One-dimensional gasdynamic simulation of electron-beam transport in a rarefied gas

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Beam transport in a rarefied gas is investigated in the one-dimensional two-fluid gasdynamic approximation with account taken of the self-consistent electric field. The conditions are analyzed for the development of beam-plasma oscillations and instabilities that cause total stopping of the beam. A quasi-one-dimensional self-consistent gasdynamic model is proposed with which to take approximate account of the transverse confinement of a real system. The results of the simulation are discussed.

INTRODUCTION

When an electron bean propagating in a rarefied gas interacts with the plasma produced by collisional ionization of the gas by the beam, intense beam-plasma oscillations can set in. As three oscillations build up, the electrons can also acquire an energy sufficient to ionize by collision of the neutral gas, and the ionization becomes avalanche-like, i.e., its rate can increase rapidly and the beam can be stopped at the distances much shorter than the electron mean free path. This process was named beam-plasma discharge (BPD).[1]

Transport of an electron beam in a gas is simultaneously accompanied by many interrelated processes, viz., ionization and excitation of neutral molecules, elastic scattering, recombination, diffusion, motion in a self-consistent electromagnetic field, as well as plasma oscillations and others. Investigation of the BPD is made complicated by the large number of these interrelated processes, by the differences in their scales, and by their essential nonlinearity as manifest in the nonlinearity of the equations that describe the system evolution. One of most important causes of the nonlinearity of the processes is the self-consistent electromagnetic field.

In theoretical investigations of beam-plasma instabilities by analytic or numerical methods, use is made of gasdynamic equations,[2,3] Monte Carlo method,[4] "large-particle" methods,[5,6] and simplified phenomenological models.[7–9] Substantial simplifications are usually introduced to facilitate the analysis. For examples, the system is assumed to be stationary,[3] spatially homogeneous.[7] or collisionless[5,6]; the self-field of the beam and plasma is disregarded[4] or approximated,[2,7–9] no account is taken of elastic collisions,[3] of the beam deceleration,[7] etc.

In view of these difficulties, the only theory developed to some degree of completeness for the BPD is a linear one that permits an estimate of the threshold of this process. Practical applications, however, of electron-beam technical apparatus intended for melting, cutting, and drilling metals with an electron beam, as well as the problem of energy transfer by an electron beam, i.e., situations in which electron-beam transport is vital, call for an investigation of the nonlinear spatial and temporal dynamics of the interaction of a beam with a gas, and for determination of the regimes in which the onset of beam-plasma instabilities does not shut off the beam current. We propose in this paper a two-fluid gasdynamic model a bounded beam-plasma system and report the results obtained with this model.

1. ONE-DIMENSIONAL GASDYNAMIC DESCRIPTION OF BEAM-PLASMA INTERACTIONS

We represent the mathematical model of the investigated beam-plasma instability in the form of one-dimensional quasilinear gasdynamic expressions for the basic physical conservation laws:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial z}=-\frac{\eta E}{\gamma^3}-\frac{\varepsilon_i}{m_0\gamma^3}(n_0-n_i)\sigma_i(u)\left(\eta=\frac{e}{m_0}\right), \quad (1)$$

$$\frac{\partial n_b}{\partial t}+\frac{\partial}{\partial z}(n_b u)=0, \quad (2)$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial z}=-\eta E-\nu v-\frac{\varepsilon_i}{m_0}(n_0-n_i)\sigma_i(v_t)\frac{v}{v_t}, \quad (3)$$

$$\frac{\partial n_e}{\partial t}+\frac{\partial}{\partial z}(n_e v)=n_b(n_0-n_i)\sigma_i(u)u+n_e(n_0-n_i)\sigma_i(v_t)v_t-\beta_r n_e(n_{i0}+n_i), \quad (4)$$

$$\frac{\partial T_e}{\partial t}+v\frac{\partial T_e}{\partial z}=-\varepsilon_i(n_0-n_i)\sigma_i(v_t)v_t\frac{1{,}5T_e}{\varepsilon_t}+2\nu\varepsilon, \quad (5)$$

and the equation for the self-consistent electric field

$$\operatorname{div}\mathbf{E}=\frac{e}{\varepsilon_0}(n_i+n_{i0}-n_b-n_e), \quad (6)$$

which in the one-dimensional case can also be written in the form

$$E(z,t)=\frac{e}{2\varepsilon_0}\int_0^l[n_i(z',t)+n_{i0}-n_b(z',t)-n_e(z',t)]\operatorname{sign}(z-z')\,dz', \quad (7)$$

$$\operatorname{sign}(x)=-1 \quad \text{at} \quad x<0,$$
$$\operatorname{sign}(x)=0 \quad \text{at} \quad x=0,$$
$$\operatorname{sign}(x)=1 \quad \text{at} \quad x>0.$$

In (1)–(7) z is the longitudinal coordinate axis along which the electronbeam propagates; $0\leqslant z\leqslant l$ is the considered region of length l, t the time, e and m_0 the absolute value of the charge and the electron rest mass, ε_0 the dielectric constant of vacuum, u and n_b the velocity and density of the

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electron beam; v, v_t, n_e, ε, ε_t, and T_e respectively the longitudinal velocity, the total mean squared velocity, the density, the longitudinal kinetic energy, the total kinetic energy, and the temperature of the plasma electrons, all connected by the equations

$$\varepsilon=\frac{m_0v^2}{2},\quad \varepsilon_t=\frac{m_0v_t^2}{2}=\varepsilon+\frac{3T_e}{2}; \tag{8}$$

n_0 and n_i are the densities of the neutral-gas molecules and of the produced positive ions, n_{i0} the density of the neutralizing ion background, ε_i the energy lost by an electron to ionization of a neutral-gas molecule, $\sigma_i(u)$ the ionization cross section as specified by the experimental curve, and ν the elastic collision frequency defined as

$$\nu=\overline{\Delta\theta^2}/(2\Delta t)=|\Delta\varepsilon/\varepsilon|/(2\Delta t),$$

where $\overline{\Delta\theta^2}$ and $|\Delta\varepsilon/\varepsilon|$ are the mean squared scattering angle and the relative energy loss of longitudinal energy of the electrons during the time Δt, β_r the recombination coefficient, E the longitudinal electric field intensity, and

$$\gamma=(1-u^2/c^2)^{-1/2}$$

the Lorentz factor (c is the speed of light). The first term in the right-hand side of (5) describes the plasma thermal-energy loss to gas ionization, and the second the conversion of the longitudinal energy into thermal via eleastic collisions. The solution (7) of Eq. (6) was written for unbounded space under the assumption that the electron beam is injected at $z=0$, and the charges that leave the region $0\leqslant z\leqslant 1$ considered are completely absorbed.

It can be seen from (1)–(7) that models can be used for the motion and the ionization losses of the beam electrons in a self-consistent field. Moreover, one can model for plasma electrons the collisions, ionization by the beam, ionization by the plasma electrons, recombination, and thermalization, i.e., the conversion of part of the translational energy into thermal. We note that the ratios v/v_t in (3) and $1.5T_e/\varepsilon_t$ in (5) take into account the fact that the plasma-electron ionization loss consists of the translational energy loss and the thermal-energy loss in accordance with Eq. (8). The excitation (inelastic collisions) and dissociation of the molecules, sticking (formation of negative ions), multiple ionization, and diffusion are not taken into account since these processes are not decisive for the investigated beam-plasma interactions. For simplicity, ν and β_r are assumed constant.

It is assumed that at the entrance into the system ($z=0$) the beam velocity is $u(0,t)u_0=(2\eta\varepsilon_{b0})^{1/2}$, where ε_{b0} (in eV is the initial energy of the beam electrons, the density is $n_b(0,t)=n_{b0}$, and the values of $v(0,t)$ and $n_e(0,t)$ are determined from the condition that the solution be smooth in second order. We assume that at the initial instant $t=0$.

$$\begin{gathered}n_b(z,0)=n_{b0}=n_{i0}=\text{const},\quad u(z,0)=u_0=\text{const},\\ n_i(z,0)=n_e(z,0)=T_e(z,0)=v(z,0)=0,\end{gathered} \tag{9}$$

i.e., the system is quasineutral, with the homogeneous beam filling the entire system volume and there are no plasma electrons. Thus, we disregard the front of the beam injection into the plasma and the associated processes, particularly the reverse current. (The reverse current flowing in the plasma upon injection of a relativistic electron beam is approximately estimated, e.g., in Ref. 2). This neglect is justified, since the characteristic ionization time $\tau_{i0}=[\sigma_i(u_0)u_0n_0]^{-1}$ in the out system is substantially longer than the passage time of the front, which is of the order of $\tau_0=l/u_0$, where τ_0 is the time of flight of an electron of velocity u_0 through the system. The beam is initially injected into the neutral gas and positive ions are graduaty accumulated and cancel the beam space charge, and the produced plasma electrons are crowded out of the system. Plasma electrons begin to accumulate only after the beam-electron space charge is completely cancelled by the positive-ion charge, i.e., when $n_{b0}=n_{i0}$.

During the start of the process, the first term of Eq. (4) for the plasma-electron beam density predominates. It describes the growth of the plasma density as a result of ionization of the neutral atoms by the plasma electrons. Next, as the total kinetic energy of the plasma electrons increases, ionization by plasma electrons, described by the second term in the right-hand side of this equation, begins to predominate.

The positive ions are assumed immobile during the times considered, i.e., it is assumed that $m_0\ll M$, where M is the positive-ion mass. Thus, the density distribution of the produced positive ions during an instant of time t is defined by the integral

$$n_i(z,t)=\int_0^t[n_b(n_0-n_i)\sigma_i(u)u+n_e(n_0-n_i)\sigma_i(v_t)v_t-\beta_rn_e(n_{i0}+n_i)]dt', \tag{10}$$

which makes Eqs. (1)–(5) and (7) a closed system.

This gasdynamic model enables us to investigate essentially nonlinear regimes of beam-plasma interaction and perform computer experiments much more rapidly than on the basis of kinetic models such as the large-particle model. The use of the two-fluid gasdynamic description precludes the possibility of reproducing multistream phenomena, including multistream instabilities and capture of the electron beam. For the same reason, the reverse motion of the electron beam is excluded and the simulation duration is limited in practice by the instant when the beam is completely stopped somewhere inside the system.

2. RESULTS

The quasilinear system (1)–(5), (7), and (10) with initial conditions (9) is solved by finite-difference methods. Figures 1 and 2 show some results of the mathematical simulation of typical regimes with parameters $n_0=1.7\cdot10^{19}\ \mathrm{m}^{-3}$, $\varepsilon_{b0}=10\ \mathrm{keV}$, $n_{b0}=0.8\cdot10^{14}\ \mathrm{m}^{-3}$ (Fig. 1) and $n_0=1.2\cdot10^{19}\ \mathrm{m}^{-3}$, $\varepsilon_{b0}=7\ \mathrm{keV}$, $n_{b0}=0.95\cdot10^{14}\ \mathrm{m}^{-3}$ (Fig. 2); the system length is $l=1\ m$. It can be seen from the figures that development of beam-plasma instability in the model under the influence of random fluctuation decrease the beam velocity u to zero within (4–5) τ_0 (curve 3 of Fig. 1 and curve 1 of Fig. 2). These results differ from the experimental ones,[10] according to which the development of intense beam-plasma oscillations begins only after the plasma-electron density n_e reaches a certain threshold value.

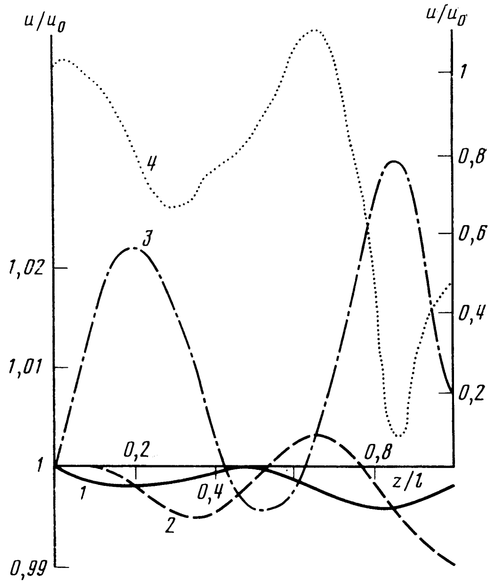

FIG. 1. Evolution of oscillations in a beam-plasma system: 1 — $t/\tau_0 = 3$; 2 — $t/\tau_0 = 3.5$; 3 — $t/\tau_0 = 4$; 4 — $t/\tau_0 = 5.38$ (the ordinate axis is on the left for curves 1–3 and on the right for curve 4).

The discrepancy between the results can be understood by analyzing the dispersion relation.

$$\frac{\omega_b^2}{(\omega-ku_0)^2}+\frac{\omega_e^2}{\omega(\omega-i\nu)}=1 \quad (i=\sqrt{-1}), \tag{11}$$

which can be easily obtained by linearizing the system (1)–(6) relative to a small harmonic perturbation with frequency ω and wave number k. In Eq. (11), $\omega_b^2 = \eta e n_{b0}/\varepsilon_0$ and $\omega_e^2 = \eta e n_{e0}/\varepsilon_0$, where n_{e0} is the dc component of the plasma-electron density. It can be shown that if elastic collisions are disregarded, i.e., at $\nu = 0$, Eq. (11) has two complex-conjugates and two real solutions if one of the plasma frequencies, ω_b or ω_e exceeds k_{u0}, i.e., $\omega_b > ku_0$ or $\omega_e > ku_0$. These are therefore the conditions for the onset of hydrodynamic two-stream instability.

The wave number k is determined by the geometry of the physical system. The minimum value of the wave number k, which is obtained according to experiment in a planar one-dimensional bounded system, is $k_{\min,l} = \pi/l$ (Ref. 11). It is also easy to show that in the case of a cylindrical beam of radius R the minimum wave number in a conducting tube is $k_{\min,R} = 2.4/R$.

A value of $k_{\min}$ limited by the transverse dimension of the beam is realized in experiment,[10] whereas in the one-dimensional model there is only one characteristic dimension, $l \gg R$, to which $k_{\min,l} \ll k_{\min,R}$ corresponds. There in the investigated model $\omega_b > k_{\min,l}\, u_0$ for the same parameters (cited above) as in the experiment, and instability sets in at any plasma density, whereas in experiment we must increase enough to meet the inequality $\omega_e > k_{\min} u_0$.

In addition, as shown by Pierce,[12] a quasineutral beam can become unstable to small fluctuations of its density. The condition for the onset of this instability in the one-dimensional case is $\omega_b > \pi u_0/l$ which coincides with the condition $\omega_b > k_{\min} u_0$ for a one-dimensional system when $k_{\min} = k_{\min,l} = \pi/l$.

The buildup of beam-plasma oscillations slows down with increasing elastic-collision frequency ν, i.e., with increase of the electron deceleration force due to these collisions. In the computer experiment, inceasing the frequency ν by the three orders increased the time to stop the beam fully by approximately three times. In this simulation an active part was played in the ionization of neutral atoms not only by the beam electrons but also by the plasma electrons whose energy increased in the beam-plasma oscillations. As a result, the growth rate of the plasma density at a certain instant of time (marked by the cross in Fig. 3) increased jumpwise by approximately 2.3 times at a plasma density comparable with the beam density.

The rapidly growing oscillations revealed by the mathematical simulation are thus the consequence of superposition of hydrodynamic and Pierce instabilities, the latter being decisive. In the strong fields resulting from the instability development, the plasma electrons, are accelerated to energies sufficient to ionize the neutral molecules (see the $n_i(z)$ distribution in Fig. 2. In addition, the beam electrons decelerated in these fields and bunched into clusters also increase the ionization of the neutral gas (corresponding to the peak of $n_i(z)$ near $z = l$).

The one-dimensional model parameters can, of course be chosen such that the instability develops only after the plasma electron density reaches a certain critical value, similar to the development of the process in the experiment. This can be achieved, e.g., by decreasing the beam-electron density. Indeed, the mathematical simulation showed that in this case the oscillations begin to grow only after the plasma electrons exceed a certain value.

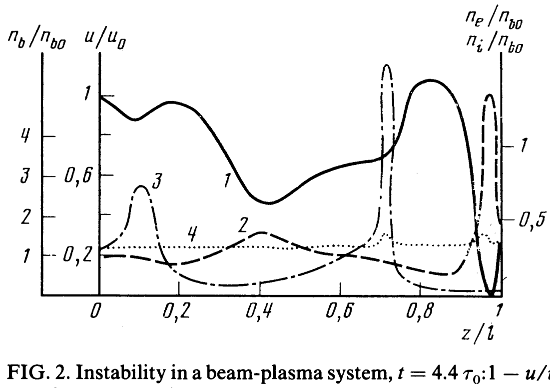

FIG. 2. Instability in a beam-plasma system, $t = 4.4\,\tau_0$: 1 — u/u_0; 2 — n_b/n_{b0}; 3 — n_e/n_{b0}; 4 — n_i/n_{b0}.

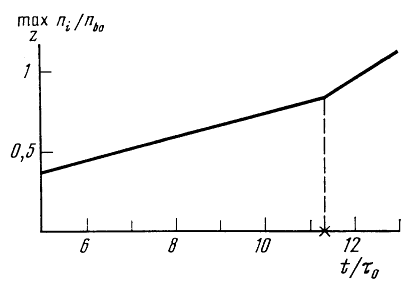

FIG. 3. Growth of density of a strongly collision-dominated plasma.

One other approach, which meets the experimental conditions, to the calculation of the critical regime of instability development would be to decrease the length of the investigated one-dimensional system. If the length of the model system is assumed approximately equal to the beam radius, i.e., $l \approx R$, then $k_{\min,l} \approx k_{\min,R}$. In this case, at the same beam parameters as in the experiment, instability will develop only after the plasma density reaches a certain critical value.

Indeed, in the numerical experiment, the instability vanished at low plasma densities. Growth of plasma-beam oscillations became observable in the system only after the plasma density exceeded the beam density by approximately one order of magnitude. To accelerate the simulation, the beam was introduced into an already prepared plasma with uniform initial density $n_e(z,0) = n_{e0} = \text{const} > 0$. An increase of the oscillation growth rate, i.e., of the instability growth rate, was observed in the course of the simulation as the plasma electron density increased. The wave numbers of the oscillations were $k = \pi/l$, i.e., the system length l was spanned by half the oscillation wavelength. The instability vanished when the neutral-gas pressure was increased onto $10^{-2} - 10^{-1}$ Torr (depending on the plasma-electron density).

3. APPROXIMATE ALLOWANCE FOR THE SYSTEM FINITE TRANSVERSE SIZE

It can be seen from the above analytic and numerical results that, at equal parameters, the evolutions of the processes in the one-dimensional model and in experiment differ substantially. The reason is that the system is bounded in the transverse direction, i.e., one more characteristic dimension is present, the beam radius $R \ll l$. Usually, particularly in the investigated system, a real beam propagates inside a metal tube. The interaction of the free electrons with those of the bounding conductor screens the space-charge field.

The reason for this is that although in real systems the transverse particle motion is usually restricted by strong uniform magnetic fields, an electric-field gradient is present. Seeking a solution of the equation for the potential

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right)+\frac{\partial^2\varphi}{\partial z^2}=-\frac{\rho}{\varepsilon_0}$$

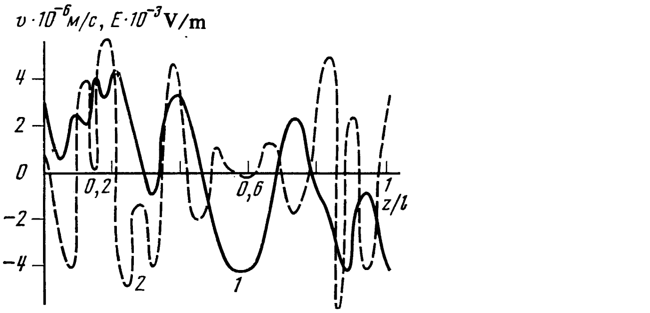

FIG. 4. Plots of $w(z,t)$ (curve 1) and $E(z,t)$ (curve 2) in a quasi-one-dimensional beam-plasma system with $l = 1$ m and $R = 0.06$ m at an instant $t = 15\,\tau_0$.

in an axisymmetric system of radius R in the form $\varphi(r,z) = \Phi(r)\chi(z)$ stipulating that $\Phi(R) = 0$ on the bounding conducting tube, we can choose the radially dependent part of the solution to be $\Phi(r) = J_0(\beta_r)$, where $\beta = 2.4/R$. We then obtain for $\chi(z)$ at fixed r, say $r = 0$, the equation

$$\chi'' - \beta^2\chi = -\rho/\varepsilon_0.$$

With the aid of the corresponding Green's function we obtain from this expressions for the potential

$$\chi(z) = -\frac{1}{2\beta\varepsilon_0}\int_0^l \rho(z')e^{-\beta|z-z'|}dz'$$

and for the z-component of the electric field

$$E = \frac{1}{2\varepsilon_0}\int_0^l \rho(z')e^{-\beta|z-z'|}\,\text{sign}(z-z')\,dz'. \qquad (12)$$

Equation (12) (Ref. 13), in contrast to (7), has under the integral sign an exponential factor that decreases rapidly away from the point z. (This exponential can also be treated as an approximate allowance for the screening of the field of a point charge in a bounded plasma.) Equation (12) for the electric field in the system thus allows us to introduce into the one-dimensional system another characteristic parameter R in addition to l. In such a model, which can be called quasi-one-dimensional, the strong screening of the space-charge field at distances larger than R makes the maximum

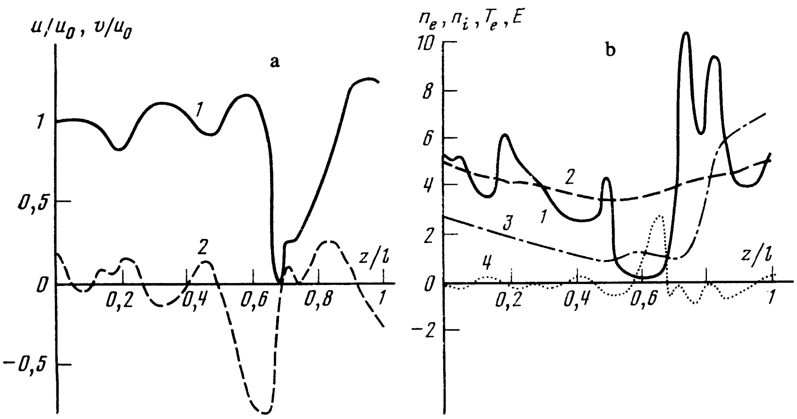

FIG. 5. Beam-plasma instability at the instant of beam-current shutoff in a quasi-one-dimensional system with $l = 1$ m, $R = 0.06$ m, $t = 29\,\tau_0$: a—curves 1—u/u_0, 2—v/u_0; b—curves 1—$n_e \cdot 10^{-14}$ m^{-3}, 2—$n_i \cdot 10^{-14}$ m^{-3}, 3—$T_e \cdot 10^{-2}$ eV, 4—$E \cdot 10^{-5}$ V/m.

characteristic wavelength in the system $\lambda_{\max} \sim R$. Hence $k_{\min} \sim 1/R$.

The simulation, whose results are shown in Figs. 4 and 5, has shown that allowance for screening gives rise in a system of length l to oscillations of wavelength $\lambda \sim R$, i.e., with wavelengths shorter than without allowance for screening (curve 2 of Fig. 4). However, to take into account a characteristic dimension $R \ll l$ the computation scheme must satisfy more stringent requirements. In particular, it is necessary in this case to increase, relative to R, the number of spatial subdivision points, and this in turn requires a longer computation time. Figure 5 shows the variation of the characteristic quantities in the system as functions of z. It can be seen from this figure that development of beam-plasma oscillations in the system gives rise, besides beam stopping, also to acceleration of part of the beam electrons to energies approximately 1.4 times their initial energy (curve 1 in Fig. 5a). We note also that although part of the plasma electrons is accelerated in the plasma oscillations to energies comparable with the beam energy, their energy is still almost half the initial energy of the beam electrons (curve 2 of Fig. 5a) and, as can be seen from the $n_e(z)$ plot (curve 1 of Fig. 5b), the high-energy fraction of the plasma electrons is insignificant.

CONCLUSIONS

We presented a self-consistent one-dimensional gasdynamic model that permits the transport of an electron beam in a rarefied plasma to be investigated without using a number of customary simplifications. Thus, e.g., in Ref. 6 no account was taken of the interaction between a beam and the plasma it produces. Allowance for this interaction shortens severalfold the two-stream instability development time. In contrast to Refs. 3 and 7 the present paper considers the self-consistent problem without assuming temporal or spatial homogeneity of the system. This allows the plasma-beam interaction to be simulated. The plasma-beam instability problem is solved in Ref. 8 in a linear approximation that is valid only at small perturbations of the physical-system parameters. Nor is account taken in that reference of the fact that real systems are laterally limited. We have shown that this neglect leads to a simultaneous rapid development of two-stream and plasma-beam instabilities during the linear and strongly nonlinear stages of the process. By foregoing these assumptions we can simulate the space and time dynamics of the process. A computer experiment was used to investigate the nonlinear interaction of an electron beam with a plasma produced by shock ionization. For a system substantially longer than the beam radius in a real experiment, the model reveals a strong plasma-beam-oscillation growth that leads to stopping of the beam after approximately several flight times of the beam through the system. An instability comprising a superposition of a Pierce and of a gasdynamic two-stream instability develops rapidly because the system model is not laterally bounded.

The proposed quasi-one-dimensional gasdynamic model modification that lets the space and time dynamics of beam-plasma interaction to be simulated assuming the system to have lateral boundaries, permits investigation of the process at model parameters that agree with experiment.

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