# Excitation of nonlinear surface waves by Gaussian light beams

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Numerical techniques have been used to study the interaction of nonlinear surface waves with Gaussian light beams incident on the surface of the nonlinear medium at grazing angles. For this purpose, the stability of the nonlinear surface waves is studied first. Specifically, the wave vector regions in which nonlinear surface waves are stable and the regions in which they are unstable are found. It is shown that both stable and unstable nonlinear surface waves can be excited by Gaussian beams with the optimum parameters (with a certain power and a certain width) which strike the interface at a given angle.

## 1. INTRODUCTION

There have been several theoretical[1–4] and experimental[5] studies of the interaction of Gaussian light beams with an interface between a linear medium and a nonlinear medium. Numerical calculations have yielded some interesting new results. In particular bistability in the response of such a system to an external field has been studied. It has been found that when a light beam is incident on the surface of a nonlinear medium the transmitted wave may break up into distinct pulses under certain conditions. The most comprehensive treatment of the interaction between a light beam and the surface of a nonlinear medium of which we are aware is that by Tomlinson *et al.*,[3] who studied in detail the paths traced out by the transmitted and reflected beams. Nevertheless, a question raised in Ref. 4 remains unanswered: Can a light beam incident on a plane interface be used to excite the nonlinear surface waves (NSWs) whose existence was predicted in Refs. 6–8? The studies in Refs. 3 and 4 essentially did not answer this question, partially because the NSWs themselves have not received much study. In particular, there has been no study of the stability of NSWs, although this question is crucial to the problem of the excitation of NSWs by external sources.

From the mathematical standpoint the problem reduces to one of finding a certain class of solutions of a nonlinear Schrödinger equation with coefficients which depend on the transverse coordinate $z$. In this sense, the problem is one of general interest. On the one hand, the absence of translational symmetry along the $z$ axis leads to the appearance of solutions of a new type: solitons localized at inhomogeneities. Nonlinear surface waves are such solitons. As the nature of the inhomogeneity along $z$ becomes more complex, the steady-state solutions also become more complex, and they increase in number.[9] We are interested in both the stability of these new formations and their properties with respect to interactions with moving wave packets. On the other hand, the absence of translational symmetry means that we cannot use the apparatus of the inverse scattering problem in its classical formulation[10] to solve the problem, so that our main tool for deriving new results at this stage is numerical simulation. Several results obtained by numerical simulation are reported in this paper.

In Section 2 we formulate the problem. In Section 3 we use numerical methods for the first study of the stability of NSWs. We show that the region of allowed values of the NSW wave vectors breaks up into subregions of stable and unstable waves. In the same section we show that for the NSWs of the unstable branch there exist two decay paths, depending on the sign of the initial perturbation: Either (1) the NSW is "ejected" into the linear medium as a light beam of definite shape (approximately Gaussian), separating from the interface at an angle greater than the critical angle for total internal reflection, or (2) it decays into an NSW of the stable branch and a beam which propagates away from the interface into the linear medium. Section 4 reports the results of numerical calculations on the excitation of NSWs by Gaussian beams. It is found that (in contrast with the conclusions reached in Ref. 3) light beams incident on the interface at grazing angles can excite both unstable and stable NSWs. Section 5 summarizes the results.

## 2. FORMULATION OF THE PROBLEM

Studies of the propagation of light beams at small angles from an interface usually employ the parabolic equation found from the wave formulation of the solution in the form of a wave with an amplitude which varies slowly along the interface.[1–4] We denote by $z$ the coordinate running normal to the interface, so that the dielectric constant of the medium in the region $z<0$ is $\varepsilon_l$, while that in the region $z>0$ is $\varepsilon = \varepsilon_0 + \alpha|E|^2$. We write the solution of the wave equation as a wave which is polarized along the $y$ axis and propagating along the $x$ axis:

$$E_y(x, z)=\alpha^{1/2}A(x, z)\exp(inx-i\omega t).$$

We are accordingly restricting the solution to the two-dimensional problem. The equation for the slowly varying amplitude $A(x,z)$ is then

$$2in\frac{\partial A}{\partial x}+\frac{\partial^2 A}{\partial z^2}-\gamma^2(z)A+\beta(z)|A|^2A=0, \quad (1)$$

where $\gamma^2(z) = n^2 - \varepsilon(z)$, the coordinates $x$ and $z$ are normalized with respect to $\lambda/2\pi$, where $\lambda$ is the wavelength of the light in vacuum, and

$$\varepsilon(z)=\begin{cases}\varepsilon_l & \text{for} \quad z<0\\ \varepsilon_0 & \text{for} \quad z>0\end{cases}, \quad \beta(z)=\begin{cases}0 & \text{for} \quad z<0\\ 1 & \text{for} \quad z>0\end{cases}. \tag{2}$$

In what follows we are assuming that the nonlinear medium is self-focusing ($\alpha > 0$). Equation (1) has integrals of motion; we will write out only one of them here:

$$I=\int_{-\infty}^{\infty}|A|^2\,dz. \tag{3}$$

For arbitrary solutions of Eq. (1) we thus have $dI/dx = 0$.

We have solved Eq. (1) numerically on an SM-4A computer. For the difference approximation of Eq. (1) we selected the Cranck-Nicholson scheme; the system of nonlinear equations was solved on the successive steps in $x$ by Newton's method combined with a matrix tridiagonal inversion along $z$. This difference scheme makes it possible to conserve the integral (3) on the grid. For a discrete representation of the solution of the complex function $A(x,z)$ we used 600 points with 300 on each side of the $z = 0$ boundary. At the boundaries of the grid at $z = \pm L$ we set the field equal to zero. Until the field decreases to small values toward these borders, the boundary conditions which we select have no effect on the solution of the problem. In cases in which the beams reach the $z = \pm L$ boundaries, they are reflected, and in the next step along $x$ we equate the field of this beam to zero, assuming that the beam has gone off to infinity. The interface between the two media ($z = 0$) was placed symmetrically between points of the grid. The difference approximation in the points nearest this interface was chosen to be the same as for points in the interior of the medium. This approach implies continuity of the function $A(x,z)$ and of its first derivative at the interface. As the initial field distribution at $x = 0$ we selected either a steady-state solution of Eq. (1) to study its stability or a Gaussian beam incident on the interface at a small angle.

### 3. NONLINEAR SURFACE WAVES AND THEIR STABILITY

At the interface between the two media, $z = 0$, under the condition $\varepsilon_l > \varepsilon_0$, there exists a solution of Eq. (1) which is stationary along $x$ and which is called a "nonlinear surface wave"[6-8]:

$$A_0(z)=\begin{cases}[2(\varepsilon_l-\varepsilon_0)]^{1/2}\exp(\varkappa z) & \text{for} \quad z<0\\ 2^{1/2}\gamma\,\mathrm{ch}^{-1}\gamma(z-z_0) & \text{for} \quad z>0\end{cases}, \tag{4}$$

where

$$\gamma=(n^2-\varepsilon_0)^{1/2}, \quad \varkappa=(n^2-\varepsilon_l)^{1/2}, \quad z_0=\frac{1}{2\gamma}\ln\frac{\gamma+\varkappa}{\gamma-\varkappa}.$$

For the steady-state solution (4), the integral in (3) becomes

$$I=\frac{\varepsilon_l-\varepsilon_0}{\varkappa}+2(\gamma+\varkappa). \tag{5}$$

Figure 1 shows this integral as a function of $n$ for the pair of values $\varepsilon_0 = 2.647$, $\varepsilon_l = 2.674$, corresponding to $CS_2$ and a particular optical glass. The integral $I$ has a minimum at a certain $n_{cr}$, which can easily be calculated from (5):

$$n_{cr}=\left(\frac{4\varepsilon_l-\varepsilon_0}{3}\right)^{1/2}.$$

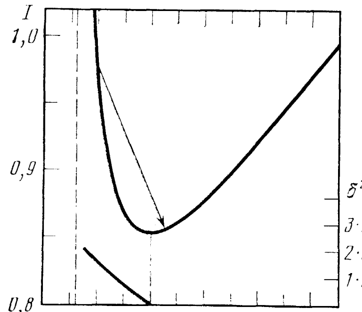


FIG. 1. The integral $I$ (the upper curve) and the square of the instability growth rate, $\delta$, versus $n$.

Here we have $I(n_{cr}) = 3[3(\varepsilon_l - \varepsilon_0)]^{1/2}$. The numerical value of $n_{cr}$ for the case above is $\sim 1.638$.

To determine the stability criterion for NSWs we numerically stimulated the solution of parabolic equation (1), specifying at $x = 0$ the steady-state solution (4) with a small additive perturbation:

$$A(x=0,z)=A_0(z)+\mu f(x=0,z). \tag{6}$$

The perturbation modes $f(x,z)$ are found from the linearized equation which is obtained by substituting (6) into our original nonlinear equation, (1):

$$2in\frac{\partial f}{\partial x}+\frac{\partial^2 f}{\partial z^2}-\gamma^2(z)f+\beta(z)A_0^2(z)(2f+f^*)=0. \tag{7}$$

For the complex function $f(x,z) = u(x,z) + iv(x,z)$, Eq. (7) reduces to a system of two partial differential equations,

$$\begin{cases}\partial u/\partial x=-L_0 v/2n,\\ \partial v/\partial x=L_1 u/2n,\end{cases} \tag{8}$$

where the operators are

$$L_0=\partial^2/\partial z^2-\gamma^2(z)+\beta(z)A_0^2(z), \quad L_1=L_0+2\beta(z)A_0^2(z).$$

System (8) has a solution in the form of exponential functions,

$$u(x,z)=u(z)\exp\delta x, \quad v(x,z)=v(z)\exp\delta x, \tag{9}$$

or trigonometric functions,

$$u(x,z)=u(z)\cos px, \quad v(x,z)=v(z)\sin px, \tag{10}$$

where real $\delta$ and $p$. The functions $u(z)$ and $v(z)$ in (9) must satisfy a system of second-order ordinary differential equations:

$$L_0 v=-2n\delta u,$$
$$L_1 u=2n\delta v.$$

This system of equations can have a set of solutions (a set of perturbation modes) with different values of $\delta$, and in this case it is difficult to find the perturbation mode which has the maximum growth rate by numerical methods. We accordingly seek the perturbation modes through a direct numerical solution of system of equations (8). If the system allows solutions of the form in (9), then an arbitrary nonzero boundary condition at $x = 0$ should give rise to the mode with the largest growth rate in the course of the evolution. In the calculations, we find exponential solutions only for val-

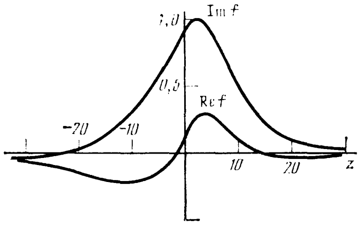


FIG. 2. Real and imaginary parts of the perturbation mode $f(z)$ with the largest growth rate for $n = 1.636$, normalized to a unit maximum of the imaginary part.

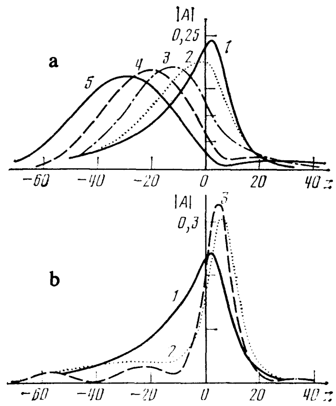


FIG. 3. Decay of an NSW of the unstable branch ($n = 1.636$) for different signs of the initial perturbation. a: $\mu < 0$. 1—$x = 0$; 2—900; 3—1200; 4—1400; 5—1600. b: $\mu > 0$. 1—$x = 0$; 2—1600; 3—3300.

ues of $n$ in the interval $\varepsilon_l^{1/2} < n < n_{cr}$. At $n > n_{cr}$ system (8) has only oscillatory solutions.

As an example we show in Fig. 2 an exponentially growing mode $u(z)$, $v(z)$ in the case $n = 1.636$. The growth rate here is $\delta \sim -.00412...$. Figure 1 shows a plot of the square of the growth rate versus $n$; we see that $n \to n_{cr}$ the quantity $\delta^2$ approaches zero.

Substituting the perturbation modes found by the method above into (6) with a small parameter $\mu$, we trace the subsequent evolution of the solution of our original nonlinear equation, (1). For a selected sign of $f(z)$ in (6), we would be equally justified in assigning either sign to the parameter $\mu$. By virtue of the asymmetry of the problem with respect to the sign of $\mu$, these two choices lead to different final results for a given initial perturbation mode $f(z)$. If we choose a negative sign for $\mu$, we find that after an initial stage of exponential growth of the perturbation mode the NSW is ejected into the linear medium as a result of the subsequent evolution. It propagates away from the surface in the form of a beam at an angle greater than the critical angle for total internal reflection. The process is illustrated in Fig. 3a. We note, however, that all of the energy of the NSW is converted into the energy of the beam. As the beam leaves the nonlinear medium, a small fraction of it is reflected backward, as can be seen clearly from curve in Fig. 3a. The energy (the part of the integral $I$) of the beam reflected into the nonlinear medium is $\sim 2$–$3\%$ of the total energy.

When we choose the positive sign for $\mu$, the growth of the perturbation leads to a shift of the field energy into the interior of the nonlinear medium, followed by the excitation of an NSW of the stable branch; simultaneously, a small fraction ($\sim 3$–$5\%$) of the energy is split off into the energy of a beam which moves away from the interface into the nonlinear medium. This process is illustrated in Fig. 3b. On curve 2 we can see part of the beam which is initially split off into the linear medium. The stable NSW is excited with a perturbation which oscillates along the $x$ axis. As the NSW propagates along the interface, a small fraction of the energy splits off into a beam moving away from the interface during each oscillation. On curve 3 in Fig. 3b we can see a sequence of two such beams. After a splitting off, the amplitude of the oscillations of the perturbation decreases, and the energy of the next beam which splits off turns out to be substantially smaller. Accordingly, as a result of the development of a perturbation with a positive $\mu$, and NSW of the unstable branch is transformed into an NSW of the stable branch. The process is shown schematically by the arrow in Fig. 1.

For an arbitrary initial perturbation, after some transients, the mode with the largest growth rate forms, and the decay of the NSW of the unstable branch proceeds in one of the two manners described above.

To conclude this section we point out that a stability criterion of the type $dI/dn > 0$ has been derived previously by Kolokolov[11] for self-focused solutions in an unbounded nonlinear medium. Our numerical results for Eq. (1) with discontinuous coefficients yields the same stability criterion. As was shown in Ref. 11 for solutions in an unbounded medium, however, the derivative $dI/dn$ does not change sign in the region of allowed values of $n$, and a soliton is always stable in the two-dimensional self-focusing problem. For surface waves, we see that the derivative changes sign at the point $n = n_{cr}$, and the region of allowed values of $n$ breaks up into subregions of stable and unstable waves. This conclusion plays a decisive role in the problem of the excitation of NSWs by light beams incident on an interface at grazing angles, since the NSWs may decay again or may exist for an unbounded time, depending on the excitation conditions.

## 4. EXCITATION OF SURFACE WAVES BY LIGHT BEAMS

Our study of the stability of NSWs yields an interesting conclusion: If the wavefront is inverted, $A \to A^*$, at $x = 1600$ in Fig. 3a (curve 5), the beam will propagate in the opposite direction. This means that a surface wave of the unstable branch can in principle be excited by a light beam of optimum shape and given density which is incident on the boundary of the nonlinear medium at a certain angle greater than the critical angle for total internal reflection. To prove this assertion, we need to discard at the same time the part of the beam which is split off into the nonlinear medium upon the wavefront inversion (phase conjugation). Our numerical simulations in fact confirm that if the field in the nonlinear medium is set equal to zero to invert the wavefront, and if the beam amplitude is increased by a factor of 1.05 for all $z < 0$ in order to conserve the integral $I$, then the beam produced in this manner excites NSWs of the unstable branch as it strikes the interface. After propagating some distance through the

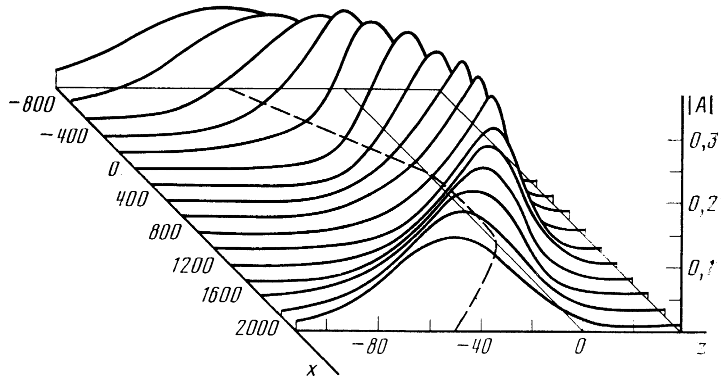


FIG. 4. Evolution of the transverse field distribution during the incidence of a Gaussian light beam with the following parameters on an interface: $I = 1.03$, $a = 25$, $z_0 = 0$, $x_0 = -2000$, $\theta = 2.57°$. The dashed line in the $x,z$ plane shows the path traced out by the maximum value of the field.

nonlinear medium, it is ejected back into the linear medium because of an instability of the NSW which is excited.

During the wavefront inversion, however, the beam which is formed has a certain shape which, although approximately Gaussian, cannot be described analytically. To study the possibility of exciting NSWs by real beams, we use a Gaussian model for the incident beam:

$$A=A_0 w \exp\{w^{-2}[-(z-z_0)^2+ia^2 n_1 (z-z_0)\sin\theta - (x-x_0)^2 \operatorname{tg}^2\theta]\}, \qquad (11)$$

where $w^2 = a^2 + 2i(x - x_0)/n_1 \cos\theta$, $a$ is the width of the beam at its narrowest point, $\theta$ is the angle of incidence, measured in this case from the interface, $n_1 = \varepsilon_l^{1/2}$, $(x_0,z_0)$ are the coordinates of the focus, and the beam amplitude $A_0$ is related to the integral $I$ by

$$I=\frac{A_0}{a}\left(\frac{\pi}{2}\right)^{1/2}. \qquad (12)$$

The parameters of the Gaussian beam are chosen so that the $z$ distribution of the field is, for all $x$, as close as possible to the field distribution of the beam leaving the nonlinear medium, shown in Fig. 3a, at the corresponding values of $x$. The parameters which we found for the Gaussian beam by this fitting procedure are listed in the Fig. 4 caption. Figure 4 itself shows the evolution of a beam with these parameters as it interacts with the interface. We see that the Gaussian beam does in fact excite NSWs of the unstable branch, with the field distribution exponentially approaching the steady-state solution. However, the unavoidable error in the simulation of the beam in Fig. 3a by a Gaussian beam has the consequence that near the steady-state solution $A_0(z)$ the perturbation does not decay to zero but is instead represented by a linear combination of positive and negative exponential functions or, equivalently, a linear combination of hyperbolic functions with the two parameters $\mu_1$ and $\mu_2$:

$$A(x, z)=A_0(z)+(\mu_1 \operatorname{ch}\delta x+\mu_2 \operatorname{sh}\delta x)f(z), \qquad (13)$$

where $f(z)$ is the perturbation mode with the highest growth rate. For simplicity we are assuming that no other modes are excited. Their presence does complicate the picture, but it does not change the fundamental conclusions of this study. It can be seen from (13) that in the excitation of NSWs of the unstable branch by Gaussian beams with an approximately optimum shape there are two possible ways in which the steady-state solution can be approached. The situation is governed by the relation between the two parameters. The solution in Fig. 4 corresponds to the situation with $\mu_1 > \mu_2$. In this case the sign of the perturbation in (13) does not change at any time during the process. After an asymptotically exponential approach to the steady-state solution, the perturbation again grows, with the same sign, and the field is ultimately ejected into the linear medium in the form of a beam. This beam may be regarded as a reflected beam, since it contains most of the beam energy. The dashed line in Fig. 4 shows the path traced out in the $(x,z)$ plane by the maximum value of the beam field during this process. We see from this figure that the light beam emerges from the nonlinear medium at a distance from the entry point several times greater than the width of the beam itself. This displacement of the reflected beam is an analog of the Goos-Haenchen effect for the case of an interface between linear and nonlinear media. We see that this displacement may be many orders of magnitude greater than a linear displacement; i.e., we are dealing with a huge effect in this case. The effect is observed even when the parameters of the Gaussian beam differ slightly from those given above. The values of the integral $I$, of the angle of incidence, and of the beam width, on the other hand, can differ by no more than 3–5% from the optimum values listed in the Fig. 4 caption. The value of $x_0$ must be chosen such that most of the field energy (up to 98%) is initially in the linear medium. In this case the result is of course independent of $x_0$. The values given here for the parameters are the optimum values for $\varepsilon_0$ and $\varepsilon_l$, chosen previously: 2.647 and 2.674, respectively. For other values of the dielectric constant, the optimum parameters will be different. A more accurate fit of the parameters would seem to be possible; a more accurate fit would lead to smaller values of $\mu_1$ and $\mu_2$ and thus an even greater Goos-Haenchen effect.

If the opposite relation holds, $\mu_1 < \mu_2$, the sign of the perturbation in (13) will change as the beam evolves after a steady-state unstable solution $A(z) = A_0(z)$ is reached, so that the subsequent process will lead to the excitation of NSWs of the stable branch, as in the case $\mu > 0$ in a study of the stability. An NSW of the stable branch is ultimately excited by the Gaussian beam. At relatively large values of $\mu_1$ and $\mu_2$, for which the parameters of the Gaussian beam are significantly different from the optimum values, or when other perturbation modes are also excited and must be taken into account in

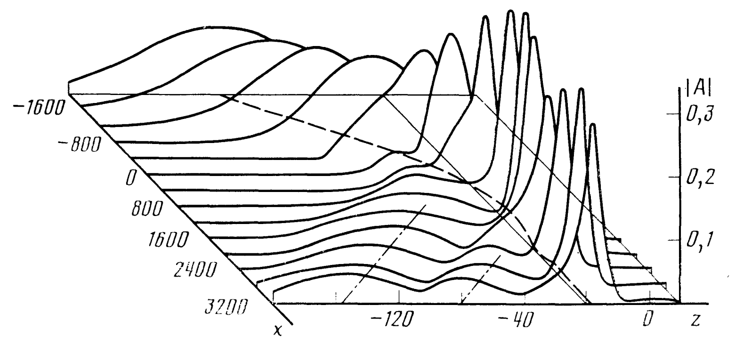


FIG. 5. The same as in Fig. 4 for the beam parameters $I = 1.1$, $a = 35$, $z_0 = 0$, $x_0 = -2000$, and $\theta = 3°$.

(13), the excitation stage for an NSW of the unstable branch may be extremely brief or missing altogether. In the latter case as a result of the interaction of the Gaussian beam with the interface the beam decays directly into an NSW of the stable branch and a beam which moves away from the interface. Figure 5 shows an example of the excitation of an NSW of the stable branch by a Gaussian beam. The parameters of the initial beam are listed in the figure caption. A necessary condition for the excitation of an NSW of the stable branch is an integral larger than in the preceding case. The energy of the initial beam must be sufficient for both the NSW and the new beam. The angle of incidence must still be near the optimum value found above. The dashed line in Fig. 5 shows the path traced out by the field maximum. We see that beyond the value $x = 2000$ the value of $z_{\max}$ corresponding to the field maximum stabilizes. We pursued the calculations up to $x = 6000$, successively equating to zero the field of the reflected beam which reach the $z = -L$ boundary. Up to these values of $x$ the excited wave remains stable; the values of $z_{\max}$ and of the part of the integral $I$ which remains in the NSW corresponds to a stationary solution with the value $n \approx 1.6382...$, i.e., to the stable branch of the NSWs.

Since the NSWs of the stable branch are initially excited with an oscillatory perturbation, as a rule, small beams may split off successively into the linear medium upon each oscillation, so that the wave reflected from the nonlinear medium will have, in addition to the main beam, a tail stretched out along the interface. The beam tail in turn breaks up into separate pulses (or streams). The splitting off of the second reflected beam can be seen clearly in Fig. 5. The dot-dashed lines in the $(x,z)$ plane show the paths traced out by the maximum value of the field of the reflected beams. The onset of the tail of the reflected wave here is somewhat analogous to the side-wave effect in the linear optics of bounded media,[12] where the reflected beam is followed by a tail of decreasing intensity which propagates at the critical angle for total internal reflection. In the linear case we know that there is no surface wave in the $S$ polarization, and the reason for the appearance of a tail in the reflected beam is diffraction back into the first medium of the transmitted beam which is grazing along the surface. In the situation under discussion here, the role of this transmitted beam is played by the NSW, and the secondary reflected beams arise because of an oscillatory relaxation to stationary solution. The "nonlinear side wave" is of a pulsating nature, stretching out over substantially greater distances along the surface and having an intensity higher than that of a linear side wave.

Let us summarize. The increase in the displacement of the reflected beam was observed previously in numerical calculations in Ref. 3. In that study, however, this effect was not explained, primarily because the authors were not familiar with the stability properties of NSWs, and they did not study this question specifically. In our own analysis we have seen that the huge Goos-Haenchen effect can be explained in a simple way in terms of the excitation of the unstable branch of NSWs. It follows in particular that by optimizing the parameters of the Gaussian beam (to satisfy the conditions $\mu_1$, $\mu_2 \to 0$), one can raise this displacement to very large values. Furthermore, one of the basic results of Ref. 3 was that it was completely impossible to excite NSWs by means of Gaussian beams. Our own analysis shows that this is not true. If the energy in a Gaussian beam exceeds the critical value required for the excitation of NSWs, the initial beam can decay into NSWs and secondary beams move away from the interface. An important point is that it is possible to excite NSWs of both the stable (!) and unstable branches. Figure 5 shows only one of the possible realizations of this process, although the excitation of NSWs would appear to be possible over a rather broad range of parameters of the Gaussian beam. Further studies will be required to resolve this question.

## 5. CONCLUSION

These numerical simulations have led to several interesting new conclusions about the behavior of light beams propagating near an interface between linear and nonlinear media.

1) The NSWs which exist at an interface may be either stable or unstable, depending on their effective refractive index.

2) There are two ways in which an NSW of the unstable branch can decay. In one case, and NSW is ejected into the linear medium, and energy propagates in the form of a beam which moves away from the interface at an angle greater than the critical angle for total internal reflection. The second type of decay results in the conversion of an NSW of the unstable branch into an NSW of the stable branch.

3) Nonlinear surface waves of both the unstable and stable branches can be excited by Gaussian light beams which are incident on a interface from a linear medium at grazing angles. This circumstance is of fundamental importances for the planning of experiments, since in this case the

excitation of surface waves does not require additional coupling elements such as frustrated-total-internal-reflection prisms or diffraction gratings on the surface. Such elements are known to be necessary for the excitation of linear surface waves.

4) The excitation of an NSW of the unstable branch by a Gaussian beam can give rise to a hugh Goos-Haenchen effect, i.e., a displacement of the reflected beam from the geometric-reflection line.

5) During excitation of an NSW of the stable branch by a Gaussian beam, the reflected wave may be of a pulsating nature; i.e., it may move away from the interface as a train of light beams.

These effects could be tested experimentally at, for example, an interface of an artificial nonlinear medium with a large nonlinear coefficient $\alpha$ such as has been used in experiments carried out to detect bistability.[5]


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