Weak link between conductors with a charge density wave

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The weak link between conductors with a charge-density wave is analyzed (the weak link is a tunnel junction or a system with a direct conductivity). The current in the system is calculated from a microscopic theory. The current has, in addition to a term determined by the product of state densities, a term proportional to the cosine of the difference between the phases of the charge-density wave. The current is also calculated for a system which contains amplitude solitons. At high voltages the current in a system with a direct conductivity is lower than the ohmic current by a constant amount.

1. INTRODUCTION

We know that the Peierls transition in a quasi-1-D conductor has much in common with a superconducting transition. At temperatures below the transition point (TP) in a Peierls conductor there is a displacement of the lattice ions, and a charge-density wave can arise. By this we mean that the electron density will be modulated in space: $\rho = \rho_0 \cos(Qr + \chi)$, where $Q = 2k_F$ is the wave vector of the charge-density wave, and χ is its phase. A gap proportional to ρ_0 appears in the excitation spectrum of the Peierls conductor. As in a superconductor, the modulus of the order parameter Δ determines the size of the energy gap, and its phase is χ. It has also been established that when there is an electric field E in a Peierls conductor the current along the conducting filaments is determined not only by quasiparticles but also by a condensate, whose role is played by a charge-density wave. While the condensate current in a superconductor is proportional to the gradient of the phase, $j_s \propto N_s \nabla\chi$ (N_s is the density of the condensate), the current due to a charge-density wave in a Peierls conductor is proportional to the time derivative of the phase: $j_w \propto \partial\chi/\partial t$. In the simplest case, that of a conductor with a static field E, the phase of the charge-density wave increases linearly over time, $\chi \propto Et$, so that j_w remains constant over time, and—in contrast with the case of a superconductor—is nonzero only if $E \neq 0$.

Josephson discovered some interesting effects when a weak link is fabricated in superconductors.[1] The weak link is produced either by an insulating film at a tunnel junction or by a local suppression of Δ (by a current flow, for example) in a direct-conductivity system. We might ask what the consequences of a weak link would be in conductors with a charge-density wave. A weak link can be fabricated in a Peierls conductor either artificially (an example might be a tunnel junction or a conductor with a contracted region) or naturally (there may be microscopic cracks or extended defects in a sample). In this paper we derive a theory for the effects which occur in a Peierls conductor containing a weak link. We examine a tunnel junction and a system consisting of two Peierls conductors connected by a narrow neck. In the calculations we assume that a 3-D ordering of the charge-density wave has occurred, so that the phases χ on different filaments are correlated.

2. TUNNEL JUNCTION

2.1 The method of a tunnel Hamiltonian

We consider a tunnel junction: two Peierls conductors separated by an insulating film. We will calculate the current in this system by the method of a tunnel Hamiltonian.[1)] In this method the tunneling is described by adding to the Hamiltonian a term which would be written in the present case as

$$\hat{H}_T = \sum_{\alpha,\beta,\mathbf{p},\mathbf{q}} [T_{\alpha\beta}(p,q)\, c_{\alpha\mathbf{p}}^{+} a_{\beta\mathbf{q}} + \text{c.c.}], \tag{1}$$

where $\hat{T} = T_0\hat{1} + T_Q\hat{\sigma}_x$ are the tunneling matrix elements; specifically, T_0 describes the case without tunneling, and T_Q describes the case of a transition from one Fermi surface to another, displaced by a vector Q from the first. Here α, $\beta = 1,2$ specify the Fermi surface; i.e., $c_{1p} = c_{\mathbf{p}+\mathbf{Q}/2}$, $c_{2\mathbf{p}} = c_{\mathbf{p}-\mathbf{Q}/2}$. The matrix elements T_0 and T_Q depend only weakly on the momenta p and q, and we will ignore this dependence. Let us examine the Green's function $\hat{G}^{ik}$ in the Keldysh technique. This function is a 4×4 matrix $G^{ik}_{\alpha\beta}$ (the superscripts are time indices). The equation obeyed by the function $\hat{G} = \hat{G}^{12} + \hat{G}^{21}$, which describes the kinetics, can be derived in precisely the same way as in Refs. 3 and 4 [see Eq. (12) in Ref. 4]. This equation is

$$\hat{G}_0^{-1}\hat{G} \equiv \left[\left(i\frac{\partial}{\partial t} - \eta - \frac{V}{2}\right)\hat{1} - (\xi\hat{\sigma}_z + \hat{\Delta})\right]\hat{G} = \hat{\Sigma}^R\hat{G} + \hat{\Sigma}\hat{G}^A, \tag{2}$$

$$\eta = \frac{1}{2}\left[\varepsilon\left(\mathbf{p}+\frac{\mathbf{Q}}{2}\right) + \varepsilon\left(\mathbf{p}-\frac{\mathbf{Q}}{2}\right)\right], \quad \xi = \frac{1}{2}\left[\varepsilon\left(\mathbf{p}+\frac{\mathbf{Q}}{2}\right) - \varepsilon\left(\mathbf{p}-\frac{\mathbf{Q}}{2}\right)\right], \quad \hat{\Delta} = \Delta(\hat{\sigma}_x\cos\chi - \hat{\sigma}_y\sin\chi).$$

Here $\varepsilon(p)$ is the energy spectrum of the metal at $T > T_P$, and V is the voltage across the barrier. The self-energy part of $\hat{\Sigma}$ stems from tunnel Hamiltonian (1). An expression for $\hat{\Sigma}$ in second order in T, in which we are interested here, will be derived below. It is convenient to use an equation for the Green's function $\hat{G}$ integrated over the variable ξ (when the vector $\mathbf{Q}$ runs parallel to the filaments we have $\xi = pQ/2m \equiv pv$). An equation can be derived for the function

$$\hat{g} = \hat{\sigma}_z\frac{i}{\pi}\int d\xi\, \hat{G} \tag{3}$$

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in precisely the same way as in Ref. 4; the result in the present case is

$$i\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial t'}\right)[\hat{\sigma}_z,\hat{g}]_+-\left[\hat{\Delta}+\left(\eta+\frac{V}{2}\right)\hat{\sigma}_z,\hat{g}\right]_- =\hat{\Sigma}^R\hat{g}+\hat{\Sigma}\hat{g}^A-\hat{g}^R\hat{\Sigma}-\hat{g}\hat{\Sigma}^A, \tag{4}$$

where

$$[\hat{a},\hat{g}]_\pm=\hat{a}\hat{g}\pm\hat{g}\hat{a}, \qquad \hat{\Delta}=i\Delta(\sigma_y\cos\chi+\sigma_x\sin\chi), \tag{5}$$

$$\hat{\Sigma}=T_0^2\hat{\sigma}_z\hat{g}\hat{\sigma}_z-T_Q^2\hat{\sigma}_y\hat{\tilde{g}}\hat{\sigma}_y-iT_0T_Q(\sigma_y\hat{g}\hat{\sigma}_z+\hat{\sigma}_z\hat{\tilde{g}}\hat{\sigma}_y),$$

and $\hat{\tilde{g}}$ is the Green's function of the other electrode. The operator $\hat{\Sigma}^{R(A)}$ is related to the function $\hat{\tilde{g}}^{R(A)}$ in an analogous way.

To find the current (I) in the system we calculate the change in the number of particles in one of the electrodes:

$$I\sim\dot{N}=a\,\mathrm{Sp}\int d\varepsilon\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial t'}\right)\hat{g}\hat{\sigma}_z$$

$$=a\,\mathrm{Sp}\int d\varepsilon[\hat{\Sigma}^R(\varepsilon_+)-\hat{\Sigma}^A(\varepsilon_-)]$$

$$\times[\hat{g}^R(\varepsilon_+)-\hat{g}^A(\varepsilon_-)](\mathrm{th}\,\varepsilon_+\beta-\mathrm{th}\,\varepsilon_-\beta). \tag{6}$$

We do not need an explicit expression for the proportionality factor a here; $\varepsilon_\pm=\varepsilon\pm V/2$. In deriving (6) we used the expressions

$$\hat{\Sigma}=(\hat{\Sigma}^R-\hat{\Sigma}^A)\,\mathrm{th}\,\varepsilon\beta, \qquad \hat{g}=(\hat{g}^R-\hat{g}^A)\,\mathrm{th}\,\varepsilon\beta,$$

where $\beta=1/(2T)$, which hold in an equilibrium state. Expression (6) can be used to find the current I in a quite general case.

We will first calculate the current by ignoring the curvature of the Fermi surfaces, impurity effects, and the possible presence of solitons in the Peierls conductors. In this case the functions $\hat{g}^{R(A)}$ are[4]

$$\hat{g}^{R(A)}(\varepsilon)=(\varepsilon\hat{\sigma}_z+\hat{\Delta})/\xi_\varepsilon^{R(A)},$$

$$\xi_\varepsilon^{R(A)}=\pm(\varepsilon^2-\Delta^2)^{1/2}\theta(|\varepsilon|-\Delta)\,\mathrm{sgn}\,\varepsilon + i(\Delta^2-\varepsilon^2)^{1/2}\theta(\Delta-|\varepsilon|). \tag{7}$$

Substituting (5) and (7) into (6), we find an expression for I:

$$I=\frac{1}{2R_N}\int d\varepsilon\,(\mathrm{th}\,\varepsilon_+\beta-\mathrm{th}\,\varepsilon_-\beta)\nu_1(\varepsilon_+)\nu_2(\varepsilon_-) \times\left\{1+\frac{T_0^2}{T_0^2+T_Q^2}\frac{\Delta_1\Delta_2}{\varepsilon_+\varepsilon_-}\cos(\chi_1-\chi_2) +\frac{T_Q^2}{T_0^2+T_Q^2}\frac{\Delta_1\Delta_2}{\varepsilon_+\varepsilon_-}\cos(\chi_1+\chi_2) +\frac{2T_0T_Q}{T_0^2+T_Q^2}\left(\frac{\Delta_1}{\varepsilon_+}\cos\chi_1+\frac{\Delta_2}{\varepsilon_-}\cos\chi_2\right)\right\},$$

$$\nu_{1,2}(\varepsilon)=[|\varepsilon|/(\varepsilon^2-\Delta_{1,2}^2)^{1/2}]\theta(|\varepsilon|-\Delta_{1,2}). \tag{8}$$

Here the $\nu_{1,2}(\varepsilon)$ are the state densities in the electrodes, and R_N is the resistance of the junction above the critical temperature. It can be seen from (8) that the current due to the last three terms in braces (curly brackets) depends on the phase in each of the electrodes. This unphysical result stems from a limited applicability of the tunnel-Hamiltonian method in this case of a spatially inhomogeneous system (with a charge-density wave). These terms actually drop out of the final result. To see this, we note that it follows from the way (1) is written in the coordinate representation that with $T_{\alpha,\beta}(p,q)$ independent of p and q we have $T(x,x')\propto T\delta(x)\delta(x')$. The tunneling thus occurs at a fixed point in space and therefore depends on the electron density at that point, i.e., on the phase of the charge-density wave. If we write $T(x,x')$ in the form $T(x,x')\propto T\delta(x-x_0)\delta(x'-x_0)$, we find a matrix $T_{\alpha\beta}$ of the form

$$\hat{T}=T_v\hat{1}+T_Q[\hat{\sigma}_x\cos Qx_0+i\hat{\sigma}_y\sin Qx_0].$$

When an average is taken over the positions of the tunneling point, i.e., over x_0, the last three terms in (8) vanish. In the next subsection of this paper we will derive this result rigorously, using a direct method to calculate I in a system with a potential barrier.

We thus see that in addition to the first, ordinary, term in the tunneling current, which is proportional to the product of the state densities ν_1 and ν_2 and the difference between distribution functions, there is another term, which is proportional to the product $\Delta_1\Delta_2\cos(\chi_1-\chi_2)$. The first term is analogous to the quasiparticle current at a Josephson tunnel junction, while the second is proportional to the imaginary part of the Josephson current,[5] $\mathrm{Im}\,I_c(V)$. Like the ordinary current I_1, the current due to the second term, I_2, is nonzero if $V\neq0$. Evaluating the corresponding integrals in (8), we find the current I for $T=0$ and $\Delta_1=\Delta_2=\Delta$:

$$I=I_1+I_2,$$

$$I_1=R^{-1}\left[(2\Delta+V)E(k)-\frac{4\Delta(\Delta+V)}{2\Delta+V}K(k)\right]\theta(k),$$

$$I_2=R^{-1}\frac{T_0^2}{T_0^2+T_Q^2}\frac{4\Delta^2\theta(k)}{V+2\Delta}K(k)\cos(\chi_1-\chi_2), \qquad k=\frac{V-2\Delta}{V+2\Delta}. \tag{9}$$

Here K and E are the complete elliptic integrals of the first and second kinds, respectively. In principle, we can make use of the dependence of the current component I_2 on the difference between the phases of the charge-density wave in the electrodes to study the motion of the charge-density wave. Let us assume that the filaments run parallel to the plane of the junction and that a current $I_\parallel$ is passed through at least one of the electrodes in the direction of the filaments. If the charge-density wave moves as a whole, the phase difference will increase in proportion to the additional current $I_\parallel$: $\chi_1-\chi_2\propto I_\parallel t$. This effect will cause the component I_2 of the tunneling current to oscillate over time. If an alternating external signal is also applied to the junction, then a resonance will be observed in a plot of $I(I_\parallel)$ at a fixed V. At this resonance, the frequencies of the natural and external oscillations are equal.

We turn now to a calculation of the voltage-current characteristic of a junction for Peierls conductors which contain a soliton band.[6] Many experiments[7] have demonstrated the existence of amplitude solitons in polyacetylene, in which there is a period doubling. On the other hand, we lack such proof for systems in which the period of the charge-density wave differs from twice and lattice period (in, for example, the transition metal trichalcogenides TaS_3 and $NbSe_3$). Measurements of the voltage-current characteristics

of tunnel junctions made from such systems will help us decide whether solitons can arise in Peierls conductors of this type. To calculate $I(V)$ in this case we should substitute into (6) an expression for the Green's functions $\hat{g}^{R(A)}$ found for the case in which there is a soliton band. To find $\hat{g}^{R(A)}$ we can use Eqs. (18) of Ref. 4. Solving the equations for $\hat{g}^{R(A)}$, and ignoring impurities and curvature of Fermi surface, we find, for $\hat{g}^R$, for example,

$$\hat{g}^R=\hat{s}\hat{g}\hat{s}^+, \qquad \hat{s}=\cos(\chi/2)+i\sigma_z\sin(\chi/2),$$

where the matrix elements of $\hat{g}$ are

$$g_{11}=-g_{22}=[\varepsilon^2+\Delta^2(x)/2-\Delta_k^2(1+k^2)/4k^2]/F(\varepsilon),$$

$$g_{12}=[\varepsilon\Delta(x)-iv\partial\Delta(x)/\partial x]/F(\varepsilon),$$

$$g_{21}=[-\varepsilon\Delta(x)-iv\partial\Delta(x)/\partial x]/F(\varepsilon),$$

$$\Delta(x)=\Delta_k\,\mathrm{sn}(x\Delta_k/kv,\ k), \qquad F(\varepsilon)=[(\varepsilon^2-\Delta_+^2)(\varepsilon^2-\Delta_-^2)]^{1/2},$$

$$\Delta_\pm=\Delta_k(1\pm k)/2k.$$

The parameter k varies from 0 to 1 and is associated with the period of the soliton lattice: In the limit $k\to 1$ we find an isolated soliton, $\Delta(x)=\Delta\tanh(\Delta x/v)$.

We now consider a sparse lattice in which the distance between solitons is substantially greater than the dimension of a soliton. The width of the soliton band, $2\Delta_-=\Delta\exp(-2\Delta/nv)$, is then small in comparison with $\Delta_+\approx\Delta$ (n is the density of solitons per unit length). We assume that the temperature satisfies the condition $\Delta_-\ll T\ll\Delta$; then at voltages $|V|<2\Delta$ we find a current which is determined by the soliton band, in addition to the small current proportional to $\exp(-\Delta/T)$, which is related to excitations with $|\varepsilon|>\Delta$. The calculations lead to

$$I_1=\theta(2\Delta_--V)\frac{V}{R}\frac{v^2n^2}{(V+2\Delta_-)T}K(k_1)$$

$$+\theta(V-\Delta+\Delta_-)\frac{2nv}{R(V+\Delta)^{1/2}[2\Delta_-(V-\Delta+\Delta_-)]^{1/4}}k_2^{1/2}K(k_2),$$

$$k_1=\left(\frac{2\Delta_--V}{2\Delta_-+V}\right)^{1/2}, \qquad k_2=\min\left\{\left(\frac{V-\Delta+\Delta_-}{2\Delta_-}\right)^{1/2},\ \left(\frac{2\Delta_-}{V-\Delta+\Delta_-}\right)^{1/2}\right\}. \tag{10}$$

If $V>2D$, the current begins to increase in accordance with (9). The first term in (10) describes transitions between states in the soliton bands, while the second describes transitions between a soliton band and states with $|\varepsilon|>\Delta$. The singularities in (9) and (10) stem from the divergence of the 1-D state density. Incorporating the transverse electron dispersion relation erases these singularities and causes a gradual increase in the tunneling current. Furthermore, the soliton band may be smeared by impurities and other defects, so that the voltage dependence of the current described by (10) would not be as sharp.

2.2 Direct method for calculating the tunneling current

We will now calculate the tunneling current (I) in this system without resorting to the method of a tunnel Hamiltonian. The method which we will use here can also be applied to other systems, to which the tunnel-Hamiltonian method is not applicable. We consider two Peierls conductors in which the filaments and the wave vector $\mathbf{Q}$ run parallel to the plane of the junction and are directed along the z axis. The conductors are separated by a potential barrier $U(x)$; in the simplest case we would have $U(x)=U_0\delta(x)$. The following functions then form a complete set of wave functions:

$$\Psi_p^{(1)}(r)=\{[e^{ipx}-e^{-ipx}(1-b_p)]\theta(-x)+b_pe^{-ipx}\theta(x)\}$$

$$\times\exp(ip_zz+ip_yy),$$

$$\Psi_p^{(2)}(r)=\Psi_p^{(1)}(-x,y,z), \tag{11}$$

$$b_p=a_p^2+ia_p, \qquad a_p=-p(m_xU_0)^{-1},$$

where m_x is the effective mass along the x axis. The functions $\Psi^{(1)}$ ($\Psi^{(2)}$) describes a particle which is incident on a δ-function potential from the left (from the right). If the potential $U(x)$ has a different form then the wave functions will also be different, but these differences will not affect the final results. A dependence of the potential barrier exclusively on x means that the reflection of particles will be specular; i.e., the components p_y and p_z will be conserved. In the tunnel-Hamiltonian method, none of the momentum components are conserved. This situation corresponds to a potential U with a roughness of an atomic scale. We therefore also introduce a random potential $U_R(r)=U_0(x)\varphi(r_\perp)$, where the function $U_0(x)$ is localized at $x=0$. We assume that the expectation value of the potential, $\langle U_R(r)\rangle$, is zero in the plane of the junction and that we have $\langle\varphi(r_\perp)\varphi(r_\perp')\rangle=\gamma(r_\perp-r_\perp')$, where the correlation function $\gamma(r_\perp)$ decays over distances on the order of interatomic distances. We expand all the Green's function in basis[2] (11):

$$\hat{G}(r,r')=\sum_{ss'}\sum_{pp'}{}'\hat{G}^{ss'}(p,p')\Psi_p^{s*}(r)\Psi_{p'}^{s'}(r'), \quad s,s'=1,2. \tag{12}$$

The current in the system can be found by determining the rate of change of the number of particles in one of the half-spaces:

$$I\sim i\,\mathrm{Sp}\int dp_\perp\int_{-\infty}^{0}dx\,\langle\hat{\dot{G}}(x,x)\rangle$$

$$=i\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial t'}\right)\mathrm{Sp}\int dp_\perp\,dp\,dp'$$

$$\times\{\langle G_{pp'}^{11}\rangle N_{p'p}^{11}+\langle G_{pp'}^{12}-G_{pp'}^{21}\rangle N_{pp'}^{12}\},$$

$$N_{pp'}^{ss'}=\int_{-\infty}^{0}\Psi_p^{s}(\Psi_{p'}^{s'})^*\,dx. \tag{13}$$

The time evolution of the Green's functions averaged in the plane of the junction, $\langle G^{11}\rangle$ and $\langle G^{12}\rangle$, is proportional to the transmission U_0^{-1}, while the product $\Psi^1\Psi^2$ also contains a factor U_0^{-1}. The second term in (13) can thus be discarded. It is not difficult to see that $N_{p'p}=2\delta(p-p')$.

We can now write an equation for $\hat{G}^{11}$, expanding in the potential U_R and retaining terms of up to second order inclusively. We find an equation of the form in (2), in which we have, for example,

$$(\hat{\Sigma}^R)^{ss'}=U_R^{ss}(\hat{G}^R)^{s_1s_2}U_R^{s_2s'}, \tag{14}$$

where

$$U_R^{ss'} = \int dr_\perp U_{pp'}^{ss'} \varphi(r_\perp) \exp[i(p_\perp - p_\perp')r_\perp] \hat{F}(z),$$

$$\hat{F}(z) = \hat{1} + \hat{\sigma}_x \cos Qz - \hat{\sigma}_y \sin Qz. \quad (15)$$

In (14) we have

$$(\hat{G}^R)_{pp'}^{ss'} \propto \delta_{ss'} \delta(p-p') \hat{G}^R(p).$$

In precisely the same way as in the preceding subsection, we can write an equation which the adjoint of (2) and then subtract one equation from the other. For the time derivative of $\mathrm{Sp}\,\hat{G}^{11}$ we find

$$i\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial t'}\right) \mathrm{Sp}\,\hat{G}_{pp}^{11} = \mathrm{Sp}\{(\hat{\Sigma}^R - \hat{\Sigma}^A)^{11} \hat{G}^{11} - (\hat{G}^R - \hat{G}^A)^{11} \hat{\Sigma}^{11}\}$$

$$= (\mathrm{th}\,\varepsilon_+ \beta - \mathrm{th}\,\varepsilon_- \beta) \int dp'\, dp_\perp'\, dr_\perp\, dr_\perp'\, \varphi_{pp'}^{12}(r_\perp) \varphi_{p'p}^{21}(r_\perp') \cdot$$

$$\times \exp[i(p_\perp - p_\perp')(r_\perp - r_\perp')]$$

$$\mathrm{Sp}\{\hat{F}(z)(\hat{G}^R - \hat{G}^A)_{p'}^{11} \hat{F}(z')(\hat{G}^R - \hat{G}^A)_{p'}^{22}\}. \quad (16)$$

We average (16) over the random potential $\varphi(r_\perp)$ and integral over p_z. We ignore the weak p_z dependence of the Fourier components $\gamma_0(p_\perp)$ and $\gamma_Q(p_\perp)$, where γ_0 corresponds to a small change in the momentum along the filaments, and γ_Q corresponds to a large change in the momentum, upon which there is a transition from one sheet of the Fermi surface to another. We now write an expression for the current in terms of the functions $\hat{g}$ introduced in the preceding subsection. From (13)–(16) we find

$$I = \tilde{a} \int dp\, dp'\, dp_y\, dp_y' |U_{0pp'}^{12}|^2 [\gamma_0(p_y - p_y') + \gamma_Q(p_y - p_y')]$$

$$\times \int d\varepsilon (\mathrm{th}\,\varepsilon_+ \beta - \mathrm{th}\,\varepsilon_- \beta) \nu_1(\varepsilon_+) \nu_2(\varepsilon_-)$$

$$\times \left[1 + \frac{\gamma_0}{\gamma_0 + \gamma_Q} \frac{\Delta_1 \Delta_2}{\varepsilon_+ \varepsilon_-} \cos(\chi_1 - \chi_2)\right], \quad (17)$$

where $\tilde{a}$ is a proportionality factor. This expression is the same as the first two terms of (8). The additional component of the tunneling current is therefore proportional to the cosine of the difference between the phases of the charge-density wave.

3. POINT CONTACT OF TWO PEIERLS CONDUCTORS

In this section we consider a system consisting of two bulk quasi-one-dimensional conductors (the banks) with a charge-density wave, which are separated by an impenetrable membrane in which there is an aperture with characteristic dimensions smaller than the correlation length. A weak link between the banks is thus provided by a direct-conductivity neck. A similar model has been used elsewhere[9] to describe a point contact between two superconductors. We again assume that the conducting filaments are directed along the z axis and parallel to the plane of the membrane (the yz plane). It is a rather difficult matter to find a general solution for the problem of the current flow in this system. We therefore consider two limting cases, that of a "clean" conductor ($\Delta\tau \gg 1$, where τ is the momentum scattering time) and that of a "dirty" conductor ($\Delta\tau \ll 1$). In the dirty limit we can use the time-dependent Ginzburg-Landau equation,[10] so that the problem can be solved rather simply. In the clean limit, in which the charge-density wave contributes substantially to the conductivity, and the related effects are more obvious, we must solve equations for the Green's functions. The results in the two cases are qualitatively similar.

3.1. The dirty limit. Approach using the Ginzburg-Landau equation

We first consider the simpler and more illustrative case of a dirty material, in which the Peierls conductor is gap-free, and we can apply to it a Ginzburg-Landau equation generalized to the time-varying case.[10] Gor'kov[10] wrote these equations for the case in which the order parameter depends on only the coordinate along the conducting chains. For our purposes, we should allow Δ to depend on all three coordinates.

Since we are dealing with the case of an aperture which is small in comparison with the correlation lengths, we need retain in the Ginzburg-Landau equation for Δ only the terms with derivatives (the situation here is analogous to the case of a point contact in a gap-free superconductor[11]). The equation for the complex order parameter Δ is therefore

$$(v^2 \partial^2/\partial z^2 + \langle v_\perp \rangle^2 \nabla_\perp^2) \Delta = 0. \quad (18)$$

Here the coefficient $\langle v_\perp \rangle^2$ of the second derivatives with respect to the coordinates transverse to the filament is the square of the group velocity $v_\perp = \partial\varepsilon/\partial p_\perp$ averaged over the momenta. It determines the value of the second term in (18) if the electron spectrum is substantialy three-dimensional [when $\langle v_\perp \rangle$ is larger than the scale sound velocity $s(m^*/m)^{1/2}$, where m is the electron mass, and m^* is the effective mass of the charge-density wave]. The vector $\mathbf{Q}$ is then tilted away from the z axis (and a period doubling occurs along the x and y axes if we can use the approximation of a strong coupling of the electrons at the filaments). In the opposite case, the coeffiicent of $\nabla_\perp^2$ is determined by the elastic of the crystal.

It is a simple matter to solve Eq. (18) in the coordinates of an oblate ellipsoid of revolution (σ, τ, φ):

$$x = a\sigma\tau, \quad y^2 = a^2(1+\sigma^2)(1-\tau^2)\cos^2\varphi,$$

$$z^2 = (va/v_\perp)^2(1+\sigma^2)(1-\tau^2)\sin^2\varphi.$$

This solution is

$$\Delta = \Delta_0 \exp\left(i\frac{\chi_1 + \chi_2}{2}\right)\left[\cos\frac{\chi_1 - \chi_2}{2} + \frac{2i}{\pi}\sin\frac{\chi_1 - \chi_2}{2}\,\mathrm{arctg}\,\sigma\right]. \quad (19)$$

We are assuming for simplicity that the aperture is elliptical (in which case we would have $\sigma = 0$). The order parameter $\Delta(\sigma)$ satisfies the boundary conditions $\Delta(-\infty) = \Delta_0 \times \exp(i\chi_2)$ and $\Delta(+\infty) = \Delta_0 \exp(i\chi_1)$.

The current density in the direction perpendicular to the filament is

$$j_\perp = \sigma_{\perp N} E_\perp (1 - d|\Delta|^2), \quad (20)$$

where $\sigma_{\perp N}$ is the transverse conductivity in the normal state, and the second term in parentheses describes the decrease in conductivity due to the Peierls transition. The form of the coefficient d depends on the transverse dispersion relation on the Fermi surface. In the limit of a small curvature of the Fermi surface, $\eta \ll 1/\tau$, we would have $d = \nu_1/2\nu$, where

$\nu = \nu_1 + \nu_2/2$, and ν_1 and ν_2 are the electron scattering frequencies respectively without a transition and with a transition between opposite parts of the Fermi surface of the quasi-one-dimensional metal. Localization effects were ignored in the derivation of this expression; this simplification is justified if, for example, the elastic scattering is a scattering by phonons.

To calculate the current through the aperture we must use Eqs. (19) and (20) in the vicinity of the aperture, $\sigma = 0$, and find the relationship between the voltage across the point contact, V, and the field $E_\perp$. After carrying out these calculations, we find the current across the contact to be

$$I = \frac{V}{R_N}\left(1 - d\Delta_0^2 \cos\frac{\chi_1 - \chi_2}{2}\right), \qquad (21)$$

where R_N is the resistance of the contact in its normal state.

It follows from (21) that the current through a point contact depends on the difference between the phases of the charge-density wave in the banks, as in the case of a tunneling contact. In the present case, this dependence stems from the circumstance that the amplitude of the order parameter near the contact depends on the difference between the phases in the banks. When a current flows parallel to the filaments in one or both banks, the relative motion of the charge-density wave in the banks will cause a change in the phase difference $\chi_1 - \chi_2$ and give rise to the time-varying effects described in Subsection 2.1.

3.2 The clean limit. Microscopic equations

Let us consider a point contact between clean conductors with a charge-density wave ($\Delta\tau \gg 1$); as before, we assume that the dimensions of the aperture are much smaller than the correlation length. To calculate the current through the contact we use the equations of Ref. 4 for the Green's functions:

$$-i\frac{\partial}{\partial t}\check{\sigma}_z\check{g} - i\frac{\partial}{\partial t'}\check{g}\check{\sigma}_z + [\check{\Delta} - (\eta + \varphi)\check{\sigma}_z - \check{\Sigma}, \check{g}]_- + \frac{i}{2}[\check{\mathbf{v}}, \nabla\check{g}]_+ = 0, \qquad (22)$$

where

$$\check{g} = \begin{pmatrix} \hat{g}^R & \hat{g} \\ 0 & \hat{g}^A \end{pmatrix}, \quad \check{\Delta} = \begin{pmatrix} \hat{\Delta} & 0 \\ 0 & \hat{\Delta} \end{pmatrix},$$

$$\check{\sigma} = \begin{pmatrix} \hat{\sigma}_z & 0 \\ 0 & \hat{\sigma}_z \end{pmatrix}, \quad \check{\Sigma} = \begin{pmatrix} \hat{\Sigma}^R & \hat{\Sigma} \\ 0 & \hat{\Sigma}^A \end{pmatrix},$$

$$\check{\mathbf{v}} = {}^1/_2(\mathbf{v}_+ - \mathbf{v}_-) + {}^1/_2\check{\sigma}_z(\mathbf{v}_+ + \mathbf{v}_-),$$

φ is the electrostatic potential, and $\mathbf{v}_\pm = \partial\varepsilon(\mathbf{p} \pm \mathbf{Q}/2)/\partial\mathbf{p}$ are the group velocities at parts of the Fermi surface displaced by the wave vector $\mathbf{Q}$ of the charge-density wave. In the elastic-collision integral

$$\check{\Sigma} = -\frac{i}{2}\int\frac{d\mathbf{k}_\perp'}{S}\left\{\nu_1(\mathbf{k}_\perp + \mathbf{k}_\perp')\check{\sigma}_z g(\mathbf{k}_\perp')\check{\sigma}_z - \frac{\nu_2(\mathbf{k}_\perp - \mathbf{k}_\perp')}{2} \times [\check{\sigma}_x\check{g}(\mathbf{k}_\perp')\check{\sigma}_x + \check{\sigma}_y\check{g}(\mathbf{k}_\perp')\check{\sigma}_y]\right\}$$

the functions ν_1 and ν_2 describe the probabilities for scattering without and with a change $2k_F$ in the longitudinal momentum. Otherwise, the notation is the same as in Subsection 2.1. We consider the case in which the curvature of the Fermi surface is smaller than the energy gap ($\eta \ll \Delta$), and for simplicity we assume that ν does not depend on $\mathbf{k}_\perp$.

We must solve Eq. (22) and then use the results found for $\hat{g}$ as a function of the coordinates to calculate the current through the aperture. System (22) is a system of linear differential equations with partial derivatives with respect to the coordinates, and a general solution is quite difficult. It can, however, be easily solved in the particular case in which the dispersion relation in the direction transverse to the filaments is described in the strong-coupling approximation with allowance for an interaction between nearest neighbors:

$$\varepsilon_\perp = \varepsilon_1 \cos a_x p_x + \varepsilon_2 \cos a_y p_y.$$

In this case the vector $\mathbf{Q}$ corresponds to a period doubling along the transverse directions, and we have $\mathbf{v}_- = -\mathbf{v}_+$. As a result, the terms with derivatives with respect to the coordinates assume the form $i\mathbf{v}_+\nabla\check{g}$. All the characteristic curves of the equations then coincide, and at a fixed value of the velocity $\mathbf{v}_+$ system (22) can be reduced to a system of ordinary differential equations which depend on the coordinate along the characteristic trajectories, $\mathbf{r} = \mathbf{v}_+ t + \mathbf{r}_0$, as was done by Kulik and Omel'yanchuk.[9] As in Ref. 9, the scale lengths in these equations are the correlation length v/Δ and the mean free path v/ν, while Δ, φ, and all the quantities which are found from g after an average is taken over $\mathbf{v}_+$ vary over small distances, on the order of the dimension of the aperture. The reason is that the number of trajectories which pass through a given point into the aperture falls off as r^{-2} as this point is removed from the aperture. Consequently, in solving Eqs. (22) along the trajectories we can assume that Δ and φ change abruptly at the aperture.

We must therefore find a solution of the equations along the trajectories by joining the solutions at the aperture and using as boundary conditions at infinity the requirement that a perturbation caused by the weak link decay with distance from the aperture.

Solving Eq. (22), we find that the perturbation decays in accordance with $\exp(i\varkappa s)$, where s is the coordinate along the trajectory. At energies $|\varepsilon_\pm| > \Delta$ [the variable ε arises from the Fourier transformation over the time difference $t - t'$ in Eqs. (22)], the index $\varkappa$ for the functions g^R and g^A is

$$\varkappa^{R(A)} = \pm\frac{1}{|v_s|}\left(2\xi_{\varepsilon_\pm}^{R(A)} + i\frac{\varepsilon_\pm\nu + \Delta^2\nu_0}{\xi_{\varepsilon_\pm}^2}\right), \qquad (23)$$

and that for the anomalous Green's function is

$$\varkappa = \pm\frac{i}{|v_s|}\left(\nu_s + \frac{2\Delta^2\nu_0}{\xi_{\varepsilon_\pm}^2}\right). \qquad (24)$$

The plus and minimum signs in (23) and (24) refer to the regions $x > 0$ and $x < 0$, respectively; $\nu_s = \nu_1 + \nu_2$, $\nu_0 = \nu_+ + \nu_2/2$, $\xi_{\varepsilon\pm}$ and $\varepsilon_\pm = \varepsilon \pm V/2$ are defined as in (6) and (7); $V = \varphi(+\infty) - \varphi(-\infty)$ is the voltage drop across the point contact; v_s is the velocity projection onto the trajectory; and Δ is the gap in the banks. In the derivation of these expressions and in the calculations below we assume that the dependence of all quantities on the average time $(t + t')/2$ is slow in comparison with frequencies on the order of Δ. We also ignore the possible existence of amplitude soli-

tons in the banks.

In the low-voltage limit, $V \ll \Delta$, the state density in the aperture is

$$\rho_a = \frac{1}{2}(g^R - g^A) = \frac{|\varepsilon|(\varepsilon^2-\Delta^2)^{1/2}}{\varepsilon^2-\Delta^2\cos^2[(\chi_1-\chi_2)/2]}\theta(|\varepsilon|-\Delta). \quad (25)$$

The state density is seen to depend on the difference between the phases in the banks.

Under the same assumptions, the function $f^R - f^A$, which determines the order parameter at the aperture, is

$$f^R - f^A = \frac{2\Delta\exp[i(\chi_1+\chi_2)/2]\cos[(\chi_1-\chi_2)/2](\varepsilon^2-\Delta^2)^{1/2}}{\varepsilon^2-\Delta^2\cos^2[(\chi_1-\chi_2)/2]} \times \theta(|\varepsilon|-\Delta). \quad (26)$$

To calculate the order parameter we need to multiply (26) by $\lambda \tanh \varepsilon\beta$, where λ is the electron-phonon interaction constant, and we need to integrate this product over ε from 0 to ∞. If $\chi_1 - \chi_2 = (2n+1)\pi$, the order parameter obviously vanishes at the aperture. On the other hand, a gap remains in the state density, according to (25). The reason for the difference between the energy gap and the modulus of the order parameter in this case is that the change in the order parameter near the aperture occurs over distances short in comparison with the correlation length, and the state density cannot adjust fast enough to keep up with changes in the order parameter.

We seek the current which flows through the aperture,

$$j \propto \int d\varepsilon\, dk_\perp v_x \operatorname{Sp} \hat{g}.$$

It follows from (22) that at the aperture we have

$$\operatorname{Sp}\hat{g} = \frac{4\xi_+\xi_-(\operatorname{th}\varepsilon_+\beta - \operatorname{th}\varepsilon_-\beta)\operatorname{sgn} v_x}{\varepsilon_+\varepsilon_- + \xi_+\xi_- - \Delta^2\cos(\chi_1-\chi_2)}, \quad (27)$$

where

$$\xi_\pm = (\varepsilon_\pm^2 - \Delta^2)^{1/2}\theta(|\varepsilon_\pm|-\Delta)\operatorname{sgn}\varepsilon_\pm.$$

Using (27), we find the total current through the contact to be

$$I = \frac{1}{R_N}\int_{-\infty}^{+\infty} d\varepsilon \frac{\xi_+\xi_-(\operatorname{th}\varepsilon_+\beta - \operatorname{th}\varepsilon_-\beta)}{\varepsilon_+\varepsilon_- + \xi_+\xi_- - \Delta^2\cos(\chi_1-\chi_2)}. \quad (28)$$

In the case under consideration here, of a Peierls conductor for which the curvature of the Fermi surface is small in comparison with Δ, the condition $T \ll \Delta$ usually holds. In this case the dependence on the phase difference which appears in the current becomes particularly sharp. For small voltages across the contact ($V \ll T \ll \Delta$), for example, the conductivity of the contact is

$$\frac{1}{R} = \frac{1}{R_N} 2e^{-\Delta/T}[1 + be^{-b}\operatorname{Ei}(b)], \quad b = \frac{\Delta}{2T}\sin^2\frac{\chi_1-\chi_2}{2}. \quad (29)$$

According to (29), a plot of $1/R$ versus $\chi_1 - \chi_2$ is a sequence of narrow peaks at the values $\chi = 2n\pi$. The maximum value of the conductivity is $(2/R_N)\exp(-\Delta/T)$, and the minimum value is $(2T/\Delta)(2/R_N)\exp(-\Delta/T)$.

At voltages $V \geqslant 2\Delta$ the conductivity increases sharply according to (28), and the small factor $\exp(-\Delta/T)$ disappears, since electrons begin to undergo transitions from states below the energy gap in one of the banks to states above the energy gap in the other bank. The dependence on the phase difference remains strong. Near the threshold, i.e., at $0 < V - 2\Delta \ll 2\Delta$, for example, the voltage dependence of the current changes from linear, $I = (4-\pi)(V - 2\Delta)$ at $\chi_1 - \chi_2 = 2n\pi$, to $I = (\pi/4\Delta)(V-2\Delta)^2$ at $\chi_1 - \chi_2 = (2n+1)\pi$.

In the limit $|V| \gg \Delta$ the voltage dependence of the current has the asymptotic behavior

$$I = \frac{1}{R_N}\left[V - \frac{3}{8}\Delta\operatorname{sgn} V - \frac{\Delta}{V}\ln\frac{|V|}{\Delta}\cos(\chi_1-\chi_2)\right]. \quad (30)$$

At high voltages V the function in (30) does not become Ohm's law but a straight line running parallel to Ohm's law; in this regard the situation is the same as at a superconducting point contact. In superconductors, however, a so-called excess current is added to Ohm's law, while in the case of a point contact between Peierls conductors the current is smaller than the Ohm's-law current, by an amount which does not depend on V. The situation can be understood by noting that the current through the contact is determined exclusively by energies $|\varepsilon \pm V/2| > 0$, and under this condition the state density near the aperture is essentially independent of the energy. Furthermore, near the energies $|\varepsilon| \approx |V/2 \pm \Delta|$, where the state density diverges as $\varepsilon/(\varepsilon^2 - \Delta^2)^{1/2}$ in the interior of one of the banks, while it is unity in the other bank, we should see a strong above-barrier reflection of electrons going from one bank to the other.

4. CONCLUSION

A calculation of the current flowing between weakly lined conductors with a charge-density wave shows that he current I contains a component I_2 which depends on the difference between the phases of the charge-density wave: $I_2 \propto \Delta_1\Delta_2\cos(\chi_1 - \chi_2)$. This result is independent of the nature of the weak link (a tunneling system or a system with direct conductivity). We thus see a analogy with the time-dependent Josephson effect in weakly linked superconductors. On the other hand, there is also a fundamental distinction: The phase in a superconductor is correlated over the entire volume of the superconductor, and the phase difference $\chi_1 - \chi_2$ is determined by the applied voltage, $\chi_1 - \chi_2 = 2eVt/\hbar$, according to the Josephson relation. In a conductor with a charge-density wave the phase difference does not depend on the current through the contact (although, as mentioned above, each of the phases χ_1, χ_2 can vary over time if there is an additional current $I_\parallel$ flowing parallel to the plane of the junction; in this case we would have $\chi_{1,2} \propto I_\parallel^{1,2} t$). Furthermore, the phase χ in a Peierls conductor fluctuates because of the presence of impurities, or it takes on different values in different regions if the sample breaks up into domains. These effects may extinguish the time-varying phenomena in these systems, but it may be that in junctions of small dimensions the current component I_2 may not average out to zero.

There is yet another circumstance which complicates an experimental observation of the generation of oscillations at junctions. It was assumed above that the filaments in the banks run parallel to each other (it was also assumed that these filaments are parallel to the plane of the junction, but it

can be shown that this assumption is not of fundamental importance). What would happen if the filaments were oriented at some finite angle with respect to each other? It turns out that the local current density will depend on the coordinates in the plane of the junction,

$$j_2 \propto \Delta_1 \Delta_2 \cos(\chi_1 - \chi_2) \cos Q(z \cos\alpha + y \sin\alpha),$$

so that the total current I_2 will vanish when an average is taken over the transverse coordinates (α is the angle between the filaments in the banks). This circumstance imposes some severe restrictions on the parameters of the system in which the time-varying effect is to be studied. The effect can apparently be seen most easily in the case in which a system of weakly linked conductors is fabricated from a common single crystal.

As for the contribution to the current from amplitude solitons, we note that this contribution does not depend on the relative arrangement of the filaments, and it should be observed both at a tunneling junction and in a system with a direct conductivity.

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[1)]The results of this section have been reported briefly elsewhere.[2]
[2)]Kulik *et al.*[8] used an expansion in basis (11) in calculating the Josephson current at a superconducting tunneling junction. They did not introduce a random potential there, however; it is taking an averge over this potential which leads to the results of the tunneling-Hamiltonian method.

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