Low-frequency oscillations in the resistive state of narrow superconductors

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Two possibilities whereby phase slip centers can generate electromagnetic oscillations of frequency appreciably lower than the Josephson frequency of each phase slip center are considered. One possibility is that the entire structure can move along the sample at a constant velocity, and the other is that the oscillations can result from inequality of the Josephson frequencies of the phase slip centers that are under unequal conditions; in particular, one of the centers may be near the sample boundary.

1. INTRODUCTION

A superconductor through which a strong enough electric current is made to flow can be in the so-called resistive state, which differs both from the purely superconducting and the purely normal states. In long narrow superconducting samples the resistive state is realized by periodic vanishing, in time, of the order parameter in a set of sample points that can form a periodic chain (see the review[1]). These points are called phase slip centers (PSC), each of which can be regarded as a Josephson junction, the average voltage drop on which $\langle V \rangle$ is connected with the frequency of the oscillations of the physical parameter by the Josephson relation $2e\langle V \rangle = \omega_J$. Each PSC exhibits typical Josephson properties when interacting with external monochromatic microwave radiation.[2,3] When a sample was irradiated in an experiment[2] by a microwave of frequency 10^9–10^{10} Hz its current-voltage characteristics (IVC) acquired the distinctive Josephson current steps. The frequency interval indicated is the typical scale of the voltage oscillations, of the order-parameter modulus, of the condensate velocity, etc., in the resistive state of thin superconductors such as tin, at temperatures $T_c - T \sim 10^{-1}$ K (see also Ref. 4).

When account is taken of all the foregoing, experimental observation of generation of electromagnetic oscillations by a sample in a resistive state, at substantially lower frequencies (10^7 Hz),[4,5] was unexpected. The appearance of oscillations with such relatively low frequencies cannot be attributed in any way to only Josephson oscillations of high frequency ω_J. A hypothesis was advanced[6] that the entire PSC structure moves as a whole along the sample, and the low frequency of the oscillations is given then by the relation $\omega = 2\pi v/l$, where l is the period of the spatial structure and v is its velocity.

We consider here two possible mechanisms for the onset of low-frequency oscillations in the resistive state of a thin superconducting sample. (We emphasize that we have in mind a frequency that is "low" compared with the Josephson frequency.) The first is connected with the motion of the structure as a whole, and the second is due to desynchronization of the neighboring PSC, so that the interaction between the PSC can cause low-frequency oscillations at the difference frequency. The frequency difference between two or several PSC can be due to a difference in the conditions at their locations; some can be, for example, close to a defect or to the sample boundary.

It must be stated that an exact description of the dynamic behavior of a superconductor is a complicated mathematical problem. At the same time, the physics of the phenomenon can be tracked in many cases using a relatively simple dynamic model. Our case is no exception in this sense. We start with a simple dynamic model of the superconductor (time-dependent Ginzburg-Landau equations), which corresponds in our particular case to the zero-gap situation.[7]

On this basis, we consider in Sec. 2 the motion of a PSC structure in a homogeneous infinitely long channel. As shown in Ref. 8, a PSC system can be visualized as a regular lattice in two-dimensional space $\{x,t\}$, at the sites of which the modulus of the order parameter is $\Delta = 0$ (x is the coordinate along the sample). An immobile system corresponds to a rectangular lattice; motion of the system deforms the unit cell, which takes the form of a parallelogram whose sides are no longer parallel to the t axis.

In Sec. 2 we shall also determine the conditions under which the structure can move, calculate the velocity of the motion v, and the period l. An immobile structure (in the sense that the PSC are produced at fixed points) can be made moving by increasing the current. In this case the system structure changes either because a new PSC enters the sample, or else the number of PSC can remain unchanged but the entire structure is set in motion. The second possibility corresponds to a lower energy dissipation j_E. This can be seen from Fig. 3 below, where on the intial section of the characteristic, at a given value of the current, a weaker electric field corresponds to a state with a moving structure.

Section 3 is devoted to the mismatch of the oscillation frequencies of neighboring PSC. We consider a dynamic model which, while incapable apparently of adequately representing the typical experimental situation, has the advantage of being exactly solvable and accounts for the basic physical features of the desynchronization effect. In this model the sample considered has a critical temperature T_c and a periodic system of defects, constituting small regions with locally increased critical temperature $T_{c1} > T_c$, distributed over the sample length. It is assumed that this system of defects is semi-infinite on an infinite sample and occupies the region $x > 0$. At temperatures in the interval $T_{c1} > T_c$ and in

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a certain range of currents, superconducting nuclei are produced near the defects.[9,10] Owing to the voltage differences between the nuclei, the order-parameter phase difference between them increases with time and leads to formation of PSC in the gaps between the nuclei. PSC located far from the edge oscillate at the Josephson frequency, which differs from that of the outermost PSC. As a result, we calculate in this section the difference beat frequency that causes the low-frequency voltage oscillations.

2. MOTION OF STRUCTURE PHASE SLIP CENTERS

As already mentioned, one possible explanation of the appearance of signals with a low-frequency spectrum is the motion of a PSC structure along a channel at a certain velocity v. All the quantities, particularly the electric field, acquire then a slow time dependence with a characteristic frequency $\omega = 2\pi v/l$, where l is the period of the structure. An estimate of the velocity of this PSC motion is given in Ref. 4, with a value 10^4–10^5 cm/s for tin.

In this section we investigate the possibility of PSC motion on the basis of the simplest model time-dependent Ginzburg-Landau equations. For the complex order parameter $\psi = \Delta\exp(i\chi)$ this system takes in dimensionless units the form

$$u\left(\frac{\partial\psi}{\partial t}+i\varphi\psi\right)-\frac{\partial^2\psi}{\partial x^2}+\psi(|\psi|^2-1)=0, \tag{1}$$

$$j=-\frac{\partial\varphi}{\partial x}+\frac{1}{2i}\left(\psi^*\frac{\partial\psi}{\partial x}-\psi\frac{\partial\psi^*}{\partial x}\right), \tag{2}$$

where φ is the scalar potential. At $u = 12$ the system (1), (2) describes a zero-gap superconductor with paramagnetic impurities.[7] The parameter u has the meaning of the ratio of the squares of the coherence length ξ^2 and the depths of penetration $l_E{}^2$ of the electric field (see, e.g., Refs. 11 and 12):

$$u=\xi^2(T)/l_E{}^2(T). \tag{3}$$

Of course, the system (1), (2) cannot describe all the details of the dynamic behavior of the superconductor, but it does have a time-periodic solution that corresponds to a PSC system in the case of physical interest $\xi(T)\ll l_E(T)$ (Ref. 12). In a real superconductor of electric-field penetration depth l_E exceeds considerably the coherence length $\xi(T)$. For this reason, just as in Ref. 12, we shall consider below the case $u\ll 1$.

Equations (1) and (2) can be written in the gauge, invariant form

$$-u\frac{\partial\Delta}{\partial t}+\frac{\partial^2\Delta}{\partial x^2}+(1-\Delta^2-Q^2)\Delta=0, \tag{4}$$

$$j=-\Delta^2Q-\frac{\partial\Phi}{\partial x}-\frac{\partial Q}{\partial t}, \tag{5}$$

$$u\Delta^2\Phi=-\frac{\partial}{\partial x}(\Delta^2Q), \tag{6}$$

where $\Phi = \varphi + \partial\chi/\partial t$; $Q = -\partial\chi/\partial x$ (in our case the vector potential $A = 0$ in view of the narrowness of the sample).

It was shown in Refs. 12 and 13 that a voltage is produced in the sample in regions of approximate size l_E (i.e., $u^{-1/2}$ in our units) between the PSC. In these regions the order parameter Δ, the potential Φ, and the superconducting current $j = -\Delta^2 Q$ are practically independent of time and are functions of only the coordinate x. A time dependence of these quantities occurs only in a rather narrow vicinity of the PSC, with dimension of the order of $x_1\sim u^{-1/4}$, so that $l_E \gg x_1$. As a result of this ratio of the characteristic scales, solution of the system (4)–(6) for the time-independent quantities, at distances x from the PSC such that $x_1\ll x\sim l_E$, was sufficient to calculate the IVC (Ref. 12). When the distance to the PSC becomes $x\ll l_E$, the following condition holds:

$$j_s=0 \quad \text{at} \quad x\ll u^{-1/2}. \tag{7}$$

In this section we consider the motion of the PSC structure as a whole, at a certain velocity v. Being interested in the current-voltage characteristic of the sample, we assume in accord with the arguments above (see also the Appendix) that it suffices for this purpose to solve the system (4)–(6) in the region $x\sim l_E$, where all the quantities will depend only on the variable $y = x - vt$. Near each PSC we assume as before satisfaction of the condition (7). Equation (4) for the self-similar solution contains v in the combination uv/l. Since u is small, we shall assume that the condition $uv/l\ll 1$ is satisfied for all the velocities considered. Over scales on the order of the distance l between the PSC, one can neglect at $l\gg 1$ also the term with the second derivative in (4). Equations (4)–(6) take the form

$$\Delta^2+Q^2=1, \tag{8}$$

$$j=-\frac{\partial\Phi}{\partial y}+v\frac{\partial Q}{\partial y}-\Delta^2Q, \tag{9}$$

$$u\Delta^2\Phi=-\frac{\partial}{\partial y}(\Delta^2Q). \tag{10}$$

Equation (8), which is valid at $x\gg 1$, leads on the basis of (7) to boundary conditions near the PSC:

$$\Delta=1 \quad \text{at} \quad 1\ll x\ll u^{-1/2}.$$

We introduce the notation

$$z=\Delta^2,\quad \mu=\Phi-vQ=\Phi+v(1-z)^{1/2},$$
$$\partial f/\partial z=[j-z(1-z)^{1/2}](3z-2)/z(1-z)^{1/2}. \tag{11}$$

In this notation, the electric field is $E = -\partial\mu/\partial y$. With the aid of (8)–(10) we can easily obtain an equation for the function $\mu(z)$:

$$\frac{\partial f}{\partial z}\frac{\partial z}{\partial\mu}=2u\mu-2uv(1-z)^{1/2}. \tag{12}$$

For the period of the structure we have

$$l=\int_{z_0}^{1}\frac{dz}{j-z(1-z)^{1/2}}\left[\frac{\partial\mu(y<y_0)}{\partial z}-\frac{\partial\mu(y>y_0)}{\partial z}\right]; \tag{13}$$

here y_0 is the point at which the potential Φ vanishes and at which $j_s = z(1-z)^{1/2}$ and $z(y)$ have extrema in accordance with (10) (Fig. 1). We note that the moving structure becomes asymmetric relative to the midpoint between two neighboring PSC.

Assuming that the point $y = \tilde{y}$ corresponds to vanishing

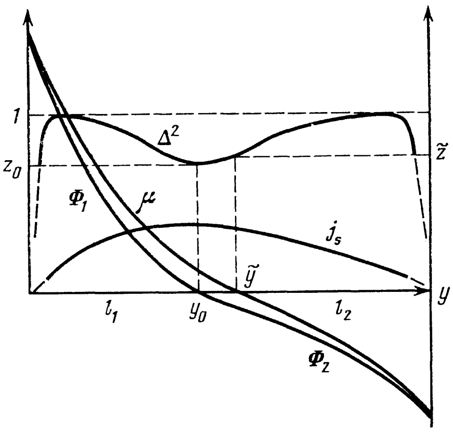

FIG. 1.

of $\mu(z)$, we can formally integrate Eq. (12):

$$f(z)=u\mu^2-2uv\int_0^{\mu}(1-z)^{1/2}\,d\mu$$

$$=u\mu^2-2uv\int_{\tilde{z}}^{z}(1-z)^{1/2}\frac{\partial\mu}{\partial z}dz, \tag{14}$$

where

$$f(z)=\int_{\tilde{z}}^{z}\frac{[j-z(1-z)^{1/2}](3z-2)}{z(1-z)^{1/2}}dz. \tag{15}$$

We consider first the case of low velocities. Expanding (14) in powers of v, we get

$$\mu(y<y_0)=\left[\frac{f}{u}+v^2(1-z_0)\right]^{1/2}$$

$$+v(1-z)^{1/2}-v\mu_0\frac{\partial\beta}{\partial\mu_0}+v^2\frac{\alpha}{\mu_0}, \tag{16}$$

$$\mu(y>y_0)=-\left[\frac{f}{u}+v^2(1-z_0)^{1/2}\right]$$

$$+v(1-z)^{1/2}-v\mu_0\frac{\partial\beta}{\partial\mu_0}-v^2\frac{\alpha}{\mu_0}. \tag{17}$$

Here

$$\mu_0=\left[\frac{f}{u}+v^2(1-z_0)\right]^{1/2},$$

$$\beta(z)=\frac{1}{\mu_0}\int_{z_0}^{z}[(1-z)^{1/2}-(1-z_0)^{1/2}]\frac{\partial\mu_0}{\partial z}dz,$$

$$\alpha(z)=\int_{z_0}^{z}\mu_0\left(\frac{\partial\beta}{\partial z}\right)^2\frac{\partial z}{\partial\mu_0}dz. \tag{18}$$

Since $\Phi=\mu+vQ=\mu-v(1-z)^{1/2}$ vanishes at $z=z_0$, we should have

$$f(z_0)=-uv^2(1-z_0).$$

This relation yields the value of $\tilde{z}$ in (15). Ultimately we have

$$\frac{f_0(z)}{u}\equiv\frac{f(z)}{u}+v^2(1-z_0)=\int_{z_0}^{z}\frac{[j-z(1-z)^{1/2}](3z-2)}{z(1-z)^{1/2}}dz, \tag{19}$$

$$\mu_0=u^{-1/2}[f_0(z)]^{1/2}. \tag{20}$$

The total voltage drop on the section between two PSC is

$$U=\mu(y<y_0)|_{z=1}-\mu(y>y_0)|_{z=1}=U_0+\delta U$$
$$=2\mu_0(z=1)+v^2\alpha(1)/\mu_0(1). \tag{21}$$

It is obvious from (18) and (19) that the coefficient of v^2 in (21) is positive. For the period of the structure we have from (13) and (16)

$$l=l_0+v^2\left\{\frac{2\alpha(z=1)}{j\mu_0(z=1)}+\int_{z_0}^{1}\frac{(3z-2)\alpha(z)\,dz}{(1-z)^{1/2}[j-z(1-z)^{1/2}]^2\mu_0(z)}\right\}. \tag{22}$$

It is easily seen that the correction to l is also positive. The integral in (22) can be easily calculated in the case when $j\to j_c$ and $uv^2\ll j-j_c$. In our case the IVC are a two-parameter family of curves in the sense that at a given current j the electric field depends on the period l of the structure and on the structure velocity v, i.e., $E=E(l,v)$. The first question that must be answered is whether a state with finite structure velocity will decrease the electric field at a given current. We put, as in Ref. 12, $z_0=2/3$ and calculate the electric field. The period of the structure is $l=l_0+\delta l$, where

$$l_0=\left(\frac{6}{u}\right)^{1/2}\ln\frac{cj_c}{j-j_c},$$

$$\delta l=\frac{2^{1/2}\eta}{9u^{1/2}}\frac{uv^2}{j-j_c}$$

$$\eta=6-8\ln 2\approx 0.45.$$

The results can be interpreted in the following manner. Assume that the channel in question is long enough to hold a large number of PSC, so that the influence of the boundary conditions on the end of the channel affects the behavior of each individual PSC relatively little. The current-voltage characteristic of such a channel is stepped (see, e.g., Ref. 2), as shown schematically in Fig. 2. Let the current flowing through the sample be in the interval $j^{(n-1)}<j<j^{(n)}$ between the $(n-1)$ st and the nth jumps of the voltage. When the current is increased above $j(n)$, the shortening of the period l_0 makes a structure with PSC in the channel impossible, an $(n+1)$ st PSC must appear, and an nth jump of the channel

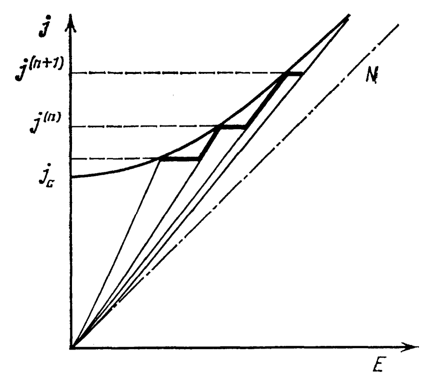

FIG. 2.

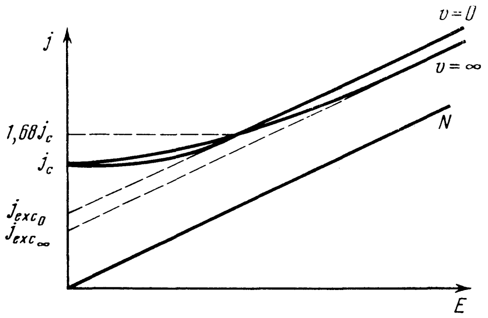

FIG. 3.

should occur. There is, however, an alternative: The PSC structure can begin to move at a velocity v such that the previous period l is preserved, and the decrease of the period by the increased current is offset by an increase of v. At currents close to j_c we have thus

$$uv^2=3^{5/2}\delta j_c/\eta,\qquad \delta j=j-j^{(n)}.$$

If the sample is infinitely long, we determine the motion of the structure by considering the average electric field

$$\bar{E}=U/l=U_0/l_0+\delta U/l_0-U_0\delta l/l_0^2=E_0+\delta E.$$

In the case $j\to j_c$, it is readily seen that $\delta E<0$. In other words, at a given current the IVC is on the left of the characteristic of the immobile structure (curve marked $v=0$ in Fig. 3), i.e., a finite structure velocity corresponds to the state of a system with a lower dissipation jE than of an immobile structure. From the viewpoint that the system tends to a state corresponding to minimum dissipation (see Ref. 10), it can be concluded that the structure velocity should increase.

Since the minimum of the electric field corresponds to finite structure velocities, we consider the case $v\gg u^{-1/2}$ (but $uv/l\ll 1$ as before). We turn to Eq. (12), which we write in the form

$$\frac{\partial f}{\partial z}\frac{1}{2u\Phi}=\frac{\partial \Phi}{\partial z}-\frac{v}{2(1-z)^{1/2}}. \qquad (23)$$

At large v there are two solutions:

$$\Phi_1=-\frac{v}{2}\int_{z_0}^{z}\frac{dz}{(1-z)^{1/2}}=v[(1-z_0)^{1/2}-(1-z)^{1/2}], \qquad (24)$$

$$\Phi_2=-\frac{\partial f}{\partial z}\frac{(1-z)^{1/2}}{uv}. \qquad (25)$$

These two solutions correspond to the sections $y<y_0$ and $y>y_0$ (see Fig. 1) at $v>0$, and conversely to $y>y_0$ and $y<y_0$ at $v<0$.

We consider below for the sake of argument the case $v>0$. The solutions Φ_1 and Φ_2 to the left and to the right of the point $y=y_0$ respectively should match at $y=y_0$. A more accurate solution of (23) in the vicinity of the point $z=z_0$ yields

$$\Phi_1=v[(1-z_0)^{1/2}-(1-z)^{1/2}]+\int_{z_0}^{z}\frac{(\partial f/\partial z)\,dz}{2u\{v[(1-z_0)^{1/2}-(1-z)^{1/2}]+(f_0/u)^{1/2}\}},$$

$$\Phi_2=-\frac{1}{2uv}\frac{\partial f}{\partial z}(1-z)^{1/2}\left[1+\frac{\partial f}{\partial z}\frac{(1-z)^{1/2}}{v(uf_0)^{1/2}}\right]^{-1}.$$

As $z\to z_0$ the functions $\Phi_{1,2}$ go over respectively into $\pm(f_0/u)^{1/2}$, and at $z-z_0\sim 1$ and $v\gg u^{-1/2}$ these expressions yield Eqs. (24) and (25).

For the period of the structure we obtain $l=l_1+l_2$ (see Fig. 1), where

$$l_1=\int_{z_0}^{1}\frac{3z-2}{2uz(1-z)^{1/2}}\frac{dz}{\Phi_1},\qquad l_2=-\int_{z_0}^{1}\frac{3z-2}{2uz(1-z)^{1/2}}\frac{dz}{\Phi_2}.$$

Since $\Phi_1\sim v$ and $\Phi_2\sim v^{-1}$, we have $l\sim v^{-1}$ and $l_2\sim v$. The structure thus becomes strongly asymmetric, with $l_2\gg l_1$. The total period is $l\approx l_2$, i.e.,

$$l=\frac{v}{2}\int_{z_0}^{1}\left[1+\frac{\partial f}{\partial z}\frac{(1-z)^{1/2}}{v(uf_0)^{1/2}}\right](1-z)^{-1/2}[j-z(1-z)^{1/2}]^{-1}dz$$

$$\approx\frac{v}{2}\int_{z_0}^{1}\frac{dz}{(1-z)^{1/2}[j-z(1-z)^{1/2}]}. \qquad (26)$$

The total voltage drop is

$$U=(\mu_1-\mu_2)|_{z=1}=(\Phi_1-\Phi_2)|_{z=1}=v(1-z_0)^{1/2}.$$

The aveage electric field $E=U/l$ is

$$E_\infty=2(1-z_0)^{1/2}\left[\int_{z_0}^{1}\frac{dz}{(1-z)^{1/2}[j-z(1-z)^{1/2}]}\right]^{-1}, \qquad (27)$$

and the frequency ω of the low-frequency non-Josephson oscillations will be

$$\omega=4\pi\left[\int_{z_0}^{1}\frac{dz}{(1-z)^{1/2}[j-z(1-z)^{1/2}]}\right]^{-1}. \qquad (27a)$$

The results are valid under the condition $uv/l\ll 1$ and $l\gg 1$, with l taken here to be the shortest scale, i.e., l_1. Since $l_1\sim 1/uv$ must have $v\ll 1/u$. These results take place therefore at

$$u^{-1/2}\ll v\ll u^{-1}.$$

We assume below that $z_0=2/3$. In the limiting case $j\to j_c$ we have

$$E_\infty=2(j-j_c)^{1/2}/3^{1/4}\pi,\qquad \omega=4\cdot 3^{1/4}(j-j_c)^{1/2}. \qquad (28)$$

In the case $j\gg j_c$ we get

$$j=E_\infty+j_{exc\infty},\qquad j_{exc\infty}=(1-z_0^2)/4(1-z_0)^{1/2}\approx 0.24, \qquad (29)$$

where $\omega=2\cdot 3^{1/2}\pi$. Comparing these expressions with the IVC of the sample at zero velocity[10]:

$$E_0=\begin{cases}0{,}32/\ln\dfrac{cj_c}{j-j_c}, & j\to j_c\\ j-j_{exc0}, & j_{exc0}\approx 0{,}3,\quad j\gg j_c,\end{cases}$$

we see that the initial section of the characteristic at high velocity of the structure lies higher than the section of the characterstics with zero velocity (curve marked $v=\infty$ in Fig. 3); the situation is reversed for large currents. A numerical calculation shows that the curves for $v=0$ and $v=\infty$ intersect at $j=1.68j_c$.

If we start from the minimum-dissipation principle, we can conclude that at $j<1.68j_c$ the PSC structure tends to acquire a higher velocity whose value is limited only by the sample length L, namely $v\sim L$. We must note, however, one very important circumstance. When solving the equations we have discarded the term with $\partial\Delta/\partial t$ in Eq. (4), since we have assumed the parameter u to be small. From the viewpoint of Eq. (4) smallness of u means rapid relaxation of Δ to its equilibrium value. It must be noted, however, that this circumstance is an artifice peculiar only to the chosen model. In real superconductors the relaxation time of Δ is long (see, e.g., Ref. 14) and is proportional to the electron-phonon collision time. This introduces into the system an additional dissipation mechanism not accounted for in the present model. As a result, the PSC structure velocity may stop growing much earlier than in our case.

To illustrate the foregoing, we consider the equation obtained for the order parameter from the microscopic theory in the case when the order parameter varies slowly in space and in time compared, respectively, with the diffusion length $(D\tau_{ph})^{1/2}$ and with the electron-phonon relaxation time τ_{ph}. In dimensionless variables the equation takes the form[15]

$$-\frac{u_0}{\Gamma}\Delta\frac{\partial\Delta}{\partial t}+\frac{\partial^2\Delta}{\partial x^2}+(1-\Delta^2-Q^2)\Delta=0, \qquad (30)$$

where $u_0 = 5.79$ and $\Gamma = (2\tau_{ph}\Delta_{GL})^{-1}$ is the so-called pair-breaking factor; $\Gamma \ll 1$. The equilibrium value of the order parameter is

$$\Delta_{\Gamma Л}=\left[\frac{8\pi^2}{7\zeta(3)}T(T_c-T)\right]^{1/2}.$$

The electric-field penetration depth is in this case

$$l_E=(u_0\Gamma)^{-1/2}\gg\xi.$$

Consider the behavior of the gap Δ in the vicinity of a PSC, where $j_s = 0$. We obtain for the self-similar solution $\Delta(x - vt)$

$$\frac{uv}{\Gamma}\Delta\frac{\partial\Delta}{\partial x}+\frac{\partial^2\Delta}{\partial x^2}+\Delta-\Delta^3=0.$$

This equation has an exact solution

$$\Delta=\text{th}\,[\alpha(x-C)], \qquad (31)$$

where α satisfies the equation

$$u_0v\alpha/\Gamma-2\alpha^2+1=0.$$

From this we get

$$\alpha_{1,2}=u_0v/4\Gamma\pm[(u_0v/4\Gamma)^2+1/2]^{1/2}.$$

The upper and lower signs correspond to the behavior of Δ on the right and left of the PSC, respectively. At $u_0v/4\Gamma \gg 1$ we have $\alpha_1 = u_0v/2\Gamma$ and $\alpha_2 = -\Gamma/4v$. Thus, to the left of the PSC the order parameter increases quite slowly into the interior of the superconducting region. Clearly, when α_2^{-1} becomes noticeably larger than l_E the order parameter cannot reach finite values and the resistive state is destroyed. This occurs at $\alpha_2^{-1} \sim l_E$, i.e., at $u_0v \sim \Gamma^{1/2}$. Thus, the maximum velocity is restricted in this case to the rather low value

$$v_{max}\sim\Gamma^{1/2}\ll1. \qquad (31a)$$

Of course, a numerical comparison of results obtained on the basis of the time-dependent Ginzburg-Landau equations with experimental data on low-frequency generation[4,5] is not very meaningful. The qualitative picture obtained, however, should apparently be observed in a real situation. According to the minimum-energy-dissipation hypothesis, which is certainly valid for certain superconductor-dynamics models,[10] at a given current the system tends to decrease the electric field E. This is achieved by simultaneously increasing the structure velocity v and its period l, as follows from (26), (27a), and (28). In the case of the simple time-dependent Ginzburg-Landau equations, the spatial period will increase right up to the length of the sample, and the velocity will also increase. For the real dynamic model (30), the increase of the structure velocity ceases to ensure a dissipation gain quite early, since the effective mechanism of the order-parameter modulus relaxation comes into play. Estimating at $j \sim j_c$, in dimensional units, the characteristic frequency $\omega = 2\pi v/l$ of the low-frequency oscillations, and using (31a) and the dimensionless units in which Eq. (30) is written,[15,1] we get

$$\omega\sim\frac{1}{\tau_{ph}}\left(\frac{T_c-T}{T_c}\right)^{1/2}, \qquad (31b)$$

which is much less, within the range of validity of (30) (Ref. 1), than the Josephson part of the oscillations at each PSC:

$$\omega_J\sim\omega(T_c\tau_{ph})^{1/2}(1-T/T_c)^{1/4}.$$

3. LOW-FREQUENCY OSCILLATIONS AS THE CONSEQUENCE OF DESYNCHRONIZATION OF INDIVIDUAL PSC

We consider in this section the other mechanism of low-frequency generation by the PSC system. As already mentioned, inequality of the Josephson frequencies of two neighboring PSC (desynchronization) can lead to low-frequency oscillations at the beat frequency. Desynchronization of two Josephson junctions was considered, for example, in Ref. 16, from which it follows that synchronization can take place even if the individual properties of the junctions are different, and to obtain desynchronization it is necessary that in a certain sense the difference between the parameters of two junctions exceed some value.

Similar phenomena can occur not only in a system of artificially produced Josephson junctions, but also in a system of freely produced PSC in the resistive state of a long superconductor. In an infinite and homogeneous superconducting channel, for example, all the PSC are synchronized. Desynchronization effects can be expected near places where the homogeneity of the sample is disturbed, particularly near its boundaries.

We study in this section the desynchronization of PSC located near the boundary of a sample occupying the region $x > 0$.

It is known that the study of the dynamics of superconductors is in general an exceedingly complicated problem. We shall use therefore a simplified model based on equations of the type (1) and (2), the same as in Refs. 9 and 10. We assume a periodic sequence of defects distributed over the length of a sample with critical temperature T_c; the defects increase the superconductivity, with a critical temperature $T_{c1} > T_c$. The length of each defect is $d \ll \xi(T)$. We assume also that the temperature range is

$$T_c<T<T_{c1}. \tag{32}$$

The analog of Eqs. (1) and (2) takes then the form

$$u\left(\frac{\partial\psi}{\partial t}+i\varphi\psi\right)-\frac{\partial^2\psi}{\partial x^2}+\psi\left[1+a\sum_{n\geqslant 0}\delta(x-nl)+|\psi|^2\right]=0,$$

$$j=E+j_s,\quad E=-\frac{\partial\varphi}{\partial x},\quad j_s=\frac{1}{2i}\left(\psi^*\frac{\partial\psi}{\partial x}-\psi\frac{\partial\psi^*}{\partial x}\right). \tag{33}$$

The chain of δ functions describes the change of the critical temperature in the region of the defects[7]

$$a=\frac{2d}{\xi_0}\frac{T_c-T_{c1}}{[T_c(T-T_c)]^{1/2}}. \tag{34}$$

The unusual sign preceding the unity in the square brackets in (33) is due to the fact that $T>T_c$.

The exposition that follows is based on the results of Refs. 9 and 10. It follows from these results that when the current decreases below a certain value j_1, islands (nuclei) of the superconducting phase are produced and are localized near the inhomogeneities described by the δ functions in Eq. (33). The amplitude of the nuclei increases with decreasing current. Just as in Refs. 9 and 10, it will be assumed that the nuclei are far from one another, $l\gg\xi$, and their overlap is small. In view of the presence of current and of normal resistance between the locations of the neighboring nuclei, a voltage drop exists and causes the phase difference between two neighboring nuclei to increase with time. This leads in turn to oscillations of the modulus of the order parameter in the overlap regions and to formation of PSC at several points between neighboring nuclei. In analogy with Ref. 10, this is precisely the picture chosen as the model for the resistive state in the present section.

The summation in (33) is over $n\geqslant 0$. A resistive state is produced in that sample region where there are stimulating inhomogeneities (i.e., at $x>0$); the region $x<0$ remains normal (Fig. 4). Thus, Eq. (33) simulates a system in which a superconducting sample in the resistive state (the region $x>0$) is joined to a normal conductor (region $x<0$). The PSC on the extreme left, located near the boundary with the normal state, is under conditions that differ from those for the remaining PSC, so that its Josephson frequency can differ from the frequencies of the remaining PSC, and it is this which leads to desynchronization. The model chosen is quite special, but offers the advantage that it can be solved exactly.

We represent the order parameter in the form

$$\psi(x,t)=\sum_n \exp[i\chi_n(t)]\varphi_n(x-nl), \tag{35}$$

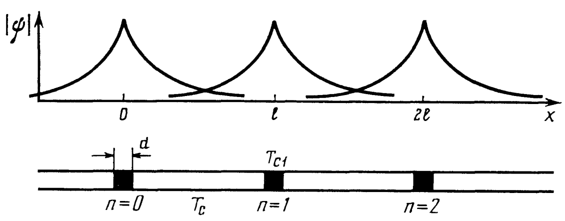

FIG. 4.

where the $\chi_n(t)$ are the phases of the individual superconducting nuclei, and the $\varphi_n(x)$ describe the form of the nth nucleus whose amplitude A_n is determined by the difference j_1-j. A solution for φ_n was obtained in Ref. 10 in a definite region of the parameters a and l for the case of an unbounded system of defects ($n\leqslant 0$). It was assumed, in particular, that $j_1\ll 1$ and $l\gg 1$ (smallness of u is not required in this case). Using the method developed in Ref. 10, we can obtain for a semi-infinite system of defects a solution similar to that of Ref. 10. Assuming that the parameters a and l are such that $uj_1l^2\ll 1$, we have

$$A_n^{-1}\varphi_n(x)=\begin{cases} e^{x}-(2i/ujl)\,e^{-x-2l}(1-\delta_{n,0}), & -l<x<0,\\ e^{-x}+(2i/ujl)\,e^{x-2l}, & 0<x<l,\\ (2i/ujl)\,e^{-x}, & l<x. \end{cases} \tag{36}$$

We take into account the interaction with only the nearest neighbors, so that the functions $\varphi_n(x)$ that describe the shape of the nucleus are the same for all $n>0$. At $n=0$ the function φ_0 differs from the others, since it has no neighbor on the left. The proportional subcritical value is

$$\Delta_n^2={}^5/_8u^2[j_1^2(n)-j^2],$$
$$j_1^2(n)=\frac{16}{5u^2}(|a|-2)+\frac{32}{5u^2}le^{-2l}(2-3\delta_{n,0}). \tag{37}$$

It is assumed here that $|a|-2\ll 1$ [$a<0$ according to (32)].

Introducing the quantities

$$g_n=2\chi_n-\chi_{n-1}-\chi_{n+1}, \tag{38}$$

and proceeding in analogy with Ref. 10, we obtain the system of equations

$$\frac{\partial g_1}{\partial t}=\gamma(\sin g_3-2\sin g_1)+q,$$
$$\frac{\partial g_2}{\partial t}=\gamma(\sin g_4-2\sin g_2),$$
$$\frac{\partial g_n}{\partial t}=\gamma(\sin g_{n+2}+\sin g_{n-2}-2\sin g_n),\quad n\geqslant 3,$$
$$\gamma=(2A^2/u^2j_1^2l^2)\,e^{-2l},\quad q=3uj_1le^{-2l}. \tag{39}$$

We have used here the values for A and j_1 at $n>0$. It follows from Ref. 10 that the electric field at the nth site, averaged over the period of the cell, can be written in the form

$$E(n,t)=j-5u^3j_1^2/8l-(j_1-j)[1+l^2e^{-2l}\cos g_n(t)]. \tag{40}$$

The last term is the average of the London current j_s.

The meaning of g_n can be understood from the following. In Fig. 4 the PSC are produced in the regions between the nuclei. The Josephson frequency is $\omega_{n+1,n}=\overline{\partial(\chi_{n+1}-\chi_n)/\partial t}$, for the PSC located between the $(n+1)$st and nth nuclei and $\omega_{n,n-1}=\overline{\partial(\chi_n-\chi_{n-1})/\partial t}$ for the PSC between the nth and the $(n-1)$st nuclei. The superior bar means time averaging. Therefore

$$\overline{\partial g_n/\partial t}=\omega_{n+1,\,n}-\omega_{n,\,n-1}. \tag{41}$$

The desynchronization is just the difference between this quantity and zero, i.e., the fact that the Josephson frequencies of two neighboring PSC are unequal.

In the approximation considered the system (36) breaks up into unconnected parts with odd and even n. At even n one can assume that all $g_n = 0$. The presence of a boundary affects only g_n with odd n. The behavior of the solution depends on the quantity

$$R=q/\gamma=6u^2j_1^2l^3/5(j_1-j). \qquad (42)$$

If $R < 1$, a static solution $\sin g_n = R$, which is stable in the small, is possible, and there is no desynchronization in accordance with (41). At $R > 1$, no static solution is possible. We consider here the simplest case $R \gg 1$, when

$$g_1=qt, \quad g_3\sim R^{-1}\cos qt\ll 1. \qquad (43)$$

We see that in this case of large $R \gg 1$, in accordance with (41), the PSC that is desynchronized from all the others is the one farthest to the left (Fig. 4). It follows from (40) that it leads to an alternating component of the total voltage across the sample:

$$V_\sim(t)=-{}^5/_8u^3j_1l^2(j_1-j)e^{-2l}\cos qt. \qquad (44)$$

The Josephson frequency $\omega_J \sim El \sim jl$, therefore the ratio of the low oscillation frequency ω to the Josephson frequency is $\omega/\omega_J \sim \exp(-2l)$. We note that generation of low-frequency oscillations in this model is possible only at currents

$$j_g<j<j_1, \qquad (45)$$

where the generation current j_g is determined from the condition $R = 1$, i.e.,

$$(j_1-j_g)/j_1={}^6/_5u^2j_1l^3. \qquad (46)$$

Since the approximation considered is suitable only at $j_1 - j \ll j_1$ Eqs. (45) and (46) are meaningful only at $j_1l^3 \ll 1$. Just as in the preceding case, the effect exists in a finite current interval.

4. CONCLUSION

Although the two considered mechanisms whereby low-frequency oscillations are generated were analyzed here using particular models of superconductor dynamics, the physical nature of these phenomena is nevertheless common. The principal statement of Sec. 2 is that the structure can move and therby decrease the dissipation. In this case the oscillations are of lower frequency than the Josephson oscillations because of the relatively slow velocity (31a) of the structure.

It should be noted that the low-frequency oscillations, as follows from our results, occur in the entire range of currents, whereas in experiment they are observed in practice at one value of the generation current.

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APPENDIX

Consider the solution of Eqs. (1) and (2). Assume that $j \gg 1$ and that the current j_s can be neglected in (2). This restriction is not fundamental and is imposed only for simplicity. The solution of the equation

$$u\frac{\partial\psi}{\partial t}-iujx\psi-\frac{\partial^2\psi}{\partial x^2}+\psi(|\psi|^2-1)=0 \qquad (A.1)$$

can then be written in the form

$$\psi(x,t)=\exp(ijvt^2/2)\sum_n \exp(injlt)R(x-vt-nl), \qquad (A.2)$$

where $R(x)$ satisfies the equation

$$\frac{\partial^2R(x)}{\partial x^2}+uv\frac{\partial R(x)}{\partial x}+(1+iujx)R(x) - \sum_{m,n}R[x-l(m-n)]R(x-ln)R^*(x-lm). \qquad (A.3)$$

At $v = 0$, Eq. (A.2) recalls the known Abrikosov solution for a vortex system in a type-II superconductor; in our case the spatial coordinate is replaced by the time. The set of localized functions $R(x)$ is then such that the gauge-invariant quantities $|\psi|$, Q, and Φ depend on the time only in a narrow region $x_1 \sim u^{-1/4}$ about the PSC, and the period of the structure is $l \sim l_E \sim u^{-1/2}$ (Ref. 12). The existence of such a set of functions $R(x)$ is equivalent to the presence of a stable time-periodic solution of Eq. (A.1). Since such a periodic solution was obtained for $v = 0$ by numerical intergration, this means that Eq. (A.3) has localized solutions at $v = 0$.

It can be assumed that Eq. (A.3) has localized solutions also at $v \neq 0$, which will mean physically motion of the structure with velocity v. If $y = x - vt$ is not too close to the PSC, then $|\psi|$, Q, and Φ will depend only on y, and this region is subject to the largest voltage drop (cf. Ref. 12).

It precisely in this sense that the solutions considered in the text can be taken to be self-similar. We note that the PSC lattice is rectangular in the coordinates $\{y,t\}$.

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