = f$ in them in the spirit of the zeroth approximation. We are also omitting small corrections $\sim \beta^2/16$ to $d(\varepsilon)$ and $v(\varepsilon)$, since they lead to only insignificant changes in the shape of the domains.

6. DOMAIN VELOCITY

To calculate the small corrections to the velocity $c = f$ we can use the van der Pol method.[32,33] For this purpose it is convenient to rewrite Eq. (40) as

$$-d(\varepsilon)\frac{d}{d\varepsilon}\tilde{C} = (f-c)\frac{\varepsilon_\xi}{\varepsilon_\xi + 1} + \varepsilon_\xi\left[\varepsilon_\xi\Gamma(\varepsilon) + \varepsilon_\xi^2\Gamma_1(\varepsilon) + \varepsilon_{\xi\xi\xi}\Gamma_2(\varepsilon) + \frac{\varepsilon_{\xi\xi\xi\xi}}{\varepsilon_\xi + 1}\Gamma_3(\varepsilon)\right], \tag{45}$$

where $\tilde{C}$ is determined by (22) and (23).

Taking the average of the right side of (45) over the period Ω_0, we find the following expression for the velocity c from the condition for the conservation of integral (22):

$$c=f+\delta(\tilde{C})=f+\alpha(\tilde{C})/\gamma(\tilde{C}), \tag{46}$$

where

$$\alpha(\tilde{C})=\int_0^{\Omega_0}\frac{\varepsilon_\xi\, d\xi}{d(\varepsilon)}\left[\varepsilon_\xi\bar{\Gamma}(\varepsilon)+\varepsilon_\xi^2\Gamma_1(\varepsilon)+\varepsilon_{\xi\xi}\Gamma_2(\varepsilon)+\frac{\varepsilon_{\xi\xi\xi}}{\varepsilon_\xi+1}\Gamma_3(\varepsilon)\right], \tag{47}$$

$$\gamma(\tilde{C})=\int_0^{\Omega_0}\frac{d\xi}{d(\varepsilon)}\frac{\varepsilon_\xi}{\varepsilon_\xi+1}. \tag{48}$$

It can be seen from (46)–(48) that the velocity c now depends on the constant $\tilde{C}$, which determines the shape of a domain. To calculate this functional dependence we need explicit expressions for $\varepsilon(\xi)$, which can be found in the limiting cases $d_0\gg 1$ and $d_0\ll 1$.

In the first case, expanding (41)–(48) in the small parameter $d_0^{-1}\ll 1$, we find the following expressions, after some straightforward manipulations:

$$\alpha(C)=2\int_{\varepsilon_2}^{\varepsilon_1}\frac{d\varepsilon}{d(\varepsilon)}\left(\frac{2}{d_0}\varphi(\varepsilon)\right)^{1/2}\left[\bar{\Gamma}(\varepsilon)+\frac{2}{d_0}\varphi(\varepsilon)\left(\Gamma_1(\varepsilon)-\frac{1}{2}\Gamma_2'(\varepsilon)\right)+\frac{1}{d_0}\varphi''(\varepsilon)\Gamma_3(\varepsilon)\right], \tag{49}$$

$$\gamma(C)=-\frac{4}{3}\int_{\varepsilon_2}^{\varepsilon_1}\frac{d\varepsilon}{d(\varepsilon)}\left(\frac{2}{d_0}\varphi(\varepsilon)\right)^{1/2}. \tag{50}$$

A straightforward analysis of the integrand in (49) shows that we have $\alpha(C)>0$ for all the permissible values of the parameters and thus $c<f$. It also follows from this integrand that the expression in brackets in (49), divided by $\beta^2 d_0/16$, does not exceed 1/2, so we have $|\delta(C)|<3\beta^2 d_0/64=6(l/8r_D)^2$. Accordingly, even in the limit $d_0\gg 1$, the velocity shift $\delta(C)$ contains, in addition to the small parameter $(l/r_D)^2$, a small numerical factor ~ 0.1. This circumstance indicates that the deviations of c from f are slight, no greater than 10%, even at the range of applicability of the gradient expansions. This is a typical situation for all Gunn systems, and it has been observed repeatedly in numerical simulations of such systems (see the bibliographies in the monographs in Refs. 6–8).

Expressions (49) and (50) simplify dramatically as $C\to C_-$, where we have $\varepsilon_1\approx\varepsilon_2\approx\varepsilon_0$ in (29), and for $\delta(C_-)$ we find

$$\delta(C_-)=-\frac{3}{2}\left[\bar{\Gamma}(\varepsilon_0)+\frac{2}{d_0}\varphi(\varepsilon_0)\left(\Gamma_1(\varepsilon_0)-\frac{1}{2}\Gamma'(\varepsilon_0)\right)+\frac{1}{d_0}\varphi''(\varepsilon_0)\Gamma_3(\varepsilon_0)\right]. \tag{51}$$

In the case $d_0\ll 1$ the domains acquire a sawtooth shape with $\Omega_0=\varepsilon_1-\varepsilon_2$; this shape is determined by (25). It is not difficult to see that $\gamma(C)$ in (48) is dominated by the region $\varepsilon_\xi<0$, which makes an exponential contribution:

$$-\gamma(C)=\left(\frac{2\pi d_0}{\varphi''(\varepsilon_0)}\right)^{1/2}\frac{1}{d(\varepsilon_0)}\exp\left(\frac{\varphi(\varepsilon_0)}{d_0}\right). \tag{52}$$

The region $\varepsilon_\xi>0$ dominates the corresponding integral (47) in this case, and for $\alpha(C)$ we find the following expression after some straightforward manipulations:

$$\alpha(C)=(\beta/4d_0)^2(F(\varepsilon_1)-F(\varepsilon_2)), \tag{53}$$

where

$$F(\varepsilon)=\ln\left(1+\frac{\varepsilon^2}{4}\right)\left[44-C-\frac{f^2}{6}\left(169+\frac{9}{4}C\right)\right]+\operatorname{arctg}(\varepsilon/2)\left[\frac{f^2}{2}-10C+\frac{f}{2}(556+27C)\right]$$
$$-\varepsilon^5\frac{f^3}{360}+\varepsilon^4\frac{f^2}{24}+\varepsilon^3\frac{f}{12}\left(f^2+\frac{13}{6}\right)+\varepsilon^2\left(\frac{f^2}{24}(235+C)-1\right)$$
$$-\varepsilon\frac{f}{12}\left(1010+3C+11\frac{f^2}{2}\right)+\frac{4C}{4+\varepsilon^2}\left(\frac{f^2}{2}-10+C\varepsilon\frac{f}{32}\right). \tag{54}$$

It follows from (52)–(54) that the correction to the velocity c in the case $d_0\ll 1$ is exponentially small, on the order of d_0^{-1}. An elementary analysis of (54) shows that the sign of the correction depends strongly on f and C, as can be demonstrated particularly simply in the case $C\to C_-$, in which we have $\varepsilon_1\approx\varepsilon_2\approx\varepsilon_0$ and

$$\alpha(C_-)=(\beta/4d_0)^2(\varepsilon_1-\varepsilon_2)F'(\varepsilon_0). \tag{55}$$

At $f\ll 1$ we have $F'(\varepsilon_0)\approx -17f/8<0$, and as $f\to 1$ we have $F'(\varepsilon_0)\approx 22\varphi(\varepsilon_0)/5>0$, so that the difference $c-f$ changes sign in the intermediate region.

In the region $d_0\ll 1$, the small parameter in expansion (47) for $\alpha(C)$ in the gradients of ε is the quantity $\beta/4d_0=2\beta(\beta r_D/4l)^2\ll 1$, so that a necessary condition for the validity of this expansion is $d_0\gg\beta$, which is equivalent to the inequality $\beta\ll(l/r_D)^{2/3}$. Since this parameter is small, we can ignore the terms of higher order in ε_ξ in expansion (40). However, this parameter is formal in nature, required only for calculating the coefficient $[\alpha(C)]$ of the exponential function in $\delta(C)$. The reason for the exponentially small value of $\delta(C)$ at $d_0\ll 1$ is the general structure of Eq. (40) for arbitrary functions $d(\varepsilon)$ and $v(\varepsilon)$. These functions determine only the specific function $\varphi(\varepsilon)$ and the value of $\varepsilon_0>1$ from the equation $f=v(\varepsilon_0)$ in the exponential factor in (52).

Consequently, the circumstance that the corrections to the domain velocity $c=f$ are exponentially small in the case of a small dimensionless diffusion coefficient $d\sim D\tau_0/l_0^2\ll 1$ is a common property of all Gunn systems to which gradient expansion (40) applies. In such systems we have $d\sim v_0 l/Vl_0\lesssim v_0 l/Vr_D\ll 1$ to the extent that $l/r_D\ll 1$ is small, since the typical electron velocity is usually $v_0\sim V$.

The procedure outlined above for calculating the corrections to c in powers of β can also be used in higher orders of perturbation theory. Because of the small numerical factors, which are typical of narrow-band systems, these corrections are relatively small, and even at $\beta\sim 1$ they lead to only small changes in c, as is particularly clear at $d<1$, where they all contain exponentially small factors.

7. CONCLUSION

The conditions for experimentally observing the Gunn effect are most favorable in systems of the semiconductor type, in which we include molecular crystals such as anthracene and tetracene,[2,3] quasi-1D organic semiconductors

such as $Cs_2(TCNQ)_3$ and (TEA) $(TCNQ)_2$ (Ref. 1), and some polymer compounds.[3] The maximum domain velocity $c=\beta Ma/4$ in these compounds should reach 10^3–10^4 cm/s and should furthermore be significantly smaller at low currents. The characteristic dimensions of the Gunn domains are $\sim r_D \sim 10^2$–10^3 Å, so that the characteristic radiation frequencies are 10^8–10^{10} Hz and lie in the microwave range.

It should be noted that the condition $T>M$ does not hold in some of these compounds since M reaches a value of 0.1 eV and exceeds the maximum permissible value $T\approx 500$ K, above which these compounds decompose. This circumstance should not, however, have any serious consequences, since the effective expansion parameter is $\beta^2/16 \ll 1$, so that the shape and velocity of the domains should not undergo any pronounced changes.

The elastic scattering of electrons by impurities in such systems does not lead to any qualitative changes in the results derived here. It can easily be incorporated in Eq. (2). For this purpose, in the $1D$ case, it is sufficient to add a term $(1/2)\nu_i(f(p)-f(-p))$ to the right side of (2) to describe elastic backscattering with a frequency ν_i. The only consequence is a replacement of B by $B+\nu_i$ in (9); in the subsequent expressions, this change leads to only a slight renormalization of j_0 and E_0:

$$\tilde{j}_0=j_0(B/(B+\nu_i))^{1/2}, \quad \tilde{E}_0=E_0((B+\nu_i)/B)^{1/2}. \tag{56}$$

The physical reason for this renormalization is the diffusive nature of the motion of an electron because of elastic scattering between elastic collisions; the result is a replacement of l by $l'=(1/(l^{-1}+l_i^{-1}))^{1/2}$. Analogous results are found when we incorporate quasielastic backscattering by phonons. In dealing with the $1D$ kinetic equation and elastic scattering we are ignoring localization effects,[34] under the assumption that the degree of one-dimensionality of the electron spectrum is not very high.

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