# Critical dynamics of uniaxial displacive ferroelectrics

V. V. Lebedev

*L. D. Landau Institute of Theoretical Physics of the Academy of Sciences of the USSR*
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Nonlinear fluctuational effects in the long-wavelength dynamics of uniaxial displacive ferroelectrics at temperatures near the phase transition are considered in the framework of an isotropic model with an oscillatory soft-mode spectrum. The important nonlinear terms in the macroscopic equations for the dynamics are singled out; these terms are due both to the self-action of the soft mode and to the interaction of the soft mode with acoustic modes. An effective action is constructed, permitting evaluation of the fluctuational corrections as a perturbation series in the interaction. It is shown that this effective action is renormalizable, and expressions are obtained for the renormalization group equations and the logarithmic laws describing the behavior of the interaction constants and speed of sound. It is shown that in the high-frequency limit the fluctuations dictate a soft-mode damping $\propto \omega$ and an acoustic damping $\propto k^2/k_z^{1/2}$. The logarithmic behavior of the coefficients in these laws are found, and the low-frequency acoustic and soft-mode damping is discussed.

## INTRODUCTION

Ferroelectrics comprise one of the main classes of materials which undergo structural phase transitions.[1,2] Recent years have seen rapid progress in experimental techniques for detecting and studying the soft modes which are associated with phase transitions in ferroelectrics. Accordingly, there is heightened interest in the theoretical description of these transitions in ferroelectrics particularly in the description of dynamical phenomena in this region.

We shall consider the dynamics of uniaxial ferroelectrics of the displacive type near the temperature of the transition from the paraelectric to the ferroelectric phase. In uniaxial ferroelectrics an important role is played by the nonlocal dipole interaction, owing to which the fluctuational corrections to the specific heat and dielectric susceptibility near the transition temperature have a logarithmic[3] rather than a power-law character as in ordinary second-order phase transitions. Recent experiments[4,5] have confirmed the $\ln^{1/3}$ law[3] for the dielectric susceptibility in uniaxial ferroelectrics of the displacive type.

In displacive ferroelectrics an important role is played by the interaction of the order parameter with acoustic modes. This interaction causes the phase transition in these materials to change from second order to first order.[6] However, in the ferroelectrics studied in these experiments[4,5] it was found that this mechanism is operative only near the very transition point, so that there is a wide fluctuation region in which the theory of Larkin and Khmel'nitskiĭ[3] works. This region is specified by the $\delta T \ll |T-T_c| \ll T_c$, where $T_c$ is the transition temperature and $\delta T$ is the thermal hysteresis associated with the first-order transition.

Acoustic vibrations in the crystal are described by the strain tensor $u_{ik}$, while the order parameter in a uniaxial displacive ferroelectric is the displacement $\xi$ of the sublattices relative to one another along the $z$ axis. We shall use an isotropic model in which the relevant part of the free energy is of the form

$$F=\frac{\lambda}{2}u_{ii}^2+\mu u_{ik}^2+\frac{\beta}{2}u_{ii}\xi^2+\frac{a_0}{2}\xi^2+\frac{b}{2}(\nabla\xi)^2+\frac{\varkappa}{2}\xi\frac{\nabla_z^2}{\nabla^2}\xi+\frac{g}{24}\xi^4. \tag{1}$$

Here the first two terms are the standard elastic energy,[7] the third term describes the interaction of the elastic strain with the order parameter $\xi$, the next two terms are the standard quadratic terms in the expansion of the free energy in powers of $\xi$, the next-to-last term describes the nonlocal dipole interaction, and the last term describes the self-action of the order parameter. All the coefficients in (1) are functions of the temperature $T$.

Near the transition temperature the parameter $a_0 \propto T-T_c$ becomes small, making for strong fluctuations of the order parameter $\xi$. It was shown in Ref. 3 that allowance for the fluctuations of $\xi$ in this situation yields logarithmic corrections to the interaction constant $g$. The situation turns out to be one of zero charge, so that at distances $r$ which are large compared to the cutoff dimension $\Lambda^{-1}$ the interaction constant $g$ goes as $L^{-1}$, where

$$L=\ln\frac{\Lambda}{\max\{a^{1/2}b^{-1/2},\, r^{-1},\, b^{-1/4}\varkappa^{1/4}z^{-1/2}\}}. \tag{2}$$

Here the constant $a$ differs from $|a_0|$ by a quantity which is determined in a self-consistent manner.[6] We note that in the temperature range $\delta T \ll |T-T_c| \ll T_c$ the difference between $a$ and $|a_0|$ is small. In order for fluctuations to be important the logarithm in (2) must be large, implying that $\Lambda b^2 \gg a$, and we shall henceforth assume that this inequality holds. We shall also adduce the estimate $\varkappa \sim b\Lambda^2$.

Fluctuations of $\xi$ also cause a renormalization of the vertex $\beta$, which in the region $\delta T \ll |T-T_c| \ll T_c$ goes as $L^{1/3}$. In addition, the presence of the vertex $\beta$ causes a renormalization of the Lamé coefficient $\lambda$, which decreases with in-

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creasing $L$. As we know,[7] for $\lambda < -{}^{2/3}\mu$ the system becomes absolutely unstable, which is indicative of a first-order phase transition. The existence of the wide logarithmic region observed in the experiments[4,5] requires that $\beta^2/\mu \ll g$. We shall henceforth assume that this inequality holds (in the short-wavelength region).

The experimental situation in regard to the soft mode in uniaxial displacive ferroelectrics near $T_c$ is not clear. Old studies[8–10] show the presence of an oscillator mode in the ferroelectric phase, with a frequency $\omega \propto (T_c - T)^{1/2}$. However, these data do not refer to to the region near $T_c$ itself. The experiment of Ref. 11 indicates the presence of a soft mode in $Pb_5Ge_3O_{11}$ near $T_c$, but judging from the frequency behavior there is apparently no wide logarithmic region. We shall proceed from a classical oscillator spectrum for the soft mode, and we shall conclude with a few words about the case of a purely damped mode.

## EFFECTIVE ACTION

For considering nonlinear elastic proceses in solids it is more convenient to use (instead of the displacement vector $\mathbf{u}$) a set of variables $X_i (i = 1,2,3)$ such that the equation $X_1(\mathbf{r}) = \text{const}$ defines the spatial position of some atom plane in the crystal (similarly for indices 2 and 3). If all three $X_i$ are fixed, we obtain equations describing the trajectory of some point in the crystal, i.e., the displacement vector $\mathbf{u}$ is determined implicitly by the equation $X_i(\mathbf{r} + \mathbf{u}) = \text{const}$. Under the natural condition that $X_i = r_i$ in the unstrained state, we obtain $\delta X_i = -u_i$ in the linear approximation.

The nondissipative part of the hydrodynamic equations for this system can be obtained most simply using a Poisson bracket formalism (see, e.g., the review by Dzyaloshinskiĭ and Volovick[12]). We obtain as a result the following nonlinear dynamical equations:

$$\begin{gathered}\partial\sigma/\partial t+\mathbf{v}\nabla\sigma=\text{c.t.},\quad \partial X_i/\partial t+\mathbf{v}\nabla X_i=0,\\ \partial\xi/\partial t+\mathbf{v}\nabla\xi=\partial E/\partial M+\text{c.t.},\qquad \partial j_i/\partial t+\nabla_k\Pi_{ik}=\text{c.t.},\\ \partial M/\partial t+\nabla(M\mathbf{v})=-\delta E/\delta\xi+\text{c.t.}\end{gathered}\tag{3}$$

Here $E$ is the energy density, $\xi$ is the specific entropy, $j_i$ is the momentum density, $\mathbf{v} = \partial E/\partial \mathbf{j}$ is the velocity, $M$ is the canonical conjugate of $\xi$, and c.t. stands for the kinetic terms. The reactive stress tensor is

$$\Pi_{ik}=\left(\mathbf{v}\mathbf{j}+M\frac{\partial E}{\partial M}-E\right)\delta_{ik}+j_iv_k+\frac{\partial E}{\partial\nabla_kX_l}\nabla_iX_l+\frac{\partial E}{\partial\nabla_k\xi}\nabla_i\xi.\tag{4}$$

By virtue of Galilean invariance we have $\mathbf{j} = \rho\mathbf{v}$, where $\rho$ is the mass density.

Among the variables in system (3), only $\xi$ is strongly fluctuating. As was shown in Ref. 13, in this situation one can incorporate nonlinear fluctuational effects by keeping only the nonlinearity in $\xi$ and linearizing the equation with respect to the remaining variables. Accordingly, we need only that part of the expansion of the energy density $E$ in these variables which is quadratic in $\xi$. We shall assume that the order parameter $\xi$ is normalized in such a way that the quadratic terms of the expansion of $E$ in powers of $M$ is equal to ${}^{1/2}M^2$. We shall use expansion (1) for the expansion in the strain tensor (and also in $\xi$). In the linear approximation one can disregard the variables $\mathbf{j}$ and $M$ and go over to second-order equations for $u_i = -\delta X_i,\xi$. In the resulting system of equations the equations for the specific entropy $\sigma$ and transverse (to the wave vector) part of $\mathbf{u}_i$ separate out and can be dropped. We thus arrive at second-order equations for the longitudinal (with respect to the wave vector) part $u_\parallel$ of the displacement vector and for the order parameter $\xi$.

We now use the technique developed in Refs. 14 and 15 to construct a distribution function of the form $e^{iI}$ for evaluating the unequal-time correlators of the fluctuating quantities. The effective action $I$ is written in the form of an integral of a local expression:

$$I=\int dt\,d^3r\,\mathscr{L}(p,p_\parallel,\xi,u_\parallel).\tag{5}$$

Here $p$ and $p_\parallel$ are auxiliary Bose fields. In the case under consideration here we can drop the dependence of the Lagrangian density $\mathscr{L}$ on the auxiliary Fermi fields $\psi$ and $\bar\psi$ introduced in Refs. 14 and 15, since the determinant which arises in the integration over these variables is equal to unity by virtue of the analytical properties of the Green functions $\langle\psi,\bar\psi\rangle$. After dropping the variables which are unimportant here, we obtain from system of equations (3) a Lagrangian density which decomposes into the following terms:

$$\mathscr{L}_r=p\left(\frac{\partial^2\xi}{\partial t^2}+a\xi-b\nabla^2\xi+\varkappa\frac{\nabla_z^2}{\nabla^2}\xi\right)+p_\parallel\left(\frac{\partial^2u_\parallel}{\partial t^2}-c^2\nabla^2u_\parallel\right),\tag{6}$$

$$\mathscr{L}_d=p\Sigma\xi+\frac{i}{2}p\Pi p+\frac{\rho}{2T}p_\parallel\Pi_\parallel\frac{\partial}{\partial t}u_\parallel+\frac{i}{2}p_\parallel\Pi_\parallel p_\parallel,\tag{7}$$

$$\mathscr{L}_{int}=-\frac{\beta}{2\rho}\nabla_ip_i\xi^2+\beta p\nabla_iu_i\xi+\frac{g}{6}p\xi^3.\tag{8}$$

Here $c = (2\mu + \lambda)^{1/2}\rho^{-1/2}$ is the longitudinal sound velocity. We note that the coefficients appearing in (6) and (8), unlike those in (1), are the adiabatic coefficients.

Expression (6) derives from the reactive part of the linear dynamical equations, expression (7) derives from the kinetic part of the same equations (with allowance for random forces), and expression (8) describes the nonlinear interaction of the long-wavelength modes. For the operators appearing in (7) we have

$$\Pi=-2\zeta T\partial^2/\partial t^2,\qquad \Sigma=-\zeta\partial^3/\partial t^3.\tag{9}$$

Here $\zeta$ is the kinetic coefficient. In the Fourier representation we thus have

$$\operatorname{Im}\Sigma(\omega)=-\frac{\omega}{2T}\Pi(\omega).\tag{10}$$

An analogous relation for the soft mode is explicitly incorporated in (7). The structure of $\Pi_\parallel$ here is identical to that of $\Pi$ in (9). The quadratic part of the Lagrangian density in (6) and (7) leads to the following expressions for the binary averages of the Fourier components ($\omega$ is the frequency and $\mathbf{k}$ the wave vector):

$$\begin{aligned}G(\omega,\mathbf{k})&=\langle\xi(\omega,\mathbf{k})\,p(-\omega,-\mathbf{k})\rangle\\ &=-i\left(\omega^2-a-bk^2-\varkappa\frac{k_z^2}{k^2}-\Sigma\right)^{-1},\end{aligned}\tag{11}$$

$$D(\omega, \mathbf{k})=\langle \xi(\omega, \mathbf{k}) \xi(-\omega, -\mathbf{k})\rangle$$
$$=-\Pi(\omega, \mathbf{k}) G(\omega, \mathbf{k}) G(-\omega, -\mathbf{k}), \tag{12}$$

$$G_{\parallel}(\omega, \mathbf{k})=\langle u_{\parallel}(\omega, \mathbf{k}) p_{\parallel}(-\omega, -\mathbf{k})\rangle$$
$$=\left(i\omega^2-ic^2k^2-\frac{\omega}{2T}\Pi_{\parallel}\right)^{-1}, \tag{13}$$

$$D_{\parallel}(\omega, \mathbf{k})=\langle u_{\parallel}(\omega, \mathbf{k}) u_{\parallel}(-\omega, -\mathbf{k})\rangle$$
$$=-\Pi_{\parallel}(\omega, \mathbf{k}) G_{\parallel}(\omega, \mathbf{k}) G_{\parallel}(-\omega, -\mathbf{k}). \tag{14}$$

Relation (10) implies the following relations the fluctuation-dissipation theorem:

$$\begin{aligned} D(\omega)&=-\frac{T}{\omega}[G(\omega)-G(-\omega)]=-\frac{2T}{\omega}\operatorname{Re} G(\omega), \\ D_{\parallel}(\omega)&=-\frac{T}{\rho\omega}[G_{\parallel}(\omega)-G_{\parallel}(-\omega)]=-\frac{2T}{\rho\omega}\operatorname{Re} G_{\parallel}(\omega). \end{aligned} \tag{15}$$

## RENORMALIZATION GROUP

Expression (8) now enables us to evaluate the fluctuational corrections, which are represented by a perturbation series in the nonlinearities in the dynamical equations. This series is represented by Feynman diagrams with the binary correlators (11)–(14) on the lines and $\beta$ and $g$ as the vertices. Analysis of the diagrams shows that there are logarithmic corrections to the interaction constants $\beta$ and $g$ and to the longitudinal sound velocity . As will become clear, in evaluating the diagrams corresponding to the accuracy of interest here, the specific form of the dissipative terms in (7) is not important. It is important only that relation (15) be satisfied and that the imaginary part of the spectrum be small compared to the real part. As we shall show below, both these conditions still remain valid when fluctuations are taken into account. The problem of renormalizing $\beta$, $g$, and $c$ can thus be considered independently. It turns out that fluctuations do not give rise to new logarithmic terms in (6)–(8), i.e., the action specified by these expressions is renormalizable. We shall consider the renormalization of this action in the single-loop approximation.

To carry out the renormalization procedure we should separate the variables appearing in (6)–(8) into slowly varying and rapidly varying parts and integrate the distribution function $e^{iI}$ over the latter, thereby shifting the ultraviolet cutoff $\Lambda$. The change of the parameters (6) and (8) during such an integration is represented by the logarithmic diagrams shown in Figs. 1–3. In these diagrams the solid line corresponds to binary average (11) or (12) (of the rapid variables), the wavy line corresponds to binary average (13) or (14), the arrow corresponds to correlator (11), and the dashed line corresponds to correlator (12). The diagrams in Fig. 1

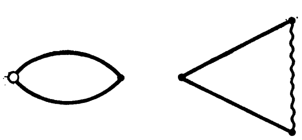

FIG. 1.

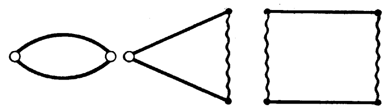

FIG. 2.

give a renormalization of the first two terms in (8) (the coefficients $\beta$ in them are renormalized in the same way), the diagrams of Fig. 2 give a renormalization of the interaction vertex $g$, and the diagrams of Fig. 3 give a renormalization of the longitudianl sound velocity.

The details of the evaluation of the diagrams are given in the Appendix; here we shall give only the final answer for the renormalization group equations:

$$\frac{\partial\beta}{\partial \ln\Lambda}=-A\left(\beta g-\frac{2\beta^3}{\rho c^2}\right), \tag{16}$$

$$\frac{\partial g}{\partial \ln\Lambda}=-3A\left(g^2-4g\frac{\beta^2}{\rho c^2}+4\frac{\beta^4}{\rho^2 c^4}\right), \tag{17}$$

$$\frac{\rho\, \partial c^2}{\partial \ln\Lambda}=-A\beta^2. \tag{18}$$

Here

$$A=T/16\pi b^{3/2}\varkappa^{1/2}. \tag{19}$$

It follows from system (16)–(18) that the true vertex for the self-action of the order parameter is

$$\gamma=A(g-3\beta^2/\rho c^2). \tag{20}$$

Equations (16)–(18) imply that this vertex $\gamma$ obeys the relation $\partial\gamma/\partial \ln\Lambda = -3\gamma^2$, from which we get

$$\gamma=\gamma_0(1+3\gamma_0 L)^{-1}. \tag{21}$$

For the above treatment to be correct it is necessary that the interaction vertex be small, and this requires that $\gamma_0 \langle 1$. In the limit that the logarithm (2) is large, the quantity $\gamma$, in accordance with (21), becomes small, ensuring the validity of the single-loop approximation. We now introduce the dimensionless (by definition) quantity

$$y=A\frac{\beta^2}{\rho c^2}\gamma^{-2/3}. \tag{22}$$

For this quantity system (16)–(18) implies an equation which is trivially integrated to yield, with allowance for (21),

$$y=y_0[1+(\gamma^{-1/3}-\gamma_0^{-1/3})y_0]^{-1}. \tag{23}$$

Now integrating (18) we find

$$c^2/c_0^2=[1+(\gamma^{-1/3}-\gamma_0^{-1/3})y_0]^{-1}. \tag{24}$$

Here $c_0$ is the bare (short-wavelength) longitudinal sound velocity.

On approach to the transition point the parameter $a$ decreases, while the logarithm (2) increases. Accordingly, $\gamma$

FIG. 3.

decreases and, according to (24), $c$ decreases as well. When the value $c^2 = {}^{4/3}c_t^2(c_t$ is the transverse sound velocity) is reached, the system becomes absolutely unstable.[7] Somewhat earlier it becomes possible to have a first order phase transition,[6] at which the system "slips through" the region of strong fluctuations. In any case, $c$ changes by no more than an order of magnitude. It follows that in order for there to be a temperature region around $T_c$ in which the interaction vertex $\gamma$ varies substantially in comparison with its bare value (and where experiments should accordingly reveal logarithmic behavior of the specific heat and dielectric constant), it is necessary that the condition $y_0 \ll 1$ be satisfied. It follows from (23) that this implies the inequality $y \ll 1$ throughout the entire temperature region, since the difference between $y$ and $y_0$ is determined by the expression on the right-hand side of (24), a factor of order unity. We note that the inequality $y_0 \ll 1$ ensures that $\beta^2/\rho c^2 g$ is small only for the bare (short-wavelength) quantities. In the low-frequency region near the transition temperature this ratio becomes of order unity, so that in system (16)–(18) one cannot neglect terms in $\beta^2$ in comparison with terms containing $g$.

## FLUCTUATIONAL CONTRIBUTION TO DAMPING

The Green functions introduced in (11) and (13) have the meaning of the generalized susceptibilities of the system with respect to external forces which can be added to the right-hand sides of the equations of system (3). The functions $G(\omega)$ and $G_\parallel(\omega)$ are thus analytic in the upper half-plane, and their poles determine the eigenvalue spectrum of the oscillations of the system, in this case the spectra of oscillations of the order parameter and longitudinal sound. Finding the poles of (11) under the assumption $\Sigma \ll \omega^2$ (see below), we obtain with allowance for (10) the spectrum of oscillations of the order parameter

$$\omega = \pm(a + bk^2 + \varkappa k_z^2/k^2)^{1/2} - i\Pi/4T. \qquad (25)$$

In an analogous way we find for sound

$$\omega = \pm ck - i\Pi_\parallel \rho/4T. \qquad (26)$$

Thus the polarization operators $\Pi$ and $\Pi_\parallel$ directly determine the damping of the modes under discussion.

The fluctuational corrections to the eigenenergy function $\Sigma$ and to the polarization operator in the leading approximation in $k/\Lambda$ are represented by two-loop diagrams. In the temperature region $\delta T \ll |T - T_c| \ll T_c$, where we can neglect the interaction effects associated with the vertex $\beta$, the leading contributions to $\Sigma$ and $\Pi$ are given, respectively, by the diagrams shown in Fig. 4. One can easily verify that with allowance for relations (15), these contributions satisfy (10). Relation (10) thus reproduces itself when fluctuations are taken into account, as is a consequence of the fluctuation-dissipation theorem.

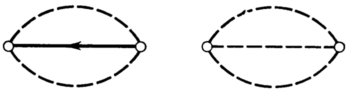

FIG. 4.

Analysis of the second diagram in Fig. 4 (see the Appendix) shows that in the high-frequency limit $\omega^2 \gg a$ it gives a fluctuational contribution

$$\Delta\Pi = \gamma^2 T|\omega|\chi(b^{1/2}k/|\omega|, \varkappa^{1/2}b^{1/2}k_z/\omega^2). \qquad (27)$$

It follows from the dispersion law (25) that the dimensionless parameters appearing in the argument of $\xi$ are of order unity. An expression for $\chi$ in the form of an integral is given in the Appendix. Although this integral is not taken explicitly (it is only for the isotropic model, anyway), we are not terribly interested in the specific form of the function $\chi$; what is important is that $\chi$ is of order unity. Expression (27) and the estimate $\chi \sim 1$ remain valid even at $|T - T_c| \sim \delta T$, where one cannot neglect the interaction effects associated with the vertex $\beta$. We thus arrive at the conclusion, which follows from the structure of (27) and the dispersion law (25), that fluctutions change the power-law behavior of the soft-mode damping from $\omega^2$ to $\omega$. This means that there is a substantial increase in the soft-mode damping in comparison with the bare damping: the fluctuational damping is smaller than the frequency by a factor $\sim \gamma^2$, but not $\sim k/\Lambda$ like the bare damping. Returning to the first diagram in Fig. 4, we note that the corresponding expression for $\mathrm{Re}\Sigma$ contains a logarithmic integration, so that

$$\mathrm{Re}\,\Sigma \sim L\,\mathrm{Im}\,\Sigma \sim \gamma\omega^2.$$

This diagram thus gives a small (proportional to $\gamma$) renormalization of the real part of the soft-mode spectrum.

The fluctuational contribution to the eigenenergy function $\Sigma_\parallel$ for the acoustic mode in the leading approximation is represented by the first diagram in Fig. 3. The real part of $\Sigma_\parallel$ gives a renormalization of the speed of sound, as discussed in the previous Section. The imaginary part of $\Sigma_\parallel$, as follows from representation (15), is related by Eq. (10) to the fluctuational contribution to the polarization operator $\Pi_\parallel$, which is represented (in the leading approximation) by the second diagram in Fig. 3. The structure of (13) and (14) is thus reproduced even when fluctuations are taken into account. This diagram is considered in the high-frequency limit in the Appendix. The leading cutoff factor in this diagram is $k_z \sim k$, because for the soft mode $k_z \sim (b/\varkappa)^{1/2}k^2 \ll k$. The evaluation described in the Appendix yields the following answer, which is valid for $k_z \gg a(b)\varkappa)^{-1/2}\gamma^{-2}$:

$$\Delta\Pi_\parallel = c^2k^2Ty\chi_\parallel/\rho\gamma^{1/3}b^{1/4}\varkappa^{1/4}|k_z|^{1/2}. \qquad (28)$$

By virtue of (26) this expression gives (to within a numerical factor $\chi_\parallel$ of order unity) the following damping-to-frequency ratio for the acoustic mode:

$$cky/\gamma^{1/3}b^{1/4}\varkappa^{1/4}|k_z|^{1/2} \sim (k/\Lambda)^{1/2}y/\gamma^{1/3}.$$

This ratio is small by virtue of the small hydrodynamic parameter, but it is much larger than the ratio $k/\Lambda$ which follows from a bare damping of the type in (9). Fluctuations thus change the character of the power-law dependence of the acoustic damping from $k^2$ to $k^2/|k_z|^{1/2}$. Note the marked anisotropy of the fluctuational damping described by (28).

Expression (27) and (28) are obtained in the high-frequency limit. In the low-frequency limit the expressions for $\Pi$ and $\Pi_{\parallel}$ assume the standard form (9). Joining this expression with (27) and (28) for $\omega \sim a^{1/2}$ and $k_z \sim a(b\varkappa)^{-1/2}\gamma^{-1/2}$, respectively, we obtain the fluctuational contributions to the kinetic coefficients as

$$\Delta\zeta \sim \gamma^2 a^{-1/2}, \tag{29}$$

$$\Delta\zeta_{\parallel} \sim y\gamma^{2/3} a^{-1/2}. \tag{30}$$

Thus, in the low-frequency region fluctuations lead to an anomalous temperature dependence of the damping both for the soft mode and for longitudinal sound.

## CONCLUSION

The above analysis has shown that fluctuations of the order parameter near the phase transition point in uniaxial displacive ferroelectrics cause a substantial modification of the long-wavelength dynamics of the system. The nonlinear fluctuational corrections lead to a logarithmic renormalization of the constants for the self-action of the order parameter and its interaction with the acoustic mode, and there is also a logarithmic renormalization of the speed of sound. These logarithmic renormalizations find a complete analogy in the static case. In addition, the fluctuations of the order parameter substantially modify the damping of both the acoustic and soft modes, leading to a change in the power-law frequency dependence of the damping in the high-frequency region and to a singular temperature dependence in the low-frequency region.

Although these conclusions were made on the basis of the isotropic model, the estimates given will also be valid for an anisotropic model in which the order parameter interacts with all the acoustic branches. In particular, the conclusion regarding the character of the high-frequency [(27) and (28)] and low-frequency [(29) and (30)] dependences of the fluctuational damping remains in force, as does the conclusion that the acoustic damping in markedly anisotropic. At small values of $k_z$ the high-frequency acoustic damping increases sharply, and at $k_z = 0$ it is determined by other cutoff factors (see Appendix). For $\omega \gg a^{1/2}$ we have the estimate

$$\rho \Pi_{\parallel}/T \sim |\omega| y\gamma^{2/3}. \tag{31}$$

In interpreting real experimental data one should keep in mind that the smallness of the above fluctuational contributions to the damping of the long-wavelength modes derives from the smallness of the coupling constants $\gamma$ and $y$. At small values of the hydrodynamic parameter $\omega/c\Lambda$ the fluctuational contributions clearly exceed the bare contributions, but at values of $\omega/c\Lambda$ that are not too small one should take both terms into account.

Let us say a few words in conclusion about the case of a purely damped soft mode. The logarithmic corrections in this case look the same as in the oscillatory case considered above. The fluctuational corrections to the soft-mode damping turn out to be small here, while the fluctuational damping of the acoustic mode is large, given by estimates (30) and (31) for all wave vectors.

## APPENDIX

Let us consider the first diagram given in Fig. 2. This diagram corresponds to the following fluctuational contribution to the interaction vertex:

$$\Delta g = 3i \int \frac{d\omega\, d^3k}{(2\pi)^4} g^2 D(\omega, \mathbf{k}) G(\omega_1 - \omega, \mathbf{k}_1 - \mathbf{k}). \tag{A.1}$$

Here $\omega_1$ and $\mathbf{k}_1$ are the external frequency and wave vector. In the renormalization group procedure they belong to the slow variables and can be neglected in relation to $\omega$ and $\mathbf{k}$ of the rapid variables. The integration over the frequency $\omega$ is most conveniently done using representation (15) together with the fact that $G(\omega)$ has singularities only in the lower half-plane. As a result, the integral over frequency can be reduced to the half-residue at the point $\omega = 0$. Taking into account that $\Sigma$ is small, we find as a result[1)]

$$\Delta g = -\frac{3T}{8\pi^2 \varkappa^{1/2} b^{3/2}} \int k^2\, dk\, dk_4 \frac{g^2}{(k^2 + k_4^2)^2}. \tag{A.2}$$

In this formula we have introduced the momentum $k_4 = (\varkappa/b)^{1/2}(k_z/k)$ goes from $-(\varkappa/b)^{1/2}$ to $(\varkappa/b)^{1/2} \sim \Lambda$. Now performing the integration over the angle in the $(k, k_4)$ plane and differentiating with respect to the upper limit in the resulting logarithmic integral, we obtain the first term on the right-hand side of (17). The first diagrams in Fig. 1 and 3 are examined in a completely analogous way; they give the first term on the right-hand side of (17). The first diagrams in Fig. 1 and 3 are examined in a completely analogous way; they give the first term on the right-hand side of (16) and the term on the right-hand side of (18).

The remaining diagrams in Figs. 1 and 2 give more awkward expressions. Dropping the frequency and wave-vector dependence of the slow variables, we obtain the following fluctuational contribution:

$$\Delta\beta = -\int \frac{d\omega\, d^3k}{(2\pi)^4} \beta^3 k^2 G(\omega) \times \left\{ G(-\omega) D_{\parallel}(\omega) + \frac{2}{\rho} G_{\parallel}(\omega) D(\omega) \right\}, \tag{A.3}$$

$$\begin{aligned} \Delta g = {} & -3 \int \frac{d\omega\, d^3k}{(2\pi)^4} \beta^2 k^2 g G(\omega) \\ & \times \left\{ \frac{3}{\rho} D(\omega) G_{\parallel}(\omega) + \frac{1}{\rho} D(\omega) G_{\parallel}(-\omega) + D_{\parallel}(\omega) G(-\omega) + D_{\parallel}(\omega) G(\omega) \right\} \\ & - \frac{6i}{\rho} \int \frac{d\omega\, d^3k}{(2\pi)^4} \beta^4 k^4 G(\omega) G_{\parallel}(\omega) \\ & \times \left\{ \frac{1}{\rho} D(\omega) G_{\parallel}(\omega) + \frac{1}{\rho} D(\omega) G_{\parallel}(-\omega) + D_{\parallel}(\omega) G(\omega) + D_{\parallel}(\omega) G(-\omega) \right\}. \end{aligned} \tag{A.4}$$

The frequency integral in these expressions is evaluated in a manner analogous to (A.1). Using representation (15), the analytic properties of $G(\omega)$ and $G_{\parallel}(\omega)$, and the symmetry of the integrand with respect to $\omega$, one can show that the integrals over $\omega$ in (A.3) and (A.4) reduce to the half-residue at the point $\omega = 0$. As a result we arrive at expressions of type (A.2) which transform like (A.2) and give the last terms in (16) and (17).

Let us now consider the fluctuational contributions to $\Pi$ from the second diagram in Fig. 4:

$$\Delta\Pi(\omega,\mathbf{k}) = \frac{1}{6A^2}\int \frac{d\omega_1\, d^3k_1\, d\omega_2\, d^3k_2}{(2\pi)^8}\gamma^2 D(\omega_1,\mathbf{k}_1)D(\omega_2,\mathbf{k}_2)D(\omega_3,\mathbf{k}_3). \tag{A.5}$$

Here $\omega_3 = \omega - \omega_1 - \omega_2$, $k_3 = k - k_1 - k_2$. The integral in (A.5) diverges linearly at the upper limit, and it must therefore be regularized by subtracting the constant to which (A.5) goes at $\omega = 0$, $k = 0$. As a result we obtain an integral which is accumulated at $\omega_1 \sim \omega_2 \sim \omega_3 \sim \omega$, $k_1 \sim k_2 \sim k_3 \sim k$. Because $\Sigma$ is small compared to $\omega^2$ in the integral (A.5), one can use the following approximate expression, obtained with allowance for (11) and (15):

$$D \approx \frac{2\pi T}{|\omega|}\delta\left(\omega^2 - bk^2 - \varkappa\frac{k_z^2}{k^2}\right). \tag{A.6}$$

In this regard it is convenient to introduce the following parametrization ($n = 1,2,3$):

$$\omega_n = \frac{\omega\zeta_n}{\cos\theta_n},\quad k_n = \frac{|\omega|}{b^{1/2}}x_n\cos\theta_n,$$

$$k_{nz} = \frac{\omega^2}{(\varkappa b)^{1/2}}x_n^2\cos\theta_n\sin\theta_n.$$

Here $\zeta_n \in (-\infty, +\infty)$, $\theta_n \in (-\pi/2, \pi/2)$, $\chi_n \in (0, \infty)$. Going over to an integration over these dimensionless variables (and over the angles $\varphi_n \in (0, 2\pi)$ in the plane perpendicular to the $z$ axis), we obtain expression (27) with the following function $\chi$:

$$\chi = \frac{1}{6\pi^4}\int d^3\zeta\, d^3\theta\, d^3\varphi\, |\zeta_1\zeta_2\zeta_3|\,\delta\left(\frac{\zeta_1}{\cos\theta_1}+\frac{\zeta_2}{\cos\theta_2}+\frac{\zeta_3}{\cos\theta_3}-1\right)$$
$$\times\delta\left(\zeta_1^2\,\mathrm{tg}^2\,\theta_1+\zeta_2^2\,\mathrm{tg}^2\,\theta_2+\zeta_3^2\,\mathrm{tg}^2\,\theta_3-(\varkappa b)^{1/2}\frac{k_z}{\omega^2}\right)$$
$$\times\delta\Big(\zeta_1^2+\zeta_2^2+\zeta_3^2+2\zeta_1\zeta_2\cos(\varphi_1-\varphi_2)+2\zeta_1\zeta_3\cos(\varphi_1-\varphi_3)$$
$$+2\zeta_2\zeta_3\cos(\varphi_2-\varphi_3)-\frac{bk^2}{\omega^2}\Big)-\dots \tag{A.7}$$

The ellipsis here denotes the same integral with $k_z$ and $k$ set equal to zero.

Let us now consider the second diagram in Fig. 3. It gives the following fluctuational contribution to the polarization operator:

$$\Delta\Pi_\parallel(\omega,\mathbf{k}) = \frac{k^2}{4\rho^2}\int\frac{d\omega_1\, d^3k_1}{(2\pi)^4}\beta^2 D(\omega_1,\mathbf{k}_1)D(\omega_2,\mathbf{k}_2). \tag{A.8}$$

Here $\omega_2 = \omega + \omega_1$, $\mathbf{k}_2 = \mathbf{k} + \mathbf{k}_1$. The expression on the right-hand side of (A.7) does not contain a logarithmic integration, and so $\beta^2$ can immediately be taken out from under the integral sign. We note that in the present case we cannot use representation (A.6) since it leads to divergences at the upper limit. The integration over frequency in (A.8) can be done by making substitution (15) and shifting the contour of integration into the upper half-plane. The integration then reduces to taking the residues at the poles of the $G$ functions (with allowance for the smallness of $\Sigma$). The result, to the required accuracy, is

$$\Delta\Pi_\parallel = \frac{\beta^2k^2T^2}{4\rho^2}\mathrm{Re}\int\frac{d^3k_1}{(2\pi)^3}\frac{i}{q_1^2}\Big\{\left[\omega+q_1^2+\frac{i\Pi_1}{4T}\right]^{-1}$$
$$\times\left[(\omega+q_1)^2-q_2^2+\frac{i}{2T}(\omega+q_1)(\Pi_1+\Pi_2)\right]^{-1}$$
$$+\left[\omega\left(\omega^2-q_2^2+\frac{i\omega}{2T}\Pi_2\right)\right]^{-1}+\dots\Big\}. \tag{A.9}$$

Here $q_1 = (bk_1^2 + \varkappa k_{1z}^2/k_1^2)^{1/2}$ and analogously for $q_2$. The ellipsis in (A.9) stands for the sum of expressions which differ from those which are written out by the replacement $\omega \to -\omega$.

The second term in the braces in (A.9) serves to cancel out the pole terms of the first term in the braces. As a result of this cancellation the integral is accumulated at $q_1 \gg |\omega_1|$; in this region the second term in (A.9) is unimportant, and the first term can be simplified. Let us first consider the general case $k_z \sim k$. Here the $\omega$ dependence of the integral can be omitted, and only the first term in the expansion of the difference $q_1^2 - q_2^2$ in powers of $k_z$ need be kept. The result is the integral

$$\Delta\Pi_\parallel = \frac{T\beta^2k^2}{8\pi^2\rho^2b^{3/2}\varkappa^{1/2}}\mathrm{Re}\int_{-\pi/2}^{\pi/2}d\theta\cos^2\theta\int_0^\infty\frac{dq_1}{\gamma^2\chi q_1^2+2i(b\varkappa)^{1/2}k_z\,\mathrm{tg}\,\theta}. \tag{A.10}$$

Here, $\sin\theta = \sqrt{\varkappa}k_{1z}/k_1q_1$, $\chi = \chi(\cos\theta, \sin\theta)$. Performing the trivial integration over $q_1$, we obtain expression (28) with the following factor:

$$\chi_\parallel = \int_0^{\pi/2}d\theta\frac{\cos^2\theta}{(\chi|\mathrm{tg}\,\theta|)^{1/2}}. \tag{A.11}$$

Let us now consider the case $k_z = 0$, or, more precisely, $|k_z| \ll \omega^2/(b\varkappa)^{1/2}$. In simplifying (A.9) in this limit one must keep in the difference $(\omega + q_1)^2 - q_2^2$ only the terms linear in $\omega$ and $k$, and in the remaining places one can omit the dependence on $\omega$ and $k$. Going over to an integration over $q_1$, $\theta$, and the angle $\varphi$ in the plane perpendicular to the $z$ axis, we obtain

$$\Delta\Pi_\parallel = \frac{T^2\beta^2k^2}{16\pi^3\rho^2\varkappa^{1/2}b^{3/2}}\mathrm{Re}\int_0^{2\pi}d\varphi\int_{-\pi/2}^{\pi/2}d\theta\cos^2\theta\int_{|\omega|}^\infty\frac{dq_1}{q_1}$$
$$[\chi\gamma^2q_1+2i(\omega+\sqrt{b}k(1-\mathrm{tg}^2\,\theta)\cos\theta\cos\varphi)]^{-1}. \tag{A.12}$$

The integration over $q_1$ is now trivial, and the integration over $\varphi$ gives a logarithmic divergence, which is cut off by the condition

$$\left|1+\frac{\sqrt{b}}{c}(1-\mathrm{tg}^2\,\theta)\cos\theta\cos\varphi\right| \gtrsim \gamma^2.$$

As a result, we obtain the expression

$$\Delta\Pi_\parallel = \frac{4T|\omega|y\gamma^{2/3}}{\pi\rho}\ln\gamma^{-1}\int_0^{\pi/2}d\theta\cos^2\theta\,\mathrm{ctg}\,\varphi_\theta. \tag{A.13}$$

Here $\varphi_\theta$ is determined implicitly by the equation

$$\cos\varphi_\theta=\frac{c}{\sqrt{b}}\frac{\cos\theta}{|\cos 2\theta|}.$$

The integration in (A.13) is over the angles $\theta$ for which this equation has a solution.

[1)]The Appendix is only concerned with the high-frequency region. Therefore, the parameter $a$ is dropped in all the formulas.


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