Effect of dynamic character of image forces on field emission

A. I. Voĭtenko, A. M. Gabovich, and V. M. Rozenbaum

Physics Institute, Ukrainian Academy of Sciences
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The image-force energy $W(z)$ near a metal surface is considered for an electron moving in an external electrostatic field F. It is shown that allowance for the spatial dispersion of the dielectric constant ε leads to absence of a divergence of $W(z)$ on the surface, and the temporal dispersion of ε leads to F-dependent dynamic corrections to $W(z)$. The dependence of the field-emission current on F in strong fields is calculated and it is shown that at large F the dynamic corrections cause deviations from the Fowler-Nordheim law, in agreement with experiment.

1. INTRODUCTION

It is known[1-3] that the tunnel current I of field emission from metals obeys the Fowler-Nordheim (FH) law

$$\lg (I/V^2) = C_1 - C_2/V, \tag{1}$$

where V is the potential difference between the electrodes, and C_1 and C_2 are independent of V. In fields $F \gtrsim 5\times 10^{-7}$ V/cm, however, the field-emission current is lower than that given by the FH law.[1-3] Although the effect is small, it is quite perceptible and calls for an explanation. Two explanations have been offered for the violation of the law (1). According to the first[4,5] the FH law was derived for the barrier formed by classical image forces having a potential.

$$W_0(z) = -e^2/4z. \tag{2}$$

Actually, however, $W(z)$ saturates at short distances z from the metal (as recently observed in experiment 6) and this leads to a barrier deformation that is particularly strongly manifest in strong fields F. The difference $\Delta W(z) = W(z) - W_0(z)$ itself does not depend on F.

According to the second explanation[3,7,8] the space charge density of the emitted electron increases in strong fields, and the electron space charge screens the field partially. This, naturally, decreases the current.

In our opinion, the need for taking the space charge into account is unquestionable. At the same time, the effect of the static corrections to $W_0(z)$ on the current should not lead to a deviation from the FH law in real fields, for even the most radical change of the barrier, complete elimination of the image forces, does not change the relation (1) (Refs. 2 and 3). Naturally, this statement calls for verification. This is the subject of the present paper, as well as of an earlier one for the case of emission from semiconductors.[9]

On the other hand, as indicated in Ref. 10, when tunneling is considered account must be taken of the dynamic effects connected with the slow response of the metal plasma to the field of an electron moving in an electrostatic field. The results of the corrections to $W_0(z)$, which are nonadiabatic in a parameter inversely proportional to the squared plasma frequency, depend on F and explain, together with the influence of the space charge,[7,8] the deviation from the FH law at large F. Besides being purely utilitarian, the problem is also of theoretical significance, since dynamic image forces are now being diligently studied[11-15] and not all questions have been fully answered.

2. THE PROBLEM. CALCULATION OF DYNAMIC IMAGE FORCES

Let the interface between the metal and the vacuum be the plane xy. Thus, the problem is one-dimensional and the emission current is directed along the z axis into the vacuum ($z>0$). At absolute zero temperature the current density j is given by[1-3]

$$j = \frac{me}{2\pi^2\hbar^3}\int_{E_0}^{\mu} dE\,(\mu - E) D(E,F), \tag{3}$$

where e and m are the charge and mass of the electron, $\hbar$ is Planck's constant, μ and E_0 are the Fermi energy and the level of the bottom of the conduction band, and $D(E,F)$ is the barrier transmission. The latter, as is well known, is calculated from the Kemble formula

$$D(E,F) = \left[1 + \exp\left\{\frac{2}{\hbar}\int_{z_1}^{z_2} dz\,(2m|E + eFz - W(z)|)^{1/2}\right\}\right]^{-1}, \tag{4}$$

where z_1 and z_2 are the zero of the radicand and E is measured from the vacuum level. Both μ and E_0 in (3) are less than zero.

To carry out the calculations it is necessary to determine the law governing the interaction between the emitted electron and the electron-ion plasma of the metal. To this end we consider the Heinrichs nonrelativistic equation[16] for the potential energy of the dynamic image forces:

$$W(z(t)) = \frac{e^2}{4\pi}\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\int_0^{\infty} dk_\parallel \int_{-\infty}^{\infty} dt'\, e^{i\omega t'} \exp\{-k_\parallel[z(t)+z(t')]\}\chi(k_\parallel,\omega), \tag{5}$$

$$\chi(k_\parallel,\omega) = \frac{1-\varepsilon_s(k_\parallel,\omega)}{1+\varepsilon_s(k_\parallel,\omega)}, \tag{6}$$

$$\varepsilon_s(k_\parallel,\omega) = \left\{\frac{k_\parallel}{\pi}\int_{-\infty}^{\infty}\frac{dk_z}{(k_z^2+k_\parallel^2)\,\varepsilon(\mathbf{k},\omega)}\right\}^{-1}, \tag{7}$$

where $z(t)$ is the time-dependent electron trajectory, ω is the frequency, $k_\parallel$ and k_z are the longitudinal and transverse components of the wave vector $\mathbf{k}$ relative to the interface, and $\varepsilon(\mathbf{k},\omega)$ is the dielectric constant of the metal with allowance for the spatial and temporal dispersions.

We assume the dynamic corrections to the static image forces to be small. It will be shown below that our assumption holds for all $z>0$ owing to the presence of spatial dispersion (screening) in the metal. The smallness of the dynamic corrections allows us to expand the function $\chi(k_\parallel,\omega)$ in powers of ω, and in the absence of absorption ($\mathrm{Im}\chi=0$) this series contains only even powers of ω. The integral with respect to ω in (5) can also be represented by a series:

$$\int_{-\infty}^{\infty} d\omega \exp\{i\omega(t'-t)\}\chi(k_\parallel,\omega) = 2\pi\sum_{m=0}^{\infty}\frac{\chi^{(2m)}(k_\parallel,0)}{i^{2m}(2m)!}\delta^{(2m)}(t'-t), \tag{8}$$

where $\delta^{n}(x)$ is the nth derivative of the δ function. Retaining only the first dynamic correction, we obtain from (5), (6), and (8)

$$W(z(t)) = -\frac{e^2}{2}\int_0^\infty dk_\parallel \frac{\varepsilon_s(k_\parallel,0)-1}{\varepsilon_s(k_\parallel,0)+1}\exp\{-2k_\parallel z(t)\} + \frac{e^2}{2}\int_0^\infty k_\parallel\, dk_\parallel \frac{\varepsilon_s''(k_\parallel,0)}{[\varepsilon_s(k_\parallel,0)+1]^2}\exp\{-2k_\parallel z(t)\} [k_\parallel z'^2(t)-z''(t)], \tag{9}$$

where the prime on z denotes a derivative with respect to time, and

$$\varepsilon_s''(k_\parallel,0)=\left.\frac{d^2\varepsilon_s(k_\parallel,\omega)}{d\omega^2}\right|_{\omega=0}. \tag{10}$$

The first term of (9) corresponds to the known[16] expression for the potential energy of the polarization forces with allowance for the spatial dispersion of the medium. The second term is the sought dynamic correction, whose calculation requires knowledge of the form of $\varepsilon(\mathbf{k},\omega)$.

The simplest dielectric function of the medium, with account taken of both the spatial and the temporal dispersion, is obtained in the gasdynamic approximation[17]:

$$\varepsilon(\mathbf{k},\omega)=1-\omega_p^2/(\omega^2-\beta^2k^2), \tag{11}$$

where $\omega_p=(4\pi ne^2/m)^{1/2}$ is the plasma frequency of the electrons, and β is of the order of the Fermi velocity. The quantities $\varepsilon_s(k_\parallel,0)$ and $\varepsilon_s''(k_\parallel,0)$ are then easily calculated and expression (9) can be reduced after integration with respect to $k_\parallel$ to the form (we omit for brevity the argument t of the functions $z(t)$)

$$\begin{aligned} W(z) = &-\frac{e^2}{2\varkappa^2}\left\{\frac{1}{2z^3}+\frac{\varkappa^2}{2z}+\frac{\pi\varkappa^2}{2z}\Phi_0(2\varkappa z)-\frac{\pi\varkappa}{2z^2}\Phi_1(2\varkappa z)\right\} \\ &+\frac{e^2}{2\omega_p^2\varkappa^4}\left\{z'^2\left[\frac{45}{z^7}+\frac{9\varkappa^2}{z^5}-\frac{60\varkappa^3}{z^4}+\frac{\varkappa^4}{2z^3}+\frac{6\varkappa^5}{z^2}+\varkappa^7\right.\right. \\ &+\pi\Phi_0(2\varkappa z)\left(\frac{45\varkappa^2}{z^5}-\frac{27\varkappa^4}{2z^3}-\frac{\varkappa^6}{4z}\right) \\ &+\pi\Phi_1(2\varkappa z)\left(-\frac{45\varkappa}{z^6}+\frac{36\varkappa^3}{z^4}\right. \\ &\left.\left.-\frac{11\varkappa^5}{4z^2}-\frac{\varkappa^7}{2}\right)\right]-z''\left[\frac{15}{z^6}+\frac{9\varkappa^2}{2z^4}-\frac{20\varkappa^3}{z^3}+\frac{\varkappa^4}{2z^2}\right. \\ &+\pi\Phi_0(2\varkappa z)\left(\frac{15\varkappa^2}{z^4}-\frac{3\varkappa^4}{z^2}-\frac{\varkappa^6}{2}\right) \\ &\left.\left.+\pi\Phi_1(2\varkappa z)\left(-\frac{15\varkappa}{z^5}+\frac{21\varkappa^3}{2z^3}+\frac{\varkappa^5}{4z}\right)\right]\right\}. \end{aligned} \tag{12}$$

Here $\varkappa\equiv\omega_p/\beta$,

$$\Phi_\nu(x)\equiv H_\nu(x)-N_\nu(x), \tag{13}$$

and $H_\nu(x)$ and $N_\nu(x)$ are Struve and Neumann functions of order ν. Equation (12) has the following asymptotic forms: at $\varkappa z\ll1$

$$\begin{aligned} W(z)\approx &-\frac{e^2\varkappa}{3}\left\{1-\frac{3\varkappa z}{4}\ln(\gamma\varkappa z)-\frac{9\varkappa z}{16}\right\} \\ &+\frac{e^2\varkappa^3}{2\omega_p^2}\left\{z'^2\frac{22}{105}\left[1+\frac{315}{176}\varkappa z\ln(\gamma\varkappa z)-\frac{315\varkappa z}{1408}\right]\right. \\ &\left.-\frac{5z''}{24\varkappa}\left[1+\frac{5088}{525}\varkappa z-\frac{9}{5}\varkappa^2z^2\ln(\gamma\varkappa z)\right]\right\}+\ldots \end{aligned} \tag{14}$$

and at $\varkappa z\gg1$

$$\begin{aligned} W(z)\approx &-\frac{e^2}{4z}\left(1-\frac{1}{\varkappa z}\right) \\ &+\frac{e^2}{2\omega_p^2}\left[\frac{z'^2}{2z^3}\left(1-\frac{21}{4\varkappa z}\right)-\frac{z''}{2z^2}\left(1-\frac{7}{2\varkappa z}\right)\right]+\ldots \end{aligned} \tag{15}$$

where $\gamma=1.7810\ldots$ is the Euler constant. It follows from (14) that the potential energy $W(z)$ of the image forces does not diverge on the metal surface regardless of whether the dynamic corections are taken into account or not. The reason is the nonlocality of the response of the electron gas to the perturbation.

In the case of the equal-acceleration motion in the electrostatic field F, when $z=eFt^2/2m$, we obtain from (14) and (15)

$$W(0)=-\frac{e^2\varkappa}{3}\left(1+\frac{5e\varkappa F}{16m\omega_p^2}\right), \tag{16}$$

$$W(z)\approx-\frac{e^2}{4z}\left(1-\frac{1}{\varkappa z}-\frac{eF}{m\omega_p^2 z}\right)\quad(\varkappa z\gg1). \tag{17}$$

Let us estimate the dynamic corrections. In maximum pre-breakdown fields $F\approx10^8$ V/cm and at the typical values $\varkappa\approx1.3\cdot10^8$ cm^{-1} and $\omega_p\sim(1\text{–}3)\times10^{16}$ sec^{-1} it follows from (16) that

$$5e\varkappa F/16m\omega_p^2=0.008-0.074.$$

For weaker fields and at finite distances from the metal the corrections are even smaller. Our initial assumption that they are small is thus corroborated and there is no need for self-consistency of the calculations.

3. NUMERICAL CALCULATIONS AND DISCUSSION OF RESULTS

The field dependences of the field-emission current density j were calculated from Eqs. (3) and (4) using the following models for the image-force potential energy $W(z)$:

1) The classical expression (2).

2) An expression that takes into account only the spatial dispersion in accordance with the first term of Eq. (9), using the Lindhard quantum-mechanical equation[18] for $\varepsilon(k)$.

3) Total neglect of the image forces ($W=0$).

4) Model with allowance for the dynamic corrections. It turns out in this case that the details of the behavior of $W(z)$ near the metal surface are inessential, since the contribution

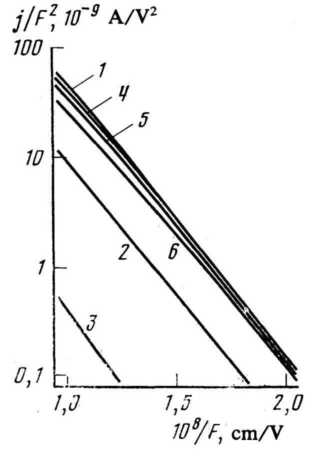

FIG. 1. Dependence of j/F^2 on F^{-1} for classical image forces (curve 1), for static image forces with allowance for spatial dispersion of the medium (curve 2), for an acute-angle barrier ($W \equiv 0$, curve 3), and for dynamic image forces at $\omega_p = 3.52 \cdot 10^{16}$ sec^{-1} (curve 4), $2 \cdot 10^{16}$ sec^{-1} (curve 5), and $1 \cdot 10^{16}$ sec^{-1} (curve 6).

made to the current by the metal electrons with the corresponding energies E is negligibly small. It suffices therefore to take into account the corrections obtained from the asymptotic Eq. (17) subject to some acceptable accuracy of the integral (3). In our case this accuracy was to 10^{-3}.

The problem parameters were chosen to be the same as for hypothetical tungsten with free electrons ($\mu = -4.5$ eV, $E_0 = -23.5$ eV). At the same time the plasma frequency ω_p was regarded as an adjustable parameter equal to $3.52 \cdot 10^{16}$, $2 \cdot 10^{16}$, and 10^{16} sec^{-1}, respectively.

The dependence of j/F^2 on F^{-1} is shown in Fig. 1. Curves 1–3 correspond to models 1)–3), and curves 4–6 take into account the dynamic corrections for different values of ω_p. We see that allowance for the static polarization corrections to the image forces (curve 2), while indeed changing the current, does not lead to violation of the FH law. On the contrary, the dynamic corrections decrease the current in the region of strong fields. This decrease is due to inertness of the electron gas of the metal to its perturbation by the moving charge. A measure of the inertness that leads to the appearance of the nonadiabatic correction is the small parameter $eF/m\omega_p^2 z^*$, where $z^* \approx z$ at large distances and $z^* \approx r_{TF}$ at short ones (r_{TF} is the Thomas-Fermi screening radius).

In a detailed analysis of actual experiments, the dynamic corrections obtained here for the field emission current should supplemented by the space-charge effects mentioned in the introduction.[3,7,8]

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