# Effect of disequilibrium of distribution functions on the Kapitza discontinuity at a superconductor-dielectric interface

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The effect of slow excitation-relaxation processes on the temperature Kapitza discontinuity at a superconductor-dielectric interface is theoretically considered at heat fluxes that are not small, when the deviations of the distribution functions from equilibrium are not small and the equations cannot be linearized. Expressions are obtained for the temperature discontinuity as a function of the heat flux. It is shown that the absolute value of the Kapitza discontinuity changes when the heat flux changes direction.

A temperature discontinuity, called the Kapitza discontinuity, occurs when heat flows through an interface between two media. A theory of this phenomenon at an interface between superfluid He II and a solid was developed by Khalatnikov.[1,2] Little[3] extended this theory to include contact between two solids having different acoustic properties (densities and sound velocities). The gist of the conclusions of the theory[1–3] is the following. The energy is carried through the interface only by phonons. Since the acoustic properties of the two media differ, the phonons have a definite probability of being reflected from the interface. As a result the relation between energy flux $I_\varepsilon$ through the interface and the temperatures on both sides of the interface is

$$I_\varepsilon = A\,(T_0^4 - T_1^4).$$

Here $A$ is a coefficient proportional to the probability of phonon passage through the interface. In the derivation of this equation it is assumed also that the phonon distribution functions on the two sides remain at equilibrium with the temperature $T_0$ and $T_1$ all the way to the interface.

Some experimental facts, however, have not yet been explained by the acoustic-mismatch theory.[1–3] One is that the Kapitza discontinuity between a superconductor and a dielectric depends on the superconductor energy gap that can be varied in experiment with a magnetic field (see, e.g., Refs. 4 and 5). The Kapitza discontinuity in the superconducting state is found to be larger than in a normal metal at the same temperature. This fact does not agree with the theory of Refs. 1–3, since the sound velocity, the density, and also the coefficient of phonon reflection from the interface, are all practically independent of the size of the energy gap. Allowance for the effect of the conduction electrons[6,7] on the coefficient of phonon transmission through the interface does not improve substantially the agreement between experiment and theory.

On the other hand, the assumption that the distribution functions of the excitations are at equilibrium all the way to the interface is generally speaking incorrect. Equilibrium distributions are established only when the distances from the interface exceed the characteristic relaxation lengths. In the immediate vicinity of the interface the excitation distribution functions are not at equilibrium and their forms depend on the relaxation mechanisms. In Ref. 8 was theoretically considered the effect of slow excitation relaxations in a superconductor on the value of the Kapitza discontinuity at a superconductor-dielectric interface. It was shown that the presence of hierarchy of relaxation lengths in the superconductor leads to the existence of a nonequilibrium region near the interface, where the quasiparticle density distribution function is

$$n_\varepsilon(z) = \exp\,[(\nu(z) - \varepsilon)/T(z)].$$

The dependence of the temperature $T$ and of the chemical potential $\nu$ on the coordinate $z$, determined from the energy-flux and particle-number conservation conditions, is found to be nonlinear. This produces an additional temperature discontinuity that depends on the size of the energy gap. This dependence is bell-shaped, and in the vicinity of the maximum the discontinuity is tens to hundreds of times larger than predicted by the acoustic-mismatch theory.[1–3]

Experimental investigations of the Kapitza discontinuity at a dielectric-superconductor interface[9] confirmed the conlusions of the theory of Ref. 8. The dependence of the discontinuity on the sample temperature is in fact as predicted in Ref. 8. The quantitative agreement is also good. The range of validity of the theory of Ref. 8, however, is limited. The point is that the entire analysis in Ref. 8 was carried in an approximation linear in the heat flux $I_\varepsilon$, when the chemical potential $\nu$ and the temperature discontinuity $\delta T$ are much smaller than the unperturbed value of $T$, i.e., when [Ref. 8]

$$\delta T/T \ll T/\Delta. \tag{1}$$

Condition (1) can be used for a practical estimate of the suitability of Ref. 8. For example, at $T = 0.2$ K, i.e., $T/\Delta \approx 0.1$, the discontinuity $\delta T$ should be much less than 0.02 K.

Condition (1) can be written in the form of a constraint on the flux $I_\varepsilon$ (Ref. 8):

$$|I_\varepsilon| \ll \frac{4}{\pi^2\sqrt{\pi}}\left(\frac{l_{im}}{l_>}\right)^{1/2}\frac{p_F^2\Delta^3}{\hbar^3 w}\left(\frac{T}{\Delta}\right)^{9/4} e^{-2\Delta/T}. \tag{2}$$

Here $p_F$ is the Fermi momentum, $\Delta$ the half-width of the energy gap, $w$ the speed of sound, $T$ the temperature, $l_{\rm im}$ the mean free path of the quasiparticles relative to collisions with impurities, and $1_>$ the mean free path of the phonons having an energy higher than $2\Delta$. Substitution in (2) the parameters of the superconducting aluminum used in Ref. 9 ($w = 5\cdot 10^3$ m/sec, $p_F = 1.8\cdot 10^{-24}$ J·sec/m, $\Delta = 2.065$ K, $l_{\rm im}$

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$= 1.7\cdot 10^{-5}$ m, $l_> = 5\cdot 10^{-6}$ m) we obtain a condition for the suitability of an analysis linear in $I_\varepsilon$ in the form

$$|I_\varepsilon| \ll 5\cdot 10^2 (T/\Delta)^{9/4} e^{-2\Delta/T}. \tag{3}$$

At sufficiently small $T/\Delta$ the condition (3) is met only for very small energy fluxes. For fluxes and temperature discontinuities measured in practice, the linear theory of Ref. 8 does not hold.

We consider in this paper the effect of the disequilibrium of the distribution functions at the interface on the Kapitza discontinuity in the case of fairly large energy fluxes, when conditions (1) and (2) are not satisfied. Just as in Ref. 8, we shall assume that the energy is carried inside the superconductor by quasiparticles, and the energy transport by phonons is negligibly small.

The onset of the additional temperature discontinuity can be explained qualitatively as follows. Energy is transported in a quasiparticle gas by two mechanisms: 1) a nonzero flux of quasiparticles, each of which carries a definite energy; 2) additional energy transport because particles moving in opposite directions carry unequal energies. Under conditions of usual heat conduction, the former contribution exceeds the latter substantially. In the case considered here, however, when the energy is carried out through the interface primarily by thermal phonons and the quasiparticle flux at the interface is zero, the principal role is played in the region of the interface by the second component, and the temperature corresponding to a given flux should change much more rapidly. The nonlinearity of the temperature as a function of coordinate is in fact the cause of the additional discontinuity.

At $T \ll \Delta$ the largest relaxation length is the one in which an equilibrium particle number $a$ sets in.[8] The fast scattering of the quasiparticles by low-frequency phonons establishes near the interface a quasiparticle distribution function

$$n(\varepsilon_p) = \exp\left[(\nu(z) - \varepsilon_p)/T(z)\right], \tag{4}$$

and the equilibrium values of $\nu$ and $T$ are established over a length $a$. To find $\nu(z)$ and $T(z)$ we use the energy-flux and particle-number conservation laws [Eqs. (9) and (19) of Ref. 8]:

$$\sum_{\mathbf{p},\sigma} \varepsilon_\mathbf{p} \frac{\partial}{\partial z}\left(D\frac{|\xi_\mathbf{p}|}{\varepsilon_\mathbf{p}}\frac{\partial}{\partial z} n(\varepsilon_\mathbf{p})\right) = 0, \tag{5}$$

$$\sum_{\mathbf{p},\sigma} \frac{\partial}{\partial z}\left(D\frac{|\xi_\mathbf{p}|}{\varepsilon_\mathbf{p}}\frac{\partial}{\partial z} n(\varepsilon_\mathbf{p})\right) = 2\sum_{\substack{\mathbf{q},i\\ \omega>2\Delta}} J_{ph}{}^s, \tag{6}$$

where $\varepsilon_p = (\xi_p^2 + \Delta^2)^{1/2}$, $\xi_p = p^2/2m - \eta$, $\eta$ is the chemical potential of the normal metal, $\mathbf{p}$ and $\mathbf{q}$ the momenta of the quasiparticles and phonons, $\sigma$ the spin of the quasiparticles, $\omega$ and $i$ the phonon frequency and polarization, $D = \frac{1}{3}v_F^2\tau_{\text{im}}$ the diffusion coefficient in the normal metal (we consider only the case when the quasiparticle pulse relaxation time $\tau_{\text{im}}$ is shorter than the remaining relaxation times), $v_F$ the Fermi velocity, and $J_{\text{ph}}{}^s$ the operator of phonon scattering by the quasiparticles.

Substituting (4) in (5) and (6) and introducing the notation

$$\frac{1}{2N(0)}\sum_{\mathbf{p},\sigma}\frac{|\xi_\mathbf{p}|}{\varepsilon_\mathbf{p}} n(\varepsilon_\mathbf{p}) = Te^{(\nu-\Delta)/T} \equiv \mu, \tag{7}$$

where $N(0) = mp_F/\pi^2\hbar^3$ is the density of states on the Fermi surface in the normal metal, we get

$$\frac{\partial^2}{\partial z^2}(\mu(\Delta+T)) = 0, \quad \frac{\partial^2\mu}{\partial z^2} = \alpha T^{1/2}\mu\left(\frac{\mu^2}{T^2} - e^{-2\Delta/T}\right),$$
$$\alpha \equiv 6\Delta^{5/2}/Dmp_F\pi^{1/2}w^2 l_>. \tag{8}$$

A derivative with respect to $z$ will hereafter be designated by a prime.

We solve the problem in the following geometry: the $z$ axis is directed from the superconductor into the dielectric, their interface corresponds to $z = a > 0$, and the origin is a point inside the superconductor, at which it can already be assumed that $\nu = 0$, i.e., the usual heat conduction regime is realized. The uncertainty in the choice of this point will not affect the results. Since

$$(\mu(\Delta+T))' = \frac{1}{2N(0)}\sum_{\mathbf{p},\sigma}|\xi_\mathbf{p}|n'(\varepsilon_\mathbf{p}) = -\frac{I_\varepsilon}{2DN(0)}, \tag{9}$$

where $I_\varepsilon$ is the energy flux, we can rewrite (8), introducing $J_\varepsilon = I_\varepsilon/2DN(0)$, in the form

$$(\mu-\mu_0)(\Delta+T_0) + \mu(T-T_0) = -J_\varepsilon z. \tag{10}$$

The complete system of equations of the problem is

$$\mu'' = \alpha T^{1/2}\mu(\mu^2/T^2 - e^{-2\Delta/T}), \tag{11}$$

$$(\mu-\mu_0)(\Delta+T_0) + \mu(T-T_0) = -J_\varepsilon z, \tag{12}$$

$$\mu'(0) = -J_\varepsilon/\Delta, \quad \mu'(a) = 0, \quad T(0) = T_0,$$
$$\mu(0) = \mu_0 = T_0 e^{-\Delta/T_0}. \tag{13}$$

Equations (13) are boundary conditions due to the fact that $\mu'$ is proportional to the quasiparticle flux at the given point; this flux is equal to zero at the interface and to the energy flux per quasiparticle in the interior of the superconductor.

It follows from (10) that if

$$|J_\varepsilon|a/\mu_0\Delta \ll 1, \tag{14}$$

we have $(\mu-\mu_0)/\mu_0 \ll 1$. It is this which makes the transformation from $\nu$ and $T$ to the variable $\mu$ advantageous, namely, $\mu$ can be regarded as practically independent of $z$ whereas $\nu$ and $T$ change significantly.

Using (8), we rewrite (11) in the form

$$-\frac{T''\mu}{\Delta} - \frac{2T'\mu'}{\Delta} = \alpha T^{1/2}\frac{\mu_0^3}{T_0^2}\left[\frac{T_0^2}{T^2} - \exp\left(\frac{2\Delta(T-T_0)}{TT_0}\right)\right]. \tag{15}$$

The second term on the left in (15) can be neglected compared with the right-hand side if

$$(J_\varepsilon\lambda)^2 \ll \left(\frac{T}{T_0}\right)^{1/2}\frac{T_0^2\mu_0^2}{4}\begin{cases}\left(\dfrac{T_0}{T}\right)^2 & \text{at } T<T_0,\\ \exp\left(\dfrac{2\Delta(T-T_0)}{TT_0}\right) & \text{at } T>T_0,\end{cases} \tag{16}$$

where $\lambda = (T_0^{3/2}e^{2\Delta/T_0}/2\alpha\Delta^2)^{1/2}$ is the same characteristic length as in Ref. 8.

1) Consider the case $T < T_0$ (i.e., $I_\varepsilon > 0$). Condition (16) is then satisfied if

$$(J_\varepsilon\lambda/\mu_0 T_0)^2 \ll 1. \tag{17}$$

The nondimensional form of (15) is

$$\frac{2\Delta}{T_0}\xi''_{\tilde z^2} = \xi^{1/2}\exp\left[\frac{2\Delta}{T_0}\left(1-\frac{1}{\xi}\right)\right] - \xi^{-3/2}, \tag{18}$$

where $\xi = T/T_0$ and $\tilde z = z/\lambda$. The boundary conditions obtained from (13) for (18) are

$$\xi'_{\tilde z}(0) = 0, \tag{19}$$

$$\xi'_{\tilde z}(a) = -J_\varepsilon\lambda/\mu_0 T_0. \tag{20}$$

We then obtain from (18)

$$\frac{J_\varepsilon\lambda}{\mu_0 T_0}\frac{2\Delta}{T_0} = \left\{\frac{8\Delta}{T_0}\left[\left(1+\frac{T_0\delta}{2\Delta}\right)^{-1/2} - 1\right] + 2\left[\exp\frac{\delta}{1+T_0\delta/2\Delta} - 1\right]\right\}^{1/2},$$

$$\delta = \frac{2\Delta}{T_0}\frac{T(a)-T_0}{T_0}. \tag{21}$$

2) At $T > T_0$ ($I_\varepsilon < 0$) the condition (16) can be shown to be satisfied in the entire range (14). We have then in lieu of (21)

$$\frac{J_\varepsilon\lambda}{\mu_0 T_0}\frac{2\Delta}{T_0} = -\left\{\frac{8\Delta}{T_0}\left[\left(1+\frac{T_0\delta}{2\Delta}\right)^{-1/2} - 1\right] + 2\left(1+\frac{T_0\delta}{2\Delta}\right)^{5/2}\left[\exp\frac{\delta}{1+T_0\delta/2\Delta} - 1\right]\right\}^{1/2}. \tag{22}$$

At $\delta \ll 2\Delta/T_0$ expressions (21) and (22) are transformed into

$$\frac{J_\varepsilon\lambda}{\mu_0 T_0}\frac{2\Delta}{T_0} = \pm[2(e^\delta - \delta - 1)]^{1/2}. \tag{23}$$

3) At $T < T_0$ ($I_\varepsilon > 0$), in the region where $\delta \gg 1$, the second term on the right in (15) can be neglected compared with the first. The analysis can then be carried out in a region where (17) is violated, i.e., $J_\varepsilon\lambda/\mu_0 T_0 \gtrsim 1$. Equation (11) then becomes

$$\mu'' = \alpha\mu^3/T^{3/2}. \tag{24}$$

If (14) is satisfied we have $\mu \approx \mu_0$, and using (10) we can rewrite (24) in the form

$$\frac{2\Delta}{T_0}\xi''_{\tilde z^2} = \xi^{-3/2}, \tag{25}$$

whence, allowing for the boundary conditions (19) and (20), we get

$$\frac{T_0}{8\Delta}\left(\frac{J_\varepsilon\lambda}{\mu_0 T_0}\frac{2\Delta}{T_0}\right)^2 = C + \left(1+\frac{T_0\delta}{2\Delta}\right)^{-1/2}. \tag{26}$$

From the condition that the solutions (23) and (26) be joined in the region $1 \ll \delta \ll 2\Delta/T_0$, we get $C = -1$. Ultimately,

$$\frac{J_\varepsilon\lambda}{\mu_0 T_0}\frac{2\Delta}{T_0} = \left\{\frac{8\Delta}{T_0}\left[\left(1+\frac{T_0\delta}{2\Delta}\right)^{-1/2} - 1\right]\right\}^{1/2}. \tag{27}$$

We note that (27) is a particular case of (21) in this range.

It follows from (25) that the distance from the interface, $a$, at which the usual heat-conduction energy-transport regime sets in, is of the order of $\lambda(2\Delta/T_0)^{1/2}$. We can then rewrite (27) in the form

$$J_\varepsilon a/\mu_0 T_0 = 2[(T_0/T(a))^{1/2} - 1]^{1/2}.$$

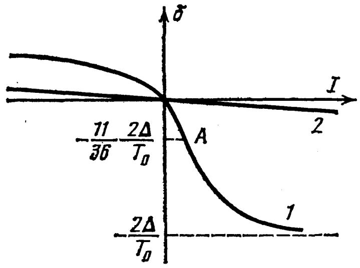


FIG. 1. Curve 1—dependence of the normalized additional temperature discontinuity $\delta$ on the normalized energy flux $I$ (see the text): A—inflection point of curve; curve 2—temperature "rise" over the length $\lambda$ vs the energy flux in the case of ordinary heat conduction.

The condition (14) becomes

$$(T_0/T(a))^{1/4} \ll \Delta/T_0,$$

and is satisfied at practically all temperatures.

Figure 1 shows a plot (curve 1) of the normalized temperature discontinuity

$$\delta = (2\Delta/T_0)\,[T(a) - T_0]/T_0$$

vs the normalized energy flux:

$$\frac{J_\varepsilon\lambda}{\mu_0 T_0}\frac{2\Delta}{T_0} = \frac{I_\varepsilon\lambda}{\varkappa_e T_0}\frac{2\Delta^3}{T_0^3} \equiv I,$$

where

$$\varkappa_e = 2DN(0)\,e^{-\Delta/T_0}\Delta^2/T_0$$

is the electronic heat conduction. From the plot and from (23) it can be seen that the nonlinear deviations become significant at $\delta \approx 1$, i.e., at

$$|T(a) - T_0|/T_0 \approx T_0/2\Delta \ll 1,$$

whereas for the usual Kapitza discontinuity the nonlinearity manifests itself only for a noticeable change of temperature

$$|T(a) - T_0|/T_0 \approx 1.$$

Figure 1 shows also a plot (curve 2) of the temperature "rise" over the length $\lambda$ vs the flux $I_\varepsilon$ in the case of ordinary heat conduction:

$$\frac{2\Delta}{T_0}\frac{(T(\lambda) - T_0)_{\text{heat}}}{T_0} = -\frac{I_\varepsilon\lambda}{\varkappa_e T_0}\frac{2\Delta}{T_0}.$$

The "rise," comparable with the discontinuity, builds up

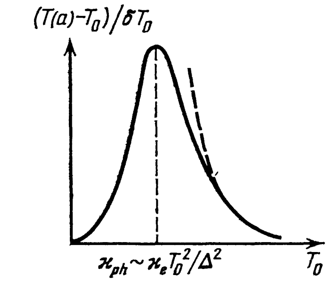


FIG. 2. Additional Kapitza discontinuity vs temperature at low energy fluxes (linear case), with (solid line) and without (dashed) allowance for the phonon heat transport; $\delta_{T_0}$ is the usual Kapitza discontinuity.

over a length

$$\lambda(\Delta/T_0)^2 \gg \lambda.$$

Curve 1 of Fig. 1 shows how allowance for the nonlinearity influences the dependence of the discontinuity on the temperature $T_0$ for a given $I_\varepsilon$. With decreasing $T_0$, the value of

$$\frac{|I_\varepsilon|\lambda}{\varkappa_e T_0}\frac{2\Delta^3}{T_0^3}$$

increases. At $I_\varepsilon < 0$, i.e., when heat flows from the dielectric into the superconductor, $\delta$ is smaller than that predicted by the linear theory.[8] It is larger at $I_\varepsilon > 0$.

The theory developed is not valid at small $T_0$, since energy transport by phonons becomes substantial. Modifying the linear analysis[8] by adding to the energy-flux conservation law [Eq. (9) of Ref. 8] the term $(\varkappa_{ph} T')'$ corresponding to the phonon energy flux ($\varkappa_{ph}$ is the phonon heat conductivity), we obtain the following expression:

$$T(a)-T_0=-I_\varepsilon\lambda_{ph}(\varkappa_e+\varkappa_{ph})^{-1}\left(\frac{T_0^2}{\Delta^2}+\frac{\varkappa_{ph}}{\varkappa_e}\right)^{-1}$$

$$=-I_\varepsilon\lambda\Delta^2(\varkappa_e T_0^2)^{-1}\left(1+\frac{\varkappa_{ph}}{\varkappa_e}\right)^{-3/2}\left(1+\frac{\varkappa_{ph}}{\varkappa_e}\frac{\Delta^2}{T_0^2}\right)^{-1/2}, \qquad (28)$$

where $\lambda_{ph}$ is the characteristic length over which the temperature changes:

$$\lambda_{ph}=\lambda\left[\left(1+\frac{\varkappa_{ph}}{\varkappa_e}\frac{\Delta^2}{T_0^2}\right)\Big/\left(1+\frac{\varkappa_{ph}}{\varkappa_e}\right)\right]^{1/2}.$$

A plot of (28) is shown in Fig. 2, where the dashed curve shows the dependence of $T(a)-T_0$ on $T_0$ at $\varkappa_{ph}=0$ (Ref. 8). When account is taken of the phonon transport, the linear analysis is valid up to large values of the flux $|I_\varepsilon|$, as follows from condition (1).


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