Landau-level distribution of electrons moving with velocities exceeding that of light in the medium

B. E. Nemstov and V. Ya. Éĭdman

Scientific Research Radiophysics Institute
(Submitted 29 March 1984)
Zh. Eksp. Teor. Fiz. **87**, 1192–1199 (October 1984)

The evolution of a beam over the orbital energy levels is investigated on the basis of the probabilities found for the transfer of electrons along the Landau levels due to their emission in the regions of the anomalous and normal Doppler effects. Emission with spin flip is considered. It is shown that inverted population of the spin energy levels is possible because of anomalous Doppler emission.

It was noted some time ago by Ginzburg and Frank[1] that a system emitting at anomalous Doppler frequencies and moving in a medium does not go to a lower energy level, as is the case for normal Doppler frequencies, but to a higher one. Thus, there is hope that this system will require an inverted population of the energy levels. However, in this process, the principal role is played by the competition of the radiated anomalous and normal Doppler frequencies. For classical systems, this process was considered earlier by Ginzburg and one of us,[2] but, so far as we know, the problem has not been solved in such a formulation for quantum systems. In this connection, it should be noted that communications have appeared very recently to the effect that it has become possible to use the anomalous Doppler effect for the creation of a maser (see Refs. 3 and 4).

In connection with what has been said above, there is interest in considering a specific quantum system in which anomalous Doppler frequencies can be emitted, and with this as the example we can investigate the singular features that appear. In the present paper, we have concentrated on a simple system of widespread practical interest—that of electrons moving in a uniform constant magnetic field in a medium that is characterized by the dielectric constant ε. It turns out that the higher energy levels of the considered system (the Landau levels) become indeed populated in the steady state, thanks to emission of anomalous Doppler frequencies. The higher levels are more populated the smaller the ratio of the probability of emission of the normal Doppler frequencies to the probability of emission of the anomalous Doppler frequencies. Moreover, it has been possible to determine in the system considered the time evolution of the distribution of the electrons over the Landau levels. The spread in the transverse momentum is connected here not with collisions of electrons with one another but with the emission of photons. For steady-state distributions, we can introduce the concept of temperature and other thermodynamic quantities. A very significant feature has been revealed by analysis of emission of electron spins. It turns out that conditions are possible in which, thanks to emission with change in the spin direction of a majority, most electrons go over as a result of the anomalous Doppler emission, to a state with the spin oriented against the direction of the external field $\mathbf{H}_0$ (inverted population of the electron-spin levels. It was known earlier that, for classical systems, the analogous situation is impossible for transverse waves in an isotropic medium (see Ref. 2).

1. ORBITAL TRANSITIONS IN THE EMISSION OF MOVING ELECTRONS

Let charged particles with spin 1/2 (electrons), placed in a homogeneous and constant magnetic field $\mathbf{H}_0$, move in a medium with a dielectric constant $\varepsilon > 1$. The energy levels of the particle, neglecting its interaction with the phonons (the Landau levels), are well known:

$$E_k=\hbar\omega_H\left(k+\frac{1}{2}\right)+\frac{p_z^2}{2m}+s\frac{\hbar\omega_H}{2}. \tag{1}$$

Here $\omega_H = |e|H_0/mc$ is the gyrofrequency of the electrons, p_z is the momentum of the particle along the z axis, and the natural number k characterizes the oribtal energy of the particle in the magnetic field. The last term on the right side of Eq. (1) describes the energy of the particle magnetic moment that is coupled with the spin when placed in the magnetic field. If the magnetic moment is directed along the field, then $s = -1$; in the opposite orientation, $s = +1$. We assume that in its initial state the electron is at the level k, the magnetic moment is arbitrarily oriented, and the longitudinal moment $p_z = p_0$. Furthermore, we shall assume that photons are absent from the medium in the initial state. Thus, the initial state of the system is characterized bya ψ function of the form

$$\psi^{(0)}=\psi_k(\mathbf{r})\exp\left(-i\frac{E_kt}{\hbar}\right)\prod_{\lambda,\, i=1,2}\psi_{n_{\lambda_i}=0}(q_{\lambda_i})\exp\left(-\frac{i\omega_{\lambda_i}t}{2}\right). \tag{2}$$

Here $\psi_k(\mathbf{r})$ is the eigenfunction of a particle with energy E_k,[5] $\psi_{n_{\lambda_i}=0}(q_{\lambda_i})$ are the eigenfunctions of the photons of the medium in the initial state,[6] q_{λ_i} are the generalized coordinates, $\omega_{\lambda_i} = c\varkappa_{\lambda_i}/\varepsilon^{1/2}$, $\varkappa_{\lambda_i}$ are the wave numbers of the photons. The Hamiltonian of the interaction of the particle with the field has the form

$$\hat{H}_{int}=-\frac{e}{mc}\left(\left(\hat{\mathbf{p}}-\frac{e}{c}\mathbf{A}_0\right)\hat{\mathbf{A}}\right)-\frac{e}{mc}(\boldsymbol{\sigma}\hat{\mathbf{H}}),$$
$$\hat{\mathbf{A}}=\sum_{\lambda,i}\left[\frac{2\pi\hbar c^2}{\varepsilon\omega_{\lambda_i}V}\right]^{1/2}\mathbf{e}_{\lambda_i}[\hat{a}_{\lambda_i}^+\exp(-i\varkappa_{\lambda_i}\mathbf{r})+\hat{a}_{\lambda_i}\exp(i\varkappa_{\lambda_i}\mathbf{r})], \tag{3}$$

where $\hat{\mathbf{A}}$ is the vector potential operator, $\mathbf{A}_0$ is the vector

 0038-5646/84/100682-05$04.00

potential of the uniform field, $\mathbf{e}_{\lambda_i}$ is the unit polarization vector of the photons, $\boldsymbol{\sigma}$ are Pauli matrices, $\hat{\mathbf{H}} = \operatorname{curl} \mathbf{A}$ is the magnetic field operator, V is the volume of the system, $\hat{a}^+_{\lambda_i}, \hat{a}_{\lambda_i}$ are the creation and annihilation operators of the photons.

In first-order perturbation theory, the interaction operator (3) acts on a spinor of the form $\binom{\psi^{(0)}}{0}$; $\psi^{(0)}$ is given by Eq. (2). It is not difficult to show that the interaction-Hamiltonian part that changes the orbital energy of the particles in the case of unchanged orientation of the spin is

$$\hat{H}_1 = -\frac{e}{mc}\left(\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}_0\right)\hat{\mathbf{A}}\right) - \frac{e}{mc}\hat{\sigma}_z \hat{H}_z. \tag{4}$$

The interaction Hamiltonian that changes the orientation of the spin is

$$\hat{H}_2 = -\frac{e}{mc}(\hat{\sigma}_x \hat{H}_x + \hat{\sigma}_y \hat{H}_y). \tag{5}$$

We first calculate the probability of the process in which the electron, emitting a photon, transforms from level k to level $k+1$.

The probability of emission of a photon of energy $\hbar\omega$ at angle ϑ to the field $\mathbf{H}_0$ is defined as $|C_{k,k+1}|^2$. At a photon polarization $\mathbf{e}_{\lambda_i}$ in the yz plane, $\vartheta = \vartheta(0, \sin\vartheta, \cos\vartheta)$, and $\mathbf{e}_{\lambda_i} = 0, \cos\vartheta, \sin\vartheta)$, we have from (4):

$$i\hbar \frac{dC^{(1)}_{k,k+1}}{dt} = -i\frac{e\hbar}{m}\left[\frac{2\pi\hbar}{\varepsilon\omega V}\right]^{1/2} \exp\left\{i\left(\frac{E_{k+1}-E_k}{\hbar}+\omega\right)t\right\}$$

$$\times \int_{-\infty}^{\infty} \chi_{k+1}(y)\exp(-i\varkappa_y y)\left(-i\frac{p_0}{\hbar}\sin\vartheta + \cos\vartheta\frac{\partial}{\partial y}\right)\chi_k(y)\,dy,$$

where $E_{\lambda+1}$ is the energy in the state $k+1$, χ_{k+1} is the wave function of the electron in the magnetic field corresponding to the Landau levels.[5] For a photon polarization $\mathbf{e}_{\lambda_i} = (1, 0, 0)$, we have

$$i\hbar\frac{dC^{(2)}_{k,k+1}}{dt}$$

$$= \left(\frac{2\pi\hbar}{\varepsilon\omega V}\right)^{1/2} e\omega_H \exp\left\{i\left(\frac{E_{k+1}-E_k}{\hbar}+\omega\right)t\right\}$$

$$\times\int_{-\infty}^{\infty} \chi_{k+1}(y)\, y \exp(-i\varkappa_y y)\chi_k(y)\,dy.$$

Using the last two formulas and summing over all possible states of the photons we find the probability of transition of an electron from the level k to the level $k+1$ in the dipole approximation:

$$\frac{dW_{k,k+1}}{dt} = \frac{e^2(k+1)}{2m\omega_H c^2 v}\int\left[\frac{(\beta^2 n^2\omega - \omega - \omega_H)^2}{\beta^2 n^2} + \omega_H^2\right]d\omega$$

$$\equiv \nu(k+1), \tag{6}$$

$$(\omega+\omega_H)/\beta n\omega < 1, \quad \beta = v/c, \quad n = \sqrt{\varepsilon}.$$

The finite transition probability of the system to the level $k+1$ is connected with the possibility of the emission of waves at the anomalous Doppler frequencies.[1,2]

Similar calculations for the transition of the system to the level $k-1$ lead to the following expression for the transition probability:

$$\frac{dW_{k,k-1}}{dt} = \frac{e^2 k}{2m\omega_H c^2 v}\int\left[\frac{(\beta^2 n^2\omega - \omega + \omega_H)^2}{\beta^2 n^2} + \omega_H^2\right]d\omega \equiv \mu k, \tag{7}$$

$$|\omega - \omega_H|/\beta n\omega < 1.$$

The first term in the integrands of (6) and (7) corresponds to emission of a photon with polarization in the plane $(\mathbf{H}_0, \mathbf{k})$ while the second corresponds to polarization of the photon perpendicular to the magnetic field. We note that (7) transforms, as $v \to 0$ and at $\varepsilon = 1$, into the well-known expression for the probability of emission of an electron in a magnetic field:

$$\frac{dW_{k,k-1}}{dt} = \frac{4e^2 k\omega_H^2}{3mc^3}. \tag{8}$$

From Eq. (8), in the quasiclassical approximation ($\hbar\omega_H k = mv_\perp^2/2$) we obtain an expression for the intensity of the magnetic bremsstrahlung in a vacuum:

$$\frac{dI}{dt} = \frac{2e^2\omega_H^2 v_\perp^2}{3c^3}.$$

In order to obtain classical quantities, for example the radiation energy per unit time and the work performed by the radiation field to change the transverse momentum of the electron, it is necessary to multiply the integrands of Eqs. (6) and (7) by $\hbar\omega$ and $\hbar\omega_H$, respectively. The expressions obtained here for the case $k \gg 1$ transform into the corresponding equations for the total and the vibrational part of the work of the emission field for an electron moving along a helix.[2]

2. EVOLUTION OF THE BEAM

Indications of the possibility of complex dynamics of a multilevel system when account is taken of the anomalous Doppler effect were given in Ref. 8. In the present work we shall show, using (6) and (7), that we can investigate completely the dynamics and the stationary distribution of a beam of electrons over the Landau levels with allowance for the anomalous and normal Doppler effects.

The equation determining the change in the population of particles over the orbital levels has the form

$$\frac{dn_k}{dt} = \mu(k+1)n_{k+1} + \nu k n_{k-1} - \nu(k+1)n_k - \mu k n_k, \tag{9}$$

where n_k is the number of particles in level k. The first term is connected with the transition $k+1 \to k$ due to emission via the normal Doppler effect, the second term corresponds to the possibility of transition from level $k-1$ to k due to the anomalous Doppler effect. The last two terms on the right side of Eq. (9) determine the decrease in the population of level k due to the anomalous and normal Doppler effects, respectively. In the derivation of Eq. (9), it was taken into account that in the dipole approximation, when the wavelength of the emission is significantly greater than the Larmor radius of the electrons, only the transition probabilities to adjacent levels are different from zero. The system (9) is an infinite set of coupled equations. The stationary solution of (9) is easily found:

$$n_k = \alpha(\nu/\mu)^k. \tag{10}$$

This distribution is normalized if $\nu < \mu$, i.e., transitions due to the normal Doppler effect are predominant relative to transitions upward. In this case, the normalization constant $\alpha = N_0(1 - \nu/\mu)$, where N is the total number of particles of the beam.

The stationary distribution corresponds to the "transverse temperature" of the beam $T = \hbar\omega_H/\ln(\mu/\nu)$.

It turns out to be possible to investigate the evolution of the beam in time over the orbital energy levels. For this purpose, we seek solution of Eqs. (9) in the form

$$n_k = \oint_C W(\rho, \tau)\rho^k \, d\rho. \tag{11}$$

Here $\tau = \mu t$ and C is the closed integration contour. (From the condition $\Sigma n_k < \infty$ it follows that $|\rho| < 1$.) Assuming $W(\rho,\tau)$ to be a single-valued function of ρ, we obtain the following equation from (9):

$$\frac{\partial W(\rho,\tau)}{\partial \tau} = (1-\rho)\frac{\partial}{\partial \rho}(\rho - x)W(\rho,\tau), \quad x \equiv \nu/\mu. \tag{12}$$

Equation (12) should satisfy the initial conditions. At $t = 0$, let there be n_0 particles on the zeroth level ($k = 0$), n, on the first, n_2 on the second, and so on. To these initial conditions, there corresponds

$$W(\rho, 0) = \frac{1}{2\pi i}\left(\frac{n_0}{\rho} + \frac{n_1}{\rho^2} + \frac{n_2}{\rho^3} + \ldots\right).$$

Equation (12) can be solved by the method of characteristics. As a result, we obtain

$$W(\rho,\tau) = \frac{1}{\rho - x}\eta\left(\frac{\rho - x}{\rho - 1}\exp(-\tau(x-1))\right),$$

where η is an aribtrary function that should be determined from the initial conditions. We consider some specific examples.

1. Let all the particles be on the level $k = 0$ at $t = 0$. The solution of Eq. (12) that satisfies this initial condition has the form

$$W(\rho,\tau) = \frac{n_0(x-1)\exp(-\tau(x-1))}{2\pi i(\rho-\rho_0)[x - \exp(-\tau(x-1))]}, \quad \rho_0 = \frac{x[1-\exp(-\tau(x-1))]}{x - \exp(-\tau(x-1))}. \tag{13}$$

It is seen from (13) that $W(\rho,\tau)$ is a single-valued analytic function. The pole $\rho = \rho_0$ lies in the interval from 0 to 1 at arbitrary τ and x. The dependence of n_k on τ is easily determined [see (13) and (11)]:

$$n_k = \frac{n_0(x-1)\exp(-\tau(x-1))}{x - \exp(-\tau(x-1))}\left\{\frac{x[1-\exp(-\tau(x-1))]}{x-\exp(-\tau(x-1))}\right\}^k.$$

At $\nu/\mu < 1$, $\tau \to \infty$, the distribution n_k reaches the stationary value (10). At $x > 1$, the stationary distribution is not realized and the population of the k th level falls off according to the law

$$n_k = n_0\frac{x-1}{x}\exp(-\tau(x-1)).$$

2. We now consider the case in which at $t = 0$ the particles are distributed over the Landau levels according to an exponential law, i.e.,

$$W(\rho, 0) = \frac{1}{2\pi i}\frac{N_0(1-\gamma)}{\rho - \gamma} \equiv \frac{N_0(1-\gamma)}{2\pi i\rho}\sum_{k=0}^{\infty}\left(\frac{\gamma}{\rho}\right)^k, \quad \gamma < 1.$$

The change in the population with time in this case has the form

$$n_k = \frac{N_0(x-1)\exp(-\tau(x-1))}{\xi - \exp(-\tau(x-1))}\left[\frac{\xi - x\exp(-\tau(x-1))}{\xi - \exp(-\tau(x-1))}\right]^k,$$

$$\xi = \frac{x-\gamma}{1-\gamma}.$$

3. CHANGE OF SPIN ORIENTATION DUE TO EMISSION

The dynamics of a beam of particles over the orbital energy levels was considered above. At the same time, the possibility of the emission of waves in the region of the anomalous Doppler effect leads to the result that the magnetic moments initially aligned along the field can reverse their orientation as a consequence of the emission. By the same token, the anomalous Doppler emission is a mechanism that increases the internal energy of the magnetic moments. To find the emission probability with change of orientation of the spin, one should take into consideration the Hamiltonian of the interaction (5). The probability amplitude of a process in which an electron emitting a photon of frequency ω_{λ_i} and polarization $\mathbf{e}_{\lambda_i}$ changes spin orientation and momentum is given by

$$i\hbar\frac{dC_{\lambda_i}}{dt} = -\frac{e\hbar}{2mc}\int \varphi^{*(1)}(\hat{H}_x + i\hat{H}_y)\psi^{(0)}\, d\mathbf{q}\, d\mathbf{r}, \tag{14}$$

$\psi^{(0)}$ is given by Eq. (2), $\varphi^{(i)}$ is an eigenfunctions of an electron single photon system in the state λ_i. Integration is carried out over all the generalized coordinates of the photons and over the coordinates of the particle.

For definiteness, let the polarization vector of the photon lie in the smae plane as $\varkappa_{\lambda_i}$ and $\mathbf{H}_0$. Taking it into account also that the annihilation operator acting on $\psi^{(0)}$ yields zero, we arrive at the relation

$$\frac{dC_{\lambda_i}}{dt} = \frac{e}{2m}\left(\frac{2\pi\hbar}{\varepsilon\omega_{\lambda_i}V}\right)^{1/2}\varkappa_{\lambda_i} \times \exp\left\{i\left(\omega_{\lambda_i} + \omega_H - \varkappa_z v + \frac{\hbar\varkappa_z^2}{2m}\right)t - \frac{\hbar\varkappa_\perp^2}{4m\omega_H}\right\}, \quad \varkappa_\perp^2 = \varkappa_x^2 + \varkappa_y^2. \tag{15}$$

In obtaining (15), we made use of the law of momentum conservation along the z axis, $p_0 = p_{1z} + h\varkappa_z$, which follows automatically from the calculation of the integral over z in Eq. (14). The probability amplitude of the considered process $|C_{\lambda_i}|^2$ is easily determined from (15):

$$\frac{d|C_{\lambda_i}|^2}{dt} = \frac{e^2\pi^2\hbar}{m^2\varepsilon\omega_{\lambda_i}V} \times \exp\left(-\frac{\hbar\varkappa_\perp^2}{2m\omega_H}\right)\delta\left(\omega_{\lambda_i} + \omega_H - \varkappa_z v + \frac{\hbar\varkappa_z^2}{2m}\right). \tag{16}$$

In obtaining Eq. (16), we used the formula

$$\lim_{t\to\infty}\frac{\sin \xi t}{\xi}=\pi\delta(\xi).$$

The fact that the argument of the δ function is equal to zero expresses the law of energy conservation in the particle with spin + photon system. Actually, if the magnetic moment of the electron is directed along the field in the initial state, then

$$E_0=\hbar\omega_H k+p_0^2/2m.$$

The energy of the system in the final state, i.e., after the spin flip, is

$$E_1=\hbar\omega_H(k+1)+p_{1z}^2/2m,$$

where p_{1z} is the momentum of the paricle along the z axis after emission. Using the law of momentum conservation along the z axis, we obtain from the equality $E_0=E_1+\hbar\omega$ the following expression:

$$\omega_{\lambda_i}+\omega_H-\varkappa_z v+\hbar\varkappa_z^2/2m=0. \tag{17}$$

As $\hbar\to 0$, the usual classical condition for emission at the anomalous Doppler frequencies follows from (17). The last term of the left side of Eq. (17) describes the recoil that the electron experiences during emission. To investigate the possible effects connected with the recoil, we shall first consider the medium without dispersion. Such an approximation will be valid if the dielectric constant changes little over the entire range of waves emitted by the spin. The angles between the initial velocity **v** and the wave vector $\varkappa$ are easily determined:

$$\cos\vartheta_{1,2}=\frac{m}{\hbar\varkappa}\left\{v\mp\left[v^2-2\frac{\hbar\varkappa}{m}\left(\frac{c}{\varepsilon^{1/2}}+\frac{\omega_H}{\varkappa}\right)\right]^{1/2}\right\}. \tag{18}$$

In contrast to the classical case, account of the loss can lead to the result that the emission with spin flip at a given freqency can propagate at two different angles.

A simple analysis of Eq. (18) shows that the emission sets in at

$$mv\left(\frac{v}{2}-\frac{c}{\varepsilon^{1/2}}\right)>\hbar\omega_H-\frac{mc^2}{2\varepsilon},\quad v>\frac{c}{\varepsilon^{1/2}},$$

and if $mv(v/2-c\varepsilon^{-1/2})<h\omega_H$, then a contribution to the total energy of emission is made only by the root ϑ_1. The total energy emitted by the electron per unit time is then

$$\frac{dW_1}{dt}=\frac{\mu_0^2}{c^2}\int_{\omega_1}^{\omega_2}\varepsilon\omega^3\exp\left(-\frac{\hbar\omega^2\varepsilon\sin^2\vartheta_1}{2m\omega_H c^2}\right)\times\left[v^2-\frac{2\hbar}{m}(\omega+\omega_H)\right]^{-1/2}d\omega$$

$$\equiv\int_{\omega_1}^{\omega_2}f(\omega,\vartheta_1)\,d\omega; \tag{19}$$

$$\omega_{1,2}=\frac{cm}{\sqrt{\varepsilon}\hbar}\left\{v-\frac{c}{\sqrt{\varepsilon}}\mp\left[\left(v-\frac{c}{\sqrt{\varepsilon}}\right)^2-\frac{2\hbar\omega_H}{m}\right]^{1/2}\right\},$$

$$\mu_0=\frac{|e|\hbar}{2mc}.$$

The angle ϑ_1 is determined from (18) where we must set $\varkappa=\omega\sqrt{\varepsilon}/c$. With increase in the velocity, or more precisely at $mv(v/2-c/\sqrt{\varepsilon})>h\omega_H$, a contribution to the field of emission from the root ϑ_z appears, and the intensity of emission is determined by the sum of integrals of type (19), but with other limits of integration:

$$\frac{dW_2}{dt}=\int_{\omega_1}^{\omega_0}f(\omega,\vartheta_1)\,d\omega+\int_{\omega_2}^{\omega_0}f(\omega,\vartheta_2)\,d\omega. \tag{20}$$

Here ω_0 is the limiting frequency of emission, which follows from the relation

$$\hbar\omega_0+\hbar\omega_H=mv^2/2.$$

The physical meaning of the limiting frequency ω_0 is that the particle comes to rest upon emission of a quantum of this frequency. The fact that the power spectrum has an integrable singularity near ω_0 turns out to be nontrivial. As has already been noted, when account is taken of the recoil in the emission process, a complex Doppler effect in the medium without dispersion also becomes possible. It follows from condition (18) that, in general, two frequencies are radiated at a specified angle ϑ:

$$\omega^{(1,2)}=\frac{cm}{\sqrt{\varepsilon}\hbar\cos^2\vartheta}\left\{\left(v\cos\vartheta-\frac{c}{\sqrt{\varepsilon}}\right)\pm\left[\left(v\cos\vartheta-\frac{c}{\sqrt{\varepsilon}}\right)^2-\frac{2\hbar\omega_H\cos^2\vartheta}{m}\right]^{1/2}\right\}.$$

We now consider the other limiting case, in which the dispersion properties of the medium become effective earlier than the recoil effect. In this case we can neglect the term $\hbar\varkappa_z^2/2m$, in (16) and (17), and the expression for the intensity of the radiation takes the form

$$\frac{dW_3}{dt}=\frac{\mu_0^2}{c^2 v}\int\varepsilon\omega^3\,d\omega,$$
$$(\omega+\omega_H)/\beta n\omega<1,\quad \mathbf{e}_{\lambda_i}=\{0,-\sin\vartheta,\cos\vartheta\}. \tag{21}$$

Below, we shall need an expression similar to (21) for the intensity of radiation with polarization perpendicular to $\mathbf{H}_0$:

$$\frac{dW_4}{dt}=\frac{\mu_0^2}{v^3}\int\omega(\omega+\omega_H)^2\,d\omega \tag{22}$$

[the region of integration is the same as in (21)].

The inverse process of radiation with rotation of the magnetic field along the field is considered in analogous fashion. The formulas obtained here differ from (19)–(22) only by the replacement of ω_H by $-\omega_H$.

For investigation of the dynamics of spins moving in a medium, it is necessary to know the probability of radiation with rotation of the spin. The corresponding formulas are easily obtained by noting that they differ from the expressions (19)–(22) by a factor $1/\hbar\omega$ under the integral sign. As a result, we obtain the following expresions for the total probability per unit time of radiation with rotation of the spin against and along the field:

$$a=\frac{\mu_0^2}{\hbar v^3}\int[(\omega+\omega_H)^2+\beta^2n^2\omega^2]\,d\omega,\quad \frac{\omega+\omega_H}{\beta n\omega}<1, \tag{23}$$

$$b=\frac{\mu_0^2}{\hbar v^3}\int[(\omega-\omega_H)^2+\beta^2n^2\omega^2]\,d\omega,\quad \frac{|\omega-\omega_H|}{\beta n\omega}<1. \tag{24}$$

Let n_1 and n_2 be the number of electrons with magnetic moment along and against the field, respectively. The equa-

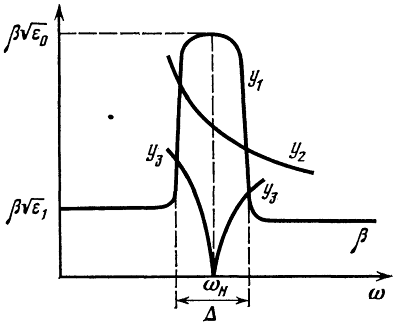

FIG. 1. Refractive-index frequency dependence that leads to inversion of spins $y_1 = \beta\sqrt{\varepsilon(\omega)}$, $y_2 = 1 + \omega_H/\omega$, $y_3 = |1 - \omega_H/\omega|$.

tions describing the dynamics are

$$\frac{dn_1}{dt} = -an_1 + bn_2, \quad n_1 + n_2 = N_0.$$

Their solutions are

$$n_1 = n_{10}\exp(-(a+b)t) + \frac{N_0 b}{a+b}[1 - \exp(-(a+b)t)].$$

The steady ratio of populations is $n_1/n_2 \equiv b/a$. If the ratio $b/a < 1$, then inversion of the spin-system population is possible.

We shall describe briefly the simple case in which an inverted population of spin energy levels occurs in a magnetic field. We shall assume that $n(\omega)$ is described by the cure shown in the figure, i.e., the quantity $n(\omega)$ is significantly larger than unity only in a narrow interval near the gyrofrequency ($\omega_H \gg \Delta$, $\beta\sqrt{\varepsilon_0} > 1$). Then, at

$$\beta\sqrt{\varepsilon_0} > 2 + \frac{\Delta}{2\omega_H} \approx 2, \quad \omega_H < \frac{\Delta}{2\beta\sqrt{\varepsilon_1}}$$

the limits of integration in (23) and (24) are identical and S

$$a = \frac{\mu_0^2\omega_H^2\Delta}{\hbar v^3}(4 + \beta^2\varepsilon_0),$$

$$b = \frac{\mu_0^2\omega_H^2\Delta}{\hbar v^3}\beta^2\varepsilon_0, \quad \frac{b}{a} = \left(1 + \frac{4}{\beta^2\varepsilon_0}\right)^{-1},$$

i.e., the probability of emission of anomalous Doppler frequencies in this case is actually greater than the probability of emission of normal Doppler frequencies, and inversion of the electron spin levels takes place. It is interesting to note that, as follows from Ref. 2, in a classical system, no buildup of oscillations due to the anomalous Doppler effect is possible in the emission of transverse waves in an isotropic medium. At the same time, in the considered quantum system of spin interaction with radiation, a situation in which the action of the anomalous Doppler emission in the isotropic medium predominates over the normal Doppler emission is quite possible.

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Translated by R. T. Beyer