Magnetically induced birefringence in cubic crystals

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(Submitted 30 March 1984)
Zh. Eksp. Teor. Fiz. **87**, 1038–1045 (September 1984)

An experimental study is made of the birefringence in GaAs crystals in the exciton region of the spectrum in the presence of a magnetic field. The birefringence caused by the magnetic field is proportional to the first power of the field. The experimental results are explained on the basis of a magnetically induced spatial dispersion of the permittivity tensor. The observed effect is due to the presence of a linear term in the valence-band Hamiltonian. The corresponding constant is found to be $\hbar k_0 = 7 \cdot 10^{-10}$ eV·cm.

1. INTRODUCTION

Although there have been many experimental and theoretical studies of the various physical aspects of excitons in semiconductors, the nature of exciton states which are degenerate as a result of a complex structure of the energy bands is still attracting attention. Magnetooptic measurements are a powerful method for studying various properties of these states: the magnetic field splits the degenerate levels into sublevels, each with its own oscillator strength and polarization. In crystals which lack a center of inversion, allowance for spatial dispersion of the permittivity in the presence of an external magnetic field gives rise to new effects. One of these is the well-known effect of inversion of the magnetic field (see, e.g., Ch. III, Sec. 9 of Ref. 1). In addition, the magnetic field induces a birefringence which is linear in the wave vector of the light.[2]

The present paper reports a study of the birefringence in GaAs crystals near the fundamental absorption edge in the presence of a magnetic field.

2. PHENOMENOLOGICAL TREATMENT

Phenomenologically, the magnetically induced birefringence can be described as follows. Sufficiently far from resonance absorption lines the permittivity tensor $\varepsilon(\omega, \mathbf{q}, \mathbf{H})$ can be expanded in a series; for crystals lacking a center of inversion we have, in the approximation linear in the magnetic field $\mathbf{H}$ and light wave vector $\mathbf{q}$ (Ch. III, Sec. 9 of Ref. 1)

$$\varepsilon_{ij}(\omega, \mathbf{q}, \mathbf{H}) = \varepsilon_{ij}(\omega) + \gamma_{ijk} H_k + A_{ijlm} H_l q_m. \tag{1}$$

In expression (1) the third-rank pseudotensor γ, which is antisymmetric in the indices i, j, describes the Faraday effect (see, e.g., Sec. 82 of Ref. 3).

According to the generalized symmetry principle for kinetic coefficients[3]

$$\varepsilon_{ij}(\omega, \mathbf{q}, \mathbf{H}) = \varepsilon_{ji}(\omega, -\mathbf{q}, -\mathbf{H}). \tag{2}$$

It follows that the pseudotensor A_{ijlm} is symmetric in the indices i, j:

$$A_{ijlm} = A_{jilm}. \tag{3}$$

By virtue of the Hermitian character of the permittivity $\varepsilon_{ij}(\omega, \mathbf{q}, \mathbf{H})$, the pseudotensor A_{ijlm} is real. Pseudotensor A_{ijlm} can be written as the sum of parts which are symmetric and antisymmetric in the indices l and m:

$$A_{ijlm} = A^{s}_{ijlm} + A^{a}_{ijlm}. \tag{4}$$

For crystals of the cubic symmetry class T_d, the pseudotensors A^{s}_{ijlm} and A^{a}_{ijlm} each have only one independent nonzero component:

$$A^{s}_{xxyy} = A^{s}_{yyzz} = A^{s}_{zzxx} = -A^{s}_{yyxx} = -A^{s}_{zzyy} = -A^{s}_{xxzz} = A, \tag{5}$$

$$A^{a}_{xyxy} = e_{xyz} g_{zxy} = g. \tag{6}$$

Here e_{xyz} is the completely antisymmetric unit tensor, and the coordinate axes are chosen along the four-fold crystallographic axes $\{100\}$.

According to relations (5) and (6), for crystals of cubic class T_d the permittivity tensor $\varepsilon(\omega, \mathbf{q}, \mathbf{H})$ given by expression (1) becomes

$$\varepsilon_{ij}(\omega, \mathbf{q}, \mathbf{H}) = \varepsilon(\omega)\delta_{ij} + \gamma_{ijk} H_k + A^{s}_{ijlm} H_l q_m \delta_{ij}\delta_{lm} + [\mathbf{H} \times \mathbf{q}]_s g_{sij}. \tag{7}$$

One is readily convinced that magnetically induced birefringence occurs, for example, for $\mathbf{H} \| [1\bar{1}0]$ and $\mathbf{q} \| [110]$:

$$\varepsilon_{\parallel} = \varepsilon - (A + 2g) H_x q_x, \qquad \varepsilon_{\perp} = \varepsilon + 2A H_x q_x. \tag{8}$$

3. EXPERIMENTAL RESULTS AND DISCUSSION

The experiments were done at $T = 2$ K in a superconducting solenoid. The geometry of the experiment is shown in Fig. 1. At $\mathbf{H} = 0$ we observed (in crossed polaroids) the natural birefringence, which is due to a second-order effect in the spatial dispersion and is characteristic of the exciton region of the spectrum.[4] When the magnetic field was turned on, the birefringence decreased, becoming practically zero at fields of the order of 27 kOe. As the magnetic field was increased further we observed a rise of the effect (Fig. 2).

The particular geometry of the experiment provides optimum conditions for observation of the natural birefringence, since the angle between the polarization direction of the light and one of the axes of the permittivity tensor is 45°. In the presence of a magnetic field the direction of the dielec-

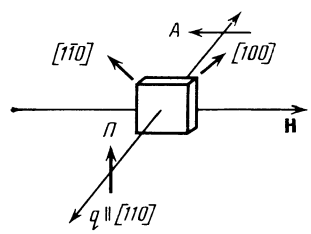

FIG. 1. Geometry of experiment.

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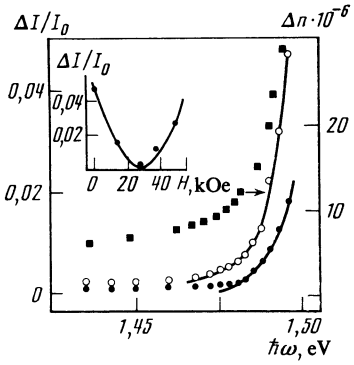

FIG. 2. Dispersion relations Δn (0) and ΔI (**H**)/I_0 for **H** = 0 (light circles) and **H** = 13 kOe (dark circles). The inset shows a plot of ΔI (**H**)/I_0 against **H** at $\hbar\omega = 1.496$ eV.

tric axes depends on the magnitude and direction of **H**. For the particular experimental geometry the relative intensity of the light passing through the crossed polaroids is

$$\frac{\Delta I(\mathbf{H})}{I_0} = \left(\frac{\pi L}{\lambda}\right)^2 \Delta n^2(\mathbf{H}, \theta)\cos^2 2\theta. \tag{9}$$

Here I_0 is the intensity of the incident light, L is the optical path length, and the angle θ specifies the direction of the dielectric axes with respect to the crystallographic axis $[1\bar{1}0]$, i.e.,

$$\operatorname{tg} 2\theta = \frac{4g}{3A+2g}\left[1-\frac{\Delta n(0)}{\Delta n(\mathbf{H})}\right]^{-1}; \tag{10}$$

further,

$$\Delta n(\mathbf{H}, 0) = \Delta n(\theta)\cos 2\theta - \Delta n(\mathbf{H})\left(\cos 2\theta + \frac{4g}{3A+2g}\sin 2\theta\right), \tag{11}$$

where

$$\Delta n(H) = \frac{1}{2\sqrt{\varepsilon}}(3A+2g)\frac{|\mathbf{H}|\,|\mathbf{q}|}{2\sqrt{2}}. \tag{12}$$

As will be shown in Sec. 4, for GaAs the quantities A and g are positive, and $A = 10g$.

The natural birefringence $\Delta n(0)$ in the exciton region of the spectrum is given by the difference $\gamma_3 - \gamma_2$ of the Luttinger parameters, and for GaAs we have $\Delta n(0) > 0$.[5]

According to relation (10), at a certain value of the magnetic field the dielectric axes are rotated by 45° and thus coincide with the polarization direction of the light. Here the birefringence should vanish. In the present experiments the corresponding value of the magnetic field is 27 kOe. Using the experimental data and relations (9)–(12), one can construct the magnetic field dependence of the quantity Δn(**H**) given by expression (12). The result is shown in Fig. 3. It is seen that the field dependence is linear, i.e., the results cannot be ascribed to the Cotton-Mouton effect.

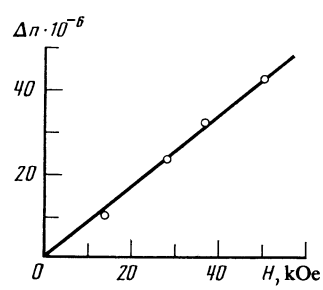

FIG. 3. Plot of Δn(**H**) against the magnetic field **H** at $\hbar\omega = 1.496$ eV.

4. THEORY

From a microscopic standpoint the appearance of effects which are of first order in the magnetic field and spatial dispersion in media lacking a center of inversion is due to the presence of the corresponding terms in the effective Hamiltonian describing the exciton ground state. This Hamiltonian can be constructed by the method of invariants (see Ref. 6, Ch. IV, Secs. 25 and 26 of Ref. 7, and also Ref. 8), and to first approximation in **H** and **q** for crystals of cubic symmetry class T_d we have

$$\mathcal{H}_{ex} = \mathcal{H}_{ex}(0) + \mathcal{H}_{ex}(\mathbf{H}, \mathbf{q}) + \mathcal{H}_{ex}(\mathbf{q}) + \mathcal{H}_{ex}(\mathbf{H}), \tag{13}$$

where

$$\mathcal{H}_{ex}(0) = E_0 + \mathcal{H}_{\text{exch}} \tag{14}$$

$$\mathcal{H}_{ex}(\mathbf{H}, \mathbf{q}) = B_1([\mathbf{H}\times\mathbf{q}]_x\{J_y, J_z\} + \text{c.p.}) + B_2[H_x q_x(J_y^2 - J_z^2) + \text{c.p.}], \tag{14a}$$

$$\mathcal{H}_{ex}(\mathbf{q}) = C_q(q_x[J_x(J_y^2 - J_z^2)] + \text{c.p.}), \tag{14b}$$

$$\mathcal{H}_{ex}(\mathbf{H}) = g_c\mu_B\boldsymbol{\sigma}\mathbf{H} - 2\mu_B[\tilde{k}\mathbf{J}\mathbf{H} + \tilde{q}(H_x J_x^3 + H_y J_y^3 + H_z J_z^3)]. \tag{14c}$$

In these formulas $\mathbf{J}(J_x, J_y, J_z)$ is the total-angular-momentum operator, $\{J_i, J_j\} = \frac{1}{2}(J_iJ_j + J_jJ_i)$, $\boldsymbol{\sigma}$ and g_c are the spin operator and g factor of a conduction electron, $\tilde{k}$ and $\tilde{q}$ are the analogs of the Luttinger parameters for an exciton, $\mathcal{H}_{\text{exch}}$ is the Hamiltonian of the exchange interaction, and c.p. stands for a cyclic permutation of x, y, z.

The constants B_1 and B_2 appearing in relation (14a) should be expressed in terms of the constants characterizing the electron and hole energy spectra. For this purpose let us write the exciton Hamiltonian for the effective-mass method in the presence of a magnetic field[9,10]

$$\mathcal{H} = \mathcal{H}^c\left(\mathbf{p} + \frac{e}{c}\mathbf{A}(\mathbf{r}) + \alpha\hbar\mathbf{q}\right) - \mathcal{H}^v\left(-\mathbf{p} + \frac{e}{c}\mathbf{A}(\mathbf{r}) + \beta\hbar\mathbf{q}\right) - \frac{e^2}{\varepsilon r}. \tag{15}$$

Hamiltonian (15) is written in the coordinates of the relative and translational motions. For crystals of class T_d we have

$$\mathcal{H}^c(\mathbf{k}) = \mathbf{k}^2/2m_e + g_c\mu_B\boldsymbol{\sigma}\mathbf{H}, \tag{16}$$

$$-\mathcal{H}^v(\mathbf{k}) = \frac{1}{m_0}\left[\left(\gamma_1 + \frac{5}{2}\gamma_2\right)\frac{\mathbf{k}^2}{2} - \gamma_2(\mathbf{k}\mathbf{J})^2 - 2\gamma_3(\{k_x, k_y\}\{J_x, J_y\} + \text{c.p.})\right] + k_0(k_x[J_x(J_y^2 - J_z^2)] + \text{c.p.}) - 2\mu_B[k\mathbf{J}\mathbf{H} + q(H_x J_x^3 + H_y J_y^3 + H_z J_z^3)]. \tag{17}$$

Here m_e is the electron mass (γ_1, γ_2, γ_3, k, q) are the Luttinger parameters,

$$\alpha=m_e/(m_e+m_h),$$

$$\beta=m_h/(m_e+m_h), \tag{18}$$

$$m_h=m_0/\gamma_1. \tag{19}$$

Because we are considering only the case of a uniform magnetic field, the vector potential can be chosen in the form $\mathbf{A}(\mathbf{r})=\frac{1}{2}(\mathbf{H}\times\mathbf{r})$. Without loss of generality we shall assumed that the magnetic field is directed along the z axis. Let us keep only the terms which are linear in the wave vector $\mathbf{q}$ and write Hamiltonian (15) in the form

$$\mathcal{H}=\mathcal{H}_s+\mathcal{H}', \tag{20}$$

where $\mathcal{H}_s$ is the spherically symmetric part of Hamiltonian (15):

$$\mathcal{H}_s=\mathbf{p}^2/2\mu_0-e^2/\varepsilon r, \tag{21}$$

$$1/\mu_0=1/m_e+\gamma_1/m_0, \tag{22}$$

$$\mathcal{H}'=\mathcal{H}_d+\mathcal{H}_k+\mathcal{H}(\mathbf{q})+\mathcal{H}(\mathbf{H})+\mathcal{H}(\mathbf{H},\mathbf{q}). \tag{23}$$

In expression (23) the Hamiltonians $\mathcal{H}_d$ and $\mathcal{H}_k$, $\mathcal{H}_s$, describe an exciton in the absence of magnetic field:

$$\mathcal{H}_d=\frac{\gamma_2}{m_0}\left[\frac{5}{4}\mathbf{p}^2-(\mathbf{pJ})^2\right]-\frac{2\gamma_3}{m_0}[p_xp_y\{J_x,J_y\}+\text{c. p.}], \tag{24}$$

$$\mathcal{H}_k=-k_0(p_x[J_x(J_y^2-J_z^2)]+\text{c.p.}), \tag{25}$$

$\mathcal{H}(\mathbf{q})$ and $\mathcal{H}(\mathbf{H})$ are, respectively, the terms linear in $\mathbf{q}$ and $\mathbf{H}$:

$$\mathcal{H}(\mathbf{q})=\beta\hbar\Big\{k_0(q_x[J_x(J_y^2-J_z^2)]+\text{c. p.})-\frac{2\gamma_2}{m_0}\Big[\frac{5}{4}\mathbf{qp}-(q_xp_xJ_x^2+q_yp_yJ_y^2+q_zp_zJ_z^2)\Big]+\frac{2\gamma_3}{m_0}[q_xp_y\{J_x,J_y\}+\text{c. p.}]\Big\}. \tag{26}$$

$$\mathcal{H}(\mathbf{H})=k_0\frac{eH}{2c}(x[J_y(J_x^2-J_z^2)]-y[J_x(J_y^2-J_z^2)])+\frac{eH}{c}\Big\{\Big(\frac{1}{2m_e}-\frac{\gamma_1}{m_0}-\frac{5}{4}\frac{\gamma_2}{m_0}\Big)(xp_y-yp_x)+\frac{\gamma_2}{m_0}(xp_yJ_y^2-yp_xJ_x^2)+\frac{\gamma_3}{m_0}[(xp_x-yp_y)\{J_x,J_y\}+xp_z\{J_y,J_z\}-yp_z\{J_x,J_z\}]-\frac{1}{m_0}\Big[kJ_z+qJ_z^3+\frac{1}{2}g_e\sigma_z\Big]\Big\}. \tag{27}$$

The bilinear part of Hamiltonian $\mathcal{H}(\mathbf{H},\mathbf{q})$ is given by the expression

$$\mathcal{H}(\mathbf{H},\mathbf{q})=2\beta\mu_BH\Big\{\Big(\frac{\gamma_1}{2}+\frac{5}{4}\gamma_2\Big)(xq_y-yq_x)-\gamma_2(xq_yJ_y^2-yq_xJ_x^2)-\gamma_3[(xq_x-yq_y)\{J_x,J_y\}+xq_z\{J_y,J_z\}-yq_z\{J_x,J_z\}]\Big\}. \tag{28}$$

Following Refs. 10 and 11, we shall treat Hamiltonian $\mathcal{H}'$ as a perturbation. This approach is valid[12] in the weak-magnetic-field region to $\gamma=0.4$, where $\gamma=R_0^*/\mu_BH$. It is known, however, that the predictions of the theory give good qualitative agreement with experiment even at considerably larger magnetic fields.[13]

In crystals of class T_d the exciton ground state Z_{12}, coupled to the valence band Γ_8 and the conduction band Γ_6, is eightfold degenerate. The eigenfunctions of Hamiltonian $\mathcal{H}_s$ are the spinors

$$\left|\begin{bmatrix}n\\k\end{bmatrix},l,m,i\right\rangle=\left|\begin{bmatrix}n\\k\end{bmatrix},l,m\right\rangle\begin{vmatrix}\delta_{1i}\\\delta_{2i}\\\delta_{3i}\\\delta_{4i}\\\delta_{5i}\\\delta_{6i}\\\delta_{7i}\\\delta_{8i}\end{vmatrix}, \tag{29}$$

where the first factor on the right stands for ordinary hydrogen-like functions (the index n refers to the discrete spectrum, the variable k to the continuum), and δ_{ij} is the Kronecker delta. The corresponding eigenvalues of the discrete spectrum are

$$E_n=-R_0^*/n^2, \qquad R_0^*=\mu_0e^4/2\hbar^2\varepsilon^2. \tag{30}$$

Appropos the linear (in $\mathbf{q}$ and $\mathbf{H}$) corrections to the exciton ground state $|1s\rangle$, we note that the bilinear part of Hamiltonian $\mathcal{H}(\mathbf{H},\mathbf{q})$ does not contribute in the first approximation. A nonvanishing contribution appears in the second order of peturbation theory in $\mathcal{H}_k$ and $\mathcal{H}(\mathbf{H},\mathbf{q})$ and also in $\mathcal{H}(\mathbf{H})$ and $\mathcal{H}(\mathbf{q})$. After straightforward calculations we obtain

$$B_1=\frac{8}{3}\frac{\mu_B}{R_0^*}\hbar k_0\frac{m_0}{M}G\Big(3\frac{\gamma_3}{\gamma_1}-1\Big), \tag{31}$$

$$B_2=-8\frac{\mu_B}{R_0^*}\hbar k_0\frac{m_0}{M}G\frac{\gamma_3}{\gamma_1}, \tag{32}$$

where

$$G=\frac{1}{4}\mathbf{S}_{n=2}^{\infty}\frac{\langle 1s|r/a_0^*|np\rangle\langle np|1s\rangle}{1-1/n^2}. \tag{33}$$

and $a_0^*=\varepsilon\hbar^2/\mu_0e^2$ is the effective Bohr radius, $M=m_e+m_h$, the symbol $\mathbf{S}$ denotes summation over the discrete spectrum and integration over the continuous spectrum, and the constant G equals 0.375 (Ref. 10). The presence of the bilinear terms $\mathcal{H}_{ex}(\mathbf{H},\mathbf{q})$ in the effective exciton Hamiltonian for cubic crystals lacking a center of inversion is due to terms which are linear in the momentum in the valence-band Hamiltonian (17). By experimentally determining the constants B_1 and B_2 one can obtain information about the value of the corresponding constant k_0. Up till now there have been no data reported on the value of k_0 for GaAs.

Having established the connection between the coefficients B_1 and B_2 and the band constants, let us use effective Hamiltonian (13) to consider the behavior of the exciton ground state Z_{12} in a magnetic field, which induces spatial dispersion.

The representation $\Gamma_8 \times \Gamma_6$ according to which the ground state Z_{12} transforms is reducible:

$$\Gamma_8 \times \Gamma_6 = \Gamma_3 + \Gamma_4 + \Gamma_5. \tag{34}$$

The dipole allowed state Γ_5 is threefold degenerate, and the states Γ_3 and Γ_4 are dipole forbidden. The singlet state Γ_5 and triple states (Γ_3, Γ_4) are split by the exchange interaction. The basis functions of the singlet state transform like x, y, z under the transformations of the cubic group.[8]

The presence in Hamiltonian (13) of terms $\mathscr{H}_{ex}(\mathbf{H},\mathbf{q})$ which are bilinear in the wave vector $\mathbf{q}$ and magnetic field $\mathbf{H}$ leads to an additional splitting of the degenerate exciton state Z_{12} and a mixing of the states Γ_5, Γ_3, and Γ_4.

Let us limit discussion to the dipole-allowed state Γ_5 and set $\tilde{q} = 0$. For $\mathbf{H}\|[100]$ and $\mathbf{q}\|[001]$ the corresponding submatrix of Hamiltonian (13) is of the form

$$\mathscr{H}_{ex} = \begin{vmatrix} E_0 & 0 & \frac{1}{2} B_1 H_x q_z \\ 0 & E_0 & -i\mu_B g H_x \\ \frac{1}{2} B_1 H_x q_z & i\mu_B g H_x & E_0 + \Delta_{LT} \end{vmatrix}, \tag{35}$$

where $E_0 = R_0^* - \delta$, the constant δ is related[14] to the terms $\mathscr{H}_d$ and $\mathscr{H}_k$ Hamiltonian (23), $g = g_c/2 + 2\tilde{k}$, and Δ_{LT} is the longitudinal-transverse splitting.

It follows from the form of matrix (35) that the bilinear term $\mathscr{H}_{ex}(\mathbf{H},\mathbf{q})$ causes a mixing of the optically active state $|x\rangle$ with the longitudinal state $|z\rangle$. For $\mathbf{H}\|[1\bar{1}0]$, $\mathbf{q}\|[110]$ the submatrix of Hamiltonian (13) in the basis

$$\frac{1}{\sqrt{2}}(|x\rangle + |y\rangle), \quad \frac{1}{\sqrt{2}}(|x\rangle - |y\rangle), |z\rangle$$

is of the form

$$\mathscr{H}_{ex} = \begin{vmatrix} E_0 + \Delta_{LT} - (B_1 + B_2) H_x q_x & 0 & i\sqrt{2}\,\mu_B g H_x \\ 0 & E_0 + (B_1 - B_2) H_x q_x & 0 \\ -i\sqrt{2}\,\mu_B g H_x & 0 & E_0 + 2B_2 H_x q_x \end{vmatrix}. \tag{36}$$

According to (36), the spectrum of Coulomb excitons is given by the expressions

$$\varepsilon_{1T} = E_0 + (B_1 - B_2) H_x q_x, \tag{37}$$

$$\varepsilon_{2T} = E_0 + \frac{\Delta_{LT}}{2} - \frac{(B_1 - B_2)}{2} H_x q_x - \Delta\varepsilon,$$

$$\varepsilon_L = E_0 + \frac{\Delta_{LT}}{2} - \frac{(B_1 - B_2)}{2} H_x q_x + \Delta\varepsilon, \tag{38}$$

$$\Delta\varepsilon = \left[\left(\frac{\Delta_{LT}}{2} - \frac{(B_1 + 3B_2)}{2} H_x q_x \right)^2 + 2\mu_B^2 g^2 H_x^2 \right]^{1/2}. \tag{39}$$

It follows from expressions (37)–(39) that for the configuration $\mathbf{H}\|[1\bar{1}0]$, $\mathbf{q}\|[110]$ there is an effect of inversion of the magnetic field: The frequencies of Coulomb excitons change when the direction of the magnetic field is reversed.

To evaluate the phenomenological quantities g and A appearing in relation (7), we use the approximate expression for the permittivity tensor in the vicinity of an isolated resonance (see, e.g., Ch. IV, Sec. 13 of Ref. 1):

$$\varepsilon_{\alpha\beta}(\omega, \mathbf{q}) = \varepsilon_0(\omega)\delta_{\alpha\beta} - \frac{4\pi}{\omega^2 V} \sum_{l=1}^{3} \frac{\langle 0|j^\alpha|l\rangle\langle l|j^\beta|0\rangle}{\hbar\omega - E_l(\mathbf{q})}. \tag{40}$$

Here the matrix elements of the current operator $\mathbf{j}$ are evaluated using the eigenfunctions of a mechanical exciton of momentum $\mathbf{q}$ and energy $E_l(\mathbf{q})$; the subscript l enumerates the Γ_5 exciton states, the function $|0\rangle$ describes the ground state of the crystal; the background part of the permittivity $\varepsilon_0(\omega)$ includes the nonresonant terms and the contribution of all the other exciton states. Then, limiting discussion to the bilinear term (14) in Hamiltonian (13), we obtain

$$g = {}^1/_2 \Delta_{LT} \varepsilon_0 B_1 [(\hbar\omega - E_0)^2 - (\mu_B g H)^2]^{-1}, \tag{41}$$

$$A = -\Delta_{LT} \varepsilon_0 B_2 [(\hbar\omega - E_0)^2 - (\mu_B g H)^2]^{-1}. \tag{42}$$

We note that the role of the linear terms $\mathscr{H}_{ex}(\mathbf{q})$ and $\mathscr{H}_{ex}(\mathbf{H})$ given by (14b) and (14c), at least under the present experimental conditions, is small. For example, the corresponding relative contribution is proportional to the ratio

$$\hbar k_0 \mu_B / B_{1,2} (\hbar\omega - E_0 + \Delta_{st}) \sim 10^{-2}.$$

For GaAs we have $\gamma_1 = 6.89$ and $\gamma_3 = 2.9$ (Ref. 15), and so $B_1 > 0$. Consequently, g and A are positive, with

$$g/A = -B_1/2B_2 = (3 - \gamma_1/\gamma_3)/6 = 0.1.$$

A processing of the experimental data with the values of Δ_{LT}, ε_0, ε, and E_0 from Ref. 15 yields the following values for the constants: $B_1 = 1.5 \cdot 10^{-11}$ eV·cm/T, $B_2 = 7.5 \cdot 10^{-11}$ eV·cm/T. Using relations (31) and (32), we obtain $\hbar k_0 = 7 \cdot 10^{-10}$ eV·cm for the constant k_0.

According to relation (11), the frequency dependence of the magnetically induced birefringence $\Delta n(\mathbf{H},\theta)$ is determined by the dispersion of g, A, and $\Delta n(0)$. Under the conditions of the present experiment the energy deficit is rather large: $\hbar\omega - E_0 \gg \mu_B g H$, since $\hbar\omega - E_0 \sim 10^{-2}$ eV and $\mu_B g H \sim 10^{-4} H$ eV/T. As a result, the frequency dependence of g and A, like that of the natural birefringence $\Delta n(0)$,[5] has a resonance character of the form $(\hbar\omega - E_0)^{-2}$, shown by the solid curves in Fig. 2. As we see, near the resonance there is good agreement with the experimental results.

In closing, we wish to thank L. V. Keldysh and V. L. Bonch-Bruevich for helpful discussions, N. N. Sibel'din for the opportunity to do the experiment, and N. V. Zamkovets for assistance in the experiment.

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Translated by Steve Torstveit