Excitation spectrum of incommensurable magnetic structures and neutron scattering

Yu. A. Izyumov and V. M. Laptev

Institute of Metal Physics, Ural Scientific Center of the Academy of Sciences of the USSR, Sverdlovsk
(Submitted 25 April 1984; resubmitted 13 August 1984)
Zh. Eksp. Teor. Fiz. **88**, 165–179 (January 1985)

As a rule, incommensurable structures are observed in crystals which have a center of inversion, so that the energy expansion does not contain Lifshitz invariants linear in the derivatives. The excitation spectra of such systems are studied over a wide range of temperatures—in the low-temperature region, where the dynamics of the magnetic order parameter is of the nature of a precession of the magnetic moment and is governed by the Landau-Lifshitz equation, and in the neighborhood of the phase transition, where the dynamics includes precessional and oscillatory motion. We find the dispersion curves for the excitation of the two basic modulated structures—a simple spiral and a longitudinal spin wave—and obtain the temperature dependence of these curves. The spectrum calculated in the neighborhood of the phase transition joins with the spin-wave spectrum of the low-temperature region. In the other limit—the case of strong anisotropy—the precession of the magnetic moment is suppressed, and the dynamics of the magnetic fluctuations is like the dynamics of the order-parameter fluctuations at a structural transition. In the case of weak anisotropy the fluctuation spectrum exhibits gaps of a dynamical nature due to precession of the magnetic moment in the periodic field of the magnetic structure. The size of these gaps depends on a dimensionless parameter which characterizes the relative share of precession and oscillations in the dynamics of the magnetic order parameter. We calculate the response of the system in terms of two-time classical Green functions and use the result to obtain the complete neutron-scattering pattern in (ω, q) space and the temperature evolution of this pattern.

1. INTRODUCTION

The overwhelming majority of incommensurable structures in crystals—structures of the simple-spiral (SS) or static longitudinal spin wave (LSW) types—result from a modulation of the initial ferromagnetic or antiferromagnetic stucture, whose symmetry does not admit Lifshitz invariants which are linear in the derivatives. This is because the symmetry group of the crystals which exhibit incommensurable (modulated) structures contains inversion.[1] Thus the situation in magnetism is contrary to that at structural phase transitions to incommensurable phases, where Lifshitz invariants typically do appear in the Ginzburg-Landau functional.[2] The physical mechanism for the long-wavelength modulation of magnetic structures, as a rule, is competition between the positive and negative exchange interactions between nearest and next-nearest neighbors in the crystal.

The structure of the incommensurable magnetic phase in the presence of Lifshitz invariants was established by Dzyaloshinskiĭ,[3] who showed that in this case the incommensurable phase can be described by a lattice of solitons inserted into the commensurable phase (see also Ref. 4). The excitation spectrum of this phase was studied in Refs. 5 and 6, where it was shown that the periodic potential that governs the motion of fluctuations of the soliton lattice is a single-band potential, and the excitation spectrum of the system is described by a Lamé equation with band index $n = 1$.

In this paper we report a study of the excitation spectrum of modulated magnetic structures in systems without Lifshitz invariants, where the modulation stems from the exchange interaction. We shall start from a phenomenological approach based on an expansion of the energy in powers of the spatial derivatives of the order parameter under the assumption that the wave vector of the modulation is small compared to the reciprocal lattice vectors of the crystal, and we shall consider a wide range of temperatures, including the low-temperature region, where the order parameter changes slowly with temperature, and the immediate vicinity of the phase transition.

In the low-temperature region the dynamics of the magnetic order parameter is governed by the Landau-Lifshitz equation, which describes the inhomogeneous precession of the magnetic moment at a constant modulus of the local magnetic-moment vector. As the phase transition is approached the modulus softens, and the dynamics of the magnetic order parameter is no longer determined solely by the precession but by oscillatory motion as well. Thus the dynamics of the order parameter near T_c is richer for magnetic systems than for systems with a structural phase transition. The contribution of the two modes of motion (precessional and oscillatory) at an antiferromagnetic homogeneous resonance was studied in Refs. 7 and 9, but, as we shall see, the superposition of these two modes of motion is manifested more clearly for incommensurable magnetic structures, since it occurs against the background of a periodic structure which leads to a band picture for the fluctuation spectrum.[9,10]

Further, a quantum calculation of the spin-wave spectrum of a simple spiral in the exchange approximation has shown[11] that there are two Goldstone modes: one at $\mathbf{q} = 0$, due to the breaking of the symmetry of the ground state with

 0038-5646/85/010095-09$04.00

respect to rotations, and a second at $\mathbf{q} = \mathbf{k}$ (where $\mathbf{k}$ is the wave vector of the spiral), due to the breaking of the translational symmetry. It has been shown [9,10,12] that the presence of a Goldstone mode (phason) at $\mathbf{q} = \mathbf{k}$ is a general property of incommensurable structures. From this point of view the spin wave in a simple spiral in the neighborhood of $\mathbf{q} = \mathbf{k}$ is also a phason. In discussing the dynamics of the magnetic order parameter near T_c we shall make wide use of the general ideas of the theory of the order-parameter dynamics at a structural phase transition to an incommensurable phase. We shall investigate the dispersion curves of the excitation spectrum of the two basic incommensurable structures, *SS* and *LSW*, evaluate the response function, and analyze the neutron inelastic scattering cross section and its evolution with temperature over a wide region of (ω, q) space.

We shall assume that our structure is a long-wavelength modulation of the initial ferromagnetic structure, and we shall write the functional for the magnetic energy of the crystal (taken to be uniaxial for the sake of definiteness) in the form

$$\Phi = \int d\mathbf{r}\{r\mathbf{M}^2+\gamma\partial_z\mathbf{M}\partial_z\mathbf{M}+\alpha\partial_z^2\mathbf{M}\partial_z^2\mathbf{M}+\gamma_\perp(\partial_x\mathbf{M}\partial_x\mathbf{M} + \partial_y\mathbf{M}\partial_y\mathbf{M})+u\mathbf{M}^4 + wM_z^2+w_\perp(M_+^n+M_-^n)\}. \quad (1.1)$$

Here $\mathbf{M}$ is the magnetic-moment density vector, and $M_\pm = M_x \pm iM_y$ are its angular projections. The first five terms derive from the exchange, and the last two allow for anisotropy, with the number n (equal to four or six) indicating the order of the anisotropy in the basal plane. The signs of the coefficients of the gradient terms, $\gamma<0$, $\alpha>0$, and $\gamma_\perp>0$, correspond to structural inhomogeneity in the z direction only; in other words, the wave vector of the structure should be directed along the z axis: $\mathbf{k} = (0, 0, k)$.

The equilibrium distribution of the magnetic-moment density is determined from the equation $\delta\Phi/\delta\mathbf{M} = 0$ and, depending on the sign of the anisotropy w, corresponds to two different magnetic structures, described by the spatial dependence of the vector $\mathbf{M} = \mathbf{M}^0(z)$:

$$w<0: \quad M_+^0=0, \quad \hat{L}_\parallel M_z^0+2u(M_z^0)^3=0; \quad (1.2)$$

$$w>0: \quad M_z^0=0, \quad \hat{L}M_+^0+2u(M_+^0)^2M_-^0+nw_\perp(M_-^0)^{n-1}=0, \quad (1.3)$$

where $\hat{L}$ is the fourth-order differential operator

$$\hat{L}=L(\partial_z)=\alpha\left(\partial_z^2-\frac{\gamma}{2\alpha}\right)^2-\gamma_\perp(\partial_x^2+\partial_y^2)-\Delta, \quad (1.4)$$

and $\hat{L}_\parallel$ differs from $L_\parallel$ by the replacement $\Delta \to \Delta_\parallel$, where

$$\Delta=-r+\frac{\gamma^2}{4\alpha}, \quad \Delta_\parallel=-r+\frac{\gamma^2}{4\alpha}-w. \quad (1.5)$$

The solution of equations (1.2) and (1.3) can be written in the form of harmonic series,[13] with the amplitudes and wave vector found by minimizing functional (1.1). We thus get

$w<0$:

$$M_z^0=M_1\cos kz+M_3\cos 3kz+\ldots,$$
$$M_1=\left(\frac{2\Delta_\parallel}{3u}\right)^{1/2}, \quad M_3=-\frac{uM_1^3}{2L(3\mathbf{k})}, \quad M_5\sim M_1^5,\ldots, \quad (1.6)$$
$$k^2=-\frac{\gamma}{2\alpha}\left[1-72\frac{M_3^2}{M_1^2}+\ldots\right];$$

$w>0$:

$$M_+^0=M_1e^{ikz}+M_{n-1}e^{-i(n-1)kz}+\ldots,$$
$$M_1=\left(\frac{\Delta}{2u}\right)^{1/2}, \quad M_{n-1}=\frac{-nM_1^{n-1}w_\perp}{L((1-n)\mathbf{k})-L(\mathbf{k})}, \quad (1.7)$$
$$k^2=-\frac{\gamma}{2\alpha}\left[1-n(n-1)^2(n-2)\frac{M_{n-1}^2}{M_1^2}+\ldots\right].$$

In these expressions $L(\mathbf{q})$ is the eigenvalue of the operator $\hat{L}$, namely

$$L(\mathbf{q})=\alpha\left(q_z^2+\frac{\gamma}{2\alpha}\right)^2+\gamma_\perp(q_x^2+q_y^2)-\Delta. \quad (1.8)$$

Thus in the exchange approximation a pure *LSW* or *SS* structure arises, with a wave vector $k=(-\gamma/2\alpha)^{1/2}$. Allowance for anisotropy gives rise to multiple harmonics in the spatial distribution of the magnetic moment; here the multiple is determined by the order of the anisotropy, and the amplitudes are attenuated by a factor $\sim wM_1^{n-2}$ for plane anisotropy and by a factor $\sim uM_1^3$ for axial anisotropy.[13]

Let us now find the spectrum of linear excitations above the vacuum determined by relations (1.6) and (1.7), considering separately the low-temperature region and the neighborhood of the phase transition.

2. EXCITATION SPECTRUM FAR FROM THE PHASE TRANSITION

At low temperatures, where the modulus of the local magnetic moment is nearly constant, the motion of the moment is described by the Landau-Lifshitz equation

$$\dot{\mathbf{M}}=\rho\left[\mathbf{M}\frac{\delta\Phi}{\delta\mathbf{M}}\right], \quad (2.1)$$

and the linearized version of this equation describes spin waves. For functional (1.1) it separates into the following three equations:

$$\dot{M}_\pm=\pm 2i\rho[M_z\hat{L}M_\pm-M_\pm\hat{L}_\parallel M_z+nw_\perp M_zM_\mp^{n-1}], \quad (2.2)$$
$$\dot{M}_z=i\rho[M_+\hat{L}M_--M_-\hat{L}M_++nw_\perp(M_+^n-M_-^n)], \quad (2.3)$$

where $\hat{L}$ and $\hat{L}_\parallel$ are the differential operators defined in (1.4).

Let us investigate the spin wave spectrum for the spiral structure specified by relations (1.7). The linearized equations for $\mathbf{m}$ ($\mathbf{M} = \mathbf{M}_0 + \mathbf{m}$) can be written in the form

$$\dot{m}_+'=2i\rho(M_1+M_{n-1}e^{-inkz})[L(\mathbf{k})-L_\parallel(\partial_z)]m_z, \quad (2.4)$$
$$\dot{m}_z=-i\rho M_1[L(\partial_z+ik)-L(\mathbf{k})]m_+' + i\rho M_{n-1}e^{inkz}[L(\partial_z+ik)-(1-n)L((1-n)\mathbf{k}) - nL(\mathbf{k})]m_+'+\text{c.c.}, \quad (2.5)$$

where the transverse components of the magnetic moment have been subjected to the gauge transformation

$$m_{\pm}=e^{\pm ikz}m_{\pm}', \tag{2.6}$$

which eliminates the periodic potential from the fundamental harmonic of the spiral magnetic structure in the equations for $\mathbf{m}$. If there is no anisotropy, this transformation turns Eqs. (2.4) and (2.5) into differential equations with constant coefficients, giving a frequency spectrum

$$\omega(\mathbf{q})=2\rho M_1\{[L(\mathbf{q})-L(\mathbf{k})+w] \times[(L(\mathbf{q}+\mathbf{k})+L(\mathbf{q}-\mathbf{k}))/2-\mathbf{L}(\mathbf{k})]\}^{1/2}. \tag{2.7}$$

Expression (2.7) corresponds precisely to the spin wave spectrum of the simple spiral structure in the quantum theory[11] and represents a Goldstone branch at momentum $\mathbf{q}=0$ [corresponding, by virtue of relation (2.6), to a momentum $\mathbf{q}=\pm\mathbf{k}$ in the laboratory system of coordinates]. In a purely exchange approximation ($w=0$) there is a second Goldstone branch at momentum $\mathbf{q}=\pm\mathbf{k}$ (i.e., at momentum $\mathbf{q}=0$ in the laboratory system).

The anisotropy in the basal plane generates the periodic potential described by the terms $e^{\pm inkz}$ in Eqs. (2.4) and (2.5); this potential should lead to discontinuities in spectrum (2.7) at the edges of the magnetic Brillouin zone, i.e., at momenta $\mathbf{q}$ such that $q_z=pnk/2$ ($p=0,\ \pm1,\ \pm2,\ \ldots$). The values of the discontinuities can be found by writing the solution for $\mathbf{m}$ from Eqs. (2.4) and (2.5) in Bloch form

$$\mathbf{m}=e^{i\mathbf{qr}}\sum_{l=-\infty}^{\infty}\mathbf{C}_l e^{ilnkz} \tag{2.8}$$

and using the two-wave approximation. For example, for momenta q_z near $nk/2$ it is sufficient (provided that the anisotropy $w_\perp$ is small) to keep only two terms, C_0 and C_{-1}, in the equations for the coefficients C_1. This leads to a gap

$$\delta\omega^2|_{\mathbf{q}=n\mathbf{k}/2}=4\rho^2M_1M_{n-1}\left[L\left(\frac{n\mathbf{k}}{2}\right)-L(\mathbf{k})+w\right] \times\left[(n-1)(L((1-n)\mathbf{k})-L(\mathbf{k})) -2L\left(\left(\frac{n}{2}-1\right)\mathbf{k}\right)\right].$$

In an analogous way one can show that for successive gaps

$$\delta\omega^2|_{\mathbf{q}=p n\mathbf{k}/2}\sim(M_1M_{n-1})^p,$$

but to actually calculate the gaps it would be necessary to first find the contribution of the next harmonics to the spiral magnetic structure (1.7).

The appearance of band structure in the spin-wave spectrum reflects the general rule for motion in a periodic potential. The band character of the spin-wave spectrum in the spiral magnetic structure due to a relativistic interaction described by a Lifshitz invariant of the form $\mathbf{M}\cdot(\nabla\times\mathbf{M})$ in a crystal lacking a center of inversion was established in Ref. 14.

In an *LSW* structure the spin-wave spectrum cannot be obtained using Eq. (21), since it does not hold for systems in which the modulus $M^2(\mathbf{r})$ of the local magnetic moment changes rapidly in space—as is just the case for a *LSW* structure. Nevertheless it is clear that here too the spectrum of magnetic excitations should have a band character on account of the periodicity of the structure itself. The excitation spectrum of the *LSW* structure near the phase transition is obtained below on the basis of a Lagrangian formalism.

3. EXCITATION SPECTRUM NEAR THE PHASE TRANSITION POINT

The expression for the magnetic energy in (1.1) is the Ginzburg-Landau functional if one sets $r\sim T-T_c^0$, where T_c^0 is the "bare" phase transition temperature. The temperature dependence of M_1 in Eqs. (1.6) and (1.7) is standard:

$$w>0:\quad M_1\sim(T_c-T)^{1/2},\quad T_c=T_c^0+\gamma^2/4\alpha; \tag{3.1}$$

$$w<0:\quad M_1\sim(T_{c\parallel}-T)^{1/2},\quad T_{c\parallel}=T_c^0+\gamma^2/4\alpha+|w|. \tag{3.2}$$

For reasons of symmetry, in a uniaxial crystal the two components M_+ and M_- of the magnetic moment transform according to a two-dimensional irreducible representation, while M_z transforms according to a one-dimensional irreducible representation. The two representations form one exchange multiplet,[1] which is split by the uniaxial anisotropy. Expression (1.1) can be treated as a functional for coupled order parameters. This coupling is not manifested in equilibrium magnetic structures (which arise when either M_z^0 or $M_\pm^0$ is nonzero), but it is very important for the dynamics of the order parameter. To write down the dynamical equations for the order parameter we construct the Lagrangian $\mathscr{L}=T-\Phi$, where the kinetic part T for a magnetic system must include[8] an invariant which is linear in $\dot{m}_\alpha$:

$$T=\int d\mathbf{r}\{\mu(\dot{m}_+\dot{m}_-+\dot{m}_z^2)+i\sigma M_z(M_+\dot{m}_--M_-\dot{m}_+)\}. \tag{3.3}$$

Because of this invariant the equation of motion will contain a term describing the local precession of the magnetic moment about its equilibrium position in the magnetic structure. As to the two phenomenological parameters σ and μ which describe the dynamics of the magnetic order parameter, one of them, σ, is directly related to the gyromagnetic ratio, while the microscopic nature of μ is not fully understood; it is clear only that it describes the inertia associated with oscillations of the size of the order parameter. The dynamics of order-parameter fluctuations described by a vector $\mathbf{m}$ is governed by the equation

$$\frac{\delta\mathscr{L}}{\delta m_\alpha}-\frac{d}{dt}\frac{\delta\mathscr{L}}{\delta\dot{m}_\alpha}=\frac{\delta F}{\delta\dot{m}_\alpha}\quad(\alpha=+,-,z), \tag{3.4}$$

where F is the dissipative function[15,16]

$$F=\int d\mathbf{r}\nu(\dot{m}_+\dot{m}_-+\dot{m}_z^2). \tag{3.5}$$

Let us first study the case $w>0$, when a spiral structure arises. The equilibrium value M^0 of the order parameter is given by relations (1.7) with allowance for (3.1). We shall assume that the anisotropy in the basis plane is negligible. Lagrange's equations for the fluctuations on top of the structure $M_\pm^0=M_1e^{\pm ikz}$ (the simple spiral) are of the form

$$\mu\ddot{m}_\pm \pm i\sigma M_1 e^{\pm ihz}\dot{m}_z + (\tilde{L}+4uM_1^2)\,m_\pm + 2uM_1^2 e^{\pm 2ihz} m_\mp = -\nu\dot{m}_\pm,$$

$$\mu\ddot{m}_z - \frac{i}{2}\sigma M_1 (e^{ihz}\dot{m}_- - e^{-ihz}\dot{m}_+) + (\tilde{L}+2uM_1^2+w)\,m_z = -\nu\dot{m}_z. \tag{3.6}$$

They include two types of periodic potentials, which can be eliminated by gauge transformation (2.6). The frequencies are found from the characteristic equation

$$\det \begin{vmatrix} \mu\omega^2 - (L(\mathbf{q}+\mathbf{k}) + 4uM_1^2) & -2uM_1^2 & -\sigma\omega M_1 \\ -2uM_1^2 & \mu\omega^2 - (L(\mathbf{q}-\mathbf{k}) + 4uM_1^2) & \sigma\omega M_1 \\ -\sigma\omega M_1 & \sigma\omega M_1 & 2\mu\omega^2 - 2(L(\mathbf{q}) + 2uM_1^2 + w) \end{vmatrix} = 0, \tag{3.7}$$

which is a cubic equation in ω^2:

$$(\omega^2-\omega_+^2)(\omega^2-\omega_-^2)(\omega^2-\omega_z^2) - \varepsilon\frac{\Delta}{2\mu}\omega^2\left[\omega^2 - \frac{1}{2}(\omega_+^2+\omega_-^2) - \frac{\Delta}{\mu}\right] = 0. \tag{3.8}$$

In this equation $\omega_\pm$ and ω_z are defined by the relations

$$\mu\omega_\pm^2 = \alpha(q_z^4+4k^2q_z^2) + \Delta \pm (\Delta^2+16\alpha^2k^2q_z^6)^{1/2} + \gamma_\perp(q_x^2+q_y^2), \tag{3.9}$$

$$\mu\omega_z^2 = \alpha(q_z^2-k^2)^2 + w + \gamma_\perp(q_x^2+q_y^2) \tag{3.10}$$

and we have introduced the dimensionless parameter $\varepsilon = \sigma^2/\mu u$, which characterizes the ratio of the precessional motion of the magnetic moment to the oscillatory motion.

The quantities given in (3.9) and (3.10) are the eigenfrequencies of fluctuations in the limit of large uniaxial anisotropy w, when the role of the σ term in the dynamical equations is unimportant. In this case the motion of the transverse components of the order parameter $m_\pm$ separates from the motion of the longitudinal component m_z and the precession of the magnetic moment about the direction of its average value, which lies in the basis plane, is reduced by a factor $\sigma M_1\omega/w$. If the anisotropy is small, the branches $\omega_\pm^2$ and ω_z^2 can cross, and gaps can appear on account of the σ term, which intermingles these oscillations. We note that the resulting hybridization has a dynamic character. It is easily shown that in the vicinity of the crossings $\omega_\pm^2(\mathbf{q}) = \omega_z^2(\mathbf{q})$ the gap $\delta\omega_\pm$ between the hybridized branches is approximately

$$\delta\omega_\pm^2 = (\varepsilon\Delta/\mu)^{1/2}\omega_z. \tag{3.11}$$

The fluctuation spectrum thus turns out to have a complex pattern (Fig. 1). It depends on three energy parameters (of dimension $\mu\omega^2$), αk^4, w, and Δ, and one dimensionless parameter, ε. For a given material all the parameters are fixed and only $\Delta \sim T_c - T$ can vary. One can see by formally taking the limit $\Delta \to \infty$ in equation (3.7) that one of the frequencies coincides with the spin-wave frequency (2.7), which follows from the Landau-Lifshitz equation if one takes $\rho = 1/2\sigma M_1^2$, while the other two frequencies lie very high (they contain a factor of Δ) and are unphysical. On the other hand, the limit $\Delta \to \infty$ corresponds to a transition to low temperatures, where the modulus of the magnetic moment is constant, and under these conditions the Landau-Lifshitz equation is valid. The characteristic equation (3.7) thus ensures the joining of the two limiting cases—the case of low temperatures, where the dynamics of the magnetic order parameter is precessional, and the case of large anisotropy w, where the precession of the order parameter is completely suppressed and the dynamics is purely oscillatory, as at the structural phase transitions studied by Golovko and Levanyuk.[10]

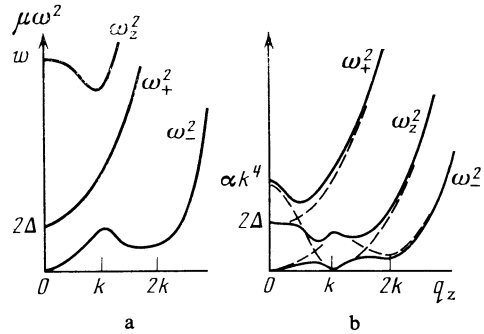

FIG. 1. Fluctuation spectrum in a simple spiral in the limit of large uniaxial anisotropy (1) and in the absence of anisotropy (b); $q_x = q_y = 0$.

In the general case the ratio of the precessional motion to oscillatory motion is characterized by the parameter ε, which one can estimate as follows. The two quantities which appear in the expression for ε are in order of magnitude $u \sim kT_c/\mu_B^4$ and $\sigma \sim 1/\rho\mu_B^2$, where ρ is the gyromagnetic ratio ($\rho \sim \mu_B/\hbar$), while μ_B is the Bohr magneton. It follows that $\varepsilon \sim (\mu\rho^2 kT_c)^{-1}$. Using the experimental value of ε determined from the dispersion curves, one can estimate μ.

Let us now turn to the case $w < 0$. The equilibrium value of the order parameter is determined by relations (1.6). The equations of motion for the transverse and longitudinal fluctuation of the order parameter separate:

$$\mu\ddot{m}_\pm \pm 2i\sigma M_z^0 m_\pm + (\tilde{L}_\parallel + 2uM_z^{02} - w)\,m_\pm = -\nu\dot{m}_\pm, \tag{3.12}$$

$$\mu\ddot{m}_z + (\tilde{L}_\parallel + 6uM_z^{02})\,m_z = -\nu\dot{m}_z. \tag{3.13}$$

Unlike the spiral case, the periodic potential cannot be eliminated from both equations, since in the equation for the longitudinal oscillations the periodic potential is generated by the term[13]

$$6uM_z^{02} = 2\Delta_\parallel - 2\sum_{l=1}^{\infty} t_l \cos(2lkz), \tag{3.14}$$

where

$$t_1 = -{}^3/_2 uM_1^2 + \ldots,\quad t_2 = -3uM_1M_3 + \ldots,\quad t_3 = -{}^3/_2 uM_3^2 + \ldots, \tag{3.15}$$

and the equation for the transverse components contains two types of terms, M_z^0 and M_z^{02}. It follows that the band spectrum of the longitudinal excitations has discontinuities at momenta $q_z = pk$, while that of the transverse excitations

has discontinuities at $q_z = pk/2$ as well (the latter are due to the σ terms in the energy).

Differential equations (3.12) and (3.13) yield the following finite-difference equations for the coefficients in the Bloch function (2.8):

$$[\mu\omega^2 - L_\parallel(\mathbf{q}+l\mathbf{k}) - 2\Delta_\parallel] C_l^z + \sum_p t_p C_{l-2p}^z = 0, \tag{3.16}$$

$$\left[\mu\omega^2 - L_\parallel(\mathbf{q}+l\mathbf{k}) - \frac{2}{3}\Delta_\parallel + w\right] C_l^+ + \sigma\omega \sum_p M_p C_{l-p}^+ + \frac{1}{3}\sum_p t_p C_{l-2p}^+ = 0. \tag{3.17}$$

The two-wave approximation is then used to find the form of the spectrum in the vicinity of the momenta $\mathbf{q} = \mathbf{k}$, $2\mathbf{k}$, and $\mathbf{k}/2$ for longitudinal oscillations:

$$\mu\omega_{1z}^2 = 4\alpha k^2 (q_z - k)^2 + \gamma_\perp (q_x^2 + q_y^2), \quad \mu\omega_{2z}^2 = 2\Delta_\parallel + 4\alpha k^2 (q_z - k)^2 + \gamma_\perp (q_x^2 + q_y^2), \quad |q_z - k| \ll k, \tag{3.18}$$

$$\mu\omega_{1,2z}^2 = L_\parallel(2\mathbf{k}) + 2\Delta_\parallel \pm \left[\Delta_2^2 + \left(\frac{dL_\parallel(\mathbf{q})}{dq_z}\right)^2 (q_z - 2k)^2\right]^{1/2} + \gamma_\perp (q_x^2 + q_y^2),$$

$$|q_z - 2k| \ll k, \quad \Delta_2 = \left| t_2 - \frac{t_1^2}{8\alpha k^4} \right| \sim (T_{c\parallel} - T)^2; \tag{3.19}$$

while for transverse oscillations

$$\mu\omega_{1,2\perp}^2 = \mu\omega_0^2 + \frac{1}{3}\varepsilon\Delta_\parallel \pm \left(\frac{1}{9}\varepsilon^2\Delta_\parallel^2 + \frac{2}{3}\varepsilon\Delta_\parallel \mu\omega_0^2\right)^{1/2}, \quad \left| q_z - \frac{k}{2} \right| \ll k; \tag{3.20}$$

$$\mu\omega_0^2(\mathbf{q}) = L_\parallel(\mathbf{q}) + {}^2/_3\Delta_\parallel + |w|,$$
$$\mu\omega_{1\perp}^2 = \alpha(q_z^2 - k^2)^2 + |w| + \gamma_\perp(q_x^2 + q_y^2), \tag{3.21}$$
$$\mu\omega_{2\perp}^2 = \alpha(q_z^2 - k^2)^2 + |w| - {}^2/_3\Delta_\parallel + \gamma_\perp(q_x^2 + q_y^2), \quad |q_z - k| \ll k.$$

The longitudinal branches in the vicinity of $q_z = k$ are thus the phase (Goldstone) and amplitude branches of the order-parameter fluctuations; these branches are well known in the theory of structural phase transitions.[9,10] At momenta $q_z = pk$ the spectrum exhibits discontinuities with a width Δ_p which falls off exponentially with the band index:

$$\Delta_p \sim (T_{c\parallel} - T)^p \sim \exp\left\{-p \ln \frac{T_{c\parallel}}{T_{c\parallel} - T}\right\}; \tag{3.22}$$

such behavior is also characteristic for the order-parameter dynamics at structural phase transitions. Specific to the magnetic system is a transverse branch, where the usual dynamics of the two-component order parameter becomes coupled to the precession of the magnetic moment. This leads to a discontinuity $\sim\varepsilon \max\{\Delta_\parallel, (\Delta_\parallel \alpha k^4)^{1/2}\}$ in the spectrum at $q_z = k/2$. At $q_z = k$ the Goldstone branch in the transverse spectrum does not exist, on account of the anisotropy. The soft mode ω_1^2 at $\mathbf{q} = \mathbf{k}$ loses stability when $2/3\,\Delta_\parallel = |w|$, and at the corresponding point in temperature there is a phase transition from the *LSW* structure to a structure in which the perpendicular components are also ordered with the same wave vector. The general picture of the spectrum is shown in Fig. 2.

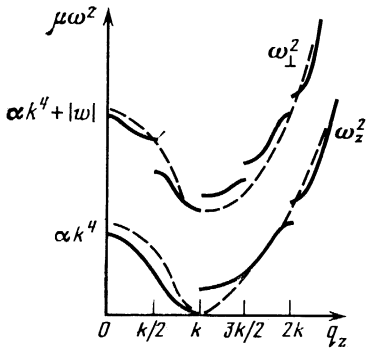

FIG. 2. Fluctuation spectrum in the "longitudinal spin wave" structure.

4. CALCULATION OF THE RESPONSE

We shall use the two-time Green's functions for classical quantities,[17] which is what the components m_α of the order parameter are in this case:

$$G_{\alpha\beta}(\mathbf{r}t, \mathbf{r}'t') = -\theta(t-t')\langle\{m_\alpha(\mathbf{r}t), m_\beta(\mathbf{r}'t')\}\rangle. \tag{4.1}$$

Here $\{\ldots,\ldots\}$ is the Poisson bracket, $\theta(t-t')$ is the step function, and $\langle\ldots\rangle$ denotes a statistical average using Hamiltonian $\mathscr{H}$:

$$\langle\ldots\rangle = \int e^{-\mathscr{H}/k_B T}(\ldots)\,d\Gamma \Big/ \int e^{-\mathscr{H}/k_B T}\,d\Gamma,$$

where $d\Gamma$ is the volume element in phase space. The Fourier component of the fluctuation correlator in the variable $t - t'$ is expressed in terms of the corresponding Green function by means of the spectral relation[17]

$$\langle m_\alpha(\mathbf{r}) m_\beta(\mathbf{r}')\rangle_\omega = (k_B T/\omega)\, 2\,\mathrm{Im}\, G_{\alpha\beta}(\mathbf{r}, \mathbf{r}', \omega). \tag{4.2}$$

To actually write down the equation of motion

$$\frac{d}{dt} G_{\alpha\beta} = -\delta(t-t')\langle\{m_\alpha, m_\beta\}\rangle - \theta(t-t')\langle\{\dot{m}_\alpha, m_\beta\}\rangle \tag{4.3}$$

we must use the Hamiltonian formalism, i.e., introduce generalized momenta $p_\pm = \delta\mathscr{L}/\delta\dot{m}_\pm$ and $p_z = \delta\mathscr{L}/\delta\dot{m}_z$. Using these relations and (3.3), we express the velocity in terms of the momentum

$$\dot{m}_\pm = \frac{1}{\mu}(p_\pm \mp i\sigma M_z M_\pm), \quad \dot{m}_z = \frac{1}{2\mu} p_z \tag{4.4}$$

and construct the Hamiltonian

$$\mathscr{H} = \int d\mathbf{r}\,(p_+\dot{m}_- + p_-\dot{m}_+ + p_z\dot{m}_z) - \mathscr{L}. \tag{4.5}$$

For the simple spiral structure ($w > 0$) in the harmonic approximation $\mathscr{H}$ is represented by the following quadratic form of the generalized coordinates and momenta:

$$\mathscr{H} = \int d\mathbf{r}\left\{\frac{1}{\mu}\left(p_+p_- + \frac{1}{4}p_z^2\right) + \frac{i\sigma}{\mu} m_z (M_-^0 p_+ - M_+^0 p_-)\right.$$

$$+\frac{\sigma^2}{\mu}M_+^0M_-^0m_z^2+m_+(\hat{L}+4uM_+^0M_-^0)m_-$$

$$+u(M_+^{02}m_-^2+M_-^{02}m_+^2)$$

$$+m_z(\hat{L}+2uM_+^0M_-^0+w)m_z\}. \tag{4.6}$$

Owing to relation (4.4) the Green function $G_{\alpha\beta}$ is coupled with the function

$$F_{\alpha\beta}(\mathbf{r}t, \mathbf{r}'t')=-\theta(t-t')\langle\{p_\alpha(\mathbf{r}t), m_\beta(\mathbf{r}'t')\}\rangle, \tag{4.7}$$

and the equations of motion for these functions in matrix form are

$$\left(\frac{d}{dt}-i\frac{\sigma}{\mu}B\right)G=\frac{1}{\mu}AF,$$
$$\left(\frac{d}{dt}+\frac{\nu}{\mu}-i\frac{\sigma}{\mu}B^+\right)F=-\delta(t-t')\delta(\mathbf{r}-\mathbf{r}')-KG, \tag{4.8}$$

where all the matrices are 3×3; A is a diagonal matrix with elements 1, 1, and 1/2 on the diagonal, B has two nonzero elements $B_{13}=-M_+^0$ and $B_{23}=M_-^0$, and

$$K=\begin{pmatrix} \hat{L}+4uM_+^0M_-^0 & 2uM_+^{02} & -i\frac{\sigma}{\mu}\nu M_+^0 \\ 2uM_-^{02} & \hat{L}+4uM_+^0M_-^0 & i\frac{\sigma}{\mu}\nu M_-^0 \\ 0 & 0 & 2\hat{L}+2\left(2u+\frac{\sigma^2}{\mu}\right)M_+^0M_-^0+2w \end{pmatrix}. \tag{4.9}$$

The damping in Eqs. (4.8) was introduced[16] with the aid of Hamilton's equation with the dissipative function (3.5):

$$\dot{p}_\pm=-\delta\mathcal{H}/\delta m_\mp-\delta F/\delta\dot{m}_\mp. \tag{4.10}$$

Equations (4.8) contain two types of periodic potentials: $\sim e^{\pm 2ikz}$ and $\sim e^{\pm ikz}$, both of which can be eliminated by gauge transformation of the matrices G and F:

$$G'(\mathbf{r}\mathbf{r}'t)=U^+(z)G(\mathbf{r}\mathbf{r}'t)U(z') \tag{4.11}$$

with a diagonal matrix $U(z)$ having elements e^{ikz}, e^{-ikz}, and 1 on the diagonal. For the Fourier transform of the matrix G' we obtain the explicit expression

$$G'(\mathbf{q}\mathbf{q}'\omega)=M^{-1}(\mathbf{q})\delta(\mathbf{q}-\mathbf{q}'), \tag{4.12}$$

where the 3×3 matrix $M(\mathbf{q})$ is the same as that in Eq. (3.7) except that $\mu\omega^2$ must be replaced by $\mu\omega^2+i\nu\omega$ in the diagonal terms. The function G of interest differs from G' by a shift of the momenta by the wave vector of the structure. This relationship is most simply expressed by the relation

$$G_{\alpha\beta}(\mathbf{q}\mathbf{q}'\omega)=G_{\alpha\beta}'(\mathbf{q}+\alpha\mathbf{k}, \mathbf{q}'-\beta\mathbf{k}, \omega), \tag{4.13}$$

in which $\alpha, \beta=+, -, z$ with the index z in the arguments of the function $G'_{\alpha\beta}$ set identically to zero.

Since it is extremely awkward to invert the 3×3 matrix, we shall give the explicit form for the limiting case of large anisotropy w, when the σ terms in (3.11) can be neglected. In this case the Green function matrix can be represented as two diagonal blocks:

$$G_\perp(\mathbf{q}\mathbf{q}'\omega)$$
$$=\begin{pmatrix} \frac{\Omega}{D(\mathbf{q}+\mathbf{k})}\delta(\mathbf{q}-\mathbf{q}') & \frac{\Delta}{D(\mathbf{q}+\mathbf{k})}\delta(\mathbf{q}-\mathbf{q}'+2\mathbf{k}) \\ \frac{\Delta}{D(\mathbf{q}-\mathbf{k})}\delta(\mathbf{q}-\mathbf{q}'-2\mathbf{k}) & \frac{\Omega}{D(\mathbf{q}-\mathbf{k})}\delta(\mathbf{q}-\mathbf{q}') \end{pmatrix}$$

$$\Omega=\mu\omega^2+i\nu\omega-\Delta-\alpha(q_z^2-k^2)^2-\gamma_\perp(q_x^2+q_y^2), \tag{4.14}$$

and

$$G_{zz}(\mathbf{q}\mathbf{q}'\omega)=\frac{\delta(\mathbf{q}-\mathbf{q}')}{2[\mu\omega^2+i\nu\omega-\mu\omega_z^2(\mathbf{q})]}, \tag{4.15}$$

$$D(\mathbf{q})=[\mu\omega^2+i\nu\omega-\mu\omega_+^2(\mathbf{q})][\mu\omega^2+i\nu\omega-\mu\omega_-^2(\mathbf{q})], \tag{4.16}$$

and the frequencies $\omega_\pm$ and ω_z are given by (3.9).

In the case of the *LSW* structure the equations for the matrix of Green functions (4.1) and (4.7) are also of form (4.8), only now the 3×3 matrices β and K are diagonal. This makes the matrix G diagonal, with diagonal elements obeying the following equations [cf. (3.12) and (3.13)]

$$(\mu\omega^2+i\nu\omega+2\sigma\omega M_z^0-\hat{L}_\parallel+w-2uM_z^{02})G_{+-}=\delta(\mathbf{r}-\mathbf{r}'), \tag{4.17}$$

$$2(\mu\omega^2+i\nu\omega-\hat{L}_\parallel-6uM_z^{02})G_{zz}=\delta(\mathbf{r}-\mathbf{r}'). \tag{4.18}$$

(The equation for G_{-+} is obtained from (4.17) by changing the sign of the term containing σ).

Recognizing that the potential $\sim M_z^{02}$ is periodic, we transform to the Fourier representation for the function G_{zz}:

$$G_{zz}(\mathbf{r}, \mathbf{r}')=\int d\mathbf{q}_\perp d\mathbf{q}_\perp'\int_{-k}^{k}dq_z\int_{-k}^{k}dq_z'$$
$$\times\sum_{pp'}\exp\{i(\mathbf{q}+2p\mathbf{k})\mathbf{r}-i(\mathbf{q}'+2p'\mathbf{k})\mathbf{r}'\}G_{pp'}^z(\mathbf{q}, \mathbf{q}'). \tag{4.19}$$

For the function G_{+-} one must take into account that the periodicity is given by the vector $\mathbf{k}$ and not $2\mathbf{k}$, and we shall denote the corresponding Fourier component by $G_{pp'}^{\perp}(\mathbf{q},\mathbf{q}')$.

With allowance for (3.14) the differential equation for G_{zz} is converted to a finite-difference equation in the band indices p and p':

$$\sum_p 2\{[\mu\omega^2+i\nu\omega-L_\parallel(\mathbf{q}+2p\mathbf{k})-2\Delta_\parallel]\delta_{lp}+t_{l-p}\}G_{pp'}=\delta_{lp'}. \tag{4.20}$$

The solution of this equation can be found for separate regions of (ω, q) space by using a special perturbation theory. Let us write Eq. (4.20) in the form of a matrix Dyson equation

$$G=G^0+G^0VG, \tag{4.21}$$

where V is defined as the off-diagonal part of the matrix G^{-1} and has elements $V_{lp}=t_{l-p}$ $(l\neq p)$, $V_{ll}=0$, while G^0 is defined as the diagonal matrix $G^0_{pp'}=G^0_p\delta_{pp'}$.

Let ω and $\mathbf{q}$ be such that G_p^{0-1} is much smaller than G^{0-1} with $l\neq p$, i.e., we are located near a pole of the function G_p^0 for fixed p. Then, regrouping terms in (4.21), we can write any matrix element of the function G in the form

$$G_{ln}=\bar{G}_{ln}+J_{l,p}G_{pp}J_{n,p}, \tag{4.22}$$

where

$$J_{l,p}=G_l^0\Big[V_{lp}+\sum_m{}'V_{lm}G_m{}^0V_{mp}+\dots\Big] \qquad (l\neq p),$$

$$J_{p,p}=1, \tag{4.23}$$

$$G_{pp}=1/(G_p^{0-1}-T_p), \quad T_p=\sum_m{}'V_{pm}G_m{}^0V_{mp}+\dots, \tag{4.24}$$

$$\bar{G}_{ln}=G_l{}^0V_{ln}G_n{}^0+\sum_m{}'G_l{}^0V_{lm}G_m{}^0V_{mn}G_n{}^0+\dots, \quad \bar{G}_{pp}=0 \tag{4.25}$$

(the prime means that the term with $m=p$ is to be omitted from the summation).

From here it is clear that for small V we have perturbation series for both the corrections T_p to the pole of the Green function of $J_{l,p}$ (this correction determines the residue at the pole) and for the nonpolar contribution G_{ln} to the matrix element of the Green function. It follows from what we have said that

$$\operatorname{Im}G_{ln}=J_{l,p}(\operatorname{Im}G_{pp})J_{n,p}. \tag{4.26}$$

Of course, this perturbation theory has meaning only when the damping is sufficiently small that the imaginary part of the pole G_p^0 is much less than the real part.

Applying these relations to Eq. (4.20), we can write, in the limit $\nu\to 0$, a relation which is suitable over all of (ω, q) space except on the boundary of the Brillouin zone:

$$\operatorname{Im}G_{ll}{}^z(\mathbf{q},\mathbf{q},\omega)=-\frac{\pi}{2\mu\omega}\sum_p J_{l,p}^{z2}\,[\delta(\omega-\omega_z(\mathbf{q}+2p\mathbf{k}))+\delta(\omega+\omega_z(\mathbf{q}+2p\mathbf{k}))], \tag{4.27}$$

$$J_{l,p}^z\sim(T_{c\|}-T)^{|l-p|}, \tag{4.28}$$

where the frequency ω_z corresponds to the single-wave approximation to Eq. (3.16).

Near the zone boundary $q_z=pk$ the two quantities G_p^0 and $G_{p'}^0$ can simultaneously have nearby poles. Then they must be joined into a single matrix block; formulas (4.22)–(4.26) remain valid in this case, only now the quantities G_p^0 in them are matrices. This corresponds to the two-wave approximation. For reasons of space, we shall not write out the explicit form of the Green functions near the zone boundary.

5. INELASTIC NEUTRON SCATTERING

Let us briefly discuss the possibility of studying the dynamics of the fluctuations of the magnetic order parameter by the methods of neutron spectroscopy. The cross section for scattering with momentum transfer $\varkappa$ and energy transfer ω in the classical limit $(\omega\ll k_BT)$ is given by[18]

$$\frac{d^2\sigma}{d\varkappa\,d\omega}\sim\frac{k_BT}{\omega}\int d\mathbf{q}\,\delta(\mathbf{q}-\varkappa)\times\Big\{(1+e_z^2)\frac{1}{2}\operatorname{Im}[G_{+-}(\mathbf{q}\mathbf{q}\omega)+G_{-+}(\mathbf{q}\mathbf{q}\omega)]+(1-e_z^2)\operatorname{Im}G_{zz}(\mathbf{q}\mathbf{q}\omega)\Big\}, \tag{5.1}$$

where $\mathbf{e}$ is the unit scattering vector.

Let us first consider the case of a *LSW* in the limit of weak damping. Using expression (4.27) for the matrix elements of the Green fuctions in the single-wave approximation, we find a formula for the cross section in regions of (ω,q) space that are far from the Brillouin zone boundary:

$$\frac{d^2\sigma}{d\varkappa\,d\omega}\sim\frac{k_BT}{\omega^2}\sum_{l,p}\int d\mathbf{q}_\perp\int dq_z\{(1+e_z^2)J_{l,p}^{\perp 2}\,\delta(-\varkappa+\mathbf{q}+l\mathbf{k})\times[\delta(\omega+\omega_\perp(\mathbf{q}+p\mathbf{k}))+\delta(\omega-\omega_\perp(\mathbf{q}+p\mathbf{k}))]+(1-e_z^2)J_{l,p}^{z2}\,\delta(-\varkappa+\mathbf{q}+2l\mathbf{k})[\delta(\omega+\omega_z(\mathbf{q}+2p\mathbf{k}))+\delta(\omega-\omega_z(\mathbf{q}+2\mathbf{k}))]\}. \tag{5.2}$$

Here the index p is the number of the energy branch and l is the number of the Brillouin zone: the coefficients J^2 determine the scattering intensity on the pth branch with allowance for Umklapp processes. The temperature dependence on the factor J_{lp}^z is determined by relation (4.28) for longitudinal excitations, while for transverse excitations $J_{lp}^\perp\sim(T_{c\|}-T)^{|l-p|/2}$.

The corresponding cross section for modes with momenta $\mathbf{q}$ close to the zone edges can be obtained by using the Green's function of the two-wave approximation. For example, for scattering by transverse modes near the Brillouin zone boundary ($q_z=k/2-\delta$) one should take into account the hybridization of the energy branches with indices $p=0$ and $p=-1$; this hybridization gives rise to a gap in the spectrum [see (3.20)], and we get

$$\frac{d^2\sigma}{d\varkappa\,d\omega}\sim\frac{k_BT}{\omega^2}\sum_l\int d\mathbf{q}_\perp\int dq_z(1+e_z^2)\,(J_{l,0}^{\perp 2}+J_{l,-1}^{\perp 2})\times\delta(-\varkappa+\mathbf{q}+l\mathbf{k})\sum_{\nu=1,2}[\delta(\omega+\omega_\nu(\mathbf{q}))+\delta(\omega-\omega_\nu(\mathbf{q}))], \tag{5.3}$$

where the index $\nu=1,2$ specifies one of the two hybridized branches (3.20). Experimental observation of this gap would yield the value of the parameter ε.

In the case of the simple spiral let us first consider the limit of strong uniaxial anisotropy $(w\gg\alpha k^4, \Delta)$, when the Green's function of the transverse and longitudinal components separate. Using their explicit form (4.14) and (4.15) in

Eq. (5.1) and neglecting damping, we write the cross section for scattering by low-lying transverse modes as

$$\frac{d^2\sigma_\perp}{d\varkappa\, d\omega} \sim \frac{k_B T}{\omega^2} \sum_{\nu=+,-} \int d\mathbf{q}\,(1+e_z^2)\,[\delta(\varkappa-\mathbf{q}-\mathbf{k})A_\nu(\mathbf{q}) + \delta(\varkappa-\mathbf{q}+\mathbf{k})A_{-\nu}(\mathbf{q})]\,[\delta(\omega-\omega_\nu(\mathbf{q}))+\delta(\omega+\omega_\nu(\mathbf{q}))], \tag{5.4}$$

where the frequencies $\omega_\pm(\mathbf{q})$ are determined by expressions (3.9), and the intensities of the spectral lines are determined by the factor

$$A_\pm = 1 \pm 4\alpha k q_z^3/(\Delta^2+16\alpha^2 k^2 q_z^6)^{1/2}.$$

For $T \to T_c$ one of these factors $A_\pm(\mathbf{q})$ is close to two, while the other is small—of order $\Delta^2 \sim (T_c - T)^2$.

It is important to elucidate the role of damping for the lowest-frequency modes, primarily for the Goldstone mode. As is clear from (5.1) and (4.14), damping is manifested in scattering at $\varkappa = \pm \mathbf{k}$. In the vicinity of this $\varkappa$ and at small $\mu\omega^2 \ll \Delta$ the general expression (5.1) reduces to the familiar spectral function for a damped oscillator[19]:

$$\frac{d^2\sigma}{d\varkappa\, d\omega} \sim (1+e_z^2)\,k_B T \frac{\nu}{(\mu\omega^2-4\alpha k^2\delta^2)^2+\nu^2\omega^2}, \qquad \delta = |\varkappa \mp \mathbf{k}|. \tag{5.5}$$

In the overdamped-oscillator regime ($\nu^2 > 8\alpha k^2 \delta^2 \mu$) the behavior of the Goldstone mode is relaxational. If $\nu^2 \gtrsim \mu\Delta$ then the amplitude branch $\omega_+(\mathbf{q})$ will also be overdamped. We note that an analogous expression could have been obtained for the scattering by the Goldstone branch ω_z in an *LSW* structure with an angular factor $(1 - e_z^2)$.

Let us now consider the more interesting case of small anisotropy: $w \lesssim \alpha k^4$. Far from the points of intersection of the bare frequencies $\omega_\pm^2$ and ω_z^2 the cross section (5.1) can be evaluated using formula (4.27), and the result can be put in a form analogous to (5.2):

$$\frac{d^2\sigma}{d\varkappa\, d\omega} \sim \frac{k_B T}{\omega^2} \sum_{lp} \int d\mathbf{q}\,[1-(-1)^l e_z^2]\,J_{l,p}^2\,\delta(-\varkappa+\mathbf{q}+l\mathbf{k}) \times [\delta(\omega-\omega_p(\mathbf{q}))+\delta(\omega+\omega_p(\mathbf{q}))], \tag{5.6}$$

where the summation is over the three indices $l, p = 1, -1, 0$ which enumerate the branches of the spectrum. The quantities $J_{l,p}$ evaluated with formula (4.23) depend on the temperature through Δ:

$$J_{0,1}(\mathbf{q}) = J_{1,0}(\mathbf{q}) = J_{0,-1}(-\mathbf{q}) = J_{-1,0}(-\mathbf{q}) = \left(\frac{\varepsilon\Delta}{8\alpha}\right)^{1/2} \left[q_z^2 (q_z+2k)^2 + \frac{\gamma_\perp}{\alpha}(q_x^2+q_y^2) \right]^{1/8} [k(2q_z+k)(2q_z^2+2q_z k-k^2)-w]^{-1},$$

$$J_{-1,1}(\mathbf{q}) = J_{1,-1}(-\mathbf{q}) = \frac{\Delta}{8\alpha k q_z^3}\left[1+\varepsilon\frac{q_z^2(q_z+2k)^2+\gamma_\perp(q_x^2+q_y^2)/\alpha}{k(2q_z+k)(2q_z^2+2q_z k-k^2)-w}\right] \tag{5.7}$$

where $J_{00} = J_{1,1} = J_{-1,-1} = 1$.

Expressions (5.7) show that each branch of the spectrum at fixed $\varkappa$ is manifested three times in the scattering—at momenta $\mathbf{q}$, $\mathbf{q}+\mathbf{k}$, and $\mathbf{q}-\mathbf{k}$, with intensities of order 1, $T_c - T$, and $(T_c - T)^2$ for the transverse oscillations and of order 1 and $T_c - T$ for the longitudinal oscillations. The unusual linear temperature dependence is due to the σ terms. Their importance as is seen from (5.7), begins to grow strongly at $q_z = \pm k_2$, but then formulas (5.7) are no longer valid and one must use the two-wave approximation.

The hybridization of the branches ω_z^2 and ω_-^2 in the vicinity of $q_z = k/2$ gives rise to a dynamic gap (3.11); here the cross section for scattering by the two hybridized branches does not contain the small factor $T_c - T$. By observing the gap and comparing with formula (3.11), one could determine the parameter ε. Thus the parameter ε, a measure of the relative contribution of the precessional motion of the order parameter in comparison to the oscillatory motion, can be determined from the corresponding splitting of the branches at the momentum $\mathbf{k}_2$ in the two cases—*LSW* and *SS*. Of course, this is possible only under the condition that the damping is small compared to the widths of these gaps, which vary with temperature in accordance with $\delta\omega^2 \propto (T_c - T)^{1/2}$. The gaps at $\mathbf{k}$, $3\mathbf{k}/2$, and $2\mathbf{k}$, which are due to the periodic potential of the structure, will be of higher orders $(T_c - T)^p$ and will therefore be smeared out at sufficiently large indices p even if the first gap is resolved.

We have not taken the crystal lattice into account here, since we have assumed that $k \ll b$, where $\mathbf{b}$ is an arbitrary vector of the reciprocal lattice of the crystal. The lattice can be incorporated in all the formulas for the scattering cross section by replacing $\varkappa$ with $\varkappa + \mathbf{b}$.

6. CONCLUSION

We have considered the dynamics of the order-parameter fluctuations for the particular case of magnetic structures which are modulations of the simple ferromagnetic structure, where the order parameter is the vector representing the spontaneous magnetic moment. In more complicated cases, when the chemical cell of the crystal contains several atoms or when it is an antiferromagnetic structure that is modulated, one must take as the order parameter the mixing coefficients of the basis functions of the irreducible representation of the symmetry group. It can be shown that in the majority of known crystals, magnetic phase transitions to incommensurable structures are described by Ginzburg-Landau functionals of the following forms, with one-component ζ or two-component (η, ξ) order parameters:

$$\Phi_1 = \int dz\{r_1\zeta^2 + u\zeta^4 + \gamma(\partial_z\zeta)^2 + \alpha(\partial_z^2\zeta)^2\}, \tag{6.1}$$

$$\Phi_2 = \int dz\{r_2(\eta\xi) + u(\eta\xi)^2 + \gamma\partial_z\eta\,\partial_z\xi + \alpha\partial_z^2\eta\,\partial_z^2\xi + w(\eta^n+\xi^n)\}, \quad n=\xi^*. \tag{6.2}$$

The functionals Φ_1 and Φ_2 correspond to the functional (3.1) of the present paper in the limit of strong anisotropy, which splits the exchange multiplet into a singlet and doublet.

However in a certain respect Φ_1 and Φ_2 are more general than functional (3.1), since the components of the order parameter can describe structures which are microscopically more complex.

Since expressions (6.1) and (6.2) together [i.e., with allowance for interaction terms of the type $\xi^2(\eta, \xi)$] are mathematically equivalent to (3.1), the physical conclusions reached from them regarding the dynamics of the order parameter are extremely general. In particular, in the presence of strong anisotropy (very different r_1 and r_2) the σ invariant cannot appear in the kinetic part of the Lagrangian, i.e., the precession of the magnetic moment of the order parameter will be suppressed, and the dynamics of the magnetic order parameter will be similar to that of the order parameter at a structural phase transition. In the presence of a strong exchange interaction ($|r_1 - r_2| \ll r_1, r_2$) the dynamics of the magnetic order parameter is governed simultaneously by the oscillatory and precessional motion. The relative contribution of these motions can be determined experimentally, as we have shown above, with the aid of neutron spectroscopy. Here one could use different crystals exhibiting *SS* and *LSW* structures.[1] Crystals of the spinel type ($HgCr_2S_4$, $ZnCr_2Se_4$, Eu, $TbMn_2$) are examples of crystals with weak anisotropy, and rare earth metals (Dy, Tb, Ho, Er) are examples of crystals with strong anisotropy.

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Translated by Steve Torstveit