Vortices with half-integral circulation in ^{3}He–A superfluid

G. E. Volovik and M. M. Salomaa

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR, and Low Temperature Laboratory, Technical University, Helsinki, Finland

(Submitted 19 November 1984)
Zh. Eksp. Teor. Fiz. **88**, 1656–1663 (May 1985)

A method is proposed for experimentally observing exotic vortices in rotating ^{3}He–A which have a half-integral number of superfluid velocity circulation quanta. The existence of such vortices in ^{3}He–A is made possible by the presence of a discrete combined symmetry (gauge transformation + spin rotation) which nontrivially couples the superfluid and magnetic properties of the liquid. This symmetry gives rise to hybrids which consist of a vortex combined with a disclination in the field of the magnetic anisotropy vector. Because of the spin-orbit (dipole) energy, topological solitons terminate on them if the superfluid is unconfined, and the hybrids are energetically unfavorable. If the fluid is confined between parallel planes separated by less than the dipole length, the dipole energy will be neutralized when a sufficiently strong magnetic field is applied normal to the plates; in this case, vortices with half-integral circulation can coexist with other types of vortices. The half-integral vortices should give rise to a distinctive NMR signal that distinguishes them from singular one-quantum and nonsingular two-quantum vortices.

1. INTRODUCTION

Because of the nontrivial violation of gauge and rotational symmetry in the superfluid phases of ^{3}He, the properties of the quantized vortices in these liquids are considerably more interesting than in He II. The exotic behavior of the vortices can be traced to the existence of combined symmetries in the A, B, and A_1 phases which mix the superfluid, liquid-crystal, and magnetic properties of the ordered liquids. The combined gauge-rotation symmetry in ^{3}He–A (cf. the review in Ref. 1) is responsible for the "nonsingular" vortices with two superfluid velocity circulation quanta ($p = 2$) which have been observed in NMR experiments[2,3] in rotating liquids. (We recall that the circulation quantum in superfluid ^{3}He is equal to h/M, where $M = 2m_3$ is the case of the two ^{3}He atoms forming a Cooper pair.) These vortices have a liquid-crystal texture which induces a continuous distribution of the curl $\nabla \times \mathbf{v}_s$ of the superfluid velocity in the vortex.

Because ^{3}He–B is invariant under a combined rotation of the orbital and spin subsystems (cf. the review in Ref. 4), the system has a nonzero intrinsic magnetic moment which is frozen into the cores of the quantized vortices and manifests itself as an unusual gyromagnetic effect that can also be observed in NMR experiments.[5]

There is an additional type of combined symmetry in ^{3}He–A which couples the superfluid and magnetic properties. This symmetry, which is discrete, gives rise to distinctive "hybrid" lines in ^{3}He–A in which a half-integral vortex ($p = 1/2$) is combined with a disclination in the field of the magnetic anisotropy vector $\mathbf{d}$ with a half-integral Franck index $m = 1/2$ (cf. Ref. 6). In terms of its effects on the properties of the elementary excitations in ^{3}He–A (fermions, boson collective modes, point-like topological objects, hedgehogs), these semivortex-semidisclination hybrids are reminiscent of the hypothetical singular lines in the grand unified theory, around which the electric charge or parity of elementary particles changes sign.[7] Moreover, topological constraints imply that neither the $p = 1/2$ vortex nor the $m = 1/2$ disclination comprising the hybrid can exist separately from one another; we thus have a possible topological mechanism for quark confinement. The experimental observation of these linear defects in ^{3}He–A would thus be of great interest.

We will show that semivortex-semidisclination hybrids should be present in ^{3}He–A when the superfluid is rotated between parallel plane plates separated by less than the dipole length $\xi_d \sim 10^{-3}$ cm. The orbital anisotropy vector $\mathbf{l}$ is specified in this geometry, and when a strong magnetic field $H > 50$ G is applied normal to the plates there will be a range of temperatures for which the singular lines are energetically more advantageous than vortices with one or two circulation quanta. These objects can be identified by the characteristic behavior of the satellite vortex peak in the NMR signal. If the magnetic field is obliquely incident on the plates, topological solitons are formed which bridge pairs of semivortices, extending from one member of the pair and terminating at the other. These solitons localize the spin waves and are responsible for the satellite vortex peak. The soliton diameter, the distance between the paired vortices, and the associated frequency and intensity of the vortex peak are very sensitive to the inclination angle of the field.

2. VORTICES WITH $p = 1/2$ IN A PARALLEL PLANE GEOMETRY

The order "parameter" in superfluid ^{3}He is a 3×3 matrix $A_{\alpha i}$ whose elements consist of the series expansion coefficients of the wave function for the pair with respect to eigenstates of orbital moment $L = 1$ and spin moment $S = 1$ (these states correspond to the different projections of L, S). In terms of the real unit vector $\mathbf{d}$ and the complex vector

$\Delta' + i\Delta''$ (where we assume that $\Delta'\Delta'' = 0$, $|\Delta'| = |\Delta''| = 1$), we can express the matrix for the A-phase in the form

$$A_{\alpha i} \sim d_\alpha (\Delta_i' + i\Delta_i'') e^{i\Phi}. \qquad (2.1)$$

This matrix describes a Cooper pair with zero spin projection on the **d** axis and unit orbital projection on the **l** axis ($\mathbf{l} = \Delta' \times \Delta''$). This state is not invariant under spin and orbital rotations which reorient **d** and $\Delta' + i\Delta''$, respectively, nor is it invariant under gauge transformations that change the global phase Φ of the Cooper pairs in the Bose condensate. However, it is invariant under two combined symmetry transformations, one continuous and the other discrete.

The continuous symmetry is of the form $\Phi \to \Phi + \alpha$, with a simultaneous rotation of the orbital variables Δ' and Δ'' by the angle α about the **l** axis. This symmetry is responsible for the unusual superfluid properties of ^{3}He–A (cf. Ref. 1 for details); they are a manifestation of the rotational nature of the superfluid velocity

$$\mathbf{v}_s = \frac{\hbar}{M} (\nabla\Phi + \Delta_i' \nabla \Delta_i''), \qquad (2.2)$$

which is invariant under the symmetry transformation.

The discrete symmetry is of the form $\Phi \to \Phi + \alpha$, $\mathbf{d} \to -\mathbf{d}$ and is responsible for the existence of vortex-spin-disclination hybrids[6] of the form

$$\Phi = \varphi/2, \quad \mathbf{d} = \mathbf{a}\cos m\varphi + \mathbf{b}\sin m\varphi, \quad \Delta' + i\Delta'' = \text{const}, \qquad (2.3)$$

e.g., here φ is the azimuthal angle in a cylindrical coordinate system with z axis parallel to the vortex axis, and the unit vectors **a** and **b** are mutually orthogonal. The phase Φ changes by π when we go around the linear hybrid. The change in the sign of the order parameters $A_{\alpha i}$ is offset by the reversal of the field **d** around the line, provided the index m of **d** is half-integral. The field **d** is thus analogous to a disclination of half-integral index in nematic liquid crystals (cf. Ref. 8). The superfluid velocity is equal to

$$\mathbf{v}_s = \frac{\hbar}{M} \nabla\Phi = \frac{\hat{\varphi}\hbar}{2Mr},$$

and the corresponding circulation is

$$\oint \mathbf{v}_s \, d\mathbf{r} = \frac{h}{2M},$$

i.e., $p = 1/2$ in units of the circulation quantum h/M.

This vortex is one of three topologically distinct types that can exist in ^{3}He–A (Ref. 6); the other two are singular with $p = 1$ and nonsingular with $p = 2$ (cf. Refs. 1 and 2). In order to find out which type of vortex makes up the periodic structure that forms when the vessel continuing the liquid is rotated at angular velocity $\mathbf{\Omega}$, we must examine the free energy functional F in the rotating system. For ^{3}He–A, F is given by the general expression (cf., e.g., Ref. 1)

$$F = \frac{1}{2}\rho_s^{\parallel}(\mathbf{l}, \mathbf{v}_s - [\mathbf{\Omega r}])^2 + \frac{1}{2}\rho_s^{\perp}[\mathbf{l}, \mathbf{v}_s - [\mathbf{\Omega r}]]^2$$
$$+ \frac{\hbar}{M} C(\mathbf{v}_s - [\mathbf{\Omega r}], \operatorname{rot}\mathbf{l}) - \frac{\hbar}{M} C_0 (\mathbf{l}, \mathbf{v}_s - [\mathbf{\Omega r}])(\mathbf{l}\operatorname{rot}\mathbf{l})$$
$$+ \frac{1}{2}\left(\frac{\hbar}{M}\right)^2 \{K_1(\nabla\mathbf{l})^2 + K_2(\mathbf{l}\operatorname{rot}\mathbf{l})^2 + K_3[\mathbf{l}, \operatorname{rot}\mathbf{l}]^2\}$$
$$+ \frac{1}{2}\left(\frac{\hbar}{M}\right)^2 \rho_{sp}^{\parallel}((\mathbf{l}\nabla)\mathbf{d})^2$$
$$+ \frac{1}{2}\left(\frac{\hbar}{M}\right)^2 \rho_{sp}^{\perp}\left\{([\mathbf{l}\nabla]d_\alpha)^2 - \frac{1}{\xi_d^2}(\mathbf{ld})^2 + \frac{1}{\xi_H^2}\frac{[\mathbf{dH}]^2}{H^2}\right\}. \qquad (2.4)$$

Here the notation $[\mathbf{AB}]$ or $[\mathbf{A}, \mathbf{B}]$ denotes the vector product; $\rho_s^{\parallel}$ and $\rho_s^{\perp}$ are the components of the superfluid density tensor parallel and normal to **l**, respectively; the coefficients K_1, K_2, and K_3 describe the distortion energy of the liquid-crystal field **l**—more precisely, the transverse bending, the twisting, and the longitudinal bending, respectively (cf. Ref. 8); $\rho_{sp}^{\parallel}$ and $\rho_{sp}^{\perp}$ are the components of the spin rigidity tensor parallel and normal to **l**; the coefficients C and C_0 determine the superfluid current $\delta F/\delta \mathbf{v}_s$ as a function of $\nabla \times \mathbf{l}$. For $T \approx T_c$, for which the Ginzburg-Landau functional can be used, these coefficients are related by[9]

$$\rho_s^{\parallel} = \rho_{sp}^{\parallel} = {}^1/_2\rho_s^{\perp} = {}^1/_2\rho_{sp}^{\perp} = 2K_1 = 2K_2 = {}^2/_3K_3 = C_0 = 2C,$$

which breaks down for T far from T_c. The parameters $\xi_d \sim 10^{-3}$ and $\xi_H \sim \xi_d$ $(H/25\ \text{Gauss})^{-1}$ are the dipole and magnetic lengths for the spin-orbit (dipole) interaction of the fields **d**, **l** and the uniaxial magnetic anisotropy energy, respectively (**d** is the magnetic axis).

The minimum energy in He II corresponds to a periodic vortex structure with circulation number as small as possible, i.e., $p = 1$. This is because the energy density F is proportional to p; indeed, F is equal to the density of vortices

$$n_V = \frac{2\Omega}{p}\frac{M}{h}, \qquad (2.5)$$

multiplied by the vortex energy $\propto p^2$ per unit length:

$$E_V = \pi\rho_s p^2 \left(\frac{\hbar}{M}\right)^2 \ln\frac{r_\Omega}{\xi} \qquad (2.6)$$

($r_\Omega \sim n_v^{-1/2}$ is the distance between the vortices, and the coherence length ξ is determined by the diameter of the vortex core). Thus,

$$F(\text{HeII}) = p\rho_s \frac{\hbar}{M}\Omega \ln\frac{r_\Omega}{\xi}.$$

The rule $F \propto p$ does not hold in ^{3}He–A because of the complicated interaction among the different degrees of freedom in the system. For unconfined systems, vortices with the minimum circulation $p = 1/2$ cannot coexist with $p = 1$ and $p = 2$ vortices, because the latter are always associated with a large dipole energy. Indeed, unlike **d** the vector **l** is uniquely determined and cannot change sign around any line; **l** therefore cannot be parallel everywhere to **d** in a system of vortices with $p = 1/2$, as would be required for the dipole interaction. A planar soliton thus extends outward from each vortex with $p = 1/2$ (Refs. 6 and 10), and the energy of the soliton is proportional to its volume, in which the dipole energy is not saturated.

In order for vortices with $p = 1/2$ to exist, the dipole energy must somehow be neutralized. One possible method (which has yet to be carried out experimentally) is to increase the angular velocity to $\Omega \sim 100$ rad/s. The distance r_Ω

between the vortices will then be comparable to the dipole length ξ_d, so that the hydrodynamic energy of the vortices is roughly equal to the dipole energy and vortices with $p = 1/2$ are energetically viable. An alternative method is to form a periodic structure consisting of pairs of $p = 1/2$ vortices in a liquid rotating at the slower velocities $\Omega \sim 1$ rad/s typical in experiments; in this case the distance between paired vortices should be $\sim \xi_d$. Each pair will then behave like an isolated vortex with $p = 1$. However, the energy for the class of vortices with $p = 1$ is known to be minimized when the field **d** is almost constant everywhere and **l** forms a disclination of integral Franck index at a distance $\sim \xi_d$ near the axis (cf. Refs. 6 and 11). Pairs of vortices with $p = 1/2$ are therefore unstable with respect to the minimum energy configuration.

The dipole energy can be eliminated more effectively by confining the ^{3}He–A between parallel plates spaced a distance $r_0 \ll \xi_d$ apart and applying a magnetic field $H \gg 25$ G along the normal **ν** to the plates, so that $\xi_H \ll \xi_d$. Under these conditions **l** is always parallel to **ν** (the z axis), and **d** is perpendicular to **ν**:

$$\mathbf{d} = \hat{\mathbf{x}} \cos \alpha + \hat{\mathbf{y}} \sin \alpha.$$

Although the dipole energy is now a maximum, it is the same for all types of vortices and therefore plays no role. The energy functional (2.4) takes the form

$$F = \frac{1}{2}\left(\frac{\hbar}{M}\right)^2 \left\{ \rho_s{}^\perp \left(\nabla_\perp \Phi - \frac{M}{\hbar}[\Omega \mathbf{r}] \right)^2 + \rho_{sp}{}^\perp (\nabla_\perp \alpha)^2 \right\} \quad (2.7)$$

in the parallel-plane geometry if $\boldsymbol{\Omega} \| \boldsymbol{\nu}$, so that Φ and **d** depend only on the coordinates transverse to **l** and **d**. We will now compare the energies F for systems of different vortices in the logarithmic approximation.

a) The energy for $p = 1$ vortices is a minimum for pure vortices for which $\Phi = \varphi$ and $\alpha = 0$ down to a distance $\sim r_0$ from the axis of the vortex. For $\rho \lesssim r_0$ it is energetically more favorable for the vector **l** to deviate from the z axis, and the vortex becomes a disclination ($\mathbf{l} = \hat{\boldsymbol{\rho}}$) of energy $\sim K_1 \ln(r_0/\xi)$ per unit length.[6] The energy density for a system of such vortices is thus given by

$$F^{(1)} = n_V^{(1)} E_V^{(1)} = \frac{\hbar}{M} \Omega \left(\rho_s{}^\perp \ln \frac{r_\Omega}{r_0} + K_1 \ln \frac{r_0}{\xi} \right). \quad (2.8)$$

b) The vortices with $p = 2$ are nonsingular of the Anderson-Toulouse-Chechetkin type[12]; the fountain-like distribution of **l** at distances $\sim r_0$ wipes out the vortex singularity near the axis. The energy for these vortices is

$$F^{(2)} = n_V^{(2)} E_V^{(2)} = 2 \frac{\hbar}{M} \Omega \rho_s{}^\perp \ln \frac{r_\Omega}{r_0}. \quad (2.9)$$

c) The solution for an isolated $p = 1/2$ vortex is given by (2.3), i.e., $\Phi = \varphi/2$, $\alpha = m\varphi$ with half-integral m. The energy is a minimum for $m = \pm 1/2$. The system of disclinations of various charges m is reminiscent of a two-dimensional plasma. In order for the system to have a finite energy, we must have equal numbers of vortices of opposite signs, i.e., the vortices with $m = 1/2$ and $m = -1/2$ must alternate; we then have

$$F^{(1/2)} = n_V^{(1/2)} E_V^{(1/2)} = \frac{\hbar}{2M} \Omega (\rho_s{}^\perp + \rho_{sp}{}^\perp) \ln \frac{r_\Omega}{\xi}. \quad (2.10)$$

We first analyze the temperature range $T \approx T_c$, for which Cross[9] has shown that $\rho_s^\perp = \rho_{sp}^\perp = 4K_1$. If we assume the values $r_\Omega \sim 10^{-2}$ cm, $r_0 \sim 10^{-4}$ cm, and $\xi \sim 10^{-5}$ cm then

$$F^{(1/2)} : F^{(1)} : F^{(2)} = 1 : 3/4 : 4/3,$$

i.e., vortices with $p = 1/2$ can compete with the $p = 1$ vortices, even though the latter are still energetically more favorable in the logarithmic approximation. A more precise analysis shows that the energy may actually be lower for the vortices with $p = 1/2$. Indeed, it is important to consider the nonlogarithmic corrections to the energy of the $p = 1$ and $p = 2$ vortices caused by the fact that the distribution of **l** is not perfectly uniform[11]; moreover, the ratio $\rho_{sp}^\perp/\rho_s^\perp$ drops to 0.5 when T decreases to $0.7T_c$ (cf. Ref. 13), which greatly decreases the relative energy of the vortices with $p = 1/2$.

3. PAIRS OF VORTICES WITH $p = 1/2$

Unconfined vortices with $p = 2$ have been detected from an additional "vortex" absorption peak in the NMR signal caused by excitation of spin waves (oscillations of the vector **d**) localized within soft vortex cores of diameter $\sim \xi_d$. The vectors **d** and **l** are not parallel inside a soft core, so that a potential well is formed which traps the spin modes. For the geometric arrangement considered in the previous section, **l** and **d** always make an angle of $\pi/2$. In this case, no spin-wave potential well is formed during longitudinal NMR excitation, so that no vortex peak is present. For transverse NMR excitation, the spatial changes in **d** will give rise to a potential well even in this geometry; however, the localized modes are not strongly excited. In order for the vortices with $p = 1/2$ to produce an intense vortex peak, the magnetic field must be directed at a small angle θ to the axis of rotation:

$$\mathbf{H} = H(\hat{\mathbf{z}} \cos \theta + \hat{\mathbf{x}} \sin \theta) \quad (3.1)$$

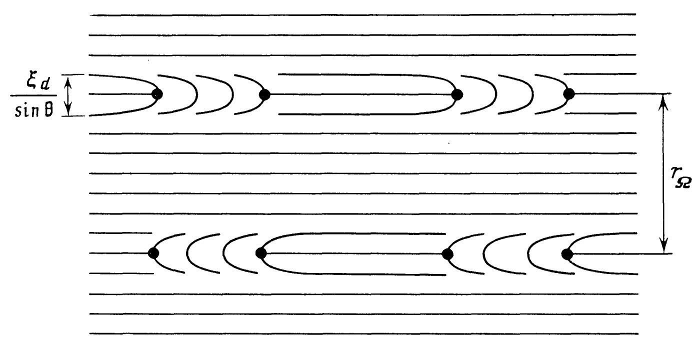

FIG. 1. Distribution of the magnetic anisotropy field **d** in a periodic structure in rotating ^{3}He–A contained between parallel plates in the plane of the figure. The structure consists of hybrids of vortices of half-integral circulation and disclinations in **d** with half-integral Franck index ($m = 1/2$ and $m = -1/2$). The points show where the vortex lines intersect the plane of the figure normal to them. The applied magnetic field makes a small angle with the normal to the plates, so that topological solitons bridging pairs of vortices form between disclinations with m of opposite signs. The field **d** outside the solitons points along the projection of the magnetic field on the plates and is responsible for the minimum in the spin-orbit (dipole) energy ($\alpha = 0$).

(recall that $\mathbf{\Omega}$, $\mathbf{v}$, and $\hat{\mathbf{z}}$ are mutually parallel). The vector $\mathbf{d}$, which is normal to $\mathbf{H}$, must then be of the form

$$\mathbf{d}=(\hat{\mathbf{z}}\sin\theta-\hat{\mathbf{x}}\cos\theta)\cos\alpha+\hat{\mathbf{y}}\sin\alpha, \tag{3.2}$$

and the energy (2.4) is given by

$$F=\frac{1}{2}\left(\frac{\hbar}{M}\right)^2\left\{\rho_s^\perp\left(\nabla_\perp\Phi-\frac{M}{\hbar}[\mathbf{\Omega},\mathbf{r}]\right)^2 + \rho_{sp}^\perp((\nabla_\perp\alpha)^2+\xi_d^{-2}\sin^2\theta\sin^2\alpha)\right\}. \tag{3.3}$$

Equation (3.3) contains a new dipole-energy term which tends to make the angle α equal to 0 or π and is not present in (2.7). The magnitude of this term depends on the length $\xi_d/\sin\theta$. If the latter exceeds the distance between the vortices ($\theta\ll\xi_d/r_\Omega$) then the dipole interaction is negligible and will not perturb the stable configuration that exists for $T\approx T_c$ (i.e., the system of vortices with $p=1/2$ and alternating charges $\pm m$).

For larger θ, a planar soliton (domain wall) forms between two adjacent vortices of opposite charges $\pm m$. The angle α changes from 0 to π within the soliton (cf. Fig. 1), which corresponds to a nontrivial element of the homotopy group (cf. Ref. 10). It differs from ordinary domain walls because it can begin and terminate on vortices with half-integral topological charge (circulation quantum). The thickness of the soliton wall is on the order of $\xi_d/\sin\theta$. The energy of the soliton is proportional to its volume, and hence also to the distance R between the vortices bridged by the soliton. We can estimate R by minimizing the energy of a vortex pair:

$$E_{\text{pair}}\approx\frac{\pi}{2}\left(\frac{\hbar}{M}\right)^2\rho_{sp}^\perp\left[\frac{\rho_s^\perp}{\rho_{sp}^\perp}\left(\ln\frac{r_\Omega}{\xi}+\ln\frac{r_\Omega}{R}\right)+\ln\frac{\xi_d}{\xi\sin\theta}+\lambda R\frac{\sin\theta}{\xi_d}\right]. \tag{3.4}$$

The first term in the square brackets is the hydrodynamic energy per unit length of a pair of vortices of equal charge separated by a distance $R<r\Omega$ from each other. The second term is the logarithmic energy of the field $\mathbf{d}$ near the disclinations that intersect the domain wall. The third term is the soliton energy per unit length of the vortex (the dimensionless parameter λ is ~ 1).

Minimizing (3.4) with respect to R, we get

$$R=\xi_d(\rho_s^\perp/\lambda\sin\theta\rho_{sp}^\perp),$$

i.e., the equilibrium distance between paired vortices is comparable to the thickness of the domain wall. We clearly cannot speak of a planar soliton under these conditions. More precisely, there exists a two-dimensional region of diameter $\sim\xi_d/\sin\theta$ near the vortex pair within which α is nonzero, and this region acts as a two-dimensional well which traps the spin waves. NMR experiments should consequently excite the spin waves localized in these wells at a frequency which is less than the fundamental absorption frequency corresponding to the edge of the continuous spectrum. For transverse NMR experiments the absorption frequency is given by the general expression

$$\omega^2=(\gamma H)^2+R_t^2\Omega_L^2\sin^2\theta-\Omega_L^2\cos^2\theta \tag{3.5}$$

(cf., e.g., Ref. 11). Here γ is the gyromagnetic ratio for the ^{3}He nucleus, Ω_L is the longitudinal NMR frequency in an unconfined geometry, and R_t is a dimensionless parameter. We have $R_t^0=1$ for the main peak, while $R_t^v<1$ for the absorption peak arising from excitation of the localized modes. For $\theta\gg\xi_d/r_\Omega$ the diameter of the potential well is $\sim\xi_d/\sin\theta$, which is comparable to the diameter of a planar soliton in the unconfined system. As in the case of a planar soliton, the parameter $R_t^v\lesssim 1$ is therefore independent of θ [the scaling dependence on θ is included in (3.5)]. In a future paper we will numerically calculate the exact value of R_t^v, which in general depends on θ. The intensity of the vortex peak can also be estimated for $\theta\gg\xi_d/r_\Omega$; it is proportional to the relative diameter of the spin-wave localization region, i.e.,

$$I^v/I^0\sim n_v\xi_d^2/\sin^2\theta. \tag{3.6}$$

CONCLUSIONS

Vortices with $p=1/2$ can coexist with $p=1$ and $p=2$ vortices if the rotating superfluid is contained between two parallel plates spaced a distance $r_0\ll\xi_d$ apart. Their presence should be detectable from the additional vortex peak in the NMR signal, to which only vortices with $p=1/2$ can contribute. For the other vortices with $p=1, 2$, the field $\mathbf{d}$ and the vector $\mathbf{l}$ are both uniformly distributed down to small distances $\sim r_0$ from the vortex axis; as a consequence, they either produce no vortex peak at all, or else the peak is so weak and close in frequency to the main peak that it cannot be detected. If a regular satellite peak is found in the NMR spectrum for ^{3}He–A, one can thus be sure that exotic vortex-disclination hybrids with half-integral circulation are present.

If the plates are brought even closer together or if thin films are used, the system will become effectively two-dimensional and a Berezinskii-Thouless-Kosterlitz vortex phase transition, accompanied by a sudden change in the density of the superfluid component,[14] should occur. This transition in ^{3}He was ascribed in Ref. 14 to depairing of vortices with $p=1$. However, if the magnetic field is normal to the film the energy $(E_V)^{1/2}$ of a $p=1/2$ vortex is less than $(E_V)^1$ for a $p=1$ vortex at all temperatures, so that the phase transition in this case should be associated with depairing of $p=1/2$ vortices; both the spin rigidity ρ_{sp} and the density ρ_s of the superfluid component should then change abruptly. This behavior reflects the hybrid nature of the linear defect (vortex-spin-disclination combination), which disrupts both the superfluid long-range order and the long-range order in the spin orientation.

The present work is the result of joint studies carried out under the Soviet-Finnish ROTA research program.

[1]G. E. Volovik, Usp. Fiz. Nauk **143**, 73 (1984) [Sov. Phys. Uspekhi **27**, 363 (1984)].
[2]P. J. Hakonen, O. T. Ikkala, S. T. Islander, O. V. Lounasmaa, and G. E. Volovik, J. Low Temp. Phys. **53**, 425 (1983).
[3]H. K. Seppälä, P. J. Hakonen, M. Krusius, T. Ohmi, M. M. Salomaa, J. T. Simola, and G. E. Volovik, Phys. Rev. Lett. **52**, 1802 (1984).
[4]M. Liu, Physica **190-100B**, 1615 (1982).
[5]P. J. Hakonen, M. Krusius, M. M. Salomaa, J. T. Simola, Yu. M. Bunkov, V. P. Mineev, and G. E. Volovik, Phys. Rev. Lett. **51**, 1362 (1983).
[6]G. E. Volovik and V. P. Mineev, Zh. Eksp. Teor. Fiz. **72**, 2256 (1977) [Sov. Phys. JETP **45**, 1186 (1977)]; Pis'ma Zh. Eksp. Teor. Fiz. **24**, 605 (1976) [JETP Lett. **24**, 561 (1976)].
[7]A. S. Schwarz, Nucl. Phys. **B208**, 141 (1982).
[8]D. G. de Gennes, *The Physics of Liquid Crystals*, Clarendon Press, Oxford (1974).
[9]M. C. Cross, J. Low Temp. Phys. **21**, 525 (1975).
[10]V. P. Mineev and G. E. Volovik, Phys. Rev. **B18**, 3197 (1978).
[11]H. K. Seppälä and G. E. Volovik, J. Low Temp. Phys. **51**, 279 (1983).
[12]P. W. Anderson and G. Toulouse, Phys. Rev. Lett. **38**, 508 (1977); V. R. Chechetkin, Zh. Eksp. Teor. Fiz. **71**, 1463 (1976) [Sov. Phys. JETP **24**, 766 (1976)].
[13]A. L. Fetter, Phys. Rev. **B20**, 303 (1979).
[14]D. L. Stein and M. C. Cross, Phys. Rev. Lett. **42**, 504 (1979).

Translated by A. Mason