Induced radiation during scattering of channeled electrons and positrons by point defects

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In scattering of channeled particles by point defects and in emission of gamma rays in the spontaneous-radiation spectral region conditions are attained where the momentum transferred to the defect is taken up by the crystal as a whole. This leads to coherent and interference effects in the radiation from the crystal defects. When the longitudinal momentum transferred is zero, an induced radiation effect appears in the transitions between the states of transverse motion.

The occurrence of spontaneous electromagnetic radiaton during the channeling and quasichanneling of electrons and positrons in crystals has now been well established (see e.g., the reviews, Refs. 1–3). It is usually assumed that the interaction of the particles with the electron-phonon subsystem of the lattice and with defects causes dechanneling and broadening of the emission band. However, along with these phenomena, upon scattering of the particles by point defects, interesting features which are unrelated to dechanneling may be observed in the radiation by channeled electrons and positrons. In this article we shall investigate qualitatively the conditions for and the nature of the effects that arise.

It is well known that the emission of γ rays as a result of scattering of free electrons by a nucleus cannot occur unless some momentum is transferred to the nucleus (henceforth we shall refer mainly to electrons, with the understanding that, unless otherwise stipulated, the general results of the theory are equally valid for positrons). In the case we are considering we have, first of all, not a free electron, but one that is governed by channeling, and second, the electron is scattered from a nucleus that is bound in the crystal lattice. A kinematical treatment shows that when a relativistic channeled electron is scattered and causes radiation from the nucleus, the longitudinal momentum transferred to the nucleus is equal to

$$\Delta p=m^2\omega/2E_e(E_e-\omega)-\Delta\varepsilon_{fi},\qquad \hbar=c=1, \tag{1}$$

where m is the electron mass, E_e and ω are the energies of the electron and the γ ray, respectively, and $\Delta\varepsilon_{fi}=\varepsilon_i-\varepsilon_f$ is the change of energy of transverse motion of the electron.

In contrast to the scattering and radiation of a free electron, expression (1) contains the term $\Delta\varepsilon_{fi}$, which is related to the change in state of transverse motion of the electron. If the momentum (1) transferred to a nucleus that is bound to the lattice satisfies the condition

$$|\Delta p|\ll 1/\delta, \tag{2}$$

(where δ is the amplitude of the thermal vibrations of the point defect), then, with overwhelming probability, the momentum will be taken up by the crystal as a whole. Condition (2) can be fulfilled in the spectral region corresponding to spontaneous radiation, i.e., when

$$\Delta\varepsilon_{fi}\approx m^2\omega/2E_e(E_e-\omega).$$

The case $\Delta p=0$ will correspond in this case to induced radiation in the transition $i\to f$.

In the reference system that is attached to the longitudinal motion of the relativistic electron the effect we are studying has a lucid physical interpretation, namely that the induced radiation arises as a result of the interaction of resonant equivalent photons of the field of the point defect with the atomic-like system of the levels of the transverse motion of the channeled electron.

The analogy of the induced radiation effect with the Mössbauer effect should be pointed out. Therefore in order to describe the radiation that occurs it is necessary to take into account the state of the defect as a whole. It is precisely the nature of the thermal vibrations of the point defect at the minimum of the potential contour of the crystal that determines the coherence conditions of the radiation.

In this paper we investigate the induced radiation in a crystal containing intrinsic interstitial atoms or impurity atoms. In §1 we determine the wave function of the electron-defect system. In §§2–4 we find the amplitude of the process and derive an expression for the number of quanta of induced radiation. In §5 we examine the effect of dechanneling on the magnitude of the effect being investigated.

§1. WAVE FUNCTION

The wave function of the "channeling electron + point defect" system obeys the stationary Dirac equation

$$(\boldsymbol{\alpha}\mathbf{p}+m\beta+\bar V+H_g+V_g)\,|\Psi\rangle=E\,|\Psi\rangle. \tag{3}$$

The Hamiltonian of the interaction in (3) contains $\bar V$, the potential part of the interaction of the electron with the crystal averaged over the longitudinal direction (the Lindhard potential of atomic strings or planes[4]) and a part V_g which describes the interaction of the electron with a point defect; H_g is the Hamiltonian of the point defect, which is located at some minimum of the potential contour of the crystal, and E is the total energy of the electron and the energy of thermal vibrations (phonon excitations) of the defect as a whole.

Writing the spinor components of the bispinor $|\Psi\rangle$ in terms of $|\psi\rangle$ and $|\psi'\rangle$, we can, in the high energy approximation, obtain from (3) the equations for $|\psi\rangle$ and $|\psi'\rangle$

$$(H_e+H_g+V_g)\,|\psi\rangle=\varepsilon\,|\psi\rangle,\qquad |\psi'\rangle=E_e^{-1}\boldsymbol{\sigma}\mathbf{p}\,|\psi\rangle, \tag{4}$$

where

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$$H_e=-\Delta/2E_e+\bar{V},\quad \varepsilon=(E^2-m^2)/2E\approx E/2.$$

In the derivation of Eqs. (4) we discarded the term quadratic in the quantity $\bar{V}+H_g+V_g$ and the term which is proportional to the gradient of the latter and which describes the interaction of the electron spin with the field of the point defect and of the crystal. The coefficients of these terms are of the order of magnitude $1/E_e$.

Let us examine the solution of Eqs. (4) without taking into account the interaction of the electron with the point defect. First we note that the longitudinal motion in the averaged potential $\bar{V}$ is described by a plane wave. Second, the large value of the longitudinal momentum in comparison with the transverse momentum allows us to replace the momentum operator in the second equation of (4) by its longitudinal value and eliminate from the wave function $|\Psi\rangle$ the constant spinor component. Therefore, the subsequent investigation will be carried out for the wave functions $|\psi\rangle$, which obey the first of Eqs. (4), which is formally identical to the Schrödinger equation

$$(H_e+H_g)|\psi_0\rangle=\varepsilon|\psi_0\rangle. \tag{5}$$

The solutions of Eq. (5) describe the states of the continuous spectrum, and for axial channeling have the following structure:

$$|\psi_0\rangle=|E,\lambda,s\rangle=(2\pi)^{-1/2}|\lambda\rangle|s\rangle\exp\{i[2E(\varepsilon-\varepsilon_\lambda-\omega_s)]^{1/2}z\}, \tag{6}$$

where

$$E\gg\varepsilon_\lambda,\ \omega_s,\qquad [2E(\varepsilon-\varepsilon_\lambda-\omega_s)]^{1/2}\approx E-\varepsilon_\lambda-\omega_s.$$

The transverse component (i.e., the one that does not depend on the coordinate z) of the electron wave function $|\lambda\rangle$ and the wave function $|s\rangle$ of the point defect as a whole obey the equation

$$\left\{-\frac{1}{2E_e}\Delta_\perp+\bar{V}\right\}|\lambda\rangle=\varepsilon_\lambda|\lambda\rangle,\quad H_g|s\rangle=\omega_s|s\rangle, \tag{7}$$

where ε_λ is the transverse energy of the electron in the state $|\lambda\rangle$ (the index λ includes the band number and the quasimomentum of the states of transverse motion in the averaged potential of the atomic strings, and ω_s is the quantized energy of thermal vibrations of the point defect as a whole.

For the following analysis it is convenient to transform Eq. (4) into an integral equation

$$|\psi^\pm\rangle=|\psi_0\rangle+G^\pm V_g|\psi^\pm\rangle. \tag{8}$$

In the representation of the Hamiltonian of (5) the Green's function of Eq. (8) in the high electron energy approximation is equal to

$$G^+(E)=-i\theta(z-z')\exp\{iE(z-z')\}\times\sum_{\lambda,s}\exp\{-i(\varepsilon_\lambda+\omega_s)(z-z')\}|\lambda\rangle|s\rangle\langle s|\langle\lambda|. \tag{9}$$

We have an analogous expression also for $G^-(E)$:

$$G^-(E)=i\theta(z'-z)\exp\{iE(z-z')\}\times\sum_{\lambda,s}\exp\{-i(\varepsilon_\lambda+\omega_s)(z-z')\}|\lambda\rangle|s\rangle\langle s|\langle\lambda|. \tag{10}$$

In most cases the interaction V_g is such that the Born series constructed by the iterations of Eq. (8) converges well. Then an approximate solution of Eq. (4) can be given in the form

$$|\psi^\pm\rangle^{(N)}=\sum_{n=0}^{N}(G^\pm V_g)^N|\psi_0\rangle,\qquad N\to\infty. \tag{11}$$

To estimate the magnitude of the effect that we are investigating, we shall use the first Born approximation in the expansion (11). The condition of applicability of this description in the case of single scattering is obvious; specifically, it is necessary that the following inequality be satisfied:

$$Z_ge^2<1,$$

where Z_g is the atomic number of the defect. If we consider emission with a small transfer of longitudinal momentum in a crystal containing point defects, then the interaction with the defects will be coherent, and the condition for the applicability of the Born approximation is different (see, e.g., Ref. 1):

$$NZ_ge^2<1, \tag{12}$$

where N is the number of defects in a length $1/\Delta p$. As $\Delta p\to 0$ for $N=N_{\max}$, it is evident that we have (generally, the lower bound of Δp is of the order of the uncertainty of $1/L\ll 1$)

$$N_{max}=\begin{cases}\pi R^2 l_g n_g, & L>l_g\\ \pi R^2 L n_g, & L<l_g\end{cases}$$

where πR^2 is the area per atom chain, L is the thickness of the crystal, l_g is the dechanneling length, and n_g is the concentration of the defects. Inequality (12) sets a limit on the possible thickness of the crystals and the defect concentration; for example, $n_g\lesssim 5\%$ for $L\gtrsim 1\ \mu$m.

§2. AMPLITUDE OF THE RADIATION

Assuming that the conditions for the applicability of the Born approximation are satisfied, we shall examine the amplitude of the radiation that arises when an electron is scattered from a single point defect. Under the conditions $E_e\gg m$ and $E_e\gg\omega$ we obtain, analogously to Ref. 5, for the amplitude of the radiation in the transition $\lambda,s\to\lambda',s'$ without change of electron spin orientation, the following:

$$\begin{gathered}\mathbf{M}=\langle\Psi_f|e^{-i\varkappa\mathbf{r}}\boldsymbol{\alpha}|\Psi_i\rangle=\mathbf{M}_0+\mathbf{M}_++\mathbf{M}_-,\\ \mathbf{M}_0=\frac{1}{E_e}\langle\psi_f|e^{-i\varkappa\mathbf{r}}\mathbf{p}|\psi_i\rangle,\quad \mathbf{M}_+=\frac{1}{E_e}\langle\psi_f|e^{-i\varkappa\mathbf{r}}\mathbf{p}G^+(E)V_g|\psi_i\rangle,\\ \mathbf{M}_-=\frac{1}{E_e}\langle\psi_f|V_g\{G^-(E')\}^*e^{-i\varkappa\mathbf{r}}\mathbf{p}|\psi_i\rangle,\\ |\psi_i\rangle=|E,\lambda,s\rangle,\quad |\psi_f\rangle=|E',\lambda',s'\rangle,\end{gathered} \tag{13}$$

where $\varkappa$ is the wave vector of the γ ray. The amplitude $\mathbf{M}_0$ in (13) describes ordinary spontaneous radiation,[3] while $\mathbf{M}_+$ and M_- describe the investigated effect. Simple calculations for axial channeling, allowing for (6), (9), and (10), show that the amplitude $\mathbf{M}_+$ and $\mathbf{M}_-$ are respectively

$$\mathbf{M}_+=\frac{-i}{2\pi E_e}\sum_{\lambda'',s''}\left\langle\lambda',s'\right|\int dz\exp(-ip_{\lambda's'}z-i\varkappa\mathbf{r})\times\mathbf{p}\exp(ip_{\lambda''s''}z)\left|\lambda'',s''\right\rangle\times\int_{-\infty}^{z}dz'\exp\{i(p_{\lambda s}-p_{\lambda''s''})z'\}\langle\lambda'',s''|V_g|\lambda,s\rangle, \tag{14}$$

$$\mathbf{M}_- = \frac{i}{2\pi E_e} \sum_{\lambda'',s''} \langle \lambda'', s'' | \int dz \exp(-ip'_{\lambda''s''}z - i\varkappa\mathbf{r})\mathbf{p}$$

$$\times \exp(ip_{\lambda s}z)|\lambda, s\rangle \int_{-\infty}^{z} dz' \exp\{i(p_{\lambda''s''} - p'_{\lambda's'})z'\}$$

$$\times \langle \lambda', s' | V_g | \lambda'', s'' \rangle, \quad (15)$$

$$p_{\lambda s} \approx E - \varepsilon_\lambda - \omega_s, \qquad p_{\lambda s}' \approx E' - \varepsilon_\lambda - \omega_s. \quad (16)$$

The operator $\mathbf{p}$ in formulas (14)–(16) operates only on the wave function of form (6) standing to the right of it.

§3. INDUCED RADIATION WITHOUT TRANSFER OF LONGITUDINAL MOMENTUM OR ENERGY TO THE DEFECT

Let us examine the induced radiation effect when the intermediate state $|\lambda'', s''\rangle$ in (14) is the same as the initial state $|\lambda, s\rangle$ and in (15) the same as the final state $|\lambda', s'\rangle$. This separation is completely admissible, since the energy levels ε_λ and ω_s as a rule are not equidistant and interference between transitions does not occur (see also §4). We shall show that, in accordance with the above discussion, the radiation process in this case occurs with conservation of longitudinal momentum and without transfer of energy to the defect.

Thus, from (14) and (15) we obtain for the case under consideration

$$\mathbf{M}_+ + \mathbf{M}_- = \frac{-i}{2\pi E_e} \langle \lambda', s' | \int dz \exp(-ip'_{\lambda's'}z - i\varkappa\mathbf{r})\mathbf{p}$$

$$\times \exp(ip_{\lambda s}z)|\lambda, s\rangle \int_{-\infty}^{z} dz' \{V_{\lambda s} - V_{\lambda' s'}\}, \quad (17)$$

where

$$V_{\lambda s} = \langle \lambda, s | V_g | \lambda, s \rangle, \qquad V_{\lambda' s'} = \langle \lambda', s' | V_g | \lambda', s' \rangle. \quad (18)$$

Because of the orthonormality of the state vectors $|s\rangle$ of the point defect as a whole, it is clear that (17) will be nonzero only for $s = s'$, since the operator $\mathbf{p}$ operates only on the states $|\lambda\rangle$. Thus, from (17) and (18) it follows that

$$\mathbf{M}_+ + \mathbf{M}_- = \frac{-i}{2\pi E_e} \delta_{ss'} \langle \lambda' | \int dz \exp(-ip'_{\lambda's}z - i\varkappa\mathbf{r})\mathbf{p}$$

$$\times \exp(ip_{\lambda s}z)|\lambda\rangle \int_{-\infty}^{z} dz' \{V_{\lambda s} - V_{\lambda' s}\}. \quad (19)$$

If condition (2) is satisfied, then we have the equality (see the Appendix)

$$\int_{-\infty}^{\infty} dz\, e^{i\Delta pz} \int_{-\infty}^{\infty} dz' \{V_{\lambda s} - V_{\lambda' s'}\}$$

$$= \pi \int_{-\infty}^{\infty} dz \{V_{\lambda s} - V_{\lambda' s}\} \delta_+(\Delta p) + O(\Delta p \delta), \quad (20)$$

the use of which allows us to obtain an expression for the longitudinal and transverse components of the radiation amplitude

$$\{\mathbf{M}_+ + \mathbf{M}_-\}_z = \frac{-i}{2} \delta_{ss'} \langle \lambda' | \exp(-i\varkappa_\perp \boldsymbol{\rho}) | \lambda \rangle$$

$$\times \int dz \{V_{\lambda s} - V_{\lambda' s}\} \delta_+(p_\lambda - p_{\lambda'}' - \varkappa_z), \quad (21)$$

$$\{\mathbf{M}_+ + \mathbf{M}_-\}_\perp = \frac{-i}{2E_e} \delta_{ss'} \langle \lambda' | \exp(-i\varkappa_\perp \boldsymbol{\rho}) \mathbf{p}_\perp | \lambda \rangle$$

$$\times \int dz \{V_{\lambda s} - V_{\lambda' s}\} \delta_+(p_\lambda - p_{\lambda'}' - \varkappa_z), \quad (22)$$

where, in accordance with (16), $p_\lambda = E - \varepsilon_\lambda$, $p'_{\lambda'} = E' - \varepsilon_{\lambda'}$.

The laws of conservation of energy $E = E' + \omega$ and of longitudinal momentum, the latter defined in (21) and (22) by the δ functions, gives the well-known expression[6] for the frequency of the radiation. In particular, for radiation in the forward direction, i.e., in the direction of motion of the particle,

$$\omega_0 \approx 2(\varepsilon_\lambda - \varepsilon_{\lambda'}) E_e^2 / m^2.$$

We note that to obtain this relation, as well as the formulas (34) it is necessary in the determination of the longitudinal momentum to retain also terms of order m/E_e.

The corresponding components of the amplitude of spontaneous radiation according to (13), as (see also, e.g., Ref. 6)

$$\{\mathbf{M}_0\}_z = \langle \lambda' | \exp(-i\varkappa_\perp \boldsymbol{\rho}) | \lambda \rangle \delta(p_\lambda - p_{\lambda'}' - \varkappa_z), \quad (23)$$

$$\{\mathbf{M}_0\}_\perp = \frac{1}{E_e} \langle \lambda' | \exp(-i\varkappa_\perp \boldsymbol{\rho}) \mathbf{p}_\perp | \lambda \rangle \delta(p_\lambda - p_{\lambda'}' - \varkappa_z). \quad (24)$$

From a comparison of (21), (22), and (23), (24) it is easy to establish that the spectral-angular and polarization characteristics of the induced radiation are identical to the corresponding characteristics of the spontaneous radiation.[2,3,6] The relation between the numbers of quanta of spontaneous and induced radiation per unit time and unit length without transfer of longitudinal momentum has the form

$$\frac{dN_g}{do\, d\omega}(\lambda, s \to \lambda', s') = \delta_{ss'} \eta \frac{dN_0}{do\, d\omega}(\lambda \to \lambda'), \quad (25)$$

where

$$\eta = \left| \frac{1}{2} \int dz (V_{\lambda s} + V_{\lambda' s}) \right|^2, \quad (26)$$

$d\omega$ is the frequency interval and do is the solid angle.

Figures 1 and 2 show the results of numerical calcula-

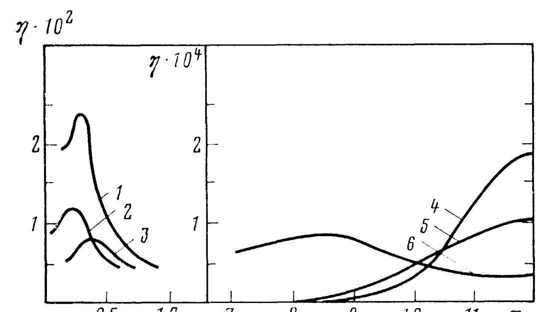

FIG. 1. Ratio of the number of induced radiation photons to the number of spontaneous radiation photons per unit time and unit length of a silicon crystal (the ⟨110⟩ axis, $Z_g = 14$). 1) e^-, 20 Mev, $3p - 1s$ transitions; 2) e^-, 5 Mev, $2p - 1s$ transition; 3) e^-, 5 Mev, $3p - 1s$ transition; 4) e^+, 20 Mev, $1 - 0$ transition; 5) e^+, 5 Mev, $1 - 0$ transition, 6) e^+, 5 Mev, $2 - 1$ transition; x_g is the distance from the defect to the atomic string in units of the Thomas-Fermi screening radius.

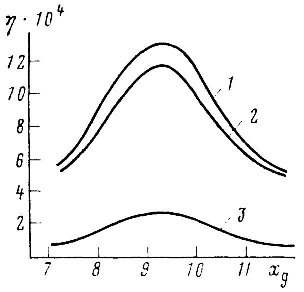

FIG. 2. The same as in Fig. 1, for 20 Mev positrons in the 2 — 1 transition but for various kinds of defects. 1) $Z_g = 81$; 2) $Z_g = 70$; 3) $Z_g = 14$.

tions from formula (26) for various transitions in axial channeling of particles. The potential of the atomic string was approximated by a Coulomb well,[7–9] and the potential for positrons by a harmonic well.[10] For the potential of the point defect we used the approximation of Moliere. We call attention to the dependence of the magnitude of the effect on the location of the defect relative to the atomic string. The maximum value of η is attained in those cases when the displacement of the defect coincides with the quasiclassical radius of the orbit of transverse motion of the electron or positron.

In the case of radiation with zero transfer of longitudinal momentum the amplitude (21) and (22) of the process does not contain a phase factor that depends on the longitudinal coordinate of the defect. If it is assumed that the displacements of the point defects relative to the crystal axis are all the same, then there will be interference of the radiation from the different defects. In this case the amplitudes of the radiation processes at the various defects located within a dechanneling length (if $L > l_g$) or within the thickness of the crystal (if $L < l_g$) will sum in phase, i.e., there will be an interference enhancement of the radiation power.

Thus, for the order of magnitude number of photons per unit time of induced radiation by channeled electrons in a crystal with a volume density n_g of defects we obtain from (25) the estimate

$$\frac{dN_g(L)}{do d\omega}(\lambda, s \to \lambda', s') \approx \delta_{ss'} N^2_{max} \eta \frac{dN_0(L)}{do d\omega}(\lambda \to \lambda'), \tag{27}$$

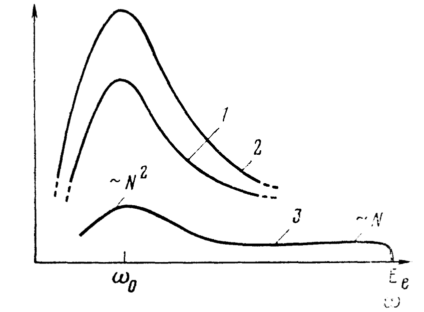

FIG. 3. Spectral density of radiation by channeled particles. 1) crystal without defects; 2) crystal with point defects; 3) difference of curves 2 and 1. N is the number of collisions of particles with defects.

where N_{max} is the maximum possible number of collisions of the electron with defects, as determined in §1.

The qualitative character of the spectrum of the radiation arising in a thin crystal with defects ($L < l_g$) during the channeling of particles is shown in Fig. 3. It can be seen from this figure, in the difference spectrum (curve 3) in the frequency region of the spontaneous radiation, that structure appears which differs from the spectrum of the Bethe-Heitler bremsstrahlung spectrum. Numerical calculations from the formulas presented here are given in §5.

§4. INDUCED RADIATION WITH TRANSFER OF MOMENTUM AND ENERGY TO A DEFECT

As was noted in §3, radiation with the transfer of longitudinal momentum to a defect is described in (14) by the terms with $\lambda'', s' \neq \lambda, s$ and in (15) by the terms with $\lambda'', s'' \neq \lambda', s'$. In this section we examine the process of induced radiation with the simultaneous excitation or relaxation of the thermal vibrations of a defect. These processes are described by the terms in (14) with $s'' \neq s$ and in (15) by the terms with $s'' \neq s'$.

For the reason stated in §3, the amplitude $\mathbf{M}_+$ will be different from zero only for $s'' = s'$ and $\mathbf{M}_-$ will be different from zero only for $s'' = s$. Thus, excluding the case that we have already considered (i.e., $\lambda'', s'' = \lambda, s$ in (14) and $\lambda'', s'' = \lambda', s'$ in (15)), we write

$$\mathbf{M}_+ = \frac{-i}{2\pi E_e} \sum_{\lambda''} \langle \lambda' | \int dz \exp(-ip'_{\lambda' s'} z - i\varkappa \mathbf{r}) \times \mathbf{p} \exp(ip_{\lambda'' s'} z) | \lambda'' \rangle \int_{-\infty}^{z} dz' \times \exp\{i(p_{\lambda s} - p_{\lambda'' s'}) z'\} V_{\lambda s; \lambda'' s'}, \tag{28}$$

$$\mathbf{M}_- = \frac{i}{2\pi E_e} \sum_{\lambda''} \langle \lambda'' | \int dz \exp(-ip'_{\lambda'' s} z - i\varkappa \mathbf{r}) \times \mathbf{p} \exp(ip_{\lambda s} z) | \lambda \rangle \int_{-\infty}^{z} dz' \times \exp\{i(p'_{\lambda'' s} - p'_{\lambda' s'}) z'\} V_{\lambda'' s; \lambda' s'}, \tag{29}$$

where $V_{\lambda s; \lambda' s'} = \langle \lambda', s' | V_g | \lambda, s \rangle$. The characteristic spatial scale δ of the variation in the matrix elements of the operator V_g in (28) and (29) (i.e., the amplitude of the thermal vibrations of the point defect) is such as always to satisfy the inequality

$$|p_{\lambda s}' - p'_{\lambda' s'}| \sim |p_{\lambda s} - p_{\lambda' s'}| \sim |\varepsilon_\lambda - \varepsilon_{\lambda'} + \omega_s - \omega_{s'}| \ll 1/\delta$$

($\delta \sim 0.1$ Å, $|\varepsilon_\lambda - \varepsilon_{\lambda'}|^{-1} \gtrsim 100$ Å). Therefore we can write

$$\int_{-\infty}^{z} dz' \exp\{i(p_{\lambda s} - p_{\lambda' s'}) z'\} V_{\lambda s; \lambda' s'} \approx \exp\{i(p_{\lambda s} - p_{\lambda' s'}) z_g\} \int_{-\infty}^{z} dz' \, V_{\lambda s; \lambda' s'},$$

where z_g is the longitudinal coordinate of the point defect. When condition (2) is satisfied, we obtain, from equality (20), for the longitudinal and transverse components of (28) and

$$\{\mathbf{M}_+\}_z = \frac{-i}{2}\sum_{\lambda''}\exp\{i(p_{\lambda s}-p_{\lambda''s'})z_g\}\langle\lambda'|\exp(-i\varkappa_\perp\rho)|\lambda''\rangle$$
$$\times\int dz' V_{\lambda s;\lambda''s'}\delta_+(p_{\lambda''}-p_{\lambda'}'-\varkappa_z), \quad (30)$$

$$\{\mathbf{M}_+\}_\perp = \frac{-i}{2E_e}\sum_{\lambda''}\exp\{i(p_{\lambda s}-p_{\lambda''s'})z_g\}\langle\lambda'|\exp(-i\varkappa_\perp\rho)\mathbf{p}_\perp|\lambda''\rangle$$
$$\times\int dz' V_{\lambda s;\lambda''s'}\delta_+(p_{\lambda''}-p_{\lambda'}'-\varkappa_z), \quad (31)$$

$$\{\mathbf{M}_-\}_z = \frac{i}{2}\sum_{\lambda''}\exp\{i(p'_{\lambda''s}-p'_{\lambda's'})z_g\}\langle\lambda''|\exp(-i\varkappa_\perp\rho)|\lambda\rangle$$
$$\times\int dz' V_{\lambda''s;\lambda's'}\delta_+(p_\lambda-p_{\lambda''}-\varkappa_z), \quad (32)$$

$$\{\mathbf{M}_-\}_\perp = \frac{i}{2E_e}\sum_{\lambda''}\exp\{i(p'_{\lambda''s}-p'_{\lambda's'})z_g\}\langle\lambda''|\exp(-i\varkappa_\perp\rho)\mathbf{p}_\perp|\lambda\rangle$$
$$\times\int dz' V_{\lambda''s;\lambda's'}\delta_+(p_\lambda-p_{\lambda''}-\varkappa_z). \quad (33)$$

From (30)–(33) it can be seen that, in agreement with the observation made in §3, there is no interference between the various terms in (14) and (15).

From the laws of conservation of energy and longitudinal momentum one can obtain expressions for the maximum frequencies of radiation in the direction of motion of the electrons:

$$\omega\approx2(\varepsilon_{\lambda''}-\varepsilon_{\lambda'}-\omega_s-\omega_{s'})E_e^2/m^2,$$
$$\omega\approx2(\varepsilon_\lambda-\varepsilon_{\lambda''}+\omega_s-\omega_{s'})E_e^2/m^2. \quad (34)$$

Thus, in the $\lambda\to\lambda'$ transition caused by scattering from a point defect with the transfer of longitudinal momentum (2) there will be generated a number of emission bands which arise in the intermediate transitions $\lambda\to\lambda''$ and $\lambda''\to\lambda'$ with simultaneous excitation or relaxation of the thermal vibrations of the point defect. The analogous effect of the conversion of crystal phonon excitations into elecromagnetic radiation has recently been studied in Ref. 11.

We call attention to the fact that each of the amplitudes (30)–(33) has a phase factor that depends on the longitudinal momentum transferred to the defect and on the coordinate z_g of the defect. These factors, generally speaking, suppress the interference of the radiation from the different defects, which was not the case for $\Delta p = 0$ [see (21) and (22)]. Interference enhancement of the radiation power in the case $\Delta p\neq0$ will occur only from those defects that are located within the range $1/\Delta p$. Moreover, the magnitude of the matrix element of the form $V_{\lambda s;\lambda's'}$ is determined by the spatial overlap of the wave functions $|\lambda,s\rangle$ and $|\lambda',s'\rangle$, and for the subbarrier states $|\lambda\rangle$ the following obvious inequality holds:

$$|V_{\lambda s;\lambda's'}|/|V_{\lambda s}|<1.$$

Keeping these circumstances in mind, we conclude that the process, studied in §3, of induced radiation by well-channeled particles is dominant.

§5. THE EFFECT OF DECHANNELING

To estimate the magnitude of the investigated effect in a crystal with defects, it is necessary to take into account the possibility of electron dechanneling during scattering from point defects and from the electron-phonon system of the crystal. In this study we limit the discussion to thin crystals for which the effect of the electron-phonon system on the dechanneling is still small and the dechanneling is due to scattering at point defects.

For the dechanneling length l_g in this case when there is a uniform distribution of defects we obtain the estimate (see, e.g., Ref. 12)

$$1/l_g\approx\sigma(\theta\geqslant\theta_c)n_g\approx(Z_ge^2/E_e)^2n_g\theta_c^{-2}, \quad (35)$$

where $\sigma(\theta>\theta_c)$ is the cross section for scattering of an electron at a point defect where the deflection through the angle θ is larger than the critical angle $\theta_c\approx|\bar{V}/E_e|^{1/2}$.

From (27) and (35) it is easy to determine the qualitative dependence of the relative magnitude of the induced radiation effect on the defect concentration. Its maximum value is attained at that defect concentration at which the dechanneling length becomes equal to the thickness of the crystal. For $L>l_g$ the relative magnitude of the effect is independent of the concentration n_g, since in this case the number of collisions of a particle with defects does not change and is equal to the maximum number possible up to the instant of dechanneling. The character of the dependence of the relative magnitude of the effect on Z_g and E_e also differs substantially in the regions $L>l_g$ and $L<l_g$. Thus, for $L>l_g$ is follows from (27) and (35) that

$$\frac{dN_g(L)}{do d\omega}\Big/\frac{dN_0(L)}{do d\omega}\sim E_e^2,$$

which is accounted for by an increase in the dechanneling length and thus by an increase in the number of possible collisions with defects as the energy increases. On the other hand, for $L<l_g$ it follows from (27) that there is a very weak dependence on the energy.

In addition, for $L>l_g$ the relative magnitude of the effect decreases with increasing Z_g:

$$\frac{dN_g(L)}{do d\omega}\Big/\frac{dN_0(L)}{do d\omega}\sim Z_g^{-4}|V_g|^2,$$

which is explained by a decrease in the dechanneling length and consequently by a decrease in the number of possible

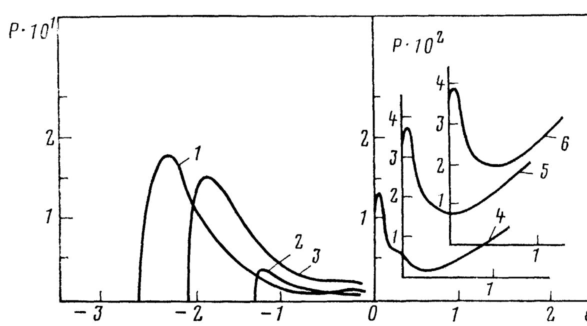

FIG. 4. Probability of dechanneling of 20 Mev electrons (1–3) and positrons (4–6) at point defects in a silicon crystal (⟨110⟩ axis). 1) $x_g = 1$, $Z_g = 14$; 2) $x_g = 2$, $Z_g = 14$; 3) $x_g = 1.4$, $Z_g = 83$; 4) $x_g = 11$, $Z_g = 14$; 5) $x_g = 9$, $Z_g = 70$; 6) $x_g = 8$, $Z_g = 81$. The transverse energy is expressed in units of the critical Lindhard energy.

TABLE I. Ratio of the number of induced radiation photons to the number of photons of spontaneous radiation by channeled particles in a silicon crystal for $N = N_{max}/2$ (particle energy 20 Mev, ⟨110⟩axis).

Transition	Z_g	x_g	N_{max}	η	$\frac{dN_g(L)}{do dw} / \frac{dN_0(L)}{do dw}$
e^-, $3p-1s$	14	1	5	$5 \cdot 10^{-3}$	0,02
e^-, $3p-1s$	14	2	25	$0{,}6 \cdot 10^{-3}$	0,08
e^-, $3p-1s$	83	1,4	7	$6 \cdot 10^{-2}$	0,56
e^+, $1-0$	14	11	70	$1{,}5 \cdot 10^{-4}$	0,18
e^+, $2-1$	14	11	70	$1{,}6 \cdot 10^{-4}$	0,18
e^+, $2-1$	70	9	27	$1{,}1 \cdot 10^{-3}$	0,19
e^+, $2-1$	81	8	25	$0{,}8 \cdot 10^{-3}$	0,12

collisions with defects. On the other hand, for $L < l_g$ it follows from (27) that

$$\frac{dN_g(L)}{do d\omega} \Big/ \frac{dN_0(L)}{do d\omega} \sim |V_g|^2.$$

To obtain numerical estimates we have carried out computer calculations for the probability of dechanneling of relativistic particles as a result of single scattering by a point defect. We considered defects consisting of isolated interstitial atoms displaced from the axis of the atomic string. In the calculations we used the Thomas-Fermi potential in the Moliere approximation for V_g, while the field of the atom chain was determined using the averaged Moliere-Erginsoy potential. The dechanneling probability was calculated with the model described in Ref. 13. At the plane ahead of the displaced atom the directon of motion of the particle was assigned with a given value of transverse energy $\varepsilon_\perp$ lying within the channeling interval. At the plane behind the displaced atom the coordinates of the scattered particle and its direction of motion relative to the atom chain were determined. From the distribution in transverse energy after scattering from the displaced atom the dechanneling probability $P(\varepsilon_\perp)$ was determined for all initial energies within the above-mentioned interval (Fig. 4).

The results of the numerical calculations allow us to estimate the relative magnitude of the effect from formula (27). The results of these estimates are shown in Table I. The maximum number of collisions with defects was determined from the relation $N_{max} \sim 1/P_{max}$.

It can be seen from Table I that first, the magnitude of the effect is great enough to be observed experimentally. Second, the effect is more marked for positrons (we note that N_{max} for positrons lies at the limit of applicability of the Born approximation). This latter result is due to the fact that the dechanneling probability is much smaller for positrons than for electrons and therefore a positron can undergo a larger number of collisions with defects before it leaves the channeling regime. The magnitude of the effect is about the same for the impurity atom sites and the intrinsic interstitial atom sites given in the table. This is due to the strong dechanneling that takes place at an impurity atom and compensates for the increase in the effect that results from the increase in $\eta \sim |V_g|^2$.

CONCLUSIONS

1. The radiation by channeled electrons and positrons during scattering with small momentum transfer at point defects has a coherent nature. Because small momentum transfers correspond to the spectral region of spontaneous radiation during channeling, in this spectral range scattering at point defects in thin crystals leads to interference enhancement of the radiation power.

2. Radiation with zero transfer of longitudinal momentum is interpreted as an effect of induced radiation in a transition between states of the transverse motion, caused by scattering at a point defect. In this case the amplitudes of the radiation from different defects displaced the same distance from the atomic string have the same phase, and as a result the power of the additional radiation is proportional to N^2_{max}. If the transferred momentum $\Delta p = 0$, but condition (2) is satisfied, then additional bands (34) appear in the radiation spectrum.

3. The magnitude of the effect depends on the location of the point defect in the channel, on its atomic number, and on the energy of the particles being scattered. If one varies the populations of the different states of the transverse motion by varying the angle of incidence of the particles on the crystal, one can locate the impurity atoms in the lattice.

4. The ratio of the number of induced photons to the number of photons of spontaneous radiation from the crystal per unit time is a maximum when the thickness of the crystal is equal to the dechanneling length and the displacement of the defect is equal to the radius of the orbit of the channeled particles. We note that the question of the variation of the absolute yield of quanta in thick crystals containing defects ($L > l_g$) requires a separate treatment.

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APPENDIX

Let us demonstrate the correctness of equality (20). We introduce the notation

$$\int_{-\infty}^{z} dz' (V_{\lambda s} - V_{\lambda' s'}) = F(z).$$

First making the limits of integration large but finite, we integrate the left hand side of (20) by parts:

$$\int_{-T}^{T} dz\, e^{i\Delta pz} F(z) = F(z)\frac{1}{i\Delta p} e^{i\Delta pz}\Big|_{-T}^{T} - \frac{1}{i\Delta p}\int_{-T}^{T} dz\, e^{i\Delta pz}\frac{\partial F(z)}{\partial z}.$$

We shall assume that the inequality $|\Delta p| \ll 1/\delta$ is satisfied, where δ is the spatial scale of the variation of the matrix elements (18), i.e., the amplitude of the thermal vibrations of the point defect in the state $|s\rangle$. Then the exponential in the integral on the right hand side can be expanded in a series and truncated at the first term. Keeping in mind that

$$\lim_{T\to\infty} F(-T) = 0,$$

we obtain

$$\int_{-T}^{T} dz\, e^{i\Delta pz} F(z) = \exp(i\Delta p z_g) F(T)\frac{1}{i\Delta p}\{e^{i\Delta pT} - 1\}.$$

In the limit as $T\to\infty$ it follows from this result that equality (20) is correct. The phase factor $\exp(i\Delta p z_g)$ can be dropped, since for $\Delta p \neq 0$ expression (20) vanishes.

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