Equations of the Ginzburg-Landau-Abrikosov-Gor'kov theory for superconductors with a state-density fine structure; effect of radiation on the upper critical field of Nb_3Sn

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The Ginzburg-Landau-Abrikosov-Gor'kov (GLAG) theory is generalized to the case in which the electron state density has an energy fine structure and the Anderson theorem is violated. The changes in the electron spectrum in a nonideal sample cause the critical field H_{c2} to behave quite differently as a function of the concentration in the dirty limit than predicted by the GLAG theory with a constant state density. Specifically, $H_{c2}(0)$ may go through a maximum as a function of the concentration, and $-(dH_{c2}/dT)_{T_c}$ may increase more slowly than linearly. The energy dependence of the electron velocity near the Fermi level in a superconductor with a fine structure in the electron density disrupts the relationship between $H_{c2}(0)$ and $(dH_{c2}/dT)_{T_c}$ predicted by the GLAG theory. Samples of Nb_3Sn have been irradiated in a reactor, and $H_{c2}(T)$ has been measured down to liquid-helium temperatures. Direct measurements reveal that the dose dependence of H_{c2} at 4.2 K is nonmonotonic (it has a maximum) and that the dose dependence of $(dH_{c2}/dT)_{T_c}$ is nonlinear, in agreement with the theoretical predictions.

INTRODUCTION

The anomalous properties of high-temperature superconductors with the A-15 structure are evidence that the electron state density $N_0(\xi)$ has a fine structure with a typical peak width $\gamma \sim 0.01$ eV (Ref. 1). The existence of a fine structure is supported by one-electron numerical calculations[2] and by models[3] of the band spectrum. A strong electron-phonon interaction promotes a localization of d electrons and increases the height of the peaks in the state density.[4] At a certain strength, this interaction may be the sole reason for the appearance of fine structure, through a polaron band contraction. On the other hand, the interaction with defects at low concentrations tends to smooth out the structural features in $N_0(\xi)$, causing a violation of the Anderson theorem in high-temperature superconductors.[5,6] Our purpose in the present study was to derive and experimentally test electrodynamic equations incorporating a fine structure in $N_0(\xi)$ and a deviation from the Anderson theorem.

As a rule,[7] the Ginzburg-Landau-Abrikosov-Gor'kov (GLAG) theory ignores any energy dependence of the state density:

$$N_0(\xi) \approx N(0). \tag{1}$$

Assumption (1) means that the state density, the velocity at the Fermi surface (v_0), and T_c itself are not affected by the introduction of a small number of defects, and such characteristics as the residual resistance and the upper critical field H_{c2} (in the dirty limit) depend linearly on the concentration.

If we introduce an energy dependence of $N_0(\xi)$ with a scale value γ, we cause a substantial renormalization[5] of $N_0(\xi)$ at a defect concentration corresponding to the condition

$$\gamma\tau \leqslant 1, \tag{2}$$

where τ is the scale time between collisions. As a result, T_c, the magnetic susceptibility,[5,6] and the electron heat capacity[8] become strong functions of the concentration, and the concentration dependence of the residual resistance becomes nonlinear.[9]

A renormalization of the electron spectrum by defects should also change the concentration dependence of the coefficients in the GLAG equations. The first attempt to incorporate a renormalization of the electron spectrum in the GLAG equations was made by Fähnle and Kronmüller,[10] who restricted their analysis to the one-dimensional approximation of linear chains for A-15 (the Labbe-Friedel model) and used standard expressions for the coefficients, found without allowance for the renormalization of the spectrum,[11] into which they substituted the renormalized values of T_c and the one-dimensional value of $N(0)$.

The systematic analysis below makes it possible to go beyond the scope of the one-dimensional approximation and also to correctly carry out an energy averaging in the coefficients. It is shown in particular that when there is a fine structure in $N_0(\xi)$ we must abandon the well-known relation between the critical field and its temperature derivative in the dirty limit,[11]

$$H_{c2}(0) = -0{,}69 T_c (dH_{c2}/dT)_{T_c}, \tag{3}$$

and the concentration dependence of $(dH_{c2}/dT)_{T_c}$ and $H_{c2}(0)$ in the dirty limit becomes nonlinear. The field $H_{c2}(0)$ may go through a maximum at a certain concentration.

The most effective method for causing controllable changes in the electron spectrum of A-15 is bombardment by neutrons and charged particles, which introduces a control-

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lable number of isoelectronic defects. Experiments by this approach have revealed a degradation of the critical current[12] and of T_c (Ref. 13) in A-15 due to a broadening of the density peak.[5,6]

The upper critical field of irradiated samples has been measured in several studies.[14] As a rule, H_{c2} has been found either by extrapolating the $J_c(H)$ dependence to a zero current or by using (3) and the measured values of $(dH_{c2}/dT)_{T=T_c}$ and T_c. In the present study we bombarded Nb_3Sn samples with neutrons in a reactor at a temperature of 60 °C. We measured H_{c2} (4.2 K) and $(dH_{c2}/dT)_{T=T_c}$ in a pulsed magnetic field. The experimental results confirm the nonlinear concentration dependence of the derivative and furnish the first direct evidence for a nonmonotonic dependence of $H_{c2}(0)$ (with a maximum). We also show that these results mean that relation (3) breaks down.

1. FINE STRUCTURE IN THE ELECTRON STATE DENSITY IN THE EQUATIONS OF THE GLAG THEORY

Let us examine the Gor'kov equations, in which an average is taken over the coordinates of the defects by the generating-functional technique[15]:

$$\left\{i\omega-\xi\hat{\sigma}_z-\frac{1}{2}[\Delta(\mathbf{r}_1)\hat{\sigma}_+ +\Delta^*(\mathbf{r}_1)\hat{\sigma}_-]\right.$$

$$\left.-n_d\Gamma^2\hat{\sigma}_z\hat{G}_\omega(\mathbf{r}_1,\mathbf{r}_1)\hat{\sigma}_z\right\}\hat{G}_\omega(\mathbf{r}_1,\mathbf{r}_2)=\delta(\mathbf{r}_1-\mathbf{r}_2), \quad (4)$$

where $\hat{\sigma}_z$ and $\hat{\sigma}_\pm$ are the Pauli matrices, n_d is the defect concentration, $m\Gamma^2/2\pi$ is the square of the scattering amplitude of an individual defect, $\omega = 2\pi T(n+1/2)$, $\xi \equiv \xi(-i\partial/\partial\mathbf{r}_1)$, and $\xi(\mathbf{p})$ is the band spectrum (the polaron effect[4] is taken into account). A magnetic field can be introduced in any stage by virtue of gradient invariance.

Because of the fine structure in $N_0(\xi)$ in an ideal sample, the constant-energy surfaces may have a complicated shape, and the functional dependence $\xi(\mathbf{p})$ will differ from an isotropic parabolic dependence. We will first solve system (4) in the spatially homogeneous case near T_c. After the standard transformations,[7] we find the following results for the Fourier components:

$$G_\omega(\mathbf{p})=\frac{1}{i\omega'-\xi}, \quad F_\omega(\mathbf{p})=-\frac{\tilde{\Delta}_\omega^*}{(i\omega'-\xi)(i\omega'^*+\xi)}, \quad (5)$$

where

$$\omega'=\omega+i\Sigma(i\omega),$$

$$\tilde{\Delta}_\omega=\Delta\left(1+n_d\Gamma^2\int d\xi\frac{N_0(\xi)}{(i\omega'-\xi)(i\omega'^*+\xi)}\right)^{-1},$$

and Σ satisfies the self-consistent equation

$$\Sigma(i\omega)=n_d\Gamma^2\int d\xi\frac{N_0(\xi)}{i\omega-\xi-\Sigma(i\omega)}. \quad (6)$$

Substituting (5) into the equation for the order parameter,[7]

$$\Delta(\mathbf{r})=V_0T\sum_\omega F_\omega^*(\mathbf{r},\mathbf{r}), \quad (7)$$

we find an implicit expression for T_c:

$$1=-V_0T_c\sum_\omega\int d\xi\frac{N_0(\xi)}{(i\omega'-\xi)(i\omega'^*+\xi)}\times\left(1+n_d\Gamma^2\int d\xi\frac{N_0(\xi)}{(i\omega'-\xi)(i\omega'^*+\xi)}\right)^{-1}, \quad (8)$$

where V_0 is the interaction constant.

We can show that Eq. (8) is the same as the Bardeen-Cooper-Schrieffer equation with a state density renormalized by defects, which was used in Ref. 5 to describe T_c as a function of the concentration and the dose. Using expression (6) for Σ, we rewrite Eq. (8) as

$$1=-\frac{V_0T_c}{n_d\Gamma^2}\sum_\omega\frac{1}{\omega}\operatorname{Im}\Sigma(i\omega). \quad (9)$$

Using the relationship between the temperature Green's function with the retarded function $G^R(\mathbf{p},\omega)$ and the advanced function $G^A(\mathbf{p},\omega)$ (Ref. 7), we can put (9) in the form

$$1=-V_0T_c\sum_{\mathbf{p}}\left\{\sum_{\omega>0}\frac{G^R(\mathbf{p},i\omega)}{i\omega}+\sum_{\omega<0}\frac{G^A(\mathbf{p},i\omega)}{i\omega}\right\}. \quad (10)$$

Replacing the discrete sums in (10) by contour integrals (Fig. 1), deforming the contours to the real axis, and making use of the analytic properties of $G^{R,A}$, we find

$$1=-\frac{V_0}{2\pi}\sum_{\mathbf{p}}\int\frac{dz}{z}\operatorname{th}\frac{z}{2T_c}\operatorname{Im}G^R(\mathbf{p},z), \quad (11)$$

where

$$G^R(\mathbf{p},z)=[z-\xi(\mathbf{p})-\Sigma(z)]^{-1}.$$

The self-energy part of $\Sigma(z)$ is found by analytically continuing $\Sigma(i\omega)$ from the upper imaginary semiaxis to the real axis; it satisfies the equation[5]

$$\Sigma(z)=n_d\Gamma^2\int d\xi\frac{N_0(\xi)}{z-\xi-\Sigma(z)}. \quad (12)$$

Using the definition of the state density,

$$N(z)=-\frac{1}{\pi}\sum_{\mathbf{p}}\operatorname{Im}G^R(\mathbf{p},z)=-\frac{1}{\pi n_d\Gamma^2}\operatorname{Im}\Sigma(z), \quad (13)$$

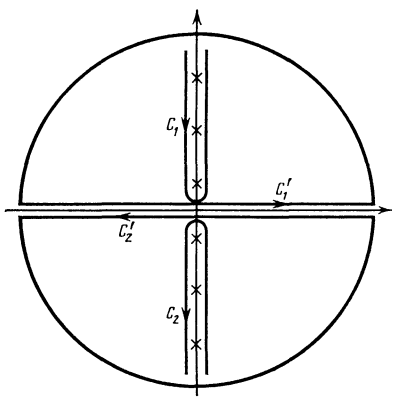

FIG. 1. Integration contours in Eq. (10). The crosses show $i\omega = i\pi T_c(2n+1)$.

we can reduce Eq. (11) to the Bardeen-Cooper-Schrieffer equation,

$$1=V_0\int\frac{dz}{2z}\,\mathrm{th}\frac{z}{2T_c}N(z). \tag{14}$$

As usual, we treat a spatial inhomogeneity in the semiclassical approximation, assuming that the coherence length is much greater than the typical electron wavelength. In the case of a fine structure the latter assumption holds if

$$T_c\ll\gamma, \tag{15}$$

which is satisfied in A-15.

Introducing the incomplete Fourier component of the Green's function in the coordinate of the relative motion $(\mathbf{r}_1-\mathbf{r}_2)$ in the approximation linear in ∇_R [$\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/2$], we find from (4) an analog of the Eilenberger equation[16]:

$$i\mathbf{v}(\mathbf{p})\nabla_R\hat{g}_\omega(\mathrm{R},\ \mathbf{p})+[\hat{\omega},\ \hat{g}_\omega(\mathbf{R},\ \mathbf{p})]_-=0, \tag{16}$$

where

$$\mathbf{v}(\mathbf{p})=\frac{\partial\xi(\mathbf{p})}{\partial\mathbf{p}},\qquad \hat{\omega}=i\omega\hat{\sigma}_z-\hat{\Delta}-n_d\Gamma^2\sum_{\mathbf{p}}\hat{g}_\omega(\mathbf{R},\mathbf{p}),$$

$$\hat{\Delta}=\begin{vmatrix}0 & \Delta(\mathbf{R})\\ -\Delta^*(\mathbf{R}) & 0\end{vmatrix},\qquad \hat{g}_\omega(\mathbf{R},\mathbf{p})=\hat{G}_\omega(\mathbf{R},\mathbf{p})\hat{\sigma}_z.$$

2. THE DIRTY LIMIT; GENERAL EXPRESSION FOR $H_{c2}(T)$

The complicated dispersion relation in a superconductor in which the electron density has a fine structure makes it difficult to solve system (16). As Usadell[17] has shown, the system (16) can be simplified considerably in the dirty limit because the constant-energy surface becomes isotropic. In the case with a fine structure, this effect makes it possible to reduce the momentum dependence $\hat{g}_\omega(\mathbf{R},\mathbf{p})$ to the single energy variable ξ. It is important to note that the region of the dirty limit is defined by the condition

$$T_c\tau\ll 1, \tag{17}$$

which, by virtue of inequality (15), clearly holds in region (2), in which there are significant changes in T_c and the other parameters because of the change in the electron spectrum.

We introduce a direct generalization of the Green's function averaged over angles on the constant-energy surface:

$$\hat{g}_\omega^{\,0}(\mathbf{R},\xi)=N_0^{-1}(\xi)\oint\frac{dS_\xi}{|\mathbf{v}(\mathbf{p})|_\xi}\hat{g}_\omega(\mathbf{R},\mathbf{p}), \tag{18}$$

where

$$N_0(\xi)=\oint\frac{dS_\xi}{|\mathbf{v}(\mathbf{p})|_\xi}$$

is the state density in the ideal crystal, and dS_ξ is an element of the constant-energy surface. We then have

$$\hat{g}_\omega(\mathbf{R},\mathbf{p})=\hat{g}_\omega^{\,0}(\mathbf{R},\xi)+\frac{\mathbf{v}(\mathbf{p})}{|\mathbf{v}(\mathbf{p})|}\hat{\mathbf{l}}_\omega(\mathbf{R},\xi), \tag{19}$$

where $\hat{\mathbf{\Gamma}}_\omega$ is small in comparison with $\hat{g}_\omega^0$. Taking the average of Eq. (16) and also the average of Eq. (16) after multiplication by $\mathbf{v}(\mathbf{p})/|\mathbf{v}(\mathbf{p})|$ over angles, we find, for a cubic lattice,

$$\frac{iv_\xi}{3}\nabla_R\hat{\mathbf{\Gamma}}_\omega(\mathbf{R},\xi)+i\omega[\hat{\sigma}_z,\hat{g}_\omega^{\,0}(\mathbf{R},\xi)]_--[\hat{\Delta},\hat{g}_\omega^{\,0}(\mathbf{R},\xi)]_-=0,$$

$$i\nabla_R\hat{g}_\omega^{\,0}(\mathbf{R},\xi)-\frac{1}{l_\xi}[\hat{g}_\omega(\mathbf{R}),\hat{\mathbf{\Gamma}}_\omega(\mathbf{R},\xi)]_-=0, \tag{20}$$

where

$$g_\omega(\mathbf{R})=\frac{1}{2\pi N_0(0)}\int d\xi N_0(\xi)g_\omega^{\,0}(\mathbf{R},\xi),$$

$$v_\xi=S_\xi/N_0(\xi),\qquad l_\xi=v_\xi\tau_0,\qquad \tau_0=(2\pi\Gamma^2n_dN_0(0))^{-1}$$

are the velocity, mean free path, and mean free time; and S_ξ is the area of the constant-energy surface.

Near the upper critical field we can restrict the analysis to the linear terms in Δ (Ref. 15) in Eqs. (20). For these terms we find from (20) and condition (7) the following results, taking into account the vector potential $\mathbf{A}(\mathbf{R})$:

$$\frac{v_\xi^2}{12\,\mathrm{Im}\,\Sigma(i\omega)}(\nabla-2ie\mathbf{A})^2F_\omega^*(\mathbf{R},\xi)+\omega F_\omega^*(\mathbf{R},\xi)$$

$$=\frac{\mathrm{Re}\,\omega'}{|\xi-i\omega'|^2}\Delta; \tag{21}$$

$$\Delta=V_0T\sum_\omega\int d\xi N_0(\xi)F_\omega^*(\mathbf{R},\xi). \tag{22}$$

If we also have $T\sim T_c$, the first term in (21) will be small[7] and can be treated as a perturbation. Determining F_ω from Eq. (21) in this manner, and substituting it into (22), we find a generalization of the Ginzburg-Landau equation for superconductors with a fine structure in the electron state density in the approximation linear in Δ:

$$(1+L)\Delta(\mathbf{R})-D(\nabla-2ie\mathbf{A})^2\Delta(\mathbf{R})=0, \tag{23}$$

where

$$L=-V_0T\sum_\omega\int d\xi\frac{N_0(\xi)}{|i\omega'-\xi|^2}\frac{\mathrm{Re}\,\omega'}{\omega}, \tag{24}$$

$$D=-\frac{V_0T}{12}\sum_\omega\int d\xi\frac{N_0(\xi)v_\xi^2\,\mathrm{Re}\,\omega'}{\omega^2\,\mathrm{Im}\,\Sigma(i\omega)\,|i\omega'-\xi|^2}. \tag{25}$$

Making use of the analytic properties of the Green's functions, as above, we can put L in the form

$$L=-V_0\int\frac{dz}{2z}N(z)\,\mathrm{th}\frac{z}{2T}. \tag{26}$$

In contrast with the ordinary Ginzburg-Landau equations, Eq. (26) contains a state density $N(z)$ renormalized by the defects. The coefficient D cannot be reduced to an expression which depends on $N(z)$ alone because of the energy dependence of the average velocity v_ξ in (20). When we have a small parameter $T_c/\gamma\ll 1$ [see (15)], expressions (25) and (26) can be simplified further. To achieve this simplification we use a general expression for $H_{c2}(T)$, which holds for arbitrary temperatures. To find it, we expand F_ω and Δ in Eqs. (21) and (22) in the eigenfunctions of the operator[18] $(\nabla-2ie\mathbf{A})^2$. Making use of the orthogonality of these eigenfunctions, we find an implicit expression for $H_{c2}(T)$:

$$1=V_0T\sum_\omega\int d\xi\frac{N_0(\xi)}{|\xi-i\omega'|^2}\left[\omega-\frac{eH_{c2}v_\xi^2}{6\,\mathrm{Im}\,\Sigma(i\omega)}\right]^{-1}\mathrm{Re}\,\omega'. \tag{27}$$

To improve the convergence of the series in (27), we add and

subtract the right side of (27) with $H_{c2}=0$. As a result we find

$$\int \frac{dz}{z} N(z)\left\{\operatorname{th}\frac{z}{2T_c}-\operatorname{th}\frac{z}{2T}\right\}=eH_{c2}T\sum_{\omega}\int d\xi$$

$$\times\frac{N_0(\xi)v_\xi^2}{|\xi-i\omega'|^2}\frac{\operatorname{Re}\omega'}{3\omega\operatorname{Im}\Sigma(i\omega)}\left[\omega-\frac{eH_{c2}v_\xi^2}{6\operatorname{Im}\Sigma(i\omega)}\right]^{-1}. \quad (28)$$

Making use of the small ratio T_c/γ and the rapid convergence of the series on the right side of (28), we can ignore the frequency dependence in the functions $\omega'(\omega)$, $\Sigma(i\omega)$ and $N(z)$, setting $\omega=0$ and $z=0$, respectively. As a result, Eq. (28) becomes

$$N(0)\ln\frac{T}{T_c}=\frac{\operatorname{Im}\Sigma(i0)}{\pi}\int d\xi\frac{N_0(\xi)}{|\xi+\Sigma(i0)|^2}$$

$$\times\left\{\psi\left(\frac{1}{2}-\frac{eH_{c2}v_\xi^2}{12\pi T\operatorname{Im}\Sigma(i0)}\right)-\psi\left(\frac{1}{2}\right)\right\}, \quad (29)$$

where $\psi(x)$ is the psi function.

Assuming $v_\xi^2=v_0^2=\text{const}$, and using (12), we find that Eq. (29) has the same form as the Maki–de Gennes equation,[18] but with a temperature T_c renormalized by defects [see (14)]:

$$\ln\frac{T_c}{T}=\psi\left(\frac{1}{2}+\frac{eH_{c2}v_0l}{6\pi T}\right)-\psi\left(\frac{1}{2}\right), \quad (30)$$

where $l=v_0\tau$, and $\tau=(2\pi\Gamma^2n_dN(0))^{-1}$ is the scale time between collisions (the renormalization of the state density has been taken into account).

In general, according to definition (20), v_ξ depends on ξ with a scale value γ, so that H_{c2} should be determined from Eq. (29). Taking the limits $T\to 0$ and $T\to T_c$ in (29), we fine, respectively,

$$H_{c2}(0)=-12\Phi_0T_c\operatorname{Im}\Sigma(i0)$$

$$\times\exp\left\{\psi\left(\frac{1}{2}\right)-n_d\Gamma^2\int d\xi\frac{N_0(\xi)\ln v_\xi^2}{|\xi+\Sigma(i0)|^2}\right\}, \quad (31)$$

$$\left.\frac{dH_{c2}}{dT}\right|_{T_c}=\frac{24\Phi_0}{\pi^2n_d\Gamma^2}\operatorname{Im}\Sigma(i0)\left\{\int d\xi\frac{N_0(\xi)v_\xi^2}{|\xi+\Sigma(i0)|^2}\right\}^{-1}, \quad (32)$$

where Φ_0 is the quantum of flux. As can be seen from a comparison of (31) and (32), the energy dependence of v_ξ leads to a violation of relation (3).

3. CALCULATION OF $H_{c2}(0)$ AND $(H_{c2}/dT)_{T_c}$ FOR A MODEL STATE DENSITY AND COMPARISON WITH THE GLAG THEORY

Expressions (31) and (32) contain the quantity $\Sigma(i0)$, which must be determined from Eq. (6). For a qualitative analysis of the concentration dependence of H_{c2} we take $N_0(\xi)$ to be a Lorentzian peak with a half-width γ and a pedestal determined by the constant C:

$$N_0(\xi)=\frac{N_0}{1+C}\left\{\frac{\gamma^2}{(\xi+\mu)^2+\gamma^2}+C\right\}, \quad (33)$$

where N_0 is the maximum value of the density, and μ is the position of the Fermi level with respect to the peak. Substituting (33) into Eq. (6), we find a quadratic equation for Σ, whose solution is

$$\Sigma(i0)=\frac{1}{2}\{\mu+i\gamma(1-Cx)-[(\mu+i\gamma(1+Cx))^2-4x\gamma^2]^{1/2}\},$$

$$x=\pi n_d\Gamma^2N_0/\gamma(1+C), \quad (34)$$

where we have introduced the dimensionless defect concentration x, whose value determines the extent of the change in T_c (Ref. 5).

Because of the deformation of the state density, the position of the Fermi level, μ, also depends on x. In the case of isoelectronic defects, in particular, radiation-induced defects, we should determine this position from the conservation of the number of valence electrons[5]:

$$\int_0^{\xi}N_0(\xi)\,d\xi=\frac{N_0}{1+C}\left(\operatorname{arctg}\frac{\mu_0}{\gamma}+\frac{\mu}{\gamma}C\right), \quad (35)$$

where $N(\xi)$ is found from Eqs. (12) and (13). Aleksandrov *et al.*[19] give an explicit expression for $N(\xi)$ for a model state density of an ideal sample as in (33). In the particular case in which the Fermi level in the ideal sample (μ_0) coincides with the density peak ($\mu_0=0$), there is no shift of the peak: $\mu=0$. We assume that the area of the constant-energy surface is independent of the energy:

$$S_\xi=S=\text{const}. \quad (36)$$

This assumption is valid for A-15 in the one-dimensional approximation. We can use approximation (36) to derive an explicit energy dependence of the velocity:

$$v_\xi=\frac{S(1+C)}{N_0}\left[\frac{\gamma^2}{(\xi+\mu)^2+\gamma^2}+C\right]^{-1}. \quad (37)$$

Substituting (37) into (32), and evaluating the integral, we find the concentration dependence of $(dH_{c2}/dT)_{T_c}$ in analytic form for $\mu_0=0$:

$$\left.\frac{dH_{c2}}{dT}\right|_{T_c}=-\frac{6\Phi_0N_0^2\gamma}{\pi^2S^2(1+C)x}\{xC-1+[(xC+1)^2+4x]^{1/2}\}^2$$

$$\times\frac{2-[C/(C+1)]^{1/2}\{1-Cx-[(1+Cx)^2+4x]^{1/2}\}}{2-[(C+1)/C]^{1/2}\{1-Cx-[(1+Cx)^2+4x]^{1/2}\}}. \quad (38)$$

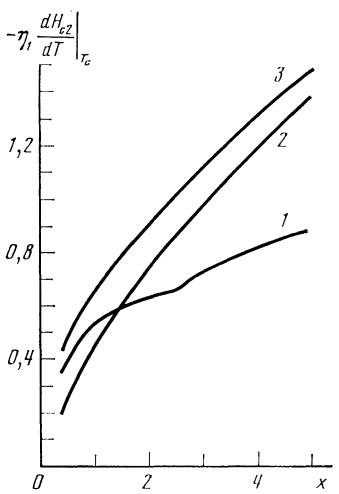

FIG. 2. Nonlinear concentration dependence of $(dH_{c2}/dT)_{T_c}$ for the parameter value $\eta_1=\pi S^2(1+C)^2/24\Phi_0N_0^2\gamma C$. 1—$C=0.4$, $\mu_0=0.2\gamma$; 2—$C=0.7$, $\mu_0=1.5\gamma$; 3—$C=0.7$, $\mu_0=0.2\gamma$.

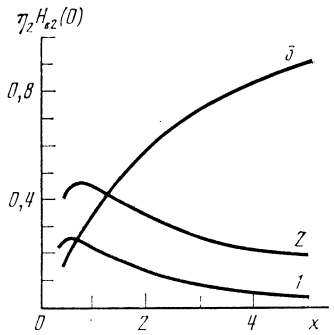

FIG. 3. Nonlinear concentration dependence of $H_{c2}(0)$ for the parameter value $\eta_2 = S^2(1+C)^2\exp[-\psi(1/2)]/12\Phi_0 N_0^2\gamma T_{c0}$. 1—$C = 0.2$, $\mu_0 = 0.2\gamma$; 2—$C = 0.4$, $\mu_0 = 0.2\gamma$; 3—$C = 0.4$, $\mu_0 = 1.5\gamma$.

Numerical calculations can be carried out for arbitrary μ_0 from Eq. (35); expressions (31), (32), (34), and (37); and Eq. (14) for T_c. Figure 2 shows the results of these calculations for certain values of μ_0 and C. The smearing of the fine structure in the state density by the defects causes a nonlinear concentration dependence of H_{c2} at $x \sim 1$. At certain values of μ_0 and C the concentration dependence of $H_{c2}(0)$ has a maximum, and $(dH_{c2}/dT)_{T_c}$ increases more slowly than in the case of a linear dependence (Figs. 2 and 3).

It is important to note that, as follows from the GLAG theory in the dirty limit, the derivative does not depend on T_c:

$$(dH_{c2}/dT)_{T_c} \sim \Phi_0/v_0^2\tau. \tag{39}$$

Consequently, the nonlinear concentration dependence of this quantity can result only from a change in the electron spectrum.

4. UPPER CRITICAL MAGNETIC FIELD IN NEUTRON-BOMBARDED Nb_3Sn; CORRESPONDENCE BETWEEN THE EXPERIMENTAL AND THEORETICAL BEHAVIOR OF H_{c2} AS A FUNCTION OF THE CONCENTRATION

The effect of bombardment by neutrons and charged particles on the upper critical field H_{c2} of A-15 superconductors was studied in Refs. 14, where the effect was measured near T_c. The experimental dose dependence $(dH_{c2}/dT)_{T=T_c}$ agrees qualitatively with the concentration dependence derived above (see Figs. 2 and 3) at high and intermediate integrated fluxes, at which the sample becomes dirty. At low fluxes the behavior of $(dH_{c2}/dT)_{T=T_c}$ depends on the initial state of the sample.

For a direct determination of the effect of neutrons on $H_{c2}(0)$, on (dH_{c2}/dT) near $T = T_c$ and at T_c, we bombarded ribbon samples of the superconductor Nb_3Sn. The samples had a dumbbell pattern. The thickness of the Nb_3Sn on the ribbon of niobium with an approximately 1.5% admixture of zirconium was 5 μm; the width of the neck in the dumbbell was ~ 0.5 mm; the length of the dumbbell was 10 mm; and the thickness of the ribbon was $\sim 50\,\mu$m. The neutrons were provided by a reactor. The samples were placed in an aluminum container, which was inserted into a water channel of the reactor. The temperature in the channel during the bombardment was 60 °C. The integrated flux of neutrons with $E_n > 1$ MeV through the sample was determined within 30% with the help of a threshold resistance detector. After bombardment, the containers were held at room temperature until their activity fell to an acceptable level for measurements. The measurements of $H_{c2}(4.2\text{ K})$ and (dH_{c2}/dT) near $T = T_c$ were carried out in a pulsed magnetic field. This field was produced in a Bitter solenoid[20]: a one-piece helix of beryllium bronze with an inside diameter 35 mm and an outside diameter 200 mm. The working part of the magnet was 29 mm in diameter and ~ 100 mm long. The magnetic field pulse was a decaying sine half-wave with a length at the base ~ 6 ms and a maximum amplitude of 300 kOe.

The measurements were carried out by a resistance method in a four-contact ac arrangement. The samples were oriented along the magnetic field. The upper critical magnetic field was determined at a fixed sample temperature from the midpoint of the transition from the superconducting state to the normal state, when the resistance reached $0.5R_0$ (R_0 is the resistance of the sample in the normal state near T_c). The amplitude of the magnetic field was chosen in the course of the measurements to arrange the transition of the sample to the normal state (the attainment of the resistance R_0) at the crest of the pulse. In the measurements of $H_{c2}(4.2\text{ K})$ the sample was immersed in liquid helium; in other cases the temperature was monitored within ± 0.1 K. The statistical error in the determination of $H_{c2}(4.2\text{ K})$ was 1%, and that in the determination of $(dH_{c2}/dT)_{T=T_c}$ was 5%.

The experimental results are summarized in Table I. We see from this table that $H_{c2}(4.2\text{ K})$ varies nonmonotonically with the dose, reaching a maximum at $\gtrsim 10^{18}$ neutron/cm^2. The derivative $(-dH_{c2}/dT)_{T=T_c}$ increases nonlinearly with the dose, in agreement with the theoretical predic-

TABLE I.

Φ, n/cm^2 ($E_n > 1$ MeV)	T_c, K	$\left(\frac{dH_{c2}}{dT}\right)_{T=T_c}$, $\frac{\text{kOe}}{\text{K}}$	$H_{c2}(4,2\text{ K})$, kOe	$\frac{H_{c2}(4,2\text{ K})}{T_c(-dH_{c2}/dT)_{T=T_c}}$
0	18,2	-24 ± 1	236±2	0,54
$1{,}3\cdot10^{17}$	18,1	-28 ± 1	238±2	0,47
$1{,}3\cdot10^{18}$	17,1	-34 ± 2	250±3	0,43
$1{,}2\cdot10^{19}$	12,6	-55 ± 3	211±2	0,3

Note. The indicated errors are statistical; the possible systematic shift of all values of H_{c2} is no more than 3%.

tions. Since the relation $H_{c2}(4.2\ \mathrm{K}) \approx H_{c2}(0)$ holds within 10%, the experimental ratio in the last column of this table shows that relation (3) does not hold over the entire dose interval.

We note in conclusion that the nonlinear concentration dependence of H_{c2} is particularly noticeable in the region

$$x \sim 1/\gamma\tau > 1, \tag{40}$$

where the ladder approximation in the impurity scattering,[7] used in the derivation of the original equations, (4), is not strictly valid (according to an estimate in Ref. 19, the value $x \approx 1$ corresponds to a neutron flux of 10^{19} n/cm^2). Physically, this fact reflects a tendency of the electrons to localize in the random field of the defects. Consequently, at high fluxes the description of the nonlinear behavior of H_{c2} by these equations is more qualitative than quantitative. Another quantitative discrepancy is the shape of the structural feature in the original state density, $N_0(\xi)$, which may be different from the Lorentzian shape in (33). The structure in $N_0(\xi)$ may be complicated by a martensitic conversion,[3] but a corresponding analysis shows that the martensitic transition is suppressed by defects much more rapidly than a significant change in T_c occurs, and the shape of the state density is approximately Lorentzian at a sufficient defect concentration. We also note that at very high fluxes, at which the uncertainty in the electron energy, $1/\tau$, is on the order of the scale frequency of the phonons, the effects associated with the blurring of the density peak may be complicated by a direct effect of defects on the polaron band contraction.[4]

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