Analytic expression for the cross section for two-phonon resonant Raman scattering of light

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We calculate the contribution made to the cross section σ_2 of two-phonon Raman scattering of light by free (disregarding Coulomb interaction) electron-hole pairs. The calculations were performed for Fröhlich interaction of electrons or holes with *LO* phonons, assuming $m_h \to \infty$ (m_h is the effective mass of the hole). Results are obtained in analytic form for the cross section σ_2 in a wide range of exciting-light frequencies ω_l.

One of the most important uses of investigations of secondary emission of light by crystals is for the analysis of the character of electron excitations and their interactions with phonons. Of great interest from this viewpoint, in particular, is multiphonon resonant Raman scattering of light (RRSL). Two-phonon scattering in semiconductors occupies a prominent place in this phenomenon and has been the object of detailed experimental and theoretical study for many years (see, e.g., the reviews[1–3]). The main reason is analysis of the changes of the intensities as well as the line shapes of two-phonon replicas on going from one material to another is easiest when the frequency ω_l of the exciting light is varied (below and above the intrinsic absorption edge), and when the temperature is varied. It is important to note that in all multiphonon RRSL processes a competition takes place between two types of intermediate states of the crystal, viz., Wannier-Mott excitons and free electron-hole pairs (EHP). If these two types of contributions to the intermediate-state cross section are identified by theoretical analysis, it becomes possible to determine the parameters of the electron excitations by comparing the experimental data with the theory.

A theoretical analysis of multiphonon RRSL has demonstrated the decisive significance of the Fröhlich interaction[4] of electrons or holes with longitudinal opticals (*LO*) phonons. A number of theoretical studies (see, e.g., Refs. 5 and 6) were made of multiphonon RRLS with participation of Wannier-Mott excitons as real intermediate states. It was shown that at $\omega_l > E_1/\hbar + N\omega_{LO}$, where E_1 is the exciton ground-state energy, ω_{LO} the *LO*-phonon frequency, and N the order of the scattering, the contribution of the hot excitons to the scattering cross section σ_N is directly proportional to the Fröhlich constant α at any number N of emitted *LO* phonons (including $N=2$). The theoretical results on the contributions of free EHP to the cross section for multiphonon RRSL of high order ($N \geqslant 4$) are given in Ref. 7 (see also the bibliography therein). For profound physical reasons, however, the calculation method used for $N \geqslant 4$, in not valid for $N=2$ and $N=3$.

The theory of two-phonon RRSL involving free EHP is the subject, in particular, of Refs. 8–11. In Refs. 9 and 10 are calculated the scattering amplitudes as functions of the phonon wave vector q (at $m/m_h \to 0$ in Ref. 9, and at arbitrary ratio of the effective masses m and m_h of the electron and hole in Ref. 10), and the cross section is expressed as an integral with respect to q. In Ref. 10 are presented numerical calculation for the $\sigma_2(\omega_l)$ dependences at m/m_h and of the damping γ of the electronic states. An explicit expression was obtained in Ref. 11 for the function $\sigma_2(\omega_l)$ in the region $\omega_l > E_g/\hbar + 2\omega_{LO}$, where E_g is the semiconductor band gap, but in Ref. 12 was used a simplified model in which the Fröhlich interaction was replaced by an interaction that does not depend on q, and an approximate method was used to calculate the contribution of the processes when one or two phonons are emitted by a hole.

Our present purpose is to obtain an analytic expression for the cross section $\sigma_2(\omega_l)$ in a wide range of ω_l in the case of Fröhlich interaction of electrons and holes with *LO* phonons. The limiting case $m/m_h \to 0$ is considered, although it must be noted[10] that the cross section σ_2 depends strongly on the ratio m/m_h.

An analytic expression for $\sigma_2(\omega_l)$ is essential, in particular, for comparison with the results of Ref. 13 in the analysis of two-phonon RRSL in a strong magnetic field.

1. CALCULATION OF THE SECOND ORDER SCATTERING AMPLITUDE

We use as the operator of the interaction of electrons (holes) with phonons the Fröhlich Hamiltonian

$$H_{int} = \sum_{\mathbf{q},j} \left(C_{j\mathbf{q}} e^{i\mathbf{q}\mathbf{r}_j} b_{\mathbf{q}} + C_{j\mathbf{q}}^{*} e^{-i\mathbf{q}\mathbf{r}_j} b_{\mathbf{q}}^{+} \right), \tag{1}$$

where

$$C_{j\mathbf{q}} = (-1)^{j} i \frac{C}{q}, \quad C = \frac{\hbar\omega_{LO}}{l} \left(\frac{4\pi\alpha l^3}{V_0} \right)^{1/2},$$
$$l = \left(\frac{\hbar}{2m\omega_{LO}} \right)^{1/2}, \quad \alpha = \frac{e^2}{2\hbar\omega_{LO} l} (\varkappa_\infty^{-1} - \varkappa_0^{-1}), \tag{2}$$

the subscripts $j=1$ and $j=2$ denote the electron and hole, respectively, $b_q (b_q^+)$ the operators of annihilation (creation) of an *LO* phonon with wave vector $\mathbf{q}$, $\mathbf{r}_j$ the radius vector of the electron or hole, e and m respectively the charge and mass of the electron, V_0 the normalization volume, and $\varkappa_0 (\varkappa_\infty)$ the static (high-frequency) dielectric constant.

We consider the region much below the Debye temperature, when the *LO*-phonon absorption can be neglected. Since we are not interested in the secondary-radiation line shape but only in the integral intensity of the scattering, we

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disregard dispersion and damping of the phonons. The second-order cross section for light scattering is defined as[1)] (Ref. 7)

$$\frac{d^2\sigma_2}{do\,d\omega_s}=\frac{V_0^2}{2\pi\hbar}\frac{\omega_s n(\omega_l)n(\omega_s)}{c^4}\left(\frac{e}{m_0}\right)^2|\mathbf{e}_s\mathbf{p}_{cv}|^2 \times F_2(\mathbf{r}=0,\mathbf{K}=0)\,\delta(\omega_l-\omega_s-2\omega_{LO}), \quad (3)$$

where o is the solid angle, ω_s the scattered-light frequency, $n(\omega)$ the refractive index of light at frequency ω, c the speed of light in vacuum, m_0 the free-electron mass, $\mathbf{p}_{cv}$ the interband matrix element of the momentum operator, $\mathbf{e}_s$ the polarization vector of the scattered light, and $F_2(\mathbf{r},\mathbf{K})$ a function that describes the distributions of the two-phonon-emitting EHP in the relative distance between the electron and hole and in the summary wave vector $\mathbf{K}$ (see Ref. 7 for an exact definition). Expression (3) emphasizes that EHP annihilation accompanied by emission of a secondary-light photon is possible if the electron and hole are at one and the same point of space, i.e. $\mathbf{r}-\mathbf{r}_1-\mathbf{r}_2=0$, and the total momentum of the pair is zero, i.e., $\mathbf{K}=\mathbf{k}_1+\mathbf{k}_2=0$, where $\mathbf{k}_j$ is the wave vector of the electron or hole. To calculate $F_2(\mathbf{r}=0,\mathbf{K}=0)$ we use a diagram technique,[7] and it is convenient to employ the expression

$$F_2(\mathbf{r}=0,\mathbf{K}=0)=2\sum_{\mathbf{q}}|A(\mathbf{q})|^2, \quad (4)$$

where $A(\mathbf{q})$ is the amplitude of the process and depends on the phonon wave vector $\mathbf{q}$. It is implied that

$$A(\mathbf{q})=\sum_{i=1}^{4}A_i(\mathbf{q}), \quad (5)$$

where $A_i(\mathbf{q})$ is the amplitude corresponding to diagram i in Fig. 1. The factor 2 in the right-hand side of (4) can be explained as follows. Let two phonons with wave vectors $\mathbf{q}$ and $-\mathbf{q}$ exist in the final state of the crystal. There exist then, besides the amplitudes shown in Fig. 1, four other amplitudes in which $\mathbf{q}$ is replaced by $-\mathbf{q}$. Since it will be shown below that $A_i(\mathbf{q})=A_i(-\mathbf{q})$, the doubling of the amplitudes leads to a factor 4. On the other hand, when summing over $\mathbf{q}$ we must introduce a factor 1/2, for otherwise one and the same final state will be taken into account twice. We get ultimately a factor 2 in Eq. (4), which takes then automatically into account all the possible diagrams for two-phonon scattering,[12] including those with crossing phonon lines. We assume hereafter that $m_h \gg m$.

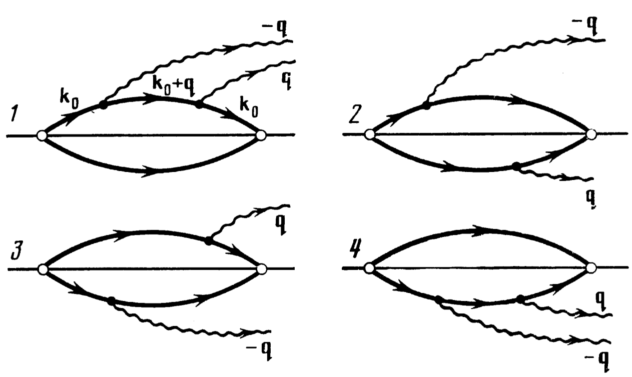

FIG. 1. Diagrams that contribute to the amplitude of two-phonon RRSL.

Let us describe in greater detail the calculation of the amplitude $A_1(\mathbf{q})$. With the aid of the diagram technique of Ref. 7 we obtain

$$A_1(\mathbf{q})=-\frac{C^2|M|}{\hbar^3V_0^{1/2}q^2}\sum_{\mathbf{k}_0}G(\mathbf{k}_0,\omega_0)G(\mathbf{k}_0+\mathbf{q},\omega_1)G(\mathbf{k}_0,\omega_2), \quad (6)$$

where

$$|M|^2=\left(\frac{e}{m_0}\right)^2\frac{2\pi\hbar}{V_0n^2(\omega_l)\omega_l}|\mathbf{e}_l\mathbf{p}_{cv}|^2, \quad (7)$$

$$G(\mathbf{k},\omega_n)=(\omega_n-\hbar k^2/2m+i\gamma_n/2)^{-1},$$

$$\omega_n=\omega_l-\frac{E_g}{\hbar}-n\omega_{LO},\qquad \gamma_n=\begin{cases}\gamma(\hbar\omega_n), & \omega_n>0\\ 0, & \omega_n<0\end{cases}, \quad (8)$$

and $\gamma(E)$ is the reciprocal lifetime of the electron in a state with kinetic energy E. We put in (8) $\gamma\to 0$ in those energy denominators of (6) which do not vanish at low values of k.

We transform in (6) to the coordinate representation of the Green's functions (cf. Ref. 8). We have

$$G(\mathbf{k},\omega_1)=\int d\mathbf{r}G(\mathbf{r})e^{-i\mathbf{kr}},$$
$$G(\mathbf{k},\omega_0)G(\mathbf{k},\omega_2)=\int d\mathbf{r}\mathcal{G}(\mathbf{r})e^{-i\mathbf{kr}}, \quad (9)$$

where

$$G(\mathbf{r})=-\frac{m}{2\pi\hbar r}e^{i\varkappa_1 r},\qquad \mathcal{G}(\mathbf{r})=\frac{m}{2\pi\hbar r}\frac{e^{i\varkappa_0 r}-e^{i\varkappa_2 r}}{\omega_0-\omega_2+i(\gamma_0-\gamma_2)/2},$$

$$\varkappa_n=\begin{cases}K_n+i/2\lambda_n, & \omega_n>0,\\ 1/2\lambda_n+iK_n & \omega_n<0,\end{cases}\qquad \lambda_n=\frac{\hbar K_n}{m\gamma_n}, \quad (10)$$

$$K_n=(2m/\hbar)^{1/2}|\tilde{\omega}_n|^{1/2},\qquad \tilde{\omega}_n={}^1/_2\omega_n[1+(1+\gamma_n^2/2\omega_n^2)^{1/2}].$$

Substituting (9) in (6) and summing over $\mathbf{k}_0$ we obtain

$$A_1(\mathbf{q})=\frac{-C^2|M|V_0^{1/2}}{\hbar^3q^2}\int d\mathbf{r}e^{i\mathbf{qr}}\mathcal{G}(\mathbf{r})G_1(-\mathbf{r}). \quad (11)$$

Calculating the contribution of diagram 4 of Fig. 1 by the same method we obtain

$$A_4(\mathbf{q})=-\frac{C^2|M|V_0^{1/2}}{\hbar^3q^2}\int d\mathbf{r}\mathcal{G}(\mathbf{r})G_1(-\mathbf{r}). \quad (12)$$

It can be shown (see Ref. 11) that in the limit as $m_h\to\infty$ the following relation holds:

$$A_2(\mathbf{q})+A_3(\mathbf{q})=-2A_1(\mathbf{q}). \quad (13)$$

With the aid of (5) and (11)–(13) we obtain as $m_h\to\infty$

$$A(\mathbf{q})=\frac{V_0^{1/2}C^2|M|m^2}{4\pi^2\hbar^5q^2(\omega_0-\omega_2)}\int\frac{d\mathbf{r}}{r^2}(1-e^{i\mathbf{qr}})e^{i\varkappa_1 r}(e^{i\varkappa_0 r}-e^{i\varkappa_2 r}). \quad (14)$$

We have neglected in (14) the quantity $\gamma_0-\gamma_2$ compared with $\omega_0-\omega_2=2\omega_{LO}$, while still assuming the electron-phonon interaction to be weak.

The integral with respect to $\mathbf{r}$ in the right-hand side of (14) can be calculated, so that we obtain an explicit dependence of the complex amplitude A on the absolute value of q. This dependence was obtained in Ref. 9 by another method. The amplitude as a function of q was calculated in Ref. 10 for an arbitrary mass ratio m/m_h. Since, however, the expression (14) for $A(\mathbf{q})$ as $m_h\to\infty$ turns out to be quite complicated (and even more so for $A(\mathbf{q})$ at arbitrary m/m_h), the integral with respect to $\mathbf{q}$ in the right-hand side of (4) has so far apparently not been calculated in explicit form. We propose another calculation method that yields explicit expression for $\sigma_2(\omega_l)$ for different range of ω_l in the limit as $m_h\to\infty$.

2. CALCULATION OF THE SECOND ORDER SCATTERING CROSS SECTION

Substituting (14) in (4) we obtain for $F_2(\mathbf{r}=0, \mathbf{K}=0)$ an expression in the form of a triple integral with respect to $\mathbf{q}$, $\mathbf{r}_1$, and $\mathbf{r}_2$. We integrate first with respect to $\mathbf{q}$, next with respect to the angles that define the directions of the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ in space, and finally with respect to the moduli r_1 and r_2. After rather lengthy calculations we get

$$F_2(\mathbf{r}=0,\mathbf{K}=0)=\frac{C^4|M|^2m^4V_0^2}{24\pi^2\hbar^{10}\omega_{LO}^2}J(\omega_l), \tag{15}$$

where

$$\begin{aligned}J(\omega_l)=&\frac{2}{\pi}\Big(\operatorname{arctg}\frac{a}{\Gamma_a}-\operatorname{arctg}\frac{b}{\Gamma_b}\\&+\operatorname{arctg}\frac{b-a}{\Gamma_a-\Gamma_b}\Big)\operatorname{Re}(\alpha_a^{-3}-\beta^{-3})\\&+\frac{2}{\pi}\operatorname{Im}\{(\alpha_a^{-1}-\beta^{-1})[(\alpha_a^*)^{-2}-(\beta^*)^{-2}]\}\\&+\frac{1}{\pi}\Big\{\ln[(a-b)^2+(\Gamma_a+\Gamma_b)^2]\\&\times\operatorname{Im}(\alpha_a^{-3}+\beta^{-3})-2\ln(2\Gamma_a)\operatorname{Im}\alpha_a^{-3}-2\ln(2\Gamma_b)\operatorname{Im}\beta^{-3}\\&+\ln\Big(\frac{a^2+\Gamma_a^2}{b^2+\Gamma_b^2}\Big)\operatorname{Im}(\alpha_a^{-3}-\beta^{-3})\Big\}.\end{aligned} \tag{16}$$

We use in (16) the notation

$$\alpha_a=a+i\Gamma_a=\varkappa_0+\varkappa_1,\quad \beta=b+i\Gamma_b=\varkappa_2+\varkappa_1. \tag{17}$$

According to (10), the quantities a, b, Γ_a, and Γ_b take different forms in different regions of the frequency ω_l. Letting $\gamma_n\to 0$, we have:

in the region $\omega_l - E_g/\hbar > 2\omega_{LO}$

$$a=\Big(\frac{2m}{\hbar}\Big)^{1/2}(\omega_0^{1/2}+\omega_1^{1/2}),$$

$$b=\Big(\frac{2m}{\hbar}\Big)^{1/2}(\omega_1^{1/2}+\omega_2^{1/2})\qquad \Gamma_a\to 0,\quad \Gamma_b\to 0; \tag{18a}$$

in the region $\omega_{LO} < \omega_l - E_g/\hbar < 2\omega_{LO}$

$$a=\Big(\frac{2m}{\hbar}\Big)^{1/2}(\omega_0^{1/2}+\omega_1^{1/2}),\quad b=\Big(\frac{2m}{\hbar}\Big)^{1/2}\omega_1^{1/2},$$

$$\Gamma_a\to 0,\quad \Gamma_b=\Big(\frac{2m}{\hbar}\Big)^{1/2}|\omega_2|^{1/2}; \tag{18b}$$

in the region $0 < \omega_l - E_g/\hbar < \omega_{LO}$

$$a=\Big(\frac{2m}{\hbar}\Big)^{1/2}\omega_0^{1/2},\quad b\to 0,\quad \Gamma_a=\Big(\frac{2m}{\hbar}\Big)^{1/2}|\omega_1|^{1/2},$$

$$\Gamma_b=\Big(\frac{2m}{\hbar}\Big)^{1/2}(|\omega_1|^{1/2}+|\omega_2|^{1/2}), \tag{18c}$$

and in the region $\omega_l - E_g/\hbar < 0$

$$a\to 0,\quad b\to 0,\quad \Gamma_a=\Big(\frac{2m}{\hbar}\Big)^{1/2}(|\omega_0|^{1/2}+|\omega_1|^{1/2}),$$

$$\Gamma_b=\Big(\frac{2m}{\hbar}\Big)^{1/2}(|\omega_1|^{1/2}+|\omega_2|^{1/2}). \tag{18d}$$

Substituting (15) in (3) and transforming to the integral scattering cross section

$$\sigma_2(\omega_l)=\int\frac{d^2\sigma_2}{do\,d\omega_s}d\omega_s\,do, \tag{19}$$

we get

$$\sigma_2(\omega_l)=\sigma_0J(\omega_l)/l^3, \tag{20}$$

where

$$\sigma_0=\frac{2\pi}{3}V_0\Big(\frac{e}{m_0c}\Big)^4|\mathbf{e}_s\mathbf{p}_{cv}|^2|\mathbf{e}_l\mathbf{p}_{cv}|^2\frac{\omega_s n(\omega_s)}{\omega_l n(\omega_l)}\alpha^2\frac{m^2l}{\hbar^4}. \tag{21}$$

3. DISCUSSION OF RESULTS

Thus, starting from the general relation (3), we obtained an expression for the cross section for two-phonon RRSL involving free EHP in the case of Fröhlich electron-phonon interaction and in the limit as $m_h/m\to\infty$. The results are valid for any frequency ω_l both below and above the intrinsic absorption edge. With the aid of (16) and (20) we obtain the following results for four different regions of the frequencies ω_l of the exciting light:

at $x>2$, where $x=(\omega_l - E_g/\hbar)/\omega_{LO}$

$$\sigma_2^{(a)}=\sigma_0[f^{-3}(x-1)-f^{-3}(x)],\qquad f(x)=|x|^{1/2}+|x-1|^{1/2}; \tag{22a}$$

at $1<x<2$

$$\begin{aligned}\sigma_2^{(b)}=\sigma_0\Big\{&\frac{(2-x)^{1/2}}{\pi}[4f^{-2}(x)+2(4x-5)\ln([2(2-x)]^{1/2}f(x))]\\&+[f^{-3}(x)-(4x-7)(x-1)^{1/2}]\\&\times\Big[1-\frac{2}{\pi}\operatorname{arctg}\Big(\frac{x-1}{2-x}\Big)^{1/2}-\frac{2}{\pi}\operatorname{arctg}\Big(\frac{x}{2-x}\Big)^{1/2}\Big]\Big\};\end{aligned} \tag{22b}$$

at $0<x<1$

$$\begin{aligned}\sigma_2^{(c)}=\frac{\sigma_0}{\pi}\Big\{&f^{-3}(x-1)\Big(2-\ln\frac{2}{f^2(x-1)}\Big)\\&+2f^{-1}(x-1)+(1-x)^{1/2}(4x-1)\\&\times\ln 2(1-x)+2x^{1/2}(4x-3)\\&\Big[\operatorname{arctg}\Big(\frac{x}{1-x}\Big)^{1/2}-\operatorname{arctg}\frac{x^{1/2}}{(2-x)^{1/2}+2(1-x)^{1/2}}\Big]\Big\};\end{aligned} \tag{22c}$$

at $x<2$

$$\begin{aligned}\sigma_2^{(d)}=\frac{2\sigma_0}{\pi}\Big\{&f^{-3}(x)\Big[1+\ln\frac{1}{2}(1+f(x)f^{-1}(x-1))\Big]+f^{-3}(x-1)\\&\times\Big[1+\ln\frac{1}{2}\Big(1+\frac{f(x-1)}{f(x)}\Big)\Big]\\&-f^{-1}(x)f^{-1}(x-1)[f^{-1}(x)+f^{-1}(x-1)]\Big\}.\end{aligned} \tag{22d}$$

Figure 2 shows the dimensionless quantity σ_2/σ_0 as a function of the dimensionless parameter $x=(\omega_l - E_g/\hbar)/\omega_{LO}$. It can be seen from this figure that σ_2/σ_0 increases with increasing x and reaches maximum values at $\omega_l = E_g/\hbar + \omega_{LO}$ and $\omega_l = E_g/\hbar + 2\omega_{LO}$, after which it decreases monotonically with increasing x at $x>2$. We note that Eq. (16) admits of finite electron-damping values γ_n. Allowance for finite n would smooth out the singularities that can be clearly seen at the points $x=0$, $x=1$, and $x=2$ in Fig. 2.

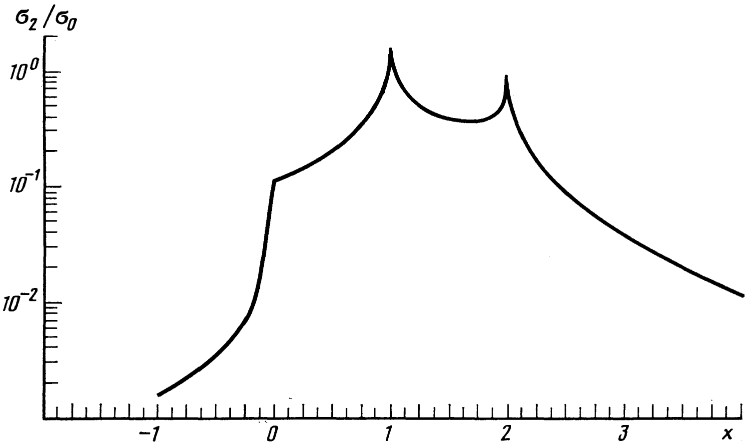

FIG. 2. Cross section ratio σ_2/σ_0 for two-phonon RRSL involving free EHP vs the dimensionless quantity $x=(\hbar\omega_l - E_g)/\hbar\omega_{LO}$ as $m_h \rightarrow \infty$.

The following physical conclusions can be drawn from the results:

1. In the limit as $\gamma \rightarrow 0$ the contribution of the free EHP to the two-phonon RRSL cross section is proportional to α^2 in all four ranges of ω_l. The reason is that the transitions are via virtual (short-lived) excited states of the crystal, in contrast to multiphonon RRSL involving Wannier-Mott excitons in the regions $\omega_l > E_1/\hbar + (N-1)\omega_{LO}$, where N is the order of the scattering, and from multiphonon RRSL at $N \geqslant 4$ involving EHP in the regions $\omega_l > E_g/\hbar + (N-1)\omega_{LO}$, when the decisive role is played by transitions via real (long-lived) intermediate states of the crystal. We note that when a strong magnetic field is turned on two-phonon scattering involving free EHP also proceeds via real intermediate states if the frequency ω_l is high enough,[13] so that the order of magnitude of the cross section σ_2 changes relative to the constant α, viz., in a strong magnetic field it is proportional to α as against α^2 when $H=0$.

2. The plot of σ_2/σ_0 vs x in Fig. 2 is not symmetric; the increase of σ_2/σ_0 with increasing x in the region $x<0$ is much faster than the decrease at $x>2$. This asymetry is attributed to the presence of "double resonance" in the region $x>2$, viz., two of the three energy denominators in the right-hand side of (6) can vanish simultaneously. It becomes clear therefore that the possibility of vanishing of the energy denominators cannot serve as a criterion that identifies the intermediate states of the transitions as virtual or real (see also Ref. 7). However, the presence of double resonance alters substantially the character of the dependence of the cross section on ω_l.

3. In the case of a Fröhlich interaction the cross section tends to a finite limit as $m_h/m \rightarrow \infty$. Comparing the present results with those of Ref. 11 we conclude that this last conclusion is typical just of the interaction (2), whereas in the case of $C_{jq}=\text{const}$ (which corresponds to the Zeyher model[12] or to electron-phonon interaction) the contribution of the free EHP to the cross section σ_2 at $m_h \gg m$ contains terms proportional to $(m_h/m)^{3/2}$ and $(m_h/m)^{1/2}$, i.e., it tends to infinity as $m_h/m \rightarrow \infty$. The contribution proportional to $(m_h/m)^{3/2}$ to the cross section is due to processes in which the phonons are emitted by holes, the quasimomenta of the holes are strongly changed after the emission of the phonons, and all the states of the holes in the region of the Brillouin zone with radius $(2m_h\omega_1/\hbar)^{1/2}$ make equal contributions (if $C_{jq}=\text{const}$).

For comparison, we consider the exciton contribution to σ_2 below the Debye temperature. In the region $\omega_l - E_1/\hbar > 2\omega_{LO}$ we have (see, e.g., Ref. 6)

$$\sigma_2(\omega_l) = A\sigma_{exc}(\omega_l)\,\frac{W_l(\hbar\omega_l - E_1 - \hbar\omega_{LO})}{\gamma_{exc}(\hbar\omega_l - E_1 - \hbar\omega_{LO})}, \qquad (23)$$

where A is a numerical factor between 1 and 2; $\sigma_{\text{exc}}(\omega_l)$ is the cross section for absorption of light of frequency ω_l accompanied by indirect production of a hot exciton; $W_l(E)$ is the probability of annihilation of an exciton of kinetic energy E, accompanied by emission of a phonon and an LO phonon; $\gamma_{\text{exc}}(E)$ is the total reciprocal lifetime of the exciton in the ground state $(n=1)$ and having a kinetic energy E; $E_1 = E_g - \Delta E$; ΔE is the exciton binding energy.

It follows from (23) that in the region $\omega_l - E_1/\hbar > 2\omega_{LO}$ we have $\sigma_2(\omega_l) \propto \alpha$, and in the region $\omega_{LO} < \omega_l - E_1/\hbar < 2\omega_{LO}$ the cross section should increase very rapidly, since the lifetime of an exciton of energy $\hbar\omega_l - E_1 - \hbar\omega_{LO}$ is determined by the weak and almost elastic scattering by acoustic phonons. If only processes with LO-phonon emission are formally taken into account, we have $\gamma_{\text{exc}} \rightarrow 0$ under the condition $\omega_{LO} < \omega_l - E_1/\hbar < 2\omega_{LO}$ and the cross section (23) tends to infinity. It can be seen from Eqs. (22) and from Fig. 2 that nothing like this takes place in scattering involving EHP. There is no fast increase of the cross section in the region $\omega_l - E_g/\hbar < 2\omega_{LO}$.

These results of the present paper can be used directly for comparison with experimental data for semiconductors in which the condition $m_h/m \gg 1$ is satisfied and the exciton effects are weak (e.g., in InSb at ω_l close to the intrinsic absorption edge) For those semiconductors in which exciton effects are distinctly observed, the expressions derived can be used to isolate the contributions of the free EHP and of the excitons to the total cross section $\sigma_2(\omega_l)$.

[1)]In Eq. (3) and below no account is taken of the electron spin. If it is assumed that the spin projection of an electron passing from the valence to the conduction band can take on two values, double summation over the intermediate states yields an additional factor 4 in the right hand side of (3).

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