Thermoelectric power of a "dirty" conductor. The role of the quantum corrections

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The "Cooperon"-type quantum corrections to the thermoelectric coefficient of "dirty" conductors are theoretically investigated. The results obtained in the present paper differ significantly from those obtained by Ting, Houghton, and Senna in their well-known paper on the same subject. Thus, the corrections $\Delta_C \eta$ to the thermoelectric coefficient η figuring in the linear relation between the current density and temperature gradient significantly differ in nature and, above all, in their dependence on the magnetic field H from the corresponding corrections to the conductivity. The experimental investigation of the $\Delta_C \eta(H)$ dependence yields information about the energy dependence of the relaxation time τ_φ. The energy and temperature dependences of the relaxation time τ_φ are also investigated in the present paper for the case of inelastic scattering of the electrons by the two-level systems that exist in amorphous conductors.

1. INTRODUCTION AND FORMULATION OF THE PROBLEM

The purpose of the present paper is to discuss the role of the quantum corrections to the thermoelectric coefficient η of a "dirty" conductor. We shall take into account the contribution to this coefficient of the so-called "fan" diagrams, whose special role has been pointed out by Gor'kov, Larkin, and Khmel'nitskiĭ,[1] as well as Abrahams and Ramakrishnan[2] (see also Langer and Neal's paper[3]). We shall consider the situation in which the role of the electron-electron interaction can be assumed to be negligibly small, and shall analyze the cases of both zero and nonzero magnetic field.

We define the coefficient η with the aid of the following relation for the current density **j**:

$$\mathbf{j}=\sigma\mathbf{E}-\eta\nabla T, \tag{1}$$

where ∇T is the temperature gradient, E is the electric field, and σ is the conductivity. In experiment, we usually measure either the coefficient η directly or the ratio η/σ, called the differential thermo-emf.

The problem of computing η in the $H = 0$ case with the aid of the Matsubara dagrammatic technique has been considered by Ting, Houghton, and Senna.[4] They arrived at the conclusion that the relative quantum correction to η coincides exactly with the correction to σ. As a result, according to Ref. 4, corrections to the differential thermo-emf should not arise at all in the approximation under consideration. In that case an experimental investigation of the coefficient η would not yield any additional information about this correction in comparison with the investigation of the conductivity.

The results of the calculation carried out in the present paper do not corroborate this result of Ref. 4. We arrived at the conclusion that, in the two-dimensional case, the "Cooper" correction $\Delta_C \eta$ to the kinetic coefficient η is not proportional to a large logarithm. There is, in the first place, a conspicuous difference between $\Delta_C \eta$ and the corresponding correction to the conductivity founr by Gor'kov, Larkin, and Khmel'nitskiĭ[1] (see also Ref. 2).

The correction $\Delta_C \eta$, like $\Delta_C \sigma$, strongly depends on the magnetic field. But again, the nature of this dependence differs from the one that was found by Altshuler, Khmel'nitzkii, Larkin, and Lee[5] and Hikami, Larkin, and Nagaoka[6] for the quantity $\Delta_C \sigma$. Moreover, we can, by studying this dependence, obtain additional information about the energy dependence of the relaxation time τ_φ figuring in the expression for the "Cooper" pole, and thus obtain additional information about the contributions of the various relaxation mechanisms to this time (for greater details, see the discussion at the end of Sec. 3).

Since the differences between the results obtained in Ref. 4 and our results are so serious, we shall give in the following sections a detailed derivation of the expression for $\Delta_C \eta$, and then discuss the possible causes of this discrepancy.

For the computation of η, we, like Ting *et al.*,[4] use the so-called Π approach. Specifically, instead of the kinetic coefficient η, we shall compute the Peltier coefficient Π, i.e., the coefficient figuring in the linear relation connecting the heat flux **Q** and the electric field **E**:

$$\mathbf{Q}=\Pi\mathbf{E}-\varkappa\nabla T, \tag{2}$$

where $\varkappa$ is the thermal conductivity. The quantities Π and η are connected by the Onsager relation

$$\eta=\Pi/T. \tag{3}$$

The advantage of this approach consists in the fact that we can use the standard linear response theory procedure to derive the expression for Π.

In the case when we can neglect the interelectron interaction, the contribution of an electron in the state Ψ to the energy flux density can be represented in the form

$$(-i\hbar/2m)\, i\hbar\,(\Psi^*\nabla\dot{\Psi}-\dot{\Psi}\nabla\Psi^*)$$

(since in the steady state the actions of the operator $i\hbar\partial/\partial t$ and the Hamiltonian yield the same result). We assume, for simplicity, that the electron spectrum is isotropic and quadratic; m is the effective mass. We shall write the Planck constant $\hbar$ only in the initial formulas, estimates, and final expressions, leaving it out in the intermediate computations. Going over to the second-quantization representation, we obtain for the energy flux density **W** the expression

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$$\mathbf{W}=\mathbf{Q}+\mu\mathbf{j}$$
$$=-\frac{i\hbar}{2m}\left\langle i\hbar\frac{\partial}{\partial t_1}[\Psi^+(x)\nabla\Psi(x_1)-\Psi(x_1)\nabla\Psi^+(x)]\right\rangle\Big|, \tag{4}$$

where the angle brackets denote thermodynamic averaging, as well as averaging over the positions of the defects scattering the electrons; μ is the chemical potential. Let us emphasize that we use in (4) the total-electron-energy operator, which we write in the form $i\hbar\partial/\partial t$, and not the free electron-energy operator $-\hbar^2\nabla^2/2m$, thereby taking account of the change that occurs in the electron spectrum in the field of the randomly disposed impurities.

Further, we shall use the representation with a fixed chemical potential, i.e., we shall measure the electron energy from the level μ.

2. COMPUTATION OF $\Delta_C\eta$ IN THE H = 0 CASE

In the Matsubara procedure the sought Cooperon correction $\Delta_C\mathbf{Q}$ is given by the graph shown in Fig. 1. To the right-hand vertex corresponds the factor $(\mathbf{p}/m)\varepsilon[\varepsilon = \pi iT(2n+1)]$, while to the left-hand vertex corresponds, as usual, the factor $(e/mc)\mathbf{p}\cdot\mathbf{A})$; here $\mathbf{A}$ is the vector potential of the electric field $\mathbf{E}$ in the div $\mathbf{A}=0$ gauge. To the graph corresponds the expression

$$\Delta_C\mathbf{Q}=\frac{2eT}{m^2c}\sum_{\varepsilon}\int(d\mathbf{p})(d\mathbf{q})(\mathbf{pA})\mathbf{p}\tilde{C}(\varepsilon,\Omega,q^2)$$
$$\times\varepsilon G(\mathbf{p},\varepsilon+\Omega)G(-\mathbf{p}+\mathbf{q},\varepsilon+\Omega)$$
$$\times G(\mathbf{p},\varepsilon)G(-\mathbf{p}+\mathbf{q},\varepsilon). \tag{5}$$

Here $\Omega = 2\pi iTm$ is the discrete frequency corresponding to the variation in time of the vector potential $\mathbf{A}$; we need to perform an analytic continuation with respect to it, and then go over to the limit $\Omega\to 0$; the G's are electron Green functions; $(d\mathbf{p})=d^u p/(2\pi\hbar)^u$, where u is the dimensionality of the space; and $\tilde{C}$ is the sum of the fan diagrams (the so-called Cooperon).

The symbol $\tilde{C}$ denotes the result of the following contraction over the spin indices (see Fig. 1):

$$\tilde{C}\equiv\tilde{C}_{\alpha\beta\beta\alpha}. \tag{6}$$

The quantity $\tilde{C}$ can be represented in the form

$$\tilde{C}=\Theta(\operatorname{Im}\varepsilon)\Theta(-\operatorname{Im}\varepsilon-\operatorname{Im}\Omega)C(-\Omega,q^2,\varepsilon)$$
$$+\Theta(-\operatorname{Im}\varepsilon)\Theta(\operatorname{Im}\varepsilon+\operatorname{Im}\Omega)C(\Omega,q^2,\varepsilon). \tag{7}$$

Let us, for simplicity, restrict ourselves to the consideration of electron scattering by point defects. If we denote by τ the lifetime of an electron in a state with a given momentum $\mathbf{p}$, then the expression for C has the form

$$C(i\Omega,q^2,\varepsilon)=\frac{1}{2\pi\nu\tau^2}\frac{1}{Dq^2-i\Omega}, \tag{8}$$

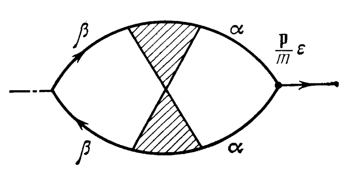

FIG. 1.

where ν is the density of electron states (without allowance for spin) and D is the diffusion coefficient. Here the quantities ν, τ, and D are assumed to be dependent on the electron energy $\varepsilon+\mu$. In the computation of the corrections to the conductivity, these quantities appear at $\varepsilon=0$. But in the computation of the correction $\Delta_C\eta$, as in the computation of the coefficient η itself, this degree of accuracy turns out to be inadequate. It is precisely the consideration of this circumstance that, to the best of our judgment, gives rise to the discrepancy between our results and the results obtained in Ref. 4 (see below).

Being interested in the steady-state thermo-emf, we should take into account the mechanisms that eliminate the divergence, as $\Omega\to 0$, of the expression (8). We shall consider two such mechanisms. First, we shall consider the spin-spin (and, in part, the spin-orbit) interaction, which has been studied for the present problem by Hikami, Larkin, and Nagaoka,[6] as well as Lee.[7] This interaction can play an important role in ordinary "dirty" metals. Second, we shall consider the inelastic scattering of the current carriers by two-level systems (TLS) (see Blake's review article[8]). This mechanism can play a role in amorphous metals.

In the first case we write the expression obtained in Ref. 6 for C in the following form, which is suitable for the subsequent computations:

$$C(i\Omega,q^2,\varepsilon)=\frac{1}{2\pi\nu\tau^2}\left[\frac{1}{Dq^2-i\Omega+1/\tau_1}+\frac{1}{2}\frac{1}{Dq^2-i\Omega+1/\tau_2}\right.$$
$$\left.-\frac{1}{2}\frac{1}{Dq^2-i\Omega+1/\tau_3}\right]. \tag{9}$$

Here

$$1/\tau_1=2(1/\tau_{s0}{}^z+1/\tau_{s0}{}^x+1/\tau_s{}^x), \tag{10}$$
$$1/\tau_2=2(1/\tau_s{}^z+2/\tau_{s0}{}^x), \tag{11}$$
$$1/\tau_3=2(1/\tau_s{}^z+2/\tau_s{}^x). \tag{12}$$

The quantities $1/\tau_s^i$ characterize spin-flip scattering:

$$1/\tau_s{}^i=2\pi\nu n_1c^2\overline{S_i{}^2}, \tag{13}$$

where n_1 is the concentration of the magnetic impurities, c is the spin-flip scattering amplitude, and $\overline{S_i^2}$ is the mean square of the ith component of the impurity's spin. The quantities $1/\tau_{s0}^i$ characterize the spin-orbit scattering:

$$1/\tau_{s0}{}^i=2\pi\nu nb^2\overline{[\mathbf{pp}']}_i, \tag{14}$$

where $ib\mathbf{p}\times\mathbf{p}'\boldsymbol{\sigma}$ is the spin-orbit scattering amplitude; the bar denotes averaging over the constant-energy surface. Accordingly, the reciprocal lifetime τ^{-1} has the additional terms

$$1/\tau_{s0}{}^z+1/\tau_s{}^z+2/\tau_{s0}{}^x+2/\tau_s{}^x. \tag{15}$$

In the second case the expression for C has the form

$$C(i\Omega,q^2,\varepsilon)=\frac{1}{2\pi\nu\tau^2}\frac{1}{Dq^2-i\Omega+1/\tau_\varphi}, \tag{16}$$

where

$$\frac{1}{\tau_\varphi}=\frac{2\pi\beta}{\hbar}Tg\left(\frac{\varepsilon}{T}\right),\quad g(x)=x\operatorname{cth}\frac{x}{2}=x\frac{N(x)}{n(x)}. \tag{17}$$

Here $N(x)$ and $n(x)$ are respectively the Planck and Fermi distribution functions; β is a dimensionless parameter characterizing the interaction of the electrons with the two-level systems (TLS):

$$\beta(\varepsilon)=(\overline{V^2}/V_0^2)Pr_c^3. \tag{18}$$

Here $\overline{V^2}$ is the mean square of the electron-TLS interaction matrix element, $V_0^{-1}=n_e^{-1}\nu$ is the density of electron states per electron (n_e is the electron concentration), and r_c is the characteristic range of the TLS-electron interaction potential. Finally, P is the density of TLS states per unit volume and unit energy range dE; this density changes little on going from one typical amorphous metal to another. The characteristic value of the dimensionless constant β for such metals is of the order of 10^{-2}–10^{-3}. The derivation of the expression (17) for τ_φ is given in Sec. 4. There we also give for reference an expression for τ_φ for the case when it is due to electron scattering by the phonons.

Notice that the denominator in (16) contains the reciprocal lifetime of the state with the given energy, i.e., the integrated part of the linearized collision operator. At this point there occurs a fundamental departure from the equation for the "diffusion" with the same energy, where the denominator of the corresponding expression would contain the linearized collision operator as such.[1)]

Let us now turn to the direct computation of the thermoelectric coefficient. Proceeding in the standard fashion, and performing an analytic continuation with respect to the variable ε, and also using the Onsager relation (3), we obtain

$$\Delta_C\eta=\frac{e}{m^2}\int(d\mathbf{p})(d\mathbf{q})\frac{d\varepsilon}{2\pi}\frac{\partial n_0}{\partial\varepsilon}p_x^2C^R(\Omega,q^2,\varepsilon)$$
$$\times G^A(\varepsilon+\Omega,\mathbf{p})G^A(\varepsilon+\Omega,-\mathbf{p}+\mathbf{q})$$
$$\times G^R(\varepsilon,\mathbf{p})G^R(\varepsilon,-\mathbf{p}+\mathbf{q}), \tag{19}$$

where C^R is the analytic continuation with respect to the variable Ω of the function $C(i\Omega)$ into the upper half-plane.

Let us neglect the small $\mathbf{q}$ term in the arguments of the Green functions. The $(d\mathbf{p})$ and $(d\mathbf{q})$ integrations are then separable. Let us set

$$\mathfrak{C}(\varepsilon)=2\pi\int(d\mathbf{q})C^R(0,q^2,\varepsilon)\nu(\varepsilon)\tau^2(\varepsilon). \tag{20}$$

Integrating over the angles, and introducing the density of states ν as a function of the variable $\xi=p^2/2m-\mu$, we obtain

$$\Delta_C\eta=\frac{2e}{umT}\int_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi}\varepsilon\frac{\partial n_0}{\partial\varepsilon}\mathfrak{C}(\varepsilon)\frac{\varepsilon+\mu}{2\pi\nu\tau^2}$$
$$\times\int d\xi\nu(\xi)[G^A(\varepsilon,\xi)G^R(\varepsilon,\xi)]^2. \tag{21}$$

But

$$G^A(\varepsilon,\xi)G^R(\varepsilon,\xi)=\frac{1}{(\varepsilon-\xi)^2+1/4\tau^2}. \tag{22}$$

Further, the ξ integral can be evaluated in the pole approximation, i.e., in the lowest order in the parameter $\hbar/\mu\tau$:

$$\int d\xi\nu(\xi)[G^A(\varepsilon,\xi)G^R(\varepsilon,\xi)]^2=4\pi\tau^3\nu(\varepsilon). \tag{23}$$

Here we assume that, to the adopted accuracy, $\nu(\varepsilon)=\nu(0)+\varepsilon\nu'(0)$, where $\nu(0)$ and $\nu'(0)$ are respectively the density of states and the energy derivative of this quantity at the Fermi level $\varepsilon=0$. As a result

$$\Delta_C\eta=\frac{4e}{T\hbar}\int_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi}\frac{\partial n_0}{\partial\varepsilon}\varepsilon D(\varepsilon)\mathfrak{C}(\varepsilon), \tag{24}$$

where, to make the notation more compact and for convenience in respect of the subsequent analysis, we have introduced a diffusion coefficient $D(\varepsilon)$ that depends on the energy ε measured from the Fermi level.

The computation carried out in the derivation of the expression (24) needs to be explained. It is necessary to indicate how the ε dependence of the quantities figuring in the formulas (23) and (24) can be taken into account. The computation of $\Delta_C\eta$ (like the computation of the quantity η itself with the aid of the diagrammatic technique) requires the evaluation of integrals over ξ of slowly varying—in the vicinity of the Fermi level—functions $f(\xi)$ multiplied by combinations of the type $G^R(\varepsilon,\xi)G^A(\varepsilon,\xi)$. The latter are always functions of the difference $x=\varepsilon-\xi$. The integrand contains the factor $\partial n_0/\partial\varepsilon$. In the case

$$T/\mu\ll 1 \tag{25}$$

of interest to us, it is a sharp function of the energy ε, even in ε. Therefore, to within the first nonvanishing term in the small parameter (25), we can expand the integrand $f(\varepsilon-x)$ in a series in powers of the small argument ε, limiting ourselves to the first two terms, and evaluate the integrals according to the rule:

$$\frac{1}{2\pi\tau}\int_{-\infty}^{\infty}dx\frac{f(\varepsilon+x)}{x^2+1/4\tau^2}=f(0)+\varepsilon f'(0)=f(\varepsilon), \tag{26}$$

since, in computing $\Delta_C\eta$, it is sufficient for us to retain in both terms on the right-hand side only the terms of lowest order in the parameter $1/\mu\tau$.

Notice that the expression (24) is found in accordance with the Cutler-Mott formula.[10]

Let us proceed to analyze the quantity $\mathfrak{C}(\varepsilon)$, (20). It is proportional to the sum of terms of the form

$$\frac{1}{D(\varepsilon)}\int\frac{(d\mathbf{q})}{q^2+L_i^{-2}(\varepsilon)}, \tag{27}$$

where in the case of the spin-orbit interaction L_i^2 is given in terms of one of the times τ_i by the formula

$$L_i^2=D(\varepsilon)\tau_i(\varepsilon). \tag{28}$$

But in the case of the interaction with the TLS we have only one term of the form (27), with

$$L_\varphi^2=D(\varepsilon)\tau_\varphi(\varepsilon,T), \tag{29}$$

where τ_φ is given by the formula (17). Let us emphasize that if the interaction with the TLS, as well as with the phonons, predominates, then we must, in computing $\Delta_C\eta$, take account of the variation of the time τ_φ over the characteristic energy scale $\varepsilon\sim T$, since it is precisely this energy region that is important in the expressions (24) and (17).

Thus far, all the formulas written out above [with the appropriate delimitation of the limits of the $(d\mathbf{q})$ integration] have been applicable to both the $u=2$ and $u=3$ cases. Below we shall focus our attention on the two-dimensional case, in which the quantum corrections of interest to us are more strongly pronounced.

Substituting as the upper limit of integration in (27) a quantity of the order of $1/l = 1/v\tau$, where $v = p/m$ is the electron velocity, we obtain, for $l \ll L$,

$$\frac{1}{2\pi D(\varepsilon)} \ln (L/l). \tag{30}$$

Substituting this expression into (24), we obtain for the contribution $\Delta_{Ci}\eta$ made by each term of the type (27) to $\Delta_C\eta$ the following expression

$$\Delta_{Ci}\eta = \frac{2e}{\pi T\hbar} \int\limits_{-\infty}^{\infty} \frac{d\varepsilon}{2\pi} \frac{\partial n_0}{\partial \varepsilon} \varepsilon \ln \left[\frac{L_i(\varepsilon)}{l} \right]. \tag{31}$$

The subsequent computation is easiest in the case of spin scattering, when the characteristic energy scale determining the dependence $L_i(\varepsilon)$ is much greater than the temperature T. In that case, in the lowest approximation in the parameter T/μ, we have after summing the expressions of the type (31):

$$\Delta_C\eta = -\frac{1}{6} \frac{eT}{\hbar} \frac{\partial}{\partial \varepsilon} \ln \left[\frac{L_1}{l} \left(\frac{L_2}{L_3} \right)^{1/2} \right] \bigg|_{\varepsilon=0}. \tag{32}$$

This formulation most clearly demonstrates the difference between the characters of the quantum corrections to the conductivity and the thermoelectric coefficient. In the first case the answer would contain the logarithm of a large argument; in the second, the energy derivative of this logarithm.

In the two-dimensional case the relaxation time τ does not, when the scattering is by "point" impurities, depend on the energy ε; therefore, from (32) we have

$$\Delta_C\eta = -\frac{eT}{12\hbar} \left[\frac{1}{\tau_1} \frac{\partial \tau_1}{\partial \varepsilon} + \frac{1}{2\tau_2} \frac{\partial \tau_2}{\partial \varepsilon} - \frac{1}{2\tau_3} \frac{\partial \tau_3}{\partial \varepsilon} \right] \bigg|_{\varepsilon=0}.$$

In the three-dimensional case the mean free path l is energy independent when the scattering is by "point" impurities. A calculation similar to the one performed above yields

$$\Delta_C\eta = -\frac{eT}{24\hbar} \left[\frac{1}{L_1^2} \frac{\partial L_1}{\partial \varepsilon} + \frac{1}{2L_2^2} \frac{\partial L_2}{\partial \varepsilon} - \frac{1}{2L_3^2} \frac{\partial L_3}{\partial \varepsilon} \right] \bigg|_{\varepsilon=0}. \tag{33}$$

In the case when the impurities are not point impurities and $u = 3$ the **q** integral in the expression for the "Cooperon" is determined by the upper limit, which, in the present case, depends on ε. Therefore, the expression for the quantum correction should look significantly more complicated than (33). It is, however, important that the expression for the difference between the zero-magnetic-field and nonzero-magnetic-field values of $\Delta_C\eta$ (and, hence, for the coefficient η itself) does not depend on the indicated limit (see the following section). Let us determine what fraction of the thermoelectric coefficient η_0 in the zeroth approximation, which is given by the expression

$$\eta_0 = \frac{\pi^2}{3} eT \frac{\partial}{\partial \varepsilon} [\nu(\varepsilon) D(\varepsilon)] |_{\varepsilon=0}, \tag{34}$$

this correction constitutes. The ratio $\Delta_C\eta/\eta_0$ is, in order of magnitude, equal to

$$\begin{aligned} &\hbar/p_F l \quad \text{for} \quad u=2, \\ &\hbar^2/p_F^2 l L_\varphi \quad \text{for} \quad u=3. \end{aligned} \tag{35}$$

Despite its relative smallness, the contribution $\Delta_C\eta$ can be separated out through its dependence on the magnetic field. This question will be considered in detail in the following section. Furthermore, the magnitude of this contribution can be significantly greater than the value given by the estimate (35) if L_i depends strongly on ε over an energy scale $\lesssim T$ through the characteristics of the scattering mechanism that determine the magnitude of L_i.

In the case of relaxation on the TLS the integral in (30) should be evaluated more accurately, since the $\tau_\varphi(\varepsilon)$ dependence is characterized by two energy scales. One of them is, according to (17), T. The other is given by the $\beta(\varepsilon)$ dependence. In the energy region of interest to us, we can represent it in the form $\beta(\varepsilon) = \beta(0) + \varepsilon\beta'(0)$, so that in order of magnitude $|\beta'(0)| \approx \beta(0)/\varepsilon_c$, where ε_c is the characteristic scale of the β variation. As a result, the expression for $\ln(\tau_\varphi/\tau)$, which figures in a formula of the type (31) for the thermo-emf, can be represented in the form

$$\ln \left(\frac{\tau_\varphi}{\tau} \right) = -\ln g \left(\frac{\varepsilon}{T} \right) - \varepsilon \frac{\beta'(0)}{\beta(0)} + \text{const}. \tag{36}$$

The integral of the first term yields, on account of its evenness, zero, and we obtain as a result the expression

$$\Delta_C\eta = \frac{eT}{12\hbar} \frac{\beta'(0)}{\beta(0)}. \tag{37}$$

We must now discuss the possible causes of the discrepancy between our results and the results obtained in Ref. 4. It is difficult for us to form an entirely definite opinion about this, since details of the calculation of the coefficient η are not given in Ref. 4. Therefore, our inference will inevitably have a conjectural character.

For example, the formula (4) of Ref. 4 can be obtained if we ignore the dependence on ε of the quantities D and τ_φ entering into the Cooperon block, but take into account the corresponding dependences at all the remaining points. But as we verified above, allowance for this dependence in the Cooper block leads to the cancellation of the logarithmic corrections to η. As a result, it turns out that, in contrast to the correction to σ (Ref. 1), the correction to η does not contain a large logarithm as a factor.

3. COMPUTATION OF $\Delta_C\eta$ IN A MAGNETIC FIELD

Let us begin by considering the purely two-dimensional situation. We shall assume that the thickness d of the film satisfies the following conditions:

$$d \ll a_H \equiv (c\hbar/|e|H)^{1/2},\ L_i. \tag{38}$$

The magnetic field H is assumed to be perpendicular to the plane of the film.

We shall proceed along the lines of the paper by Al'tshuler, Khmel'nitskii, Larkin, and Lee,[5] who consider the effect of a magnetic field on the correction to the conductivity. According to this paper, in fairly weak magnetic fields it is sufficient to consider the effect of the field only on the

Cooperon. Allowance for this effect amounts to the making of the following substitution in the Cooper block:

$$\int \frac{d^2q}{(2\pi\hbar)^2}\frac{1}{Dq^2+1/\tau_i} \to \frac{4|e|H}{2\pi c\hbar}\sum_{n=0}^{m}\left[4D\left(\frac{|e|H}{\hbar c}\right)\times\left(n+\frac{1}{2}\right)+\frac{1}{\tau_i}\right]^{-1}, \tag{39}$$

where we take as the upper limit the integral part of the quantity

$$c\hbar/2|e|Hl^2=a_H{}^2/2l^2, \tag{40}$$

and the results make sense when $m\gg 1$, i.e., when $a_H\gg l$. As a result, it turns out that the product $2\pi D(\varepsilon)\mathfrak{C}(\varepsilon)$ figuring in the expression for the thermo-emf can, by analogy with the corresponding formulas of the preceding section, be represented in the form of a sum of terms of the form

$$\sum_{n=0}^{\infty}\frac{1}{n+{}^1/_2+\gamma_i(\varepsilon)}-\sum_{n=0}\frac{1}{n+m+{}^1/_2+\gamma_i(\varepsilon)}, \tag{41}$$

where $\gamma_i=a_H^2/4L_i^2(\varepsilon)$ and the quantities $L_i(\varepsilon)$ are given by the formula (28). Proceeding in the same way as we did above, we obtain the following results.

In the case of "spin" scattering

$$\Delta_C\eta=\frac{eT}{12\hbar}\left(\Phi_1+\frac{1}{2}\Phi_2-\frac{1}{2}\Phi_3\right), \tag{42}$$

where

$$\Phi_i=[\gamma_i'\zeta(2,\ \gamma_i+{}^1/_2)+D'(\varepsilon)/D(\varepsilon)]|_{\varepsilon=0},$$

$$\zeta(q,x)=\sum_{n=0}^{\infty}(x+n)^q, \tag{43}$$

ξ being the Riemann function.

In the case of scattering by the TLS

$$\Delta_C\eta=\frac{eT}{12\hbar}\Phi_\varphi, \tag{44}$$

where

$$\Phi_\varphi=\frac{3\gamma'}{\pi^2}\int_{-\infty}^{\infty}\frac{x^2\,dx}{4\,\mathrm{ch}^2(x/2)}g(x)\zeta\left(2,\gamma g(x)+\frac{1}{2}\right)+\frac{D'(0)}{D(0)},$$

$$\gamma=\frac{2\pi a_H{}^2\beta(0)T}{4\hbar D(0)},\quad \gamma'=\gamma\left[\frac{\beta'(0)}{\beta(0)}-\frac{D'(0)}{D(0)}\right]. \tag{45}$$

The second terms in the expressions (43) and (45) are connected with the ε derivative of the upper limit of the summation over n. They do not depend on the magnetic field, since we assume that $a_H\gg l$, and ensure the $H\to 0$ transition of the expressions (42) and (44) into (32) and (37) respectively. Notice that the (H dependent) first terms in (43) and (45) have different forms. The more complicated form of (45), as compared to (43), is due to the fact that, in the case of inelastic processes, the time $\tau_\varphi(\varepsilon)$ depends essentially on the energy ε in a region of width of the order of T, and we cannot set $x=0$ in the argument of the function g. This is the situation in the case of inelastic scattering of any nature, and, thus, a detailed study of the experimental dependence of $\Delta_C\eta$ on the magnetic field allows to determine how important the inelastic processes are and precisely which of them predominate. In the absence of a magnetic field, this difference does not arise, since g is then an even function of its argument. In stong magnetic fields, i.e., for $l\ll a_H\ll L_\varphi$, we can neglect γg in comparison with 1/2 in the expression (45). As a result, the first term assumes the form

$${}^3/_{16}\pi^4\gamma'\propto H^{-1}. \tag{46}$$

In the three-dimensional case a calculation similar to the one performed above yields magnetic-field-dependent contributions differing from (43) and (45) by the replacement $\zeta(2,x)\to(l/4a_H)\zeta(3/2,x)$. There are no H-independent terms in the case of scattering by point impurities, since l does not depend on ε. If, on the other hand, l depends on ε, then within the framework of our approximations we can consider only $\Delta_C\eta(H)-\Delta_C\eta(0)=\eta(H)-\eta(0)$.

4. COMPUTATION OF τ_φ FOR THE INELASTIC PROCESSES

Let us give the derivation of the expressions (16) and (17), which give the sum of the fan diagrams with allowance for the inelastic scattering. At this scattering, we shall consider either the scattering by the phonons or the scattering resulting from the interaction of the conduction electrons with TLS. We shall in either case represent the corresponding propagator (with allowance for the vertices describing the interaction) by a wavy line (see Fig. 2a).

In the approximation in which both the electronic spectrum and the elastic properties of the medium are isotropic, so that we need to take into consideration the interaction of the electrons with only the longitudinal phonons, the propagator for these phonons can be represented in the form

$$D^R(\omega,q)=|c_q|^2[(\omega-\Omega_q+i0)^{-1}-(\omega+\Omega_q+i0)^{-1}], \tag{47}$$

where

$$|c_q|^2=\hbar\Xi^2\Omega_q/2\rho s^2.$$

Here q is the phonon wave vector, s is the velocity of (longitudinal) sound, $\Omega_q=sq$, and Ξ is the constant of the deformation potential describing the interaction of the electrons with the deformation field.

Let us explain the expression (47) for the electron-phonon vertex c_q. The relaxation time τ_φ is determined by the actual processes to which the phonons with wave vectors $q\approx q_T\equiv T/\hbar s$ contribute. In fairly good conductors the wavelength of such a phonon is much longer than the characteristic screening distance, and, hence, the averaged—over the Fermi surface—part of the deformation potential

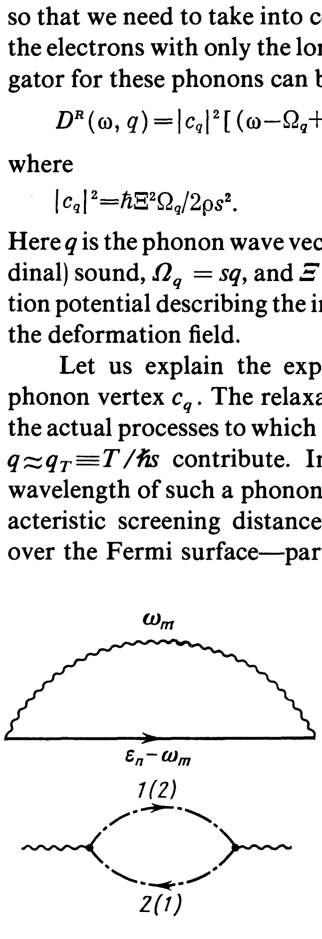

FIG. 2.

tensor $\Xi_{ik}(\mathbf{p})$ is virtually completely canceled out by the self-consistent electric field that arises in the presence of deformation. Therefore, it can be assumed that $\langle \Xi_{ik}(\mathbf{p})\rangle = 0$, where the angle brackets denote averaging over the constant-energy surface, but that $\langle \Xi^2_{ik}(\mathbf{p})\rangle \neq 0$. Accordingly, the encircling of the electron-phonon vertex by an impurity dashed line yields an additional infinitesimal. As a result, the value of the electron-phonon vertex turns out to be the same as in the pure metal.

In the specific computations of the time τ_φ, because of the interaction of the electrons with the phonons, we shall assume the quantity Ξ^2 in (47) to be isotropic, and not consider its complicated angular dependence. This allows us to obtain the correct temperature and energy dependences, as well as order-of-magnitude estimates for the quantity τ_φ.

Of greater interest to us will be the propagator describing the interaction of the electrons with the TLS. The Hamiltonian of the electron-TLS system has the form[8]

$$H=\frac{1}{2}\sigma_3\Delta-\frac{1}{2}\sigma_1\Lambda+\sum_{\mathbf{p}}\varepsilon_{\mathbf{p}}c_{\mathbf{p}}{}^+c_{\mathbf{p}}+\frac{\sigma_3}{2}\sum_{\mathbf{p},\mathbf{p}'}V_{\mathbf{p}\mathbf{p}'}c_{\mathbf{p}}{}^+c_{\mathbf{p}'}. \quad (48)$$

Here Δ is a parameter characterizing the asymmetry of the TLS, Δ is the tunneling parameter of the TLS,[2)] σ_1 and σ_3 are the Pauli matrices, which act in the space defined by the states of the TLS, $\varepsilon_{\mathbf{p}}$ is the single-particle electron energy, the $c_{\mathbf{p}}^+$ $(c_{\mathbf{p}})$ are the creation (annihilation) operators for an electron in the state $\mathbf{p}$, and $V_{\mathbf{pp}}{}'$ is the matrix element giving the change that occurs in the asymmetry Δ during transitions in the electron subsystem.

Let us perform on the states of the TLS the canonical transformation that diagonalizes the sum of the first two terms in (48):

$$\tilde{H}=\frac{1}{2}E\sigma_3+\sum_{\mathbf{p}}c_{\mathbf{p}}{}^+c_{\mathbf{p}}\varepsilon_{\mathbf{p}}+\sum_{\mathbf{p},\mathbf{p}'}\left(W^{(1)}_{\mathbf{p}\mathbf{p}'}\sigma_1+W^{(3)}_{\mathbf{p}\mathbf{p}'}\sigma_3\right)c_{\mathbf{p}}{}^+c_{\mathbf{p}'}. \quad (49)$$

Here

$$E=(\Delta^2+\Lambda^2)^{1/2},\quad W^{(1)}_{\mathbf{p}\mathbf{p}'}=\frac{1}{2}\frac{\Lambda}{E}V_{\mathbf{p}\mathbf{p}'},\quad W^{(3)}_{\mathbf{p}\mathbf{p}'}=\frac{1}{2}\frac{\Delta}{E}V_{\mathbf{p}\mathbf{p}'}. \quad (50)$$

Further, we shall use for the Pauli matrices σ_1 and σ_3 the quasifermion representation introduced by Abrikosov[11] for the description of spin systems, and used by Maleev[12] to analyze the TLS in glasses.

The interaction between the conduction electrons and the TLS can cause the atom or group of atoms forming the TLS to undergo transitions from one level to another. As a result, there arises a TLS-mediated inelastic interaction between the conduction electrons that is similar to the phonon-mediated interaction. With this interaction can be associated a propagator of the boson type. It is depicted in Fig. 2b.

In this representation a TLS is described by a Fermi propagator (the arrowed dot-dash lines in Fig. 2b). The numbers on the fermion lines indicate the TLS-level numbers, 1 and 2. Since we are interested in the inelastic processes only, we take into consideration only the term with σ_1 in the last term of the Hamiltonian (49). In this case the retarded propagator of the effective TLS-mediated interaction between the electrons (which, according to our definition, includes the $W^{(1)}$ vertices) can be represented in the form

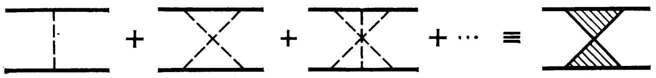

FIG. 3.

$$D^R(\omega,\mathbf{q})=-\frac{P}{4}\int_0^\infty dE\,|V_{\mathbf{q}}|^2\,\mathrm{th}\frac{E}{2T}\left[\frac{1}{\omega-E+i0}-\frac{1}{\omega+E+i0}\right]. \quad (51)$$

Let us now find out how a wavy propagator can figure in the equation for the Cooperon, i.e., the sum of all the fan diagrams. In the zeroth approximation in the inelastic scattering the answer is given by the sum of the diagrams shown in Fig. 3, which satisfy the integral equation depicted in Fig. 4.

As is evident from Ref. 1, the Cooperon corrections separate out because there occurs, when the $(d\mathbf{p}')$ integration in the last diagram in Fig. 4 is performed, almost total cancellation of the left-hand side of the equation depicted in the figure, or, more precisely, the difference between the left-hand side and the second term on the right-hand side is $Dq^3\tau C$.

The inelastic scattering leads to two types of graph modification. The modification of the first type constitutes a correction to the self-energy of the electron (Fig. 2a). Its allowance amounts to the following replacement in the electron Green function

$$1/2\tau\to 1/2\tau+1/2\tau_\varphi, \quad (52)$$

where

$$\frac{1}{2\tau_\varphi(\varepsilon,T)}=2\int\frac{d\omega\,(d\mathbf{p}')}{2\pi}\,\mathrm{Im}\,D^R(\omega,\mathbf{q})\,\mathrm{Im}\,G^R(|\varepsilon|-\omega,\mathbf{p}'-\mathbf{q})\times N(\omega)\frac{n(|\varepsilon|-\omega)}{n(|\varepsilon|)}. \quad (53)$$

In the case of electron scattering by three-dimensional phonons, we have after substituting (47) into (53)

$$\tau_\varphi^{-1}(\varepsilon,T)=bI(\varepsilon/T)T^3/\omega_D{}^2\hbar^2,$$

$$I(x)=\frac{1}{n(|x|)}\int_0^{2sp_F/T}y^2\,dy\,N(y)\left[n(|x|-y)+e^y n(|x|+y)\right], \quad (54)$$

$$b=\Xi^2mk_D{}^2/4\pi\rho s^2p\approx\mu/\rho s^2.$$

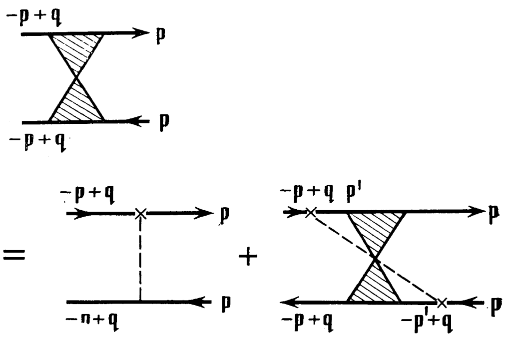

FIG. 4.

In computing the TLS-induced relaxation time τ_φ, we must take into account the fact that in metals (after the averaging over the positions of the TLS in the sample) $|V_q|^2$ does not, to a high degree of accuracy, depend on q. After substituting (51) into (53), w can easily obtain the formula (18).

The second type of modification consists in the fact that there arise graphs in which the wavy lines go into a fan (see Fig. 4). In our approximation such graphs make a small contribution. Indeed, allowance for the wavy lines in a fan leads to the appearance of pole factors of the type $[-i\Omega + Dq^2 + 1/\tau_\varphi]$, where Ω is the transmitted frequency, equal in order of magnitude to $T/\hbar$ [see (17) and (47)]. Thus, on account of the condition $T\tau_\varphi/\hbar \gg 1$, the corresponding contributions are negligibly small.

5. CONCLUSION

Thus, we see that the investigation of the thermoelectric phenomena occurring in two-dimensional dirty conductors can yield a wide variety of data, by no means identical with the data that are obtained in the experimental investigation of magnetoresistance. We have at present a wealth of experimental magnetoresistance data on dirty conductors. These data are interpreted in a number of cases from the standpoint of the theory of quantum corrections. The investigation of the thermoelectric phenomena can serve as an additional verification of the correctness of such an interpretation.

In the present paper we limited ourselves to the investigation of only one possible experimental geometry. In actual fact the calculation carried out here can, if the needs of the experiment require it, relatively easily be generalized to the case of another geometry. It is especially tempting to investigate a thermoelectric circuit consisting of two identical long thin-walled cylinders. The thermoelectric current in such a circuit should, evidently, be equal to zero. But by producing a magnetic field in one of the cylinders, we obtain a way of carrying out direct measurement of the Cooperon contribution to the thermo-emf. We should, however, note that, for the purpose of measuring the absolute values of the coefficient η, we may find it useful to study a thermoelectric circuit consisting of the dirty conductor under investigation and a superconductor.

The entire analysis performed above concerned the case of the isotropic spectrum. Let us discuss what changes will have to be made in the results obtained when we go over to the real situation. First, the relaxation time τ_φ in the expression for the diffusion coefficient will be replaced by the reciprocal operator $\hat{I}^{-1}$ for the collisions with the elastic impurities; as a result, the diffusion coefficient will become a tensor: $D(\varepsilon) \to D_{ik}(\varepsilon) = \langle v_i \hat{I}^{-2} v_k \rangle_\varepsilon$, where the angle brackets denote averaging over the constant-energy surface. In view of this, we shall have entering into the expression for L_i some average—over the directions of $\mathbf{q}$—value of the quantity $D_{ik}(\varepsilon) q_i q_k / q^2$. It is, however, important that the energy dependence of this quantity is the same as that of the component of the tensor $\hat{D}$ entering into the numerator of the integrand in (24). In consequence, as in the isotropic case, the corresponding energy dependences cancel out, and the results differ by only numerical coefficients.

Further, we should emphasize that, in the present paper, we have studied only the "diffusion-governed part" of the thermo-emf. In principle, there exists another contribution to the coefficient η resulting from the entrainment of the electrons by the phonons. In interpreting experiments, it is advisable for us to verify that the contribution from the phonon drag is negligibly small. In the first place, the temperature dependence of the main part of the thermoelectric coefficient η can serve as the criterion here.

Futhermore, we must bear the following in mind. In discussing the theory of the electron-phonon interaction, we considered, as is customarily done in such problems, the electrons to be two-dimensional and the phonons to be "three-dimensional." This corresponds to the usual physical situation: a metallic film is fastened to a bulk substrate in which phonons can propagate. The idea we wish to express here is that, to decrease the effect of the drag, we must take a substrate prepared from an amorphous dielectric (glass). The phonons in such a substrate have a short mean free path, since they are intensively scattered by the TLS. We should only bear in mind that, in this case, even when the metallic film is crystalline, the conduction electrons can interact with the dielectric's TLS located at the boundary with the metal, which can lead to a decrease in the time τ_φ.[3] Thus, from the standpoint of the magnitude of this time it does make a difference from which material (amorphous or crystalline) the substrate is made.

Thus, the study of the thermoelectric coefficient in dirty conductors can yield a variety of data on the relaxation mechanisms determining the magnitude of the quantum corrections. Of primary interest to us here is the comparison of the τ_φ values that result from the processing of experimental $\Delta_C \eta$ and $\Delta_C \sigma$ data.

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[1] This operator is written out in its explicit form in, for example, the paper by Gal'perin, Gurevich, and Parshin.[9]

[2] In many papers this quantity is denoted by Δ_0.

[3] This argument belongs to V. I. Kozub.

[1] L. P. Gor'kov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 248 (1979) [JETP Lett. **30**, 228 (1979)].

[2] E. Abrahams and T. V. Ramakrishnan, J. Non-Cryst. Solids **35/36**, 15 (1980).

[3] J. S. Langer and T. Neal, Phys. Rev. Lett. **16**, 984 (1966).

[4] C. S. Ting, A. Houghton, and J. R. Senna, Phys. Rev. B **25**, 1439 (1982).

[5] B. L. Altshuler, D. E. Khmel'nitzkiĭ, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).

[6] S. Hikami, A. I. Larkin, and Y. Nagaoka, Prog. Theor. Phys. **63**, 707 (1980).

[7] P. A. Lee, J. Non-Cryst. Solids **35**, 21 (1980).

[8] J. Blake, in: Metallic Glasses (Ed. by J. J. Gilman and H. J. Leamy), Metals Park, OH, 1978 (Russ. Transl., Mir, Moscow, 1983, p. 245).

[9] Yu. M. Gal'perin, V. L. Gurevich, and D. A. Parshin, Zh. Eksp. Teor. Fiz. **85**, 1757 (1983) [Sov. Phys. JETP **58**, 1023 (1983)].

[10] M. Cutler and N. F. Mott, Phys. Rev. **181**, 1336 (1969).

[11] A. A. Abrikosov, Physics (Long Island City, N. Y.) **2**, 21 (1965).

[12] S. V. Maleev, Zh. Eksp. Teor. Fiz. **79**, 1995 (1980); **83**, 260 (1982) [Sov. Phys. JETP **52**, 1008 (1980) **56**, 142 (1982)].

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