Theory of surface superconductivity induced by a static electric field

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An electric field normal to the surface of a superconducting semiconductor or semimetal increases the electron density near the surface over a distance equal to the Debye screening length. This density increase enhances the electron-electron interaction and thereby gives rise to a localized superconducting state near the surface. A proximity effect allows this state to penetrate deep into the interior. A systematic theory of such states is derived. In particular, the critical temperature for the onset of surface superconductivity, the scale dimension of these states, their structure, and their magnetic moment are all determined.

I. INTRODUCTION

The behavior of superconducting metals and semiconductors in external electric fields is presently the subject of active research.[1–6] An external electric field directed normal to the surface of a sample causes curvature of the bottom of the conduction band, where localized electron states may arise as a result. In addition, the density of free electrons is increased near the surface; we assume here that these electrons are degenerate. These two phenomena can give rise to a nonuniform superconducting state near the surface. In contrast with the $2D$ superconductivity which is caused by electrons which form a surface band, the superconductivity due to free electrons must be three-dimensional, while remaining localized near the surface.

Sandormirskiĭ[3] has pointed out that an electric field might increase the critical temperature for the onset of superconductivity. Kelly and Hanke[4] and Takada[5] have calculated the surface critical temperature for $2D$ superconductivity in the p-type semiconductor InAs and also in Si. There are no free electrons in such a system. The surface superconducting state in such a system is found to extend ~ 40 Å. The localized energy level lies $\Delta E \sim 0.015$ eV from the continuous spectrum.

When a system contains close-lying localized and delocalized electron (or hole) energy levels, or if a system has no localized states at all (this is apparently the most common situation), a region enriched in charge carriers (for definiteness we will speak in terms of electrons below) forms in a surface layer. This effect in turn enhances the electron-electron interaction in a narrow surface layer, with the result that a region appears in which conditions are more favorable for Cooper pairing than in the interior of a superconductor. Because of a proximity effect, such electrons induce a superconductivity deep in the interior of the sample.

The proximity effect in systems with a nonuniform electron-electron interaction has been the subject of several theoretical papers.[7–11] In the present paper we derive a theory for surface superconducting states which are induced by a static external electric field.

2. BASIC EQUATIONS

A degenerate electron gas in an electric field is conveniently analyzed in the Thomas-Fermi approximation, in which the many-particle problem is reduced to a single-particle problem through the introduction of an effective electrostatic potential which depends on the coordinate (x) normal to the surface. The one-particle energy of the electron in this case can be written

$$\varepsilon(r) = P^2/2m - e\varphi(x), \tag{1}$$

where P is the momentum of the electron, m is its effective mass, and $\varphi(x)$ is the electrostatic potential. Using the Poisson equation and the condition for equilibrium of the system, we can easily derive the following well-known result for the electric field in a degenerate electron system:

$$E(x) = E\exp(-x/l_D), \quad l_D^2 = \mu_0\varepsilon_\infty/6\pi n_0 e^2, \tag{2}$$

where μ_0 is the Fermi energy far from the nonuniformity, n_0 is the electron density in the volume, and ε_∞ is the dielectric constant.

In the same approximation, the electron state density is written

$$N_0(x) = N_\infty[1 + \varkappa\exp(-x/l_D)], \qquad \varkappa = E/E^*, \quad E^* = 12\pi e l_D n_0/\varepsilon_\infty; \tag{3}$$

where N_∞ is the state density far from the surface. The semiclassical approximation, used in the derivation of these relations, is known to hold under the condition

$$\left|\frac{em}{P^3}\frac{d\varphi}{dx}\right| \ll 1, \quad \varkappa(\mu_B/\mu_0)^{1/2} \ll 1, \quad \mu_B = e^2 n_0^{1/3}. \tag{4}$$

Everywhere below, we assume that this condition holds.

In this approximation, the system of superconducting electrons is described by the Hamiltonian

$$\hat{\mathscr{H}} = \hat{\mathscr{H}}_0 + \hat{\mathscr{H}}_{int} + \hat{\mathscr{H}}_{imp},$$
$$\hat{\mathscr{H}}_0 = \int \psi_\sigma^+ \hat{\varepsilon}(\mathbf{r})\psi_{\sigma'} d^3\mathbf{r}, \quad \hat{\mathscr{H}}_{imp} = \sum_i \int \psi_\sigma^+ V(\mathbf{r}-\mathbf{r}_i)\psi_\sigma d^3\mathbf{r}, \tag{5}$$
$$\hat{\mathscr{H}}_{int} = \Lambda\int \psi_\sigma^+(\mathbf{r})\psi_{\sigma'}^+(\mathbf{r})\psi_\sigma(\mathbf{r})\psi_{\sigma'}(\mathbf{r})d^3\mathbf{r};$$

where $V(\mathbf{r}-\mathbf{r}_i)$ is the potential of a nonmagnetic impurity at the point with coordinates $\mathbf{r}_i$, ψ_σ^+ are fermion creation operators. Λ is the strength of the electron-electron interaction, and σ is a spin variable.

The system of Gor'kov equations found from (5) by the

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standard procedure[12] takes the following form near the curve of the coexistence of the normal and superconducting phases, after an average is taken over the positions of the impurities:

$$\Psi(\mathbf{r})=\Lambda\int\langle K(\mathbf{r},\mathbf{r}')\rangle\Psi(\mathbf{r}')\,d^3\mathbf{r}'$$
$$+\Lambda\int\langle R(\mathbf{r},\mathbf{m},\mathbf{l},\mathbf{s})\rangle\Psi(\mathbf{l})\Psi^+(\mathbf{m})\Psi(\mathbf{s})\,d^3\mathbf{l}\,d^3\mathbf{s}\,d^3\mathbf{m}. \quad (6)$$

Here $\Psi(\mathbf{r})$ is the order parameter, $\langle ... \rangle$ is an average over the impurity positions, and

$$K(\mathbf{r},\mathbf{r}')=T\sum_n G_\omega(\mathbf{r},\mathbf{r}')G_{-\omega}(\mathbf{r},\mathbf{r}'), \quad (7)$$

$$R(\mathbf{r},\mathbf{l},\mathbf{m},\mathbf{s})=T\sum_n G_\omega(\mathbf{r},\mathbf{l})G_{-\omega}(\mathbf{l},\mathbf{m})G_\omega(\mathbf{m},\mathbf{s})G_{-\omega}(\mathbf{s},\mathbf{r}). \quad (8)$$

Here $\omega_n=\pi T(2n+1)$, and the G_ω are temperature Green's functions. Expanding the Green's functions in the system of functions φ_s, and introducing a coordinate-dependent electron-state density at the Fermi surface[13] in the standard way

$$[\hat{\varepsilon}(\mathbf{r})-\mu_0]\varphi_s=\xi_s\varphi_s, \quad (9)$$
$$N_0(x)=\sum_s|\varphi_s(x)|^2\delta(\xi_s), \quad (10)$$

we find, in the semiclassical approximation, a system of equations describing the behavior of a dirty semiconductor over the entire temperature range,[14] with a coordinate-dependent quantity $g=N_0(x)V$:

$$\hat{L}\Psi^+(\mathbf{r})+\frac{1}{8\pi^2T^2}\sum_{n=0}^{\infty}\left(n+\frac{1}{2}+\frac{v_0^2\tau_{tr}}{48\pi T}\Pi^2\right)$$
$$\times\prod_{i=1}^{4}\left(n+\frac{1}{2}+\frac{v_0^2\tau_{tr}}{12\pi T}P_i^2\right)^{-1}$$
$$\times\Psi^+(\mathbf{r}_1)\Psi(\mathbf{r}_2)\Psi^+(\mathbf{r}_3)|_{r_1=r_2=r_3=r}=0, \quad (11)$$

where

$$\Pi^2=(\hat{P}_1-\hat{P}_3)^2+(\hat{P}_2-\hat{P}_4)^2,\quad P_i^2=[-i\partial_i+(-1)^i(2e/c)\mathbf{A}]^2,$$
$$\hat{L}\Psi^+\equiv[\chi(\xi_T^2P^2)+(V(x)-\varepsilon^*)]\Psi^+,\quad P^2\equiv P_1^2,$$
$$\chi(z)=\tilde{\Psi}(1/2+z/2)-\tilde{\Psi}(1/2),\quad \varepsilon^*=-\ln\tilde{E},$$
$$\tilde{E}=T/1{,}14\omega_D,\quad V(x)=g_\infty^{-1}[1-\varkappa\exp(-x/l_D)],$$
$$\xi_T^2=v_0l/6\pi T,\quad \hat{P}_4=\hat{P}_1-\hat{P}_2+\hat{P}_3,\quad i=1-4,\quad \mathbf{r}=(x,y,z).$$

Here $\mathbf{A}$ is the vector potential, ω_D is the Debye frequency, g_∞ is the strength of the electron-electron interaction far from the nonuniformity, $\tilde{\Psi}$ is the digamma function, v_0 is the Fermi velocity, and l is the electron mean free path.

The boundary condition at the interface between the superconductor and the insulator has the standard form:

$$\left(-i\partial-\frac{2e}{c}\mathbf{A}\right)_x\Psi=0. \quad (12)$$

The current density is given in the "dirty" limit by

$$\mathbf{j}(\mathbf{r})=-\frac{e\tau_{tr}n_0}{4\pi mT}\sum_{n=0}^{\infty}(\hat{P}_1-\hat{P}_2)$$
$$\times\prod_{i=1}^{2}\left(n+\frac{1}{2}+\frac{v_0^2\tau_{tr}}{12\pi T}P_i^2\right)^{-1}\Psi^+(\mathbf{r}_1)\Psi(\mathbf{r}_2)|_{r_1=r_2=r}. \quad (13)$$

Here τ_{tr} is the transport time between collisions, and n_0 is the electron density.

3. CRITICAL TEMPERATURE OF THE SURFACE SUPERCONDUCTING STATE

To determine the critical temperature of the surface superconducting state we need to find the lower "energy" level of the equation $\hat{L}\Psi(r)=0$, using boundary condition (12). If there is no external magnetic field, the equation $\hat{L}\Psi(r)=0$ is put in the following form for surface states with a characteristic dimension $d\gg\xi_T$ (this assumption is justified by the solution found below):

$$-{}^1/_4\pi^2\xi_T^2\partial_x^2\Psi(x)+[V(x)-\varepsilon^*]\Psi(x)=0 \quad (14)$$

with the boundary condition

$$(\partial\Psi/\partial x)_{x=0}=0. \quad (15)$$

Equation (14) with the potential $V(x)$ given by (11) can be solved exactly after the change of variables[15]

$$e^{-x/2l_D}=u. \quad (16)$$

A general solution of (14) which is bounded in the limit $x\to\infty$ is

$$\Psi(x)\sim J_{\nu_0}[e^{-x/2l_D}(4\varkappa l_D^2/g_\infty\xi_T^2)^{1/2}], \quad (17)$$

$$\nu_0^2=\frac{4l_D^2}{\xi^2}\ln\frac{T_s}{T_\infty},\quad \xi^2=\frac{\pi^2}{4}\xi_T^2, \quad (18)$$

where J_{ν_0} is the Bessel function. The dispersion relation for determining the surface critical temperature T_s can be found from Eqs. (14) and boundary condition (15); it is

$$\nu_0/\alpha=J_{\nu_0+1}(\alpha)/J_{\nu_0}(\alpha),\quad \alpha^2=4\varkappa l_D^2/g_\infty\xi^2. \quad (19)$$

The critical temperature T_s can be determined asymptotically exactly in the two limiting cases (1) $\alpha\ll1$ and (2) $\alpha\gg1$. For $\alpha\ll1$ we find from the dispersion relation ($l_D\lesssim\xi$)

$$\frac{\Delta T}{T_\infty}\approx\frac{4}{\pi^2}\left(\frac{\varkappa l_D}{g_\infty\xi_{T_\infty}}\right)^2,\quad \Delta T=T_s-T_\infty. \quad (20)$$

The spatial part of the order parameter in this limit is

$$\Psi(x)\sim\exp(-x\nu_0/2l_D). \quad (21)$$

The characteristic dimension of a superconducting state localized near the surface is determined by

$$d_1\approx g_\infty\xi_T^2\pi^2/4\varkappa l_D\gg\xi_T \quad (22)$$

(in this case the surface superconducting state penetrates deep into the interior of the superconductor).

The parameter α plays the role of an effective electric field in this theory. Everywhere below, we will call the region with $\alpha\ll1$ the "region of weak electric fields," while the region with $\alpha\gg1$ is the "region of strong electric fields." It should be noted that the region of strong electric fields may exist even if the electric field is quite weak, while the region of weak fields can exist even if the fields are quite strong.

In the strong-field region, with $\alpha\gg1$, we find

$$\nu_0\sim\alpha \quad (23)$$

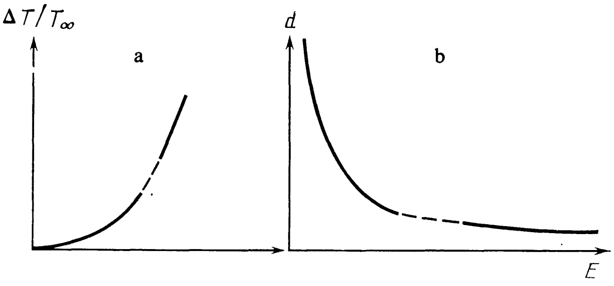

FIG. 1. a) The critical surface temperature and b) the scale length of the localized surface state as functions of the electric field.

from the dispersion relation. Here

$$\frac{\Delta T}{T_\infty}\approx\frac{\varkappa}{g_\infty},\quad d_2\approx\frac{\pi}{2}\xi_T\left(\frac{g_\infty}{\varkappa}\right)^{1/2}\gg\xi_T. \tag{24}$$

Figure 1 shows curves of the surface temperature and of the characteristic scale of a superconducting state localized near the surface versus the external electric field.

4. SURFACE CRITICAL MAGNETIC FIELD

If the transition to the superconducting state occurs in a magnetic field, there is an interval of magnetic fields in which superconductivity exists in a surface layer, while there is no superconductivity far from the surface.

The critical magnetic field at which a nonuniform superconducting state arises near the surface depends on not only the temperature but also the angle between the direction of the magnetic field and the surface of the sample. Let us consider several cases.

a) Magnetic field normal to the surface

In this case the vector potential in Eq. (11) can be chosen in the form

$$A_y=Hz \tag{25}$$

(the magnetic field is directed along the x axis). In this case the variables in (11) can be separated at low temperatures, where the relation $a_H/d_{iH}(E)\ll 1$ holds [here $a_H=(c/2eH_s)^{1/2}$, and d_{iH} is the scale dimension of the surface superconducting state in the magnetic field]. The variables can also be separated at temperatures near T_s, where the parameter d_{iH}/a_H is small. Seeking a solution of (11) in the form

$$\Psi(x,\ z)\sim\exp\left(-z^2/2a_H^2\right)\eta(x), \tag{26}$$

we find a Schrödinger equation for $\eta(x)$:

$$-\partial_x^2\eta(x)-\frac{\varkappa}{g_\infty\xi_T^2\chi'}\exp\left(-\frac{x}{l_D}\right)\eta(x)=-\frac{2e}{c}l_D^2\Delta H\eta(x),$$
$$\chi'=d\chi/dz,\quad \Delta H=H_s-H_{c2}^\infty \tag{27}$$

with the boundary condition

$$(\partial\eta(x)/\partial x)_{x=0}=0. \tag{28}$$

Working by the method described above, we can easily derive from (27) and (28) a dispersion relation for determining the surface magnetic field

$$\nu_1/\beta=J_{\nu_1+1}(\beta)/J_{\nu_1}(\beta), \tag{29}$$

$$\beta=(4\varkappa l_D^2/g_\infty\xi_T^2\chi')^{1/2},\quad \nu_1^2=8el_D^2\Delta H/c. \tag{30}$$

For $\beta\ll 1$ (weak electric fields) we have

$$\Delta H=\frac{c\varkappa^2 l_D^2}{2e\xi_T^4(\chi')^2 g_\infty^2},\quad d_{1H}=\frac{\xi_T^2\chi' g_\infty}{\varkappa}. \tag{31}$$

In the opposite limit, $\beta\gg 1$, we have

$$\Delta H=\frac{2\varkappa c}{\pi^2 g_\infty e\xi_T^2\chi'},\quad d_{2H}\approx\xi_T\left(\frac{g_\infty\chi'}{\varkappa}\right)^{1/2}. \tag{32}$$

The spatial part of the order parameter of the surface superconducting state in a magnetic field normal to the surface is also determined from Eq. (27); the result is

$$\Psi_H\sim J_{\nu_1}(e^{-x/2l_D}\beta). \tag{33}$$

b) Magnetic field parallel to the surface

In this case the variables cannot be separated in Eq. (11). A solution can be derived asymptotically exactly at temperatures $T\to T_s$ and at low temperatures by perturbation theory, since in this case there is no localization of the superconducting state along the magnetic field.

The vector potential is conveniently chosen in the form $A_y=Hx$. In this case Eq. (11) becomes

$$\chi\left[\xi_T^2\left(-\partial_x^2+\frac{4e^2}{c^2}H^2(x-x_0)^2\right]\Psi(x)+\left[\ln\frac{T}{T_\infty}\right.\right.$$
$$\left.-\frac{\varkappa}{g_\infty}\exp\left(-\frac{x}{l_D}\right)\right]\Psi(x)=0,\quad \left.\frac{\partial\Psi}{\partial x}\right|_{x=0}=0. \tag{34}$$

The scale size imposed by the magnetic field on the region occupied by the superconducting state is $\sim a_H$, while that imposed by the potential energy is $\sim d_i$.

For $a_H\ll d_i$ the potential associated with the external electric field can be treated by perturbation theory. In first order in a_H/d_i, Eq. (34) is an equation for determining the surface field H_{c3} of the superconductor in the absence of an electric field. A trial function which provides the corresponding value H_{c3} is [16]

$$\Psi_0(x)\sim\exp\left(-rx^2\right),\quad r=1.7c/eH_{c3}. \tag{35}$$

Writing the field H_s in (34) as $H_s=\Delta H+H_{c3}^\infty$, where $\Delta H=H_s-H_{c3}^\infty$, and substituting (35) into (34), we find, to first order in a_H/d_i,

$$\Delta H=0.8c\varkappa/\xi_T^2e\chi'(u_1),\quad u_1=1.2\xi_T^2eH_{c3}^\infty/c. \tag{36}$$

In the opposite limit, $a_H\gg d_i(E)$, the magnetic field can be treated by perturbation theory. Using an expression for the spatial part of the order parameter, we find

$$\delta\tau_H\equiv\frac{T_s-T_{sH}}{T_s}$$
$$=\frac{\pi^2e^2}{c^2}H^2\int_0^\infty(x-x_0)^2J_{\nu_0}^2(e^{-x/2l_D}\alpha)\,dx\bigg/\int_0^\infty J_{\nu_0}^2(e^{-x/2l_D}\alpha)\,dx. \tag{37}$$

Evaluating the corresponding integrals in (37), and minimizing $\delta\tau_H$ with respect to x_0, we find

$$\delta\tau_H \approx \frac{\pi^2}{4}\frac{e^2}{c^2} H_s^2 d_i^2 \xi_T^2. \tag{38}$$

5. STRUCTURE AND MAGNETIC MOMENT OF THE SURFACE SUPERCONDUCTING STATE

To determine the structure of the nonuniform surface state and its magnetic moment we need to use the nonlinear equation for the order parameter, (11), and expression (13) for the superconducting current density.

The nonuniformity of the electron-electron interaction and the effect of the free surface give rise to several important features in the behavior of such superconductors in a magnetic field. In our case, in which the nonuniformity of the electron-electron interaction is caused by an external electric field, the magnetic moment which arises near the boundary of the nonuniform superconducting state depends on the magnitude of this field. The scale sizes of the superconducting structures which arise near the surface also depend on the strength of the external electric field. As in Ref. 10, in determining the magnetic moment we should write the solution of nonlinear equation (11) as a function with the spatial behavior determined by the solution of the equation linear in Ψ (Ref. 17).

The magnetic characteristics and the structure of the nonuniform superconducting state depend on the temperature and on the orientation of the external magnetic field with respect to the surface.

a) Magnetic field normal to the surface

In this case, as mentioned above, a solution of Eq. (11) which is linear in the order parameter can be derived asymptotically exactly in the two temperature regions (1) $a_H/d_{iH}(E) \ll 1$ and (2) $a_H/d_{iH}(E) \gg 1$. The order parameter is found to be localized along the magnetic field. In a direction parallel to the surface, on the other hand, the nonuniform superconducting state is degenerate with respect to the positions of the centers of the orbits. Boundary condition (12) affects the relations between β and ν_1 in expression (33) for the order parameter.

We therefore seek a solution of the nonlinear equation (11) in the following form:

$$\Psi(\mathbf{r}) = \Psi_H(x)\varphi(y, z) + \Psi_1, \quad \mathbf{r} = (x, y, z) \equiv (x, \mathbf{R}), \tag{39}$$

$$\varphi(y, z) = \sum_n c_n e^{iqny}\varphi(z - z_n), \quad z_n = cqn/2eH_s, \tag{40}$$

$$\varphi(z - z_n) = \exp[-(z - z_n)^2/2a_H^2], \quad q = -i\partial_y. \tag{41}$$

Substituting (40) and (41) into expression (13) for the current density, and using Eqs. (11) and (39), we find the following expression for the Gibbs potential:

$$G = \frac{1}{8\pi}\int (\mathbf{H}(\mathbf{r}) - \mathbf{H}_e)^2 d^3\mathbf{r} - \frac{N_\infty f_1}{32\pi^2 T^2}\int |\Psi|^4 d^3\mathbf{r}, \tag{42}$$

$$f_1 = \sum_{n=0}^{\infty}\left(n + \frac{1}{2} + \xi_T^2\frac{eH}{c}\right)^{-3}; \tag{43}$$

Here $\mathbf{H}(\mathbf{r})$ is the magnetic field, and $\mathbf{H}_e$ is the field far from the surface.

Working in the usual way, we find from (42) the following expression for the magnetic moment of a surface superconducting structure:

$$\mathbf{M} = -(\Delta\mathbf{H})\gamma/4\pi(2K^2 - 1)\beta_\Delta, \tag{44}$$

where

$$K^2 = 3f_1 mc^2/8\pi n_0(e\tau_{tr}v_0\rho)^2,$$
$$\rho = \sum_{n=0}^{\infty}\left(n + \frac{1}{2} + \xi_T^2\frac{eH}{c}\right)^{-2}, \quad \Delta H = H_s - H_e, \tag{45}$$

and the quantity

$$\gamma = \left(\int_0^\infty \Psi_H^2(x)\,dx\right)^2 \Big/ \int_0^\infty \Psi_H^4(x)\,dx \tag{46}$$

serves as the scale size of the localization of the superconducting structure along the direction of the magnetic field. This length naturally depends on the applied electric field: $\gamma = \gamma(E)$. Substituting the expression for Ψ_H into (46), we find that in the case $\beta \ll 1$ we have $\nu_1 = \beta^2/2$ and $\gamma \approx d_{1H}$. At $\beta \gg 1$ we have $\nu_1 \sim \beta$. In this case we should use the asymptotic expression for the Bessel function at large values of the index and of the argument to calculate γ (in this case the integrals are dominated by the region of small values of the argument). As a result of the integration, we easily find $\gamma \sim d_{2H}$ in this limit. The spatial structure of the nonuniform surface state shown in Fig. 2 is determined by the order parameter $\Psi(\mathbf{r})$, in which we have $\beta_\Delta = 1.16$ for a triangular spatial lattice of superconducting states which are localized along the magnetic field.

b) Magnetic field parallel to the surface (weak fields)

If the magnetic field is parallel to the surface, an asymptotically exact solution can be derived at low temperatures, $a_H/d_i(E) \ll 1$, and at temperatures near the critical temperature (the region of weak magnetic fields), $a_H/d_i \gg 1$.

For $a_H/d_i \gg 1$ the magnetic field in (11) should be treated by perturbation theory, as we have already mentioned. To first order in the magnetic field, we should seek a solution of (11) in the form[18]

$$\Psi(x) = \sum_i c_i J_{\nu_i}(\alpha e^{-x/2l_D}), \tag{47}$$

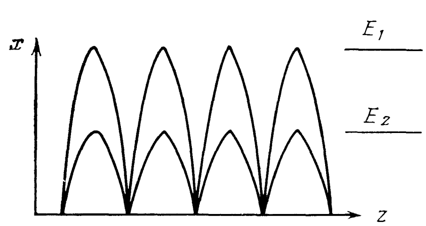

FIG. 2. Structure of a surface superconducting state in a magnetic field normal to the surface ($E, H \perp z$). The superconducting states are localized along the magnetic field. The scale size of the localization region increases with increasing electric field.

where the $J_{\nu i}$ are Bessel functions which satisfy dispersion relation (19). Substituting (47) into (11), we find, in first order in d_i/a_H,

$$\Psi(x,\ T)\approx c_0 J_{\nu_0}(\alpha e^{-x/2l_D}), \tag{48}$$

where

$$c_0=T_s(\delta\tau)^{1/2}[8\pi^2\gamma_1/7\zeta(3)]^{1/2},\quad \delta\tau=(T_{sH}-T)/T_{sH},$$
$$\gamma_1=\int_0^\infty J_{\nu_0}^2(\alpha e^{-x/2l_D})\,dx \Big/ \int_0^\infty J_{\nu_0}^4(\alpha e^{-x/2l_D})\,dx \tag{49}$$

(γ_1 is of order unity both for weak electric fields and for strong fields), and $\xi(3)$ is the zeta function.

As can be seen from expression (48) for the order parameter, the surface superconducting state is a superconducting layer with a thickness which depends on the external electric field. As the electric field (the parameter α) is increased, the thickness of the superconducting layer changes from $d_1(E)$ to $d_2(E)$.

To determine the magnetic moment of the surface superconducting state we need to substitute expression (48) for the order parameter into expression (13) for the current density and to make use of the following relationship between the magnetic vector potential and the current density:

$$\frac{d^2A_y}{dx^2}=\frac{\delta\tau}{\lambda^2}J_{\nu_0}^2(\alpha e^{-x/2l_D})A_y,$$
$$\lambda^2=7\zeta(3)mc^2/\pi\gamma_1 e^2\tau_{tr}n_0T_s. \tag{50}$$

In a type II superconductor, with $\delta\tau d_i^{\,2}(E)/\lambda^2\ll 1$, the magnetic field in (50) can be dealt with by perturbation theory. As a result we find the following expressions for the magnetic field $\mathbf{H}(x)$ and the magnetic moment $\mathbf{M}$:

$$\mathbf{H}(x)=\mathbf{H}_e-\frac{\delta\tau}{\lambda^2}\int_0^x J_{\nu_0}^2(\alpha e^{-x'/2l_D})\mathbf{H}_e x'\,dx',$$
$$\mathbf{M}=\frac{1}{4\pi}\int[\mathbf{H}(x)-\mathbf{H}_e]\,dx$$
$$=-\frac{1}{4\pi}\frac{\delta\tau}{\lambda^2}\mathbf{H}_e\int_0^\infty dx\int_0^x J_{\nu_0}^2(\alpha e^{-x'/2l_D})x'\,dx'. \tag{51}$$

The integral in (51) can be evaluated in both limiting cases — of weak and strong electric fields. In these regions of the parameters, the magnetic moment (per unit surface area) is

$$\mathbf{M}\approx-\frac{\delta\tau}{\lambda^2}\mathbf{H}_e d_i^{\,3}(E). \tag{52}$$

Figure 3 shows the function $\mathbf{M}(E)$.

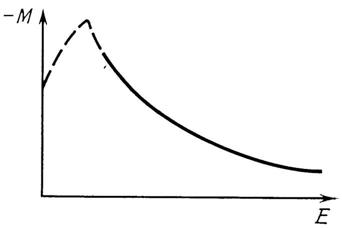

FIG. 3. Diamagnetic moment as a function of the electric field for the case in which the magnetic field is parallel to the surface.

c) Magnetic field parallel to the surface (strong fields)

In this case, seeking the order parameter Ψ in the form

$$\Psi=\Psi_0(x)e^{iqy}+\Psi_1, \tag{53}$$

by the method described in Ref. 10, we can easily show that the magnetic moment (per unit surface area) of a superconducting plate which forms near a surface is given by the following expression:

$$M\approx-a_{H_s}\Delta H/4\pi(2K^2-1),\quad \Delta H=H_s^{\parallel}-H_e, \tag{54}$$

which depends on the electric field only through the critical field $H_s^{\parallel}$.

In a magnetic field parallal to the surface, the magnetic moment therefore becomes essentially independent of the electric field as the temperature is lowered.

6. DISCUSSION OF RESULTS

Many experimental studies have now been published in which a change in the critical temperature has been linked with the existence of a strong electric field in a system.[19–23] On the other hand, there have been only two direct studies of the stimulation of superconductivity by an electric field.[24–25]

As follows from the results derived above, the shifts of the critical temperatures and magnetic fields depend on the parameters α and β, the Debye screening length, and the coherence length. From this vantage point we see that the systems studied in Refs. (24) and (25) are the least favorable for the detection of this effect, since the metals studied there have a small value of $\varkappa$ and a short screening length, combined with a long coherence length ξ. In particular, Stadler's experiment[25] used a tin film ≈ 150 Å thick in a strong electric field $E\sim 10^8$ V/cm produced by a ferroelectric (triglycine sulfate). The shift ΔT in this field was observed to be $\Delta T=1.3\cdot 10^{-3}$ K. Using this value we can easily calculate the coherence length in the film from the results derived in the present paper. For tin we find

$$E^*\approx 10^9\ \text{V/cm},\quad l_D\approx 5\ \text{Å},\quad T_c\approx 3.7\ \text{K} \tag{55}$$

and $\xi\approx 500$ Å, in order-of-magnitude agreement with the known coherence length of tin films.

We believe that the systems most favorable for the observation of the effects discussed here are superconducting semiconductors, in which a large screening length is combined with a rather small coherence length. For the classical superconducting semiconductor $SrTiO_3$, for example, with the parameter values

$$\xi\approx 100\ \text{Å},\quad \varepsilon_\infty\approx 10^4,\quad T_\infty\approx 0.4\ \text{K},\quad n_0\approx 10^{18}\ \text{cm}^{-3} \tag{56}$$

in an electric field $E\approx 10^3$ V/cm, for example, we find the temperature shift and the size of the surface superconducting region to be

$$\Delta T \approx 0.1\ \mathrm{K}, \quad d \approx 5\cdot 10^{-6}\ \mathrm{cm}. \tag{57}$$

It would be particularly interesting to make use of a ferroelectric with a ferroelectric transition temperature near the superconducting transition temperature ($T_k \approx T_c$) as the electric-field source for the effects discussed here. In this case it would become possible to vary a strong electric field over a rather broad range. For estimates we consider only the case $T_k > T_c$. In this case, the role of the electric field in all the expressions is played by the temperature-dependent electric displacement, which is given in the simplest case by[26]

$$D = 4\pi\left[a(T_k - T)/2B\right]^{1/2}; \tag{58}$$

here a and B are the coefficients in the Ginzburg-Landau expansion of the functional for the spontaneous polarizability.

The effect which we have pointed out in this paper may be observed in experiments in which the resistance of a surface layer of a superconductor is measured in an electric field perpendicular to the surface, in which the magnetic moment of this layer is measured, and in which oscillations of the magnetic moment are observed in the case in which the sample is in a cylindrical capacitor, and the surface superconducting state which arises is a closed ring.

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