Multiple Coulomb scattering of ultrarelativistic charged particles moving at small angles to crystallographic planes

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It is shown that if an ultrarelativistic charged particle intersects crystallographic planes at a sufficiently small angle α, the mean square angle of its deviation in the plane perpendicular to the crystallographic plane decreases in comparison with the case of an amorphous material or of a crystal in the case of large α. The effect depends on the temperature, and its magnitude can reach 10–20%.

It was shown long ago by Ter-Mikaelyan[1] that the ordered location of the atoms in a crystal can influence the characteristics of multiple Coulomb scattering. Subsequently it was discovered that Ter-Mikaelyan's results are valid only for very thin crystalline layers in which it is possible to neglect phenomena associated with channeling in motion of charged particles near the principal crystallographic axes or planes. The features of multiple Coulomb scattering under conditions of channeling have also been discussed in a number of studies (see for example Refs. 2 and 3 and references given in these publications). In the present note we discuss the multiple Coulomb scattering of ultrarelativistic particles traveling at a small angle α to crystallographic planes under the condition that this angle still exceeds the Lindhard critical angle α_L at which channeling is possible (i.e., we are discussing here so-called super-barrier particles which are not captured into the regime of planar channeling).

If the angle is

$$\alpha < \tilde{\alpha} \sim b^2/d^2, \tag{1}$$

where d is the distance between neighboring atoms and b is the effective thickness of the planar layer, which is determined by the screening radius and the amplitude of thermal vibrations of the nuclei, then a charged particle crossing the plane will interact in a correlated way with a large number of atoms. As is well known, under these conditions the discrete distribution of charges can be replaced by a continuous distribution which produces an average effective potential U. Usually in order of magnitude the characteristic angle is $\tilde{\alpha} \sim 10^{-2}$. On the other hand, as we have already mentioned, we shall assume that

$$\alpha \gg \alpha_L \approx (U/mc^2\gamma)^{1/2}, \tag{2}$$

where α_L is the Lindhard limiting angle,[4] m is the mass of the particle, and γ is its Lorentz factor. The Lindhard angle drops as the particle energy increases, and for electrons and positrons with energies of several GeV it is about 10^{-4}. Therefore for such particles the following double inequality is satisfied with a large margin:

$$\alpha_L \ll \alpha \ll \tilde{\alpha}. \tag{3}$$

The direction of motion of a particle in a crystal changes mainly as the result of multiple Coulomb scattering. We shall consider first scattering by an individual atom of the medium. As is well known, this reduces to elastic scattering by a screened nucleus and to inelastic collisions in which only the quantum states of the electrons change. If in elastic scattering by an atom the scattering angle η satisfies the condition $\eta < \hbar/pa_n$, where a_n is the size of the region of localization of the nucleus near the crystal lattice site and p is the momentum of the particle, then the nature of the vibrational motion of the nucleus has a high probability of not changing. On the other hand, for $\eta > \hbar/pa_n$ the probability of change in the vibrational state of the nucleus is high.

In an amorphous material the atoms are located randomly. The same thing actually applies to atoms which are near the trajectory of a particle in the crystal for $\alpha > \tilde{\alpha}$. Therefore both inelastic scattering and the two forms of elastic scattering of a particle by an atom contribute to the quantity $\overline{\eta^2}$ which characterizes multiple Coulomb scattering (in the case of heavy atoms, inelastic collisions involving scattering by the electrons can be neglected). This is not the case in a crystal if $\alpha \ll \tilde{\alpha}$. Then the combined influence of those scattering events which are not accompanied by change of the nature of motion of the electrons and the nuclei can be taken into account by means of an effective potential U which varies smoothly along the normal to the crystallographic planes but which remains random with change of the coordinates along these planes. Therefore the multiple scattering becomes azimuthally asymmetric: for deflections parallel to the crystallographic planes the result remains the same as in an amorphous medium, while in the perpendicular direction the multiple scattering is determined only by those events which lead to a change of the state of the motion of the electrons or nuclei. Consequently the rms deflection angle of a particle in the direction perpendicular to the crystallographic planes should be less than in an amorphous medium or in a crystal[1)] for $\alpha > \tilde{\alpha}$. It is clear also that everything said above applies equally to ultrarelativistic particles with charges of either sign.

According to the theory of multiple Coulomb scattering in amorphous media, and also in crystals with $\alpha > \tilde{\alpha}$, the rms angle between the initial momentum and the projection of the final momentum on any plane containing the initial straight-line particle trajectory is described (with neglect of the small contribution of scattering by electrons) by the expression

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$$\overline{\eta_\parallel^2}=\overline{\eta_\perp^2}=\frac{1}{2}\overline{\eta^2}$$

$$\approx\frac{4\pi NZ^2e^4l}{p^2c^2}\int_0^\infty F^2\left(\frac{pR_n\eta}{\hbar}\right)\left[1-\varphi\left(\frac{pa_s\eta}{\hbar}\right)\right]^2\frac{d\eta}{\eta}. \quad (4)$$

Here N is the number of atoms per unit volume, l is the thickness of the layer of material, F is the nuclear form factor, $Z\varphi$ is the atomic form factor, R_n is the radius of the nucleus, and a_s is the screening radius; the functions $F(x)$ and $\varphi(x)$ are equal to unity at $x=0$ and approach zero for $x\gg1$. We shall assume that $R_n=1.3\cdot10^{-13}A^{1/3}$ cm and $a_s=5\cdot10^{-9}/Z^{1/3}$ cm.

In crystals for $\alpha\ll\tilde{\alpha}$ the quantity $\overline{\eta_\perp^2}$ corresponding to deflections perpendicular to the crystallographic planes is determined only by processes which lead to a change of the nature of the vibrational motion of the nuclei. Therefore in this case in calculation of $\overline{\eta_\perp^2}$ it is necessary to insert in the integrand of Eq. (4) an additional factor $(1-e^{-2W})$, where

$$e^{-2W}=\exp\left(-2\frac{p^2a_n^2}{\hbar^2}\eta^2\right)$$

is the Debye-Waller factor.[2)] Consequently the decrease of $\overline{\eta_\perp^2}$ in crystals for $\alpha\ll\tilde{\alpha}$ in comparison with the same quantity in an amorphous medium is

$$\overline{\Delta\eta_\perp^2}=\frac{4\pi NZ^2e^4l}{p^2c^2}\int_0^\infty F^2\left(\frac{pR_n\eta}{\hbar}\right)\left[1-\varphi\left(\frac{pa_s\eta}{\hbar}\right)\right]^2e^{-2W}\frac{d\eta}{\eta}, \quad (5)$$

while the quantity $\overline{\eta_\parallel^2}$ which corresponds to deflections parallel to the crystallographic planes is described as before by Eq. (4). Taking it into account that $R_n\ll a_s$, we can write approximately[3)]

$$\overline{\Delta\eta_\perp^2}\approx\frac{4\pi NZ^2e^4l}{p^2c^2}\int_0^\infty\exp\left(-\frac{2p^2a_n^2\eta^2}{\hbar^2}\right)\times\left[1-\varphi\left(\frac{pa_s\eta}{\hbar}\right)\right]^2\frac{d\eta}{\eta}. \quad (6)$$

Thus, under the conditions considered the angular distribution for sufficiently thick crystals should be described by a two-dimensional Gaussian distribution with different rms deviations in the two indicated mutually perpendicular directions.

An exact analysis of Eqs. (4)–(6) requires cumbersome calculations. Qualitative estimates can be obtained in the framework of the logarithmic approximation which is customary in multiple scattering theory. It follows then from Eqs. (4) and (6) that the relative decrease is

$$\overline{\Delta\eta_\perp^2}/\overline{\eta_\parallel^2}\approx\ln\left(\frac{a_s}{a_n}\right)\Big/\ln\left(\frac{a_s}{R_n}\right). \quad (7)$$

Since the region of localization of the nucleus a_n changes with the crystal temperature T, the ratio $\overline{\Delta\eta_\perp^2}/\overline{\eta_\parallel^2}$ also will depend on the temperature.[4)] The numerical value of the parameter a_n can be determined if we know the Debye-Waller factor, or it can be found from data on the probability of the Mössbauer effect. For cubic crystals it is sufficient to use the Debye model, in which

$$a_n^2=\frac{3\hbar^2}{Mk\Theta}\left[\frac{1}{4}+\left(\frac{T}{\Theta}\right)^2\int_0^{\Theta/T}\frac{t\,dt}{e^t-1}\right], \quad (8)$$

where M is the mass of the nucleus, k is Boltzmann's constant, and Θ is the Debye temperature.[8,9]

On this basis we arrive at the following numerical estimates: for iron at $T\ll\Theta$ we have $\overline{\Delta\eta_\perp^2}/\overline{\eta_\parallel^2}\approx18\%$, and at room temperature $\overline{\Delta\eta_\perp^2}/\overline{\eta_\parallel^2}\approx10\%$; for silicon, tungsten, and diamond at $T=300$ K the values of $\overline{\Delta\eta_\perp^2}/\overline{\eta_\parallel^2}$ are respectively 9, 13, and 23%. The effect turns out to be small, but it appears to be quite observable experimentally.[5)]

In traversing a crystal the direction of motion of a particle can change also as the result of interaction with the effective potential U. However, one must keep in mind that this potential is symmetric with respect to each of the crystallographic planes. Therefore, in crossing any of them the particle is first deflected in the direction perpendicular to this plane, and then is deflected by exactly the same angle in the opposite direction. Consequently the initial direction of motion of the particle will not be altered at all if its intersections with the planes occur completely inside the crystal.

A change of the direction of motion is possible only as the result of edge effects if the distance between the particle trajectory and the nearest crystallographic plane on entry into the crystal does not coincide with the corresponding distance of leaving the sample. In order of magnitude this change of direction is $|\Delta\alpha|\sim|\Delta q|/p$, where $|\Delta q|\sim U/c\alpha$. Therefore[6)] we have $|\Delta\alpha|\sim U/mc^2\gamma\alpha\sim\alpha_L^2/\alpha$. Since $\alpha_L/\alpha\ll1$, we have $|\Delta\alpha|\ll\alpha_L$ and to an even greater degree $|\Delta\alpha|\ll\alpha$. The value of $|\Delta\alpha|$ will not depend on the sample thickness, whereas $\overline{\eta^2}$ increases with the thickness; here all of the discussion above remains valid as long as the particle satisfies the comparatively very lax condition $(\overline{\eta^2})^{1/2}\ll\alpha$. Therefore for sufficiently thick crystals it is always possible to have the case $(\overline{\eta^2})^{1/2}\gg|\Delta\alpha|$, in which the principal role is played by multiple scattering. Then the edge effects can be neglected and the decrease of multiple scattering which we have been discussing will appear in pure form. Actually this situation occurs also for rather thin crystals. For example, in silicon of thickness 100 μm in motion of a particle with energy of the order of 1 GeV at an angle $\alpha\approx10^{-2}$ to the (110) crystallographic planes we have $(\overline{\eta^2})^{1/2}\approx3\cdot10^{-4}$ and $|\Delta\alpha|\approx3\cdot10^{-6}$.

[1)]We pay particular attention to the case of ultrarelativistic particles since for them the inequalities (3) are satisfied with a large margin. However, it is clear that in a less distinct form a similar asymmetry exists also for heavy nonrelativistic particles. We note that in the report by Baryshevskiĭ and Tikhomirov[5] there is a brief mention of the existence of an additional mechanism for asymmetry of multiple Coulomb scattering, which exists for superbarrier particles moving at small angles to crystallographic axes.

[2)]Regardless of the nature of the interaction of the particle with the crystal atoms, the factor $(1-e^{-2W})$ correctly describes the probability of change of the vibrational state of the nucleus in the crystal lattice in the framework of the so-called impact approximation, in which the time of collision of the particle with an atom is much less than the period of the vibrations of the nucleus in the crystal. The latter condition is almost always satisfied.

[3)]It must be emphasized that inclusion of the contribution of elastic scat-

tering by atomic electrons to the value of $\overline{\eta_1^2}$ does not in any way affect Eqs. (5) and (6).

[4)]For the same reason the averaged potential U also will depend on the temperature (see for example Refs. 6 and 7).

[5)]A similar effect exists also for bremsstrahlung but is absent for other nuclear processes, which do not involve the Coulomb interaction.

[6)]As the result of diffraction there is an additional change of the angle, which is equal in order of magnitude to $\hbar/pd$, where d is the interplanar distance. Under the conditions considered, this change can be neglected (for $p \approx 1$ GeV/c $\hbar/pd \sim 10^{-6}$).

[1]M. L. Ter-Mikaelyan, Zh. Eksp. Teor. Fiz. **25**, 289 (1953).

[2]A. I. Akhiezer and N. F. Shul'ga, Usp. Fiz. Nauk **137**, 561 (1981) [Sov. Phys. Uspekhi **25**, 541 (1981)].

[3]V. A. Bazylev and V. V. Goloviznin, Zh. Eksp. Teor. Fiz. **82**, 1204 (1982) [Sov. Phys. JETP **55**, 700 (1982)].

[4]J. Lindhard, Mat.-Fys. Medd. Dan. Vid. Selsk. **34**, No. 14 (1965) Russ. transl. in Usp. Fiz. Nauk **99**, 249 (1969).

[5]V. G. Baryshevskiĭ and V. V. Tikhomirov, Tezisy dokladov XIII Vsesoyuzn. Soveshch. po fizike vzaimodeĭstviya zaryazhennykh chastits s kristallami (Abstracts of Papers at the Thirteenth All-Union Conf. on the Physics of Interaction of Charged Particles with Crystals), Moscow State Univ. Press, Moscow, 1983, p. 68.

[6]B. R. Appleton, C. Erginsoy, and W. M. Gibson, Phys. Rev. **161**, 330 (1967).

[7]N. F. Shul'ga, V. I. Truten', and S. P. Fomin, Preprint 80-32, Khar'kov Physico-Technical Institute, 1980.

[8]R. L. Mossbauer, report delivered at the Second All-Union Conference or Nuclear Reactions at Low and Intermediate Energies, Moscow, July, 1960. Russ. tranl. in Usp. Fiz. Nauk **72**, 658 (1960).

[9]F. L. Shapiro, Usp. Fiz. Nauk **72**, 685 (1960) [Sov. Phys. Uspekhi **3**, 881 (1961)].

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