Orientation effects in interaction of high-energy particles with strings of atoms in a crystal

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Orientation effects in scattering and emission of high-energy particles and in their nuclear interactions with atom strings in crystals are investigated. It is shown that the unusual character of the motion of above-barrier particles in the field of a continuous potential of crystal-atom strings influences these processes substantially at high energies. It is demonstrated in particular that rainbow scattering and interference effects in scattering are possible in a thin crystal and that the emission spectrum of above-barrier electrons on a string of crystal atoms has a sharp maximum at low frequency with a high emission intensity at the maximum. It is established that the yields of inelastic processes involving small impact parameters are substantially different in channeling and in above-barrier particle motion.

§1. INTRODUCTION

An important role in the motion of fast charged particles in a crystal at a small angle to one of the crystallographic axes is played by the correlations between successive collisions of the particle and the lattice atom. These correlations cause the particle motion in the crystal to differ from motion in an amorphous medium. Channeling and above-barrier motion of the particles is then possible.

In channeling, the particles move in channels made up of strings of crystal atoms, being periodically deflected at small angles to the channel axis. This phenomenon takes place when the angle ψ between the incident beam and the crystallographic (z) axis is small compared with the critical axial-channeling axis ψ_c ($\psi_c = (4Ze^2/\mathscr{E}d)^{1/2}$, where $Z|e|$ is the charge the atomic nucleus, $\mathscr{E}$ the particle energy, and d the distance between the atoms along the z axis). The features of the interaction of channeled particles with crystal atoms were investigated in many studies (see, e.g., the reviews 1–3).

In above-barrier motion, the particle collides successively with different crystal-atoms strings parallel to the crystallographic axis z. The correlations in collisions of a relativistic particle with the lattice atoms manifest themselves in this case in an interval of angles ψ considerably larger than the interval in which channeling of the particles is possible.[4,5]

This paper is devoted to a theoretical investigation of the interaction of above-barrier electrons and positrons with crystal-atom strings. We show that the special character of the motion of above-barrier relativistic electrons and positrons in the field of the continuous potential of a string of crystal atoms influences substantially the scattering and the radiation of these particles in the crystals, as well as the yield of the inelastic processes due to the small impact parameters.

At $\psi \lesssim \psi_c$, some features of the scattering and radiation of above-barrier particles on crystal-atom strings were investigated in Refs. 6–10, where it was shown in particular that in the continuous-string approximation the particles are scattered only along an azimuthal angle φ in a plane perpendicular to the string axis. The calculations were simplified in the cited papers by approximating the string potential by a function of the form $U_1(\rho) = \text{const}/\rho$, where ρ is the distance to the string axis. It is well known,[11–13] however, that the use of a function of this type for the investigation of particle scattering by isolated atoms does not explain such important scattering effects as, e.g., rainbow scattering, glory, and twisting. This function is therefore of limited use for description of particle scattering by atoms. The situation is similar also for scattering of particles by a string of crystal atoms.

A function such as $U_1(\rho)$ overestimates substantially the continuous potential of the string at short distances from its axis, so that the use of this function to describe particle motion in a crystal is justified only when the particles do not come close to the lattice-atom nuclei. It will be shown in §§3 and 4 that when the continuous potential of a string of crystal atoms is more accurately approximated the angle of scattering of the particle by the string is generally speaking not a single-valued function of the impact parameter of the string. Interference scattering effects can therefore appear in scattering of particles by an atom string. It has been established, in particular, that rainbow scattering of the particle should then occur in the crystal.

An above-barrier particle moves in an intense average field of strings of crystal atoms, and should therefore radiate intensely. This circumstance was pointed out in Ref. 16, where it was shown that at high frequencies the radiation intensity of above-barrier particles in a crystal can exceed considerably that of particles in an amorphous medium. In §5 of the present paper we investigate the radiation of above-barrier electrons and positrons in the field of a continuous potential of a crystal-atom string at low frequencies. We shall show that at $\psi < \psi_c$ the radiation spectrum of above-barrier electrons has a maximum in the low-frequency region and a high radiation intensity at the maximum. This maximum is due to peculiarities of motion of above-barrier electrons in the field of the atom string at low impact parameters of the latter.

In §6 we investigate the orientation dependence of the yield of inelastic processes due to small impact parameters, for particles moving in a crystal at a small angle to one of the crystallographic axes. It is shown that for an electron, in

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contrast to the case of particle motion along crystallographic planes,[17] the yields of the nuclear reactions in a crystal and in an amorphous medium differ in this case only because of the contribution made to the process by the channeled particles. This result shows that an experimental investigation of the orientation dependence of the yield of nuclear reactions in a crystal can provide information on the fraction of particles that move in a specific crystal under channeling conditions, and on the dechanneling of relativistic particles in this crystal.

§2. APPROXIMATION OF CONTINUOUS POTENTIAL OF CRYSTAL-ATOM STRINGS

When a fast charged particle moves in a crystal at a small angle ψ to one of the crystallographic axes (the z axis), the effective constant of the interaction between the particle and the crystal-lattice particles is large,[18] so that in a number of cases classical mechanics can be used to describe the particle motion in the crystal. We consider therefore first the motion of a fast particle in a crystal within the framework of classical mechanics.

At small ψ and large $\mathscr{E}$ the impact parameter changes little between successive collisions of the particle with the lattice atoms compared with the impact parameter itself. The particle motion in the crystal can therefore be described in the continuous-string approximation—by the crystal potential averaged along the x axis.[1-4]

In the continuous-string approximation, the particle-momentum component along the z axis is conserved. The particle trajectory in the (x, y) plane perpendicular to the z axis is then given by

$$\mathscr{E}\ddot{\boldsymbol{\rho}}=-\nabla_{\perp}U(\boldsymbol{\rho}), \quad U(\boldsymbol{\rho})=\frac{1}{L}\int dz\sum_{k} u(\mathbf{r}-\mathbf{r}_k), \tag{2.1}$$

where L is the crystal thickness, $\boldsymbol{\rho}$ the radius-vector in the (x, y) plane, $u(\mathbf{r}-\mathbf{r}_k)$ the potential energy of the particle interaction with a lattice atom located at the point $\mathbf{r}_k$, $\mathbf{r}_k=\mathbf{r}_k^0+\mathbf{u}_k$, $\mathbf{r}_k^0$ the equilibrium position of the atom in the lattice, $\mathbf{u}_k$ its thermal displacement, and $U(\boldsymbol{\rho})$ the continuous potential energy of the particle interaction with the crystal (we use a system of units with speed of light $c=1$).

The first integral of Eq. (2.1) is

$$\frac{\mathscr{E}}{2}\dot{\boldsymbol{\rho}}^2=\mathscr{E}_{\perp}+U(\boldsymbol{\rho}_0)-U(\boldsymbol{\rho}), \quad \mathscr{E}_{\perp}=\frac{\mathscr{E}\psi^2}{2}, \tag{2.2}$$

where $\boldsymbol{\rho}_0$ is the coordinate of the point of entry of the particle into the crystal. The particle trajectory in the (xy) plane in the crystal is thus determined by the transverse energy $\mathscr{E}_{\perp}(\boldsymbol{\rho}_0)=\mathscr{E}_{\perp}+U(\boldsymbol{\rho}_0)$ and by the continuous potential energy $U(\boldsymbol{\rho})$. The latter is determined in turn by the potential energy of the particle interaction with an individual crystal atom, by the crystal-lattice temperature, and by the index of the crystallographic axis near which the particle moves.

We consider by way of example the motion of relativistic electrtons and positron in a silicon crystal in an (x, y) plane perpendicular to the $\langle 111\rangle$ crystallographic axis. Figure 1 shows the exponential surfaces of the continuous potential energy $U(\boldsymbol{\rho})$ of the interaction of positrons with the lattice field in this case, calcuted in the Moliere model for the

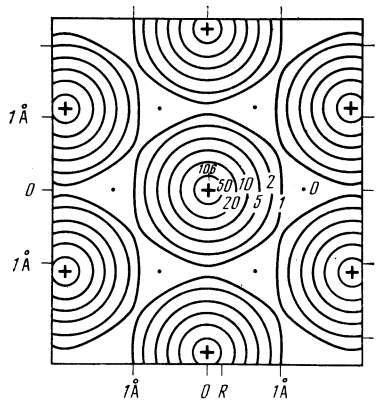

FIG. 1. Equipotential surfaces of continuous potential energy of positron interaction with a silicon-crystal field in a plane perpendicular to the $\langle 111\rangle$ axis at $T=293$ K.

potential of the atom with account taken of the thermal vibrations of the lattice atoms at $T=293$ K (the numbers on the equal-potential-energy lines are the values of $U(\boldsymbol{\rho})$ in eV; the values of $U(\boldsymbol{\rho})$ for electrons should be taken with the sign reversed).

In a field with such a potential distribution, the motion of the electrons and positrons can be either limited or unlimited (above-barrier motion).

The fraction of particles executing limited motion in the (x, y) plane is given by

$$\Phi=nd\int d\boldsymbol{\rho}_0\theta\left(U_H-\frac{\mathscr{E}\psi^2}{2}-U(\boldsymbol{\rho}_0)\right), \tag{2.3}$$

where n is the atom density, $\theta(x)$ the step function, and U_H the potential energy at the saddle point. The integration in (2.3) is carried out within the limits of the unit cell.

Equation (2.3) shows that for electrons limited motion of the particles in the (x, y) plane is possible only at $\psi<\psi_H$, where

$$\psi_H=(2U_H/\mathscr{E})^{1/2}, \quad \psi_H\ll\psi_c.$$

For electrons, according to (2.3), limited motion is possible if $\psi\lesssim\psi_c$. Figure 2 shows the dependence of Φ on the angle ψ with the $\langle 111\rangle$ crystal axis for particles incident on a silicon crystal when ψ is small [the calculations were performed for a continuous potential $U(\rho)$ in the Moliere model with account taken of the thermal vibrations of the lattice

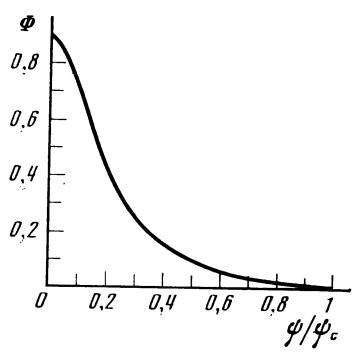

FIG. 2. Orientation dependence of electron fraction captured in axial channels when incident on a silicon crystal at small angle to the $\langle 111\rangle$ crystallographic axis.

atoms at $T = 293$ K]. The plot shows that even at $\psi < \psi_c$ the fraction of electrons trapped in a channel on entering the crystal is quite small.[1)]

Thus, in a wide range of angles ψ almost all the particles incident on the crystal will move in it above the barrier. Let us identify the singularities of the scattering and radiation of this group of particles in the field of the continuous potential of the crystal-atom strings.

We note as a preliminary that substantial changes in the trajectory of an above-barrier particle in a crystal take place at distances of its closest approach to the atom strings, where the potential gradients are a maximum and the continuous potential can be regarded as cylindrically symmetric. When considering the radiation and scattering of the particles through not too small angles we can assume in first approximation that the continuous potential inside a unit cell with center on the string axis is cylindrically symmetric:

$$U(\rho) = \begin{cases} U(\rho), & \rho \leqslant \bar{a}/2, \\ 0, & \rho > \bar{a}/2, \end{cases} \tag{2.4}$$

where $\bar{a}$ is the average distance between the atom strings.

An analysis of the scattering and radiation of above-barrier particles is greatly simplified in this approximation and can be carried out by simple methods both in classical and in quantum mechanics.

§3. CLASSICAL THEORY OF ELASTIC SCATTERING OF FAST PARTICLES BY A STRING OF CRYSTAL ATOMS

We consider elastic scattering of electrons and positrons in a relatively thin crystal whose thickness L is small compared with the particle mean free path between successive collisions with different atom strings. A particle passing through the crystal collides in this case with only one atom string.

A particle is known[4,21] to be scattered in the field $U(\rho)$ of a continuous atom-string potential only at an azimuthal angle φ in the (x, y) plane perpendicular to the string axis z. The angle ϑ of scattering of the particle by the string is connected with the azimuthal scattering angle φ by the relation

$$\vartheta = 2\psi \sin\frac{\varphi}{2}, \quad \varphi = \varphi(b) = \pi - 2b\int_{\rho_0}^{\infty} \frac{d\rho}{\rho^2}\left(1 - \frac{U(\rho)}{\mathscr{E}_\perp} - \frac{b^2}{\rho^2}\right)^{-1/2}, \tag{3.1}$$

where b is the string impact parameter (see Fig. 15 of Ref. 4) and ρ_0 is the minimum value of φ. The function $\varphi = \varphi(b)$ is called the particle deflection function in the field $U(\rho)$.[11-13]

Let us show that $\varphi(b)$, generally speaking, is not a single-valued function of the impact parameter b. To this end we consider electron and positron scattering by a silicon-crystal atom string in the case when the particles move near the $\langle 111 \rangle$ crystallographic axis and the crystal temperature is $T = 293$ K. The continuous potential $U(\rho)$ of the crystal-atom string can be approximated in this case with good accuracy (to within a percent), at both short and long distances from the string axis, by a function of the form

$$U(\rho) = U_1 \ln\left[1 + cR^2/(\rho^2 + \gamma\bar{u}^2)\right] - U_2, \quad \rho \leqslant \bar{a}/2, \tag{3.2}$$

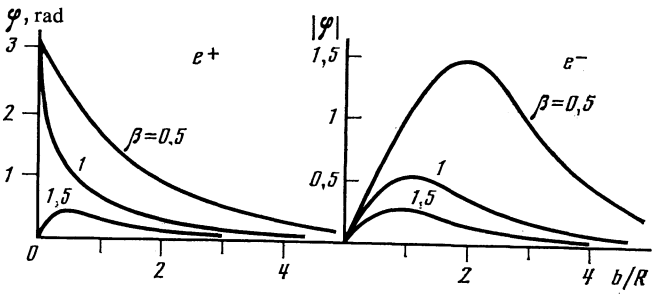

FIG. 3. Function of deflection of above-barrier positrons in the field (3.2) in silicon, $\langle 111 \rangle$ axis, at different values of $\beta = \psi/\psi_c$.

where R is the atom screening radius, $\bar{u}^2$ the mean square of the thermal vibrations of the lattice atoms, $\sqrt{\bar{u}^2} = 7.5 \times 10^{-10}$ cm (Ref. 2), $U_1 = 59$ eV, $U_2 = 3$ eV, $c = 2$, and $\gamma = 2.5$. The deflection function in a field with such a potential distribution can be obtained only by numerical methods.

Figure 3 shows the calculated deflection function of electrons and positrons in the field (3.2) at different values of $\beta = \psi/\psi_c$.

The curves show that $\varphi(b)$ for positrons at $\beta < 1$ is a single-valued function of the impact parameter b. For positrons with $\beta > 1$ and for electrons, this function is two-valued. The difference in the behavior of the positrons at $\beta < 1$ and $\beta > 1$ is due to the fact that at $\beta < 1$ the positrons cannot come close to the string axis.

Knowing the deflection function, we can calculate within the framework of classical mechanics the cross section for particle scattering by an atom string in the angle interval $(\varphi, \varphi + d\varphi)$

$$\frac{d\sigma_{cl}}{d\varphi} = L\psi \sum_i \left(\left|\frac{d\varphi}{db}\right|_i\right)^{-1}, \tag{3.3}$$

where the summation is over the single-valued branches of the function $\varphi(b)$.

The electron and positron cross sections calculated from the equations of classical mechanics in the field of the continuous potential (3.2) are shown in Fig. 4 by thin solid lines. The ordinate in this figure is the quantity $F = (LR\psi)^{-1}(d\sigma/d\varphi)$.

We see that for positrons at $\psi < \psi_c$ the cross section $d\sigma_{cl}/d\varphi$ is a monotonic function of the angle. For electrons, however, the cross section $d\sigma_{cl}/d\varphi$ differs from zero in the interval of the angles φ from 0 to φ_{max}, and as $\varphi \to \varphi_{max}$ the cross section (3.3) increases without limit. The reason is that as $\varphi \to \varphi_{max}$ the derivative of the deflection function tends to zero. Similar results are obtained also for positrons at $\psi > \psi_c$.

It is well known in optics and in atomic and nuclear physics (Refs. 11–13, 22) that if the deflection function is not a single-valued function of the impact parameter interference effects can appear in the scattering of waves and particles by water droplets and by atoms and atomic nuclei. We consider therefore now, from the standpoint of quantum mechanics, the scattering of electrons and positrons by a string of crystal atoms.

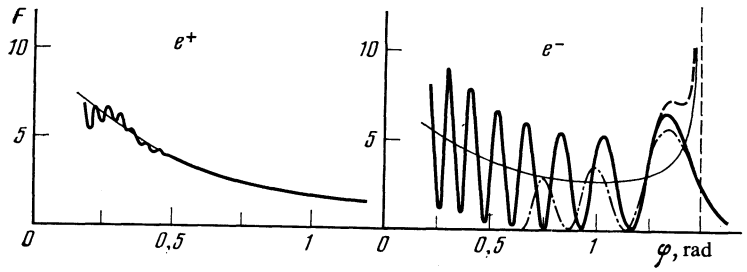

FIG. 4. Cross section for scattering of electrons and positrons with $\mathscr{E} = 0.5$ GeV in the field (3.2) of the continuous potential of a string of silicon-crystal atoms at $\psi = 0.5\ \psi_c$. Solid thick curves—quantum cross section (4.1); thin solid curves—classical cross section (3.3); dashed curve—scattering cross section (4.4) in the semiclassical approximation of quantum mechanics; dash-dot curve—cross section (4.5) for rainbow scattering of electrons by an atom string.

§4. QUANTUM THEORY OF ABOVE-BARRIER PARTICLE SCATTERING BY A STRING OF CRYSTAL ATOMS

In quantum mechanics, the cross section for scattering of fast charged particles in the continuous-potential field of a string of crystal atoms (a potential having cylindrical symmetry) can be represented in the form[23,24]

$$d\sigma/d\varphi = L\psi |f(\varphi)|^2, \tag{4.1}$$

$$f(\varphi) = \frac{1}{(2\pi i p_\perp)^{1/2}} \sum_{n=-\infty}^{\infty} (e^{2i\eta_n} - 1), \tag{4.2}$$

where $f(\varphi)$ is the scattering amplitude, $p_\perp = \mathscr{E}\psi$, and η_n are the scattering phase shifts. In the quasi-classical approximation,

$$\eta_n = \frac{\pi |n|}{2} - p_\perp \rho_0 + \int_{\rho_0}^{\infty} d\rho \left[\left(p_\perp^2 - 2\mathscr{E}U(\rho) - \frac{n^2}{\rho^2} \right)^{1/2} - p_\perp \right]. \tag{4.3}$$

The thick lines in Fig. 4 show the calculated quantum cross section of positrons and electrons in the continuous-potential (3.2).

The curves show that for positrons at $\psi < \psi_c$ the quantum scattering cross section (4.1) contains small oscillations about the cross section $d\sigma_{cl}/d\varphi$ in the region of small φ. These oscillations are due to diffraction effects in scattering of particles by an atom string.

The quantum cross section for electrons contains large oscillations about $d\sigma_{cl}/d\varphi$ in the entire angle region $\varphi < \varphi_{max}$, while in the angle region $\varphi > \varphi_{max}$ the cross section (4.2) decreases rapidly with increasing φ.

We shall show now that the strong oscillations of the quantum cross section for scattering by a crystal-atom string are due to the two-valuedness of the deflection function $\varphi(b)$. We consider for this purpose electron scattering in the semiclassical approximation of quantum mechanics,[11] i.e., going formally to $\hbar \to 0$ in (4.1). In this case the summation in (4.2) can be replaced by integration with respect to n ($n = \hbar p_\perp b$), and the stationary-phase method can be used to calculate the resultant integral.

If $\varphi \ll \varphi_{max}$, the two classical trajectories that lead to electron scattering through the same angle φ will be far enough from each other, so that the phase difference $\Delta\eta = \eta(b_1) - \eta(b_2)$ corresponding to these trajectories [where b_1 and b_2 are the roots of the equation $\varphi = \varphi(b)$] will be large. The stationary-phase method can be used in this case to calculate $f(\varphi)$ for each stationary-phase point separately. Then

$$\frac{d\sigma}{d\varphi} = L\psi \left| \sum_{k=1}^{2} f_k(\varphi) \right|^2, \tag{4.4}$$

$$f_k(\varphi) = \left(\frac{d\varphi}{db} \right)^{-1/2} \exp\{2i\eta(b_k) - ip_\perp b_k \varphi\}.$$

The scattering cross section will oscillate rapidly in this case with changing φ, owing to the interference between the amplitudes $f_k(\varphi)$. We note that after averaging over the oscillations Eq. (4.4) goes over into (3.3).

As $\varphi \to \varphi_{max}$ the two classical trajectories that lead to electron scattering through the same angle φ merge, and the phase difference $\Delta\eta$ tends then to zero. When calculating the scattering amplitude in this case it is necessary to retain the term containing the second derivative of the deflection function in the expansion of the scattering phase shift in the vicinity of the impact parameter corresponding to the maximum scattering angle φ_{max}. The scattering cross section (4.1) takes then the form

$$\frac{d\sigma}{d\varphi} = \frac{2\pi L}{\mathscr{E}B^{2/3}} |\mathrm{Ai}((\varphi - \varphi_{max})B^{-1/3})|^2, \quad B = \frac{1}{2\mathscr{E}^2\psi^2} \left| \frac{d^2\varphi}{db^2} \right|_{\varphi=\varphi_{max}}, \tag{4.5}$$

where $\mathrm{Ai}(x)$ is an Airy function.

Equation (4.5) shows that in particle scattering by an atom string in the angle region $\varphi \sim \varphi_{max}$ rainbow scattering of the particles takes place, similar to rainbow scattering and waves and particles by water droplets, by atoms, and by atomic nuclei.

The cross sections for electron scattering in the field (3.2), calculated from Eqs. (4.4) and (4.5), are shown in Fig. 4 by dashed and dash-dot curves, respectively.

The results show that in the angle region $\varphi \ll \varphi_{max}$ the cross-section oscillations are due to interference between scattering amplitudes corresponding to the two classical trajectories along which scattering through the same angle takes place, and that at $\varphi \lesssim \varphi_{max}$ we get rainbow scattering of the electron by the crystal-atom string. In the latter scattering the cross section contains oscillations at angles $\varphi > \varphi_{max}$ and decreases rapidly with increasing φ at $\varphi > \varphi_{max}$. With increasing particle energy, according to (4.5), the oscillations of the quantum scattering cross section become more frequent.

The results are similar also for positrons at $\psi > \psi_c$. Thus, quantum scattering effects can appear in scattering of fast particles by a string of crystal atoms. To observe these effects in experiment it is necessary that the beam divergence be small compared with the widths of the maxima of the

angular distribution of the particles leaving the crystals, $\Delta\psi < \psi\Delta d$, where $\Delta\varphi$ is the width of the maximum cross section for scattering along the azimuthal angle φ. It follows from (4.5) that $\Delta\varphi \sim 2(\mathscr{E}R\psi)^{-2/3}$, so that the condition on $\Delta\psi$ takes the form $\Delta\psi < 2\psi(\mathscr{E}R\psi)^{-2/3}$. It is required in addition that the crystal thickness be small enough for the particle passing through the crystal to collide with only one atom string. This condition is certainly satisfied if $L < \bar{a}/\psi$.

We note that oscillations in particle scattering along the angle φ can occur also when the particles pass through crystals thicker than $L < \bar{a}/\psi$. In that case, however, the oscillations can be due not only to quantum effects in scattering, but also to the effect, on the particle motion, of crystal planes with close-packed atoms. The last effect was considered in detail in a number of papers (see, e.g., Ref. 3) and we shall not dwell on it here. We note only that quantum oscillations of the cross section for particle scattering by an atom string should be symmetric about a plane passing through the string axis and the incident-particle momentum. Oscillations due to the influence of crystal planes on the particle motion, however, have in general no such symmetry, so that the two effects can be separated in experiment.

Oscillations of the scattering cross section of fast particles passing through thin crystals were experimentally observed in Refs. 6, 9, and 25. There, however, the orientations of crystallographic planes relative to the particle momentum were not indicated, so that additional data are needed to make clear the nature of the oscillations observed in Refs. 6, 9, and 25.

The results described above pertain to particle scattering in a relatively thin crystal ($L \ll \bar{a}^2/R\psi$). When this condition is violated, account must be taken of multiple scattering of the particles by the atom strings. This process was investigated in detail in Refs. 5 and 26, where it was shown that multiple scattering by atom strings causes a redistribution in the particle azimuthal angle φ. This redistribution suppresses the aforementioned interference effects in particle scattering by an atom string.

§5. RADIATION PRODUCED BY COLLISION OF RELATIVISTIC ELECTRONS AND POSITRONS WITH AN ATOM STRING

The results reported above show that at small ψ and at large $\mathscr{E}$ the particle motion in a crystal differs substantially from motion in an amorphous medium. Clearly, the radiation of the particles in a crystal should differ in this case from the radiation in an amorphous medium.

We shall show that the special character of motion of above-barrier electrons and positrons in the field of the continuous potential of a crystal-atom string influences the radiation from these particles at low frequencies.

It is known[18,27] that the radiation from a relativistic particle at low frequencies ω evolves in a large spatial region along the particle momentum. If the length $l = 2\mathscr{E}^2/m^2\omega$ over which the radiation is formed is large compared with the lattice constant, the inhomogeneity of the crystal potential distribution along the crystallographic axis near which the particle moves is immaterial to the radiation. To describe the particle motion and radiation in this case we can use the continuous-string approximation. If furthermore the length l is small compared with the particle mean free path between successive collisions with different crystal-atom strings, the particle radiation in the crystal is determined only by the singularities of its interaction with the field of an individual crystal-atom string. The mean value of the spectral density $d\bar{E}/d\omega$ of the energy radiated by the particle in the crystal can then be expressed in terms of the spectral density of the energy $E'(b)$ radiated in collisions with an individual atom string by the relation 16

$$\frac{d\bar{E}}{d\omega} = Ln\, d\psi \int_{-\infty}^{\infty} db \frac{dE(b)}{d\omega}. \tag{5.1}$$

The value of $dE(b)/d\omega$ is determined by the particle trajectory in the field of the crystal-atom string, and the trajectory depends in turn on the string impact parameter b.

The radiation dipolarity condition $\bar{\theta} \ll m/\mathscr{E}$, where $\bar{\theta}$ is the average angle of electron scattering by a crystal-atom string ($\bar{\theta} \sim \psi_c$), is satisfied for a wide interval of the energy $\mathscr{E}$ in the angle region $\psi \sim \psi_c$. The quantity $dE(b)/d\omega$ can then be represented in the form

$$\frac{dE(b)}{d\omega} = \frac{e^2\omega}{2\pi} \int_{\delta}^{\infty} \frac{dq}{q^2}\left[1 - 2\frac{\delta}{q}\left(1 - \frac{\delta}{q}\right)\right] |W(q)|^2, \tag{5.2}$$

$$W(q) = \int_{-\infty}^{\infty} dt\, \rho(t)\, e^{iqt},$$

where $\delta = \omega m^2/2\mathscr{E}^2$ and $\rho(t)$ is determined by Eq. (2.1).

Equations (5.1) and (5.2) show that $d\bar{E}/d\omega$ depends on the sign of the particle charge and on the relation between ψ and ψ_c. At $\psi \gtrsim \psi_c$ it is possible to expand in these equations in terms of the potential ($U/\mathscr{E}_\perp \sim \psi_c^2\psi^2$), and Eq. (5.1) yields the corresponding result of the Born theory of coherent radiation of particles in a crystal.[27,28] In this case the radiation from electrons on a string of crystal atoms is the same as that of positrons.

At $\psi \lesssim \psi_c$ the spectral radiation density can be calculated only by numerical methods.

Figure 5 shows the calculated spectral density of the radiation from electrons and positrons moving in a silicon crystal near the ⟨111⟩ axis. The ordinate is the quantity $f = L^{-1}D^{-1}d\bar{E}/d\omega$, and the abscissa is ω/ω_0, where $D = 4e^2ndRU_0^2/\pi m^2\psi_c$, $U_0 = 2Ze^2/d$ and $\omega_0 = 2\mathscr{E}^2\psi_c/m^2R$. The curves are labeled by the values $\beta = \psi/\psi_c$.

The calculations show that at $\psi < \psi_c$ the radiation spectrum of above-barrier electrons has a maximum at $\omega \sim \omega_0/3$. With increasing $\mathscr{E}$ the position of this maximum varies as $\omega_{max} \sim \mathscr{E}^{3/2}$. Comparing the obtained $d\bar{E}/d\omega$ with the spectral density of electron radiation in an amorphous medium.

$$dE_{am}/d\omega = \frac{16Z^2e^6nL}{3m^2}\ln(183Z^{-1/3}),$$

we find that at the maximum the spectral density of electron radiation in a crystal exceeds $dE_{am}/d\omega$ by a factor $R/d\psi_c \ln(183Z^{-1/3})$.

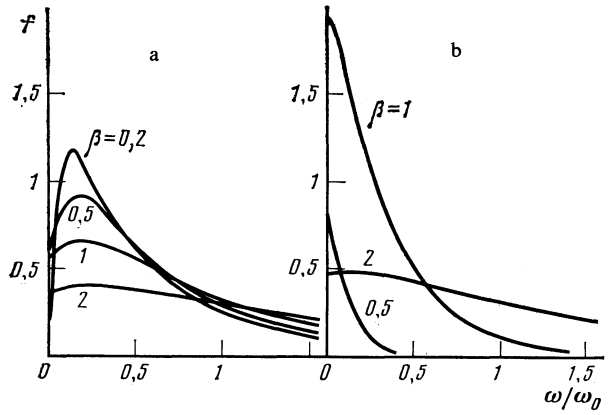

FIG. 5. Spectral density of radiation of electrons (a) and positrons (b) moving in a silicon crystal near the ⟨111⟩ axis at $T = 293$ K.

We call attention now to the fact that similar regularities occur also for radiation of channeled electrons in a crystal. The only difference is that the radiation intensity at the maximum is considerably larger for channeled electrons than for above-barrier electrons. However, the physical causes of the maxima in the radiation spectra of channeled and above-barrier electrons are different. We know[21,29,30] that in channeling the maximum is due to the periodicity of the electron motion in the channel. For above-barrier motion, as will be shown below, the maximum is due to singularities of the electron scattering by a crystal-atom string in the region of small impact parameters.

Indeed, according to (5.2), $dE(b)/d\omega$ is proportional to the square of the particle scattering angle ϑ_l^2 over the length l on which the radiation is formed:

$$\frac{dE(b)}{d\omega} \sim e^2 \frac{\mathscr{E}^2}{m^2} \vartheta_l^2. \tag{5.3}$$

Since the main contribution to (5.1) is made by the region of small impact parameters $b \lesssim R$, to analyze the spectrum of electron radiation on a string of crystal atoms we must know the dependence of ϑ_l on l at $b \lesssim R$.

At sufficiently low frequencies, when the length l exceeds considerably the path length $s \sim 2R/\psi_c$ on which the electron is acted upon effectively by the crystal-atom-string field, ϑ_l is equal to the total angle of particle scattering by the atom chain, $\vartheta_l = \vartheta(b)$. The plots of Fig. 3 show that at $\psi \lesssim \psi_c$, in the region $b \lesssim R$, the function $\varphi(b)$ decreases rapidly with decreasing b so that in this frequency region $\vartheta_l = \vartheta(b) \ll \psi_c$.

At the closest approach of the electron to the string axis the angle of deflection of the electron relative to the initial momentum, as follows from the transverse-energy conservation law, is of the order of ψ_c. The scale of the path negotiated by the electron in this case is R/ψ_c. We therefore have $\vartheta_l \sim \psi_c$ in the frequency region satisfying the condition $l \sim R/\psi_c$. If $l \ll R/\psi_c$ we obviously have $\vartheta_l \ll \psi_c$.

Thus, the quantity ϑ_l^2 and with it the radiation spectral density (5.1) should have maxima at $l \sim R/\psi_c$; this corresponds to the frequency region $\omega \sim \omega_0 = 2\mathscr{E}^2\psi_c/m^2R$.

The plots of Fig. 5b show that for positrons the quantity $d\bar{E}/d\omega$ is a monotonic function of the frequency and that the positron-radiation intensity is a maximum at $\psi \sim \psi_c$. The abrupt decrease of the positron radiation intensity at $\psi < \psi_c$ compared with the case $\psi \sim \psi_c$ is due to the fact that at $\psi < \psi_c$ the positrons do not come close to the string-atom nuclei, where the gradients of the continuous string potential are maximal.

We note in conclusion that the available experimental data[31–33] on the radiation spectra of relativistic electrons in crystals confirm the presence of a maximum in the radiation spectrum at low frequencies and the high radiation intensity at the maximum when relativistic electrons pass through a crystal along a crystallographic axis. A maximum in the radiation spectrum and a high radiation intensity at the maximum at low frequencies, as indicated above, can occur in both channeling and in above-barrier motion of particles. The experimental results,[31–33] however, pertain to rather thick crystals. For particles passing through such crystals it is necessary to take into account, besides the effects considered above, the influence of dechanneling and multiple scattering of the particles by thermal vibrations of the lattice atoms. The determination of the physical nature of the effects observed in Ref. 31–33 calls therefore for additional research.

§6. ORIENTATION DEPENDENCE OF THE YIELD OF NUCLEAR REACTIONS IN INTERACTIONS OF RELATIVISTIC PARTICLES WITH CRYSTAL-ATOM STRINGS

Above-barrier electrons moving in a crystal come close to a string axis and participate therefore in processes governed by small impact parameters. The special role of above-barrier states in electron interactions with lattice-atom nuclei (nuclear reactions) when the particles move in the crystal along crystallographic planes was pointed out in Ref. 17. It was shown there, in particular, that the yield of the nuclear reactions can in this case be substantially lower than the yield of the reactions with moving particles in an amorphous medium.

We investigate now the orientation dependence of the yield of nuclear reactions when relativistic electrons and positrons move in a crystal at a small angle ψ ($\psi \sim \psi_c$) to one of the crystallographic axes.

Collisions of an above-barrier particle with various atom strings can in this case be regarded as random. The yield of the process of interest to us in a crystal can be connected with the yield $I_a = \sigma n L$ of this process in an amorphous medium by the relation

$$I_H = I_a \psi \int_{-\infty}^{\infty} db \int_{-\infty}^{\infty} dt W(\boldsymbol{\rho}(t)), \tag{6.1}$$

where σ is the reaction cross section, $\boldsymbol{\rho} = \boldsymbol{\rho}(t)$ is the particle trajectory in the field of the continuous potential of the atom string in a plane (x, y) perpendicular to the string axis, and

$$W(\rho) = (1/2\pi\overline{u^2}) \exp[-\rho^2/2\overline{u^2}]$$

is the probability of finding the atom nucleus at a distance ρ from the string axis.

Using the equation of motion (2.2), we now easily show that for electrons

$$I_H^- = I_a, \tag{6.2}$$

and for positrons

$$I_H^+ = 2\pi I_a \int_{\rho^*}^{\infty} \rho\, d\rho W(\rho), \tag{6.3}$$

where ρ^* is determined from the relation $\mathscr{E}_\perp = \mathscr{E}\psi^2/2 = U(\rho^*)$, if $\mathscr{E}_\perp < U(0)$ and $\rho^* = 0$ if $\mathscr{E}_\perp > U(0)$.

Thus, for above-barrier electrons the yield of the process of interest to us is the same in a crystal and in an amorphous medium. For positrons, I_H^+ and I_a differ only at $\psi < \psi_c$, and in this angle region $I_H^+ < I_a$. This dependence of I_H^+ on ψ is known[1-3] to be due to the fact that at $\psi < \psi_c$ the positrons do not come close to the string axis, where the lattice-atom nuclei have a maximum density.

We consider now the orientation dependence of the yield I of nuclear reactions from a crystal for an electron beam incident at an angle $\psi \lesssim \psi_c$ to one of the crystallographic axes. In this case some of the incident-particles become axially channeled on entering the crystal. The quantity $\eta(\psi) = I^-/I_a$ can then be represented in the form

$$\eta(\psi) = [1 - \Phi(\psi)] + \int d\rho_0 \frac{1}{T(\rho_0, \psi)} \int_0^T dt\, W(\rho), \tag{6.4}$$

where $\Phi(\psi)$ is the fraction of the channeled particles, $T(\rho_0, \psi)$ the period of the oscillation of the channeled electron, ρ_0 the point of entry of the particle into the crystal, and $\rho = \rho(\rho_0, t)$ the trajectory of the channeled electron. The integration in (6.4) is over the unit-cell region ρ_0 for which axial channeling of the particles takes place at a given ψ. The value of $\Phi(\psi)$ is determined by relation (2.3)

Equation (6.4) shows that at $\psi < \psi_c$ a contribution to made not only by above-barrier electrons but also by channeled particles whose trajectories pass through the region $\rho \lesssim \sqrt{\bar{u}^2}$. Let us estimate the contribution made to η by the channeled particles in the simplest case when $\psi \to 0$. In this case all the channeled electrons pass through a region where lattice-atom nuclei are located. The main contribution to the integral with respect to time in (6.4) is made then by the values $\Delta t \sim 2\sqrt{\bar{u}^2}/\psi_c$. The period of the oscillations of the channeled particles is of the order of $T \sim 2\bar{a}/\psi_c$; the region of integration with respect to $d\rho_0$ is of the order of $\pi\bar{a}^2/4$. Therefore

$$\eta(\psi)|_{\psi=0} \sim \frac{1}{8} \frac{\bar{a}}{\sqrt{\bar{u}^2}}. \tag{6.5}$$

Thus, as $\psi \to 0$ the yield of the nuclear reactions in a crystal should exceed considerably that in an amorphous medium, with η increasing as the crystal temperature is lowered.

Figure 6 shows the results of calculation of the $\eta = \eta(\psi)$ dependence in the case when electrons are incident on a silicon crystal at a small angle ψ to the $\langle 111 \rangle$ axis. The solid curve corresponds to a crystal temperature $T = 293$ K and the dashed curve to $T = 0$ K.

The results show that the dependences of η on ψ for above-barrier and channeled particles differ substantially. An experimental investigation of the orientational dependence of the yield of the nuclear reactions in the crystal can therefore yield information on the dynamics of the motion of various particle groups in a specific crystal.

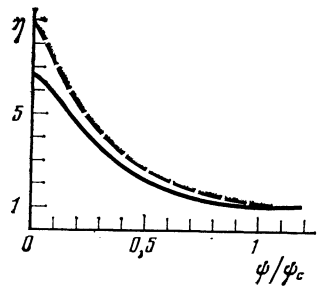

FIG. 6. Orientation dependences of the yield of nuclear reactions when a beam of electrons is incident on a silicon crystal at a small angle to the $\langle 111 \rangle$ axis and at different crystal temperatures. The solid and dashed curves correspond to $T = 293$ K and to $T = 0$ K, respectively.

Equation (6.4) can be used also in investigations of the orientation dependences of the yields of a number of other processes governed by small impact parameters (ionization of low-lying atomic shells, large-angle scattering, total electron energy loss to radiation, etc.). The only difference is that the function $W(\rho)$ has in these cases a different form.

We note in conclusion that the equations presented are valid if $\overline{\vartheta^2} \ll \psi_c^2$, where $\overline{\vartheta^2}$ is the mean squared angle of particle scattering by thermal vibrations of the lattice atoms. Since $\overline{\vartheta^2} \sim \mathscr{E}^{-2}$ and $\psi_c^2 \sim \mathscr{E}^{-1}$, at sufficiently large $\mathscr{E}$ the condition $\overline{\vartheta^2} \ll \psi_c^2$ can be satisfied even for quite thick targets.

[1)]A similar conclusion was drawn in Refs. 19 and 20 in an analysis of the $\Phi = \Phi(\psi)$ dependence in other models of a continuous potential of crystal-atom strings. In these references, however, no account was taken of the contribution made to Φ by particles trapped in a channel on trajectories passing through a region where lattice-atom nuclei are located.

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