Theory of weak parametric plasma turbulence

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(Submitted 17 January 1984)
Zh. Eksp. Teor. Fiz. **87**, 161–176 (July 1984)

We obtain a nonlinear dynamic equation for the amplitude of the potential φ of self-consistent perturbations up to and including terms in φ^3; it takes into account the finite displacements of particles in a sinusoidal pump wave field ($k\xi_E \sim 1$, $\xi_E \sim eE_0/m\omega_0^2$). Using it we construct a generalized weak turbulence theory for parametric kinetic type instabilities caused by the build-up of resonant particle oscillations; it takes into account the finite displacements of particles in the pump wave field. The quasilinear theory equations obtained are valid for $k\xi_E \sim 1$. Using the equations obtained we study the nonlinear stage and determine the saturation level of the ion cyclotron kinetic instability in the case when the relative velocity of the ions and electrons is less than or of the order of the ion thermal velocity. We estimate the turbulent heating rate of the plasma components.

1. INTRODUCTION

Kinetic parametric instabilities of a plasma in the electric field of a pump wave

$$\mathbf{E}_0(t)=\mathbf{E}_0 \sin \omega_0 t \tag{1.1}$$

are caused by the interaction of resonant electrons or ions with the beats formed by the pump wave and the unstable electrostatic oscillations (see, e.g., Refs. 1 to 4). Such instabilities arise in the high-frequency heating of a plasma by fast magneto-sonic waves with frequencies of the order of the lower hybrid frequency,[2] the ion cyclotron frequency,[3] or the electron cyclotron frequency.[4] The growth rate of such oscillations is a maximum when the displacement of a particle in the field of the pump wave $\xi_E \sim eE_0/m\omega_0^2$ is of the order of the wavelength of the unstable oscillations $k\xi_E \sim 1$. For a study of the nonlinear stage of these instabilities and of the parametric turbulence which then arises it is necessary to generalize the well known equations of the nonlinear plasma theory to the case of finite displacements ξ_E. In the present paper we obtain nonlinear equations for the Fourier transform of the potential of the oscillations $\varphi(\mathbf{r}, t)$ up to and including terms cubic in the amplitude of amplitude of the oscillations and we determine, using them, an equation for the intensity $I(\mathbf{k})$ of the oscillations in the case when the unstable waves form a wavepacket with random phases. We obtain also a generalization of the quasilinear equations for averaged distribution functions of the plasma particles for the case of finite ξ_E.

In deriving the equations for the nonlinear wave interaction and the quasi-linear equations we have used a Fourier transformation in the frame of reference frame moving with the electron (ion) velocity in the field of the pump wave (1.1) (and in a constant magnetic field $\mathbf{B}_0$ if present) which made it possible appreciably to simplify the derivation procedure and the form of the equations obtained when using the fact that the growth rate is small compared with the frequency.

We show in section 2 that the application of such a Fourier transformation to the linear material equation allows us at once to obtain a chain of difference equations for the Fourier components of the potential when there is a pump wave present and, using this chain, to obtain the linear dispersion equation and the energy balance equation. The application of the Fourier transformation in moving reference frames to the nonlinear material equation (section 3) leads to a nonlinear equation for the amplitudes of the Fourier transformation of the potential. Using this equation we give in section 4 a derivation of a kinetic equation for waves with random phases. Section 5 is devoted to the derivation of the equations of the quasilinear approximation. We give in section 6 a discussion of the results obtained and consider the application of the theory developed here to the study of ion and electron cyclotron parametric turbulence.

2. LINEAR APPROXIMATION

We obtain the equations which describe the linear stage of parametric potential instabilities of a plasma in the field (1.1) of a pump wave from the material equation for a homogeneous plasma

$$\mathbf{D}(\mathbf{r},t)=\mathbf{E}(\mathbf{r},t)+\sum_\alpha \int_0^t dt' \int d\mathbf{r}' \varepsilon_\alpha(\mathbf{r}-\mathbf{r}'; t, t')\mathbf{E}(\mathbf{r}', t') \tag{2.1}$$

and the Poisson equation

$$\operatorname{div} \mathbf{D}(\mathbf{r}, t)=0 \tag{2.2}$$

(normally these equations are obtained (see, e.g., Refs. 1, 2) from the Poisson equation for the electric field strength $\mathbf{E}$ of the self-consistent potential perturbations of the plasma in which the perturbations of the charge density are found from the solution of the linearized Vlasov equations for each kind of particle).

The external pumping field (1.1) leads to the kernel ε_α being nonstationary in time, but in a system of coordinates moving with the velocity $\mathbf{u}_\alpha(t)$ with which the charge of species α moves in the field $\mathbf{E}_0(t)$ of the pump wave, the nonstationarity of the kernel disappears. We use this fact to obtain the Fourier transformation of Eq. (2.1).

We write, as usual, the Fourier-Laplace transform of Eq. (2.1) in the form

$$\mathbf{D}(\mathbf{k},\omega)=\mathbf{E}(\mathbf{k},\omega)$$
$$+(2\pi)^{-4}\sum_\alpha \int_0^\infty dt \int d\mathbf{r} \int_0^t dt' \int d\mathbf{r}' \varepsilon_\alpha(\mathbf{r}-\mathbf{r}'; t, t')\cdot$$

 0038-5646/84/070092-09$04.00

$$\times \exp[i\omega(t-t')-i\mathbf{k}(\mathbf{r}-\mathbf{r}')]\mathbf{E}(\mathbf{r}',t')\exp[i\omega t'-i\mathbf{k}\times\mathbf{r}']. \quad (2.3)$$

We perform the Fourier transformation of the kernel ε_α in a system of coordinates moving with the velocity $\mathbf{u}_\alpha(t)$ which satisfies the equation

$$\frac{d\mathbf{u}_\alpha}{dt}=\frac{e_\alpha}{m_\alpha}\mathbf{E}_0(t)+\frac{e_\alpha}{m_\alpha c}[\mathbf{u}_\alpha\times\mathbf{B}_0]. \quad (2.4)$$

We change in Eq. (2.3) from the variables $(\mathbf{r}-\mathbf{r}',\mathbf{r}',t',t)$ to the variables $(\mathbf{r}_\alpha-\mathbf{r}'_\alpha,\mathbf{r}',t',t)$, where

$$\mathbf{r}-\mathbf{r}'=\mathbf{r}_\alpha-\mathbf{r}_\alpha'+\int_{t'}^{t}\mathbf{u}_\alpha(\tau)\,d\tau,$$

and we obtain

$$\mathbf{D}(\mathbf{k},\omega)=\mathbf{E}(\mathbf{k},\omega)+(2\pi)^{-1}\sum_\alpha\int_0^\infty dt\int_0^t dt'\,\varepsilon_\alpha(\mathbf{k},t-t') \quad (2.5)$$

$$\times\exp\left[i\omega(t-t')-i\mathbf{k}\int_{t'}^{t}\mathbf{u}_\alpha(\tau)\,d\tau\right]\mathbf{E}(\mathbf{k},t)\exp(i\omega t').$$

For the sinusoidal field (1.1) of the pump wave

$$\exp\left(-i\mathbf{k}\int_{t'}^{t}\mathbf{u}_\alpha(\tau)\,d\tau\right)=\sum_{n,p=-\infty}^{\infty}J_n(a_\alpha)J_p(a_\alpha)$$
$$\times\exp[i(n-p)(\delta_\alpha+\omega_0t')]\exp[in\omega_0(t-t')], \quad (2.6)$$

where the argument a_α of the Bessel functions $J_m(a_\alpha)$ is equal to[2]

$$a_\alpha=\left|\frac{e_\alpha}{m_\alpha}\right|\left[\left(\frac{k_\parallel E_{0\parallel}}{\omega_0^2}+\frac{\mathbf{k}_\perp\mathbf{E}_{0\perp}}{\omega_0^2-\omega_{c\alpha}^2}\right)^2+\frac{\omega_{c\alpha}^2}{\omega_0^2}\frac{(\mathbf{B}_0[\mathbf{k}\times\mathbf{E}_0])^2}{B_0^2(\omega_0^2-\omega_{c\alpha}^2)}\right]^{1/2}, \quad (2.7)$$

$$\operatorname{ctg}\delta_\alpha=\frac{\omega_0}{\omega_{c\alpha}}\left(\frac{k_\parallel E_{0\parallel}}{\omega_0^2}+\frac{\mathbf{k}_\perp\mathbf{E}_{0\perp}}{\omega_0^2-\omega_{c\alpha}^2}\right)\left(\frac{\mathbf{B}_0[\mathbf{k}\times\mathbf{E}_0]}{B_0(\omega_{c\alpha}^2-\omega_0^2)}\right)^{-1},$$

$$\omega_{c\alpha}=\frac{e_\alpha B_0}{m_\alpha c}.$$

Substituting (2.6) into (2.7) and using the Poisson equation $\mathbf{kD}(\mathbf{k},\omega)=0$, we obtain the following equation describing the linear stage of the parametric plasma instabilities:

$$\varphi(\mathbf{k},\omega)+\sum_\alpha\sum_{n,p=-\infty}^{\infty}J_n(a_\alpha)J_{n-p}(a_\alpha)e^{ip\delta_\alpha} \quad (2.8)$$
$$\times\varepsilon_\alpha(\mathbf{k},\omega+n\omega_0)\varphi(\mathbf{k},\omega+p\omega_0)=0,$$

where

$$\mathbf{E}(\mathbf{k},\omega)=-i\mathbf{k}\varphi(\mathbf{k},\omega),$$

in which the contributions ε_α from particles of the kind α to the longitudinal permittivity are given by the usual expressions.

In a number of cases it turns out to be expedient to perform the Fourier-Laplace transformation in the moving system of coordinates not only for the kernels ε_α of the material equation, but also for the fields **E** and **D**. We denote by $\vec{\mathscr{E}}(\mathbf{k},\omega)$ and $\vec{\mathscr{D}}(\mathbf{k},\omega)$ the Fourier-Laplace transforms of, respectively, the quantities **E** and **D** obtained in the $(\mathbf{r}_\alpha,t)$ system of coordinates:

$$\{\vec{\mathscr{D}}(\mathbf{k},\omega);\vec{\mathscr{E}}(\mathbf{k},\omega)\}=\frac{1}{(2\pi)^4}\int_0^\infty dt\int d\mathbf{r}_\alpha\exp(i\omega t-i\mathbf{k}\mathbf{r}_\alpha)$$

$$\times\left\{\mathbf{D}\left(\mathbf{r}_\alpha+\int^t\mathbf{u}_\alpha(\tau)\,d\tau,t\right);\mathbf{E}\left(\mathbf{r}_\alpha+\int^t\mathbf{u}_\alpha(\tau)\,d\tau,t\right)\right\}, \quad (2.9)$$

where

$$\mathbf{r}_\alpha=\mathbf{r}-\int^t\mathbf{u}_\alpha(\tau)\,d\tau.$$

We then get instead of Eq. (2.5)

$$\vec{\mathscr{D}}(\mathbf{k},\omega)=[1+\varepsilon_\alpha(\mathbf{k},\omega)]\vec{\mathscr{E}}(\mathbf{k},\omega)+(2\pi)^{-1}\sum_{\beta\neq\alpha}\int_0^\infty dt\int_0^t dt'$$

$$\times\exp\left[i\omega(t-t')-i\mathbf{k}\int_{t'}^{t}(\mathbf{u}_\beta-\mathbf{u}_\alpha)\,d\tau\right]\varepsilon_\beta(\mathbf{k},t-t')\vec{\mathscr{E}}(\mathbf{k},t')$$
$$\exp(i\omega t'). \quad (2.10)$$

To fix the ideas we shall assume that $\vec{\mathscr{E}}(\mathbf{k},\omega)$ and $\vec{\mathscr{D}}(\mathbf{k},\omega)$ are the Fourier transforms of the quantities **E** and **D** evaluated in the system of coordinates moving with the velocity $\mathbf{u}_i(t)$ of the motion of the ions of mass m_i and charge e_i in the fields $\mathbf{E}_0(t)$ and $\mathbf{B}_0$. For the sinusoidal pump wave field (1.1) we then get the following connection between $\vec{\mathscr{D}}(\mathbf{k},\omega)$ and $\vec{\mathscr{E}}(\mathbf{k},\omega)$ in a plasma consisting of electrons and one kind of ions:

$$\vec{\mathscr{D}}(\mathbf{k},\omega)=[1+\varepsilon_i(\mathbf{k},\omega)]\vec{\mathscr{E}}(\mathbf{k},\omega)$$

$$+\sum_{v,u=-\infty}^{\infty}J_v(a)J_{u+v}(a)e^{iu\delta}\varepsilon_e(\mathbf{k},\omega-v\omega_0)\vec{\mathscr{E}}(\mathbf{k},\omega+u\omega_0), \quad (2.11)$$

where the quantities a and δ are given by the relations[2]

$$a=\left\{\left[\sum_{\alpha=e,i}\frac{|e_\alpha|}{m_\alpha}\left(\frac{k_\parallel E_{0\parallel}}{\omega_0^2}+\frac{\mathbf{k}_\perp\mathbf{E}_{0\perp}}{\omega_0^2-\omega_{c\alpha}^2}\right)\right]^2+\left[\sum_{\alpha=e,i}\frac{|e_\alpha|}{m_\alpha}\frac{\omega_{c\alpha}}{\omega_0}\frac{[\mathbf{B}_0(\mathbf{kE}_0)]}{B_0(\omega_0^2-\omega_{c\alpha}^2)}\right]^2\right\}^{1/2}$$

$$\operatorname{ctg}\delta=\left[\sum_{\alpha=e,i}\frac{|e_\alpha|}{m_\alpha}\left(\frac{k_\parallel E_{0\parallel}}{\omega_0^2}+\frac{\mathbf{k}_\perp\mathbf{E}_{0\perp}}{\omega_0^2-\omega_{c\alpha}^2}\right)\right]\times\left[\sum_{\alpha=e,i}\frac{|e_\alpha|}{m_\alpha}\frac{\omega_{c\alpha}}{\omega_0}\frac{(\mathbf{B}_0[\mathbf{kE}_0])}{B_0(\omega_0^2-\omega_{c\alpha}^2)}\right]^{-1}. \quad (2.12)$$

We introduce the variable $\xi(\mathbf{k},\omega)$—the Fourier transform of the potential $\varphi(\mathbf{r},t)$ evaluated in the system of coordinates moving with the velocity $\mathbf{u}_i(t)$ satisfying (2.4) ($\vec{\mathscr{E}}(\mathbf{k},\omega)=-i\mathbf{k}\xi(\mathbf{k},\omega)$). We then get from (2.11) and the Poisson equation $\mathbf{k}\vec{\mathscr{D}}(\mathbf{k},\omega)=0$

$$[1+\varepsilon_i(\mathbf{k},\omega)]\xi(\mathbf{k},\omega)$$
$$+\sum_{v,u=-\infty}^{\infty}J_v(a)J_{u+v}(a)e^{iu\delta}\varepsilon_e(\mathbf{k},\omega-v\omega_0)\xi(\mathbf{k},\omega+u\omega_0)=0 \quad (2.13)$$

which, like (2.8) describes the linear stage of the parametric instabilities.

Between the Fourier transform $R(\mathbf{k},\omega)$ of some quantity $R(\mathbf{r},t)$ evaluated in the laboratory system of coordinates and the Fourier transform $\mathscr{R}(\mathbf{k},\omega)$ of the same quantity evaluated in the system of coordinates moving with the velocity $\mathbf{u}_i(t)$ there exist the connections

$$R(\mathbf{k},\omega)=\sum_{m=-\infty}^{\infty} J_m(a_i)e^{im\delta_i}\mathscr{R}(\mathbf{k},\omega+m\omega_0),$$
$$\mathscr{R}(\mathbf{k},\omega)=\sum_{p=-\infty}^{\infty} J_p(a_i)e^{-ip\delta_i}R(\mathbf{k},\omega-p\omega_0), \tag{2.14}$$

which are obtained by replacing in the Fourier integrals which determine $R(\mathbf{k},\omega)$ the variable $\mathbf{r}$ by $\mathbf{r}_i$ where $\mathbf{r}=\mathbf{r}_i+\int^t \mathbf{u}_i(\tau)d\tau$, or by replacing $\mathbf{r}_i$ by $\mathbf{r}$ in the Fourier integrals determining $\mathscr{R}(\mathbf{k},\omega)$. Using (2.14) for the variables $\varphi(\mathbf{k},\omega)$ and $\xi(\mathbf{k},\omega)$ we can transform Eqs. (2.8) and (2.13) into one another.

The Fourier transform $\psi(\mathbf{k},\omega)$ of the potential evaluated in the system of coordinates moving with the velocity $\mathbf{u}_e(t)$ is connected with $\xi(\mathbf{k},\omega)$ through the relations

$$\psi(\mathbf{k},\omega)=\sum_{m=-\infty}^{\infty} J_m(a)e^{im\delta}\xi(\mathbf{k},\omega+m\omega_0),$$
$$\xi(\mathbf{k},\omega)=\sum_{p=-\infty}^{\infty} J_p(a)e^{-ip\delta}\psi(\mathbf{k},\omega-p\omega_0), \tag{2.15}$$

and with $\varphi(\mathbf{k},\omega)$ through the relations

$$\varphi(\mathbf{k},\omega)=\sum_{m=-\infty}^{\infty} J_m(a_e)e^{-im\delta_e}\psi(\mathbf{k},\omega-m\omega_0),$$
$$\psi(\mathbf{k},\omega)=\sum_{p=-\infty}^{\infty} J_p(a_e)e^{ip\delta_e}\varphi(\mathbf{k},\omega+p\omega_0). \tag{2.16}$$

In the variables $\psi(\mathbf{k},\omega)$ we get instead of (2.13)

$$[1+\varepsilon_e(\mathbf{k},\omega)]\psi(\mathbf{k},\omega)+\sum_{u,v=-\infty}^{\infty} J_v(a)J_{u+v}(a)e^{-iu\delta}\varepsilon_i(\mathbf{k},\omega+v\omega_0)\psi(\mathbf{k},\omega-u\omega_0)=0. \tag{2.17}$$

Equations (2.8), (2.13), and (2.17), and also Eqs. (2.14) and (2.16) were obtained in Ref. 2; the discussion given here indicates the connection of them with the Fourier transformation in the moving system of coordinates.

Equation (2.3) (or (2.17)) is particularly convenient in the cases when the main terms occurring in $1+\varepsilon_i$ in (2.13) (or $1+\varepsilon_e$ in (2.17)) are appreciably larger than the terms $\propto \varepsilon_e$ (or $\propto \varepsilon_i$). We consider, for instance, Eq. (2.13) when

$$|\mathrm{Re}\,\varepsilon_i(\mathbf{k},\omega)|\sim 1 \gg \{|\mathrm{Im}\,\varepsilon_i(\mathbf{k},\omega)|,\ |\varepsilon_e(\mathbf{k},\omega)|\}. \tag{2.18}$$

This condition holds, for example, for the ion cyclotron instability of a plasma with hot electrons and cold ions ($T_e \gg T_i$) or for the short-wavelength ion cyclotron instability with $kv_{Ti}/\omega_{ci}\gg 1$.[3] In the zeroth approximation in the small parameter $\gamma/\mathrm{Re}\,\omega(\mathbf{k})$, where $\gamma(\mathbf{k})$ is the growth rate of the oscillations, Eq. (2.13) becomes

$$[1+\mathrm{Re}\,\varepsilon_i(\mathbf{k},\omega)]\xi(\mathbf{k},\omega)=0, \tag{2.19}$$

whence it follows that

$$\xi(\mathbf{k},\omega)=\xi(\mathbf{k})\delta(\omega-\omega(\mathbf{k})), \tag{2.20}$$

where $\omega(\mathbf{k})$ is a solution of the equation

$$1+\mathrm{Re}\,\varepsilon_i(\mathbf{k},\omega(\mathbf{k}))=0. \tag{2.21}$$

Taking the neglected terms into account we can obtain from Eq. (2.11) the following relation between $\vec{\mathscr{D}}(\mathbf{k},t)$ and the slowly varying amplitude $\vec{\mathscr{A}}(\mathbf{k},t)$ of the oscillations of the self-consistent electric field strength

$$\vec{\mathscr{E}}(\mathbf{k},t)=\vec{\mathscr{A}}(\mathbf{k},t)\exp(-i\omega(\mathbf{k})t)$$

in the form

$$\vec{\mathscr{D}}(\mathbf{k},t)\approx e^{-i\omega(\mathbf{k})t}\Big\{i\frac{\partial\vec{\mathscr{A}}(\mathbf{k},t)}{\partial t}\frac{\partial\,\mathrm{Re}\,\varepsilon_i(\mathbf{k},\omega(\mathbf{k}))}{\partial\omega(\mathbf{k})}+\vec{\mathscr{A}}(\mathbf{k},t)\Big[i\,\mathrm{Im}\,\varepsilon_i(\mathbf{k},\omega(\mathbf{k}))+\sum_{v=-\infty}^{\infty}J_v^2(a)\varepsilon_e(\mathbf{k},\omega(\mathbf{k})-v\omega_0)\Big]\Big\}. \tag{2.22}$$

Using (2.22) and the relations

$$\mathbf{E}(\mathbf{k},t)=\vec{\mathscr{E}}(\mathbf{k},t)\exp[ia_i\sin(\omega_0t+\delta_i)],$$
$$\mathbf{D}(\mathbf{k},t)=\vec{\mathscr{D}}(\mathbf{k},t)\exp[ia_i\sin(\omega_0t+\delta_i)], \tag{2.23}$$

obtained from the inverse Fourier transform of the relations (2.14) between $\vec{\mathscr{E}}(\mathbf{k},\omega)$ and $\mathbf{E}(\mathbf{k},\omega)$, and between $\vec{\mathscr{D}}(\mathbf{k},\omega)$ and $\mathbf{D}(\mathbf{k},\omega)$, we obtain the energy balance equation for the parametric kinetic instabilities. In the variables $\vec{\mathscr{E}}(\mathbf{k},t)$, $\vec{\mathscr{D}}(\mathbf{k},t)$ the energy conservation law for electrostatic perturbations $(4\pi)^{-1}\mathbf{E}\cdot\partial\mathbf{D}/\partial t=0$ has the form

$$\frac{1}{8\pi}\int d\mathbf{k}\Big\{\vec{\mathscr{E}}(\mathbf{k},t)\frac{\partial\vec{\mathscr{D}}^*(\mathbf{k},t)}{\partial t}+\vec{\mathscr{E}}^*(\mathbf{k},t)\frac{\partial\vec{\mathscr{D}}(\mathbf{k},t)}{\partial t}-ia_i\omega_0\cos(\omega_0t+\delta_i)[\vec{\mathscr{E}}(\mathbf{k},t)\vec{\mathscr{D}}^*(\mathbf{k},t)-\vec{\mathscr{E}}^*(\mathbf{k},t)\vec{\mathscr{D}}(\mathbf{k},t)]\Big\}=0. \tag{2.24}$$

Substituting (2.22) into (2.24) and averaging the equation obtained over a time interval $\gamma^{-1}\gg\Delta t\gg\omega(k)^{-1}$ we get the energy conservation equation

$$\int d\mathbf{k}\left(\frac{\partial W}{\partial t}-2\gamma(\mathbf{k})W(\mathbf{k},t)\right)=0, \tag{2.25}$$

in which W is the spectral density of the energy of the oscillations determined, taking into account the relation

$$|\mathbf{E}(\mathbf{k},t)|^2=|\vec{\mathscr{E}}(k,t)|^2=|\vec{\mathscr{A}}(\mathbf{k},t)|^2=\mathbf{k}^2|\xi(\mathbf{k},t)|^2$$

in the form

$$W(\mathbf{k})=\frac{\omega(\mathbf{k})}{8\pi}\frac{\partial\,\mathrm{Re}\,\varepsilon_i(\mathbf{k},\omega(\mathbf{k}))}{\partial\omega(\mathbf{k})}|\mathbf{E}(\mathbf{k},t)|^2, \tag{2.26}$$

while $\gamma(\mathbf{k})$ is the growth rate of the kinetic parametric instability which equals

$$\gamma(\mathbf{k})=-\Big[\mathrm{Im}\,\varepsilon_i(\mathbf{k},\omega(\mathbf{k}))+\sum_{v=-\infty}^{\infty}J_v^2(a)\,\mathrm{Im}\,\varepsilon_e(\mathbf{k},\omega(\mathbf{k})-v\omega_0)\Big]\times\Big[\frac{\partial\,\mathrm{Re}\,\varepsilon_i(\mathbf{k},\omega(\mathbf{k}))}{\partial\omega(\mathbf{k})}\Big]^{-1}. \tag{2.27}$$

3. NONLINEAR EQUATION FOR THE OSCILLATION POTENTIAL

As in the linear material equation, the motion of the particles in the external field of the pump wave leads to the appearance of nonstationarity in the nonlinear material equation. The nonlinear material equation has in this case the form[5]

$$D_i(\mathbf{r},t)=E_i(\mathbf{r},t)+\sum_\alpha\int_0^t dt_1\int d\mathbf{r}_1\varepsilon^{(\alpha)}_{ij(1)}(\mathbf{r}-\mathbf{r}_1;t,t_1)E_{j(1)}(\mathbf{r}_1,t_1)$$
$$+\sum_\alpha\int_0^t dt_1\int d\mathbf{r}_1\int_0^{t_1}dt_2\int d\mathbf{r}_2\varepsilon^{(\alpha)}_{ij(1)j(2)}(\mathbf{r}-\mathbf{r}_1;t,t_1|\mathbf{r}_1-\mathbf{r}_2;t_1,t_2)\times E_{j(1)}(\mathbf{r}_1,t_1)E_{j(2)}(\mathbf{r}_2,t_2)$$
$$+\sum_\alpha\int_0^t dt_1\int d\mathbf{r}_1\int_0^{t_1}dt_2\int d\mathbf{r}_2\int_0^{t_2}dt_3\int d\mathbf{r}_3E_{j(1)}(\mathbf{r}_1,t_1)E_{j(2)}\times(\mathbf{r}_2,t_2)E_{j(3)}(\mathbf{r}_3,t_3)$$
$$\times\varepsilon^{(\alpha)}_{ij(1)j(2)j(3)}(\mathbf{r}-\mathbf{r}_1;t,t_1|\mathbf{r}_1-\mathbf{r}_2;t_1,t_2|\mathbf{r}_2-\mathbf{r}_3;t_2,t_3)+\dots\quad(3.1)$$

In the system of coordinates moving with a velocity $\mathbf{u}_\alpha(t)$, where $\mathbf{r}=\mathbf{r}_\alpha\int^t\mathbf{u}_\alpha(\tau)d\tau$ the nonstationarity of the kernels of the material Eq. (3.1) vanishes. Performing in Eq. (3.1) a Fourier-Laplace transformation of the quantities E_j and D_i in the laboratory system of coordinates and of the kernels $\varepsilon^{(\alpha)}$ in the system of coordinates moving with the velocity $\mathbf{u}_\alpha(t)$, and using the Poisson equation $\mathbf{k}\cdot\mathbf{D}(\mathbf{k},\omega)=0$, we get the following equation for the Fourier transform of the potential $\varphi(\mathbf{k},\omega)$:

$$\varphi(\mathbf{k},\omega)+\sum_\alpha\sum_{m,p=-\infty}^{\infty}J_m(a_\alpha)J_{m-p}(a_\alpha)e^{ip\delta_\alpha}\varepsilon_\alpha(\mathbf{k},\omega+m\omega_0)\varphi(\mathbf{k},\omega+p\omega_0)$$
$$=\sum_\alpha\sum_{m,p,q=-\infty}\int d\mathbf{k}_1\,d\omega_1\,d\mathbf{k}_2\,d\omega_2V_\alpha(\mathbf{k},\omega+m\omega_0|\mathbf{k}_1,\omega_1+(m-p)\omega_0)$$
$$\times J_m(a_\alpha)e^{im\delta_\alpha}J_{m-p}(a_{\alpha1})e^{-i(m-p)\delta_{\alpha1}}J_{p-q}(a_{\alpha2})\times e^{-i(p-q)\delta_{\alpha2}}\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\delta(\omega-\omega_1-\omega_2)$$
$$\times\varphi(\mathbf{k}_1,\omega_1)\varphi(\mathbf{k}_2,\omega_2+q\omega_0)+\sum_\alpha\sum_{m,p,q,r=-\infty}^{\infty}\int d\mathbf{k}_1\,d\omega_1\,d\mathbf{k}_3\,d\omega_3\,d\mathbf{k}_4\,d\omega_4J_m(a_\alpha)$$
$$\times e^{im\delta_\alpha}J_{m-p}(a_{\alpha1})e^{-i(m-p)\delta_{\alpha1}}J_{p-q}(a_{\alpha3})e^{-i(p-q)\delta_{\alpha3}}J_{q-r}(a_{\alpha4})e^{-i(q-r)\delta_{\alpha4}}$$
$$\times W_\alpha(\mathbf{k},\omega+m\omega_0|\mathbf{k}_1,\omega_1+(m-p)\omega_0|\mathbf{k}_3,\omega_3+(p-q)\omega_0)\cdot\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_3-\mathbf{k}_4)$$
$$\times\delta(\omega-\omega_1-\omega_3-\omega_4)\varphi(\mathbf{k}_1,\omega_1)\varphi(\mathbf{k}_3,\omega_3)\varphi(\mathbf{k}_4,\omega_4+r\omega_0),\quad(3.2)$$

in which

$$\varepsilon_\alpha(\mathbf{k},\omega+m\omega_0)=k_ik_{j(1)}k^{-2}\varepsilon^{(\alpha)}_{ij(1)}(\mathbf{k},\omega+n\omega_0),$$
$$V_\alpha(\mathbf{k},\omega+n\omega_0|\mathbf{k}_1,\omega_1+p\omega_0)$$
$$=ik_ik_{j(1)}k_{j(2)}k^{-2}\varepsilon^{(\alpha)}_{ij(1)j(2)}(\mathbf{k},\omega+n\omega_0|\mathbf{k}-\mathbf{k}_1,\omega-\omega_1+(n-p)\omega_0),\quad(3.3)$$
$$W_\alpha(\mathbf{k},\omega+n\omega_0|\mathbf{k}_1,\omega_1+p\omega_0|\mathbf{k}_3,\omega_3+p_1\omega_0)$$
$$=k_ik_{j(1)}k_{j(2)}k_{j(3)}k^{-2}\varepsilon^{(\alpha)}_{ij(1)j(2)j(3)}$$
$$\times(\mathbf{k},\omega+n\omega_0|\mathbf{k}-\mathbf{k}_1,\omega-\omega_1+(n-p)\omega_0|\mathbf{k}-\mathbf{k}_1-\mathbf{k}_3,\omega-\omega_1-\omega_3+(n-p-p_1)\omega_0).$$

The matrix elements V_α and W_α in Eq. (3.2) are given by the usual expressions well known from weak turbulence theory when there is no pump wave.[6] Taking the external field (1.1) into account leads to the appearance of factors with products of a few Bessel functions $J_n(a_{\alpha i})$ with appropriate phase factors and to shifts in the frequencies ω and ω_i ($i=1,2,3,4$) by amounts which are multiples of the pump wave field frequency. The quantities a_α and δ_α are given by Eqs. (2.7) and the quantities $a_{\alpha j}$ and $\delta_{\alpha j}$ are obtained from (2.7) by the substitutions $\mathbf{k}\to\mathbf{k}_j$ ($j=1,2,3,4$).

If, however, the Fourier transformation of the quantities E_j and D_i in Eq. (3.1) is performed in the system of coordinates moving with the velocity $\mathbf{u}_i(t)$, and that of the kernels ε_α in a system of coordinates moving with the velocity $\mathbf{u}_\alpha(t)$ ($\alpha=i,e$) we get from the Poisson equation $\mathbf{k}\mathscr{D}(\mathbf{k},\omega)=0$ the following equation for the quantity $\xi(\mathbf{k},\omega)$:

$$[1+\varepsilon_i(\mathbf{k},\omega)]\xi(\mathbf{k},\omega)+\sum_{v,u=-\infty}^{\infty}J_v(a)J_{u+v}(a)e^{iu\delta}\varepsilon_e(\mathbf{k},\omega-v\omega_0)\xi(\mathbf{k},\omega+u\omega_0)$$
$$=\int d\mathbf{k}_1\,d\omega_1\,d\mathbf{k}_2\,d\omega_2V_i(\mathbf{k},\omega|\mathbf{k}_1,\omega_1)\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\delta\times(\omega-\omega_1-\omega_2)\xi(\mathbf{k}_1,\omega_1)\xi(\mathbf{k}_2,\omega_2)$$
$$+\sum_{v,p,q=-\infty}^{\infty}\int d\mathbf{k}_1\,d\omega_1\,d\mathbf{k}_2\,d\omega_2V_e(\mathbf{k},\omega-v\omega_0|\mathbf{k}_1,\omega_1)J_v(a)\times e^{-iv\delta}J_p(a_1)e^{ip\delta_1}$$
$$\times J_q(a_2)e^{iq\delta_2}\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\delta(\omega-\omega_1-\omega_2)\xi(\mathbf{k}_1,\omega_1+p\omega_0)\xi\times(\mathbf{k}_2,\omega_2-(v-q)\omega_0)$$
$$+\int d\mathbf{k}_1\,d\omega_1\,d\mathbf{k}_3\,d\omega_3\,d\mathbf{k}_4\,d\omega_4W_i(\mathbf{k},\omega|\mathbf{k}_1,\omega_1|\mathbf{k}_3,\omega_3)\delta\times(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_3-\mathbf{k}_4)$$
$$\times\delta(\omega-\omega_1-\omega_3-\omega_4)\xi(\mathbf{k}_1,\omega_1)\xi(\mathbf{k}_3,\omega_3)\xi(\mathbf{k}_4,\omega_4)$$
$$+\sum_{v,p,q,r=-\infty}^{\infty}\int d\mathbf{k}_1\,d\omega_1\,d\mathbf{k}_3\,d\omega_3\,d\mathbf{k}_4\,d\omega_4\,W_e(\mathbf{k},\omega-v\omega|\mathbf{k}_1,\omega_1|\mathbf{k}_3,\omega_3)$$
$$\times J_v(a)e^{-iv\delta}J_p(a_1)e^{ip\delta_1}J_q(a_3)e^{iq\delta_3}J_r(a_4)e^{ir\delta_4}\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_3-\mathbf{k}_4)\delta\times(\omega-\omega_1-\omega_3-\omega_4)$$
$$\times\xi(\mathbf{k}_1,\omega_1+p\omega_0)\xi(\mathbf{k}_3,\omega_3+q\omega_0)\xi(\mathbf{k}_4,\omega_4-(v-r)\omega_0),\quad(3.4)$$

in which the quantities a and δ are determined by (2.12) and the quantities a_i and δ_j are obtained from (2.12) by the substitutions $\mathbf{k}\to\mathbf{k}_j$ ($j=1,2,3,4$).

Equation (3.4) shows that the use of the variable $\xi(\mathbf{k},\omega)$ leads to the elimination of the frequency shifts and of the Bessel function products in the ionic terms and they take the same form as when there is no pump wave. Similarly, in the nonlinear equation for the quantity $\psi(\mathbf{k},\omega)$ the effect of the pump field is eliminated in all electron terms. One can obtain this equation from (3.4) by the substitution of the indexes

$i \rightleftarrows e$ and taking the summation indexes, apart from the indexes of the Bessel functions, with the opposite sign.

4. NONLINEAR EQUATION FOR THE INTENSITY OF THE OSCILLATIONS

Using Eq. (3.4) we get in the case (2.18), in the random phase approximation, a nonlinear equation for the intensity $I(\mathbf{k})$ of the oscillations in which, in contrast to the kinetic equation for the waves in the usual weak turbulence theory the finite displacements of the particles in the pump wave field (1.1) are taken into account.

Assuming the quantity $|\xi(\mathbf{k})|^2 = I(\mathbf{k}) \propto \gamma/\omega(\mathbf{k})$ to be a small parameter we find from (3.4) to a first approximation in this parameter that $\xi^{(0)}(\mathbf{k},\omega) = \xi(\mathbf{k})\delta(\omega-\omega(\mathbf{k}))$, where $\omega(\mathbf{k})$ is the solution of Eq. (2.21), and to the next approximation

$$\xi^{(1)}(\mathbf{k},\omega)=\sum_{\alpha=e,i}\sum_{v,p,q=-\infty}^{\infty}\int d\mathbf{k}_1\,d\omega_1 V_\alpha(\mathbf{k},\omega-v\omega_0|\mathbf{k}_1,\omega_1)A^{(\alpha)}_{v,p,q}\times\Big[1+\varepsilon_i(\mathbf{k},\omega)+\sum_{m=-\infty}^{\infty}J_m^2(a)\varepsilon_e(\mathbf{k},\omega-m\omega_0)\Big]^{-1}\xi^{(0)}(\mathbf{k}_1,\omega_1+p\omega_0)\xi^{(0)}\times(\mathbf{k}_2,\omega-\omega_1-(v-q)\omega_0),\tag{4.1}$$

where

$$A^{(e)}_{v,p,q}=J_v(a)e^{-iv\delta}J_p(a_1)e^{ip\delta_1}J_q(a_2)e^{iq\delta_2},\quad A^{(i)}_{v,p,q}=\delta_{v0}\delta_{p0}\delta_{q0}.\tag{4.2}$$

We multiply (3.4) by $\xi^*(\mathbf{k}',\omega')$ and average the equation thus obtained over the random phases. Using (4.1) we can express the triple correlation functions in terms of the fourfold ones which, in their turn, are in the form of a sum of products of the two-fold correlation functions. Defining the intensity $I(\mathbf{k})$ of the oscillations by the relation

$$\langle\xi(\mathbf{k},\omega)\xi^*(\mathbf{k}',\omega')\rangle=I(\mathbf{k})\delta(\omega-\omega(\mathbf{k}))\delta(\mathbf{k}-\mathbf{k}')\delta(\omega-\omega'),\tag{4.3}$$

we get after a number of manipulations the following equation for $I(\mathbf{k})$:

$$\frac{\partial I(\mathbf{k},t)}{\partial t}=\gamma(\mathbf{k})I(\mathbf{k},t)+\Gamma(\mathbf{k})I(\mathbf{k},t)-\pi\sum_{\alpha,\beta=i,e}\sum{}'\,\mathrm{Re}\int d\mathbf{k}_1\times v_\alpha(\mathbf{k},\Omega(\mathbf{k})|\mathbf{k}_1,\Omega_1(\mathbf{k}_1))v_\beta(\mathbf{k}_2,\Omega_2'(\mathbf{k}_2)|\mathbf{k},\Omega'(\mathbf{k}))\times\Big[\frac{\partial\,\mathrm{Re}\,\varepsilon_i(\mathbf{k},\omega(\mathbf{k}))}{\partial\omega(\mathbf{k})}\frac{\partial\,\mathrm{Re}\,\varepsilon_i(\mathbf{k}_2,\omega_2(\mathbf{k}_2))}{\partial\omega_2(k_2)}\Big]^{-1}\times A^{(\alpha)}_{v,p,q}A^{(\beta)*}_{v_1,p_1,q_1}I(\mathbf{k})I(\mathbf{k}_1)\delta(\Omega(\mathbf{k})-\Omega_1(\mathbf{k}_1)-\Omega_2(\mathbf{k}_2))+\frac{\pi}{2}\sum_{\alpha,\beta=i,e}\sum{}'\,\mathrm{Re}\int d\mathbf{k}_1\,v_\alpha(\mathbf{k},\Omega(\mathbf{k})|\mathbf{k}_1,\Omega_1(\mathbf{k}_1))\times v_\beta^*(\mathbf{k},\Omega'(\mathbf{k})|\mathbf{k}_1,\Omega'(\mathbf{k}_1))\Big[\frac{\partial\,\mathrm{Re}\,\varepsilon_i(\mathbf{k},\omega(\mathbf{k}))}{\partial\omega(\mathbf{k})}\Big]^{-2}A^{(\alpha)}_{v,p,q}A^{(\beta)*}_{v_1,p_1,q_1}I(\mathbf{k}_1)I(\mathbf{k}_2)\delta(\Omega(\mathbf{k})-\Omega_1(\mathbf{k}_1)-\Omega_2(\mathbf{k}_2))\tag{4.4}$$

(Σ' denotes summation over v, p, q, v_1, p_1, q_1 from $-\infty$ to $+\infty$ with the condition that $v-p-q=v_1-p_1-q_1$).

In Eq. (4.4) $\gamma(\mathbf{k})$ is the linear growth rate (2.27),

$$\Gamma(\mathbf{k})=\sum_{\alpha=e,i}\sum{}''\Big[\frac{\partial\,\mathrm{Re}\,\varepsilon_i}{\partial\omega(\mathbf{k})}\Big]^{-1}\mathrm{Im}\int d\mathbf{k}_1 B^{(\alpha)}_{v,p,q,r}\times w_\alpha(\mathbf{k},\Omega(\mathbf{k})|\mathbf{k}_1,\Omega_1(\mathbf{k}_1)|\mathbf{k},\Omega(\mathbf{k})+(v-q)\omega_0)I(\mathbf{k}_1)+\sum_{\alpha,\beta=i,e}\sum{}'\,\mathrm{Im}\,\mathrm{P}\int d\mathbf{k}_1\times A^{(\alpha)}_{v,p,q}A^{(\beta)*}_{v_1,p_1,q_1}v_\alpha(\mathbf{k},\Omega(\mathbf{k})|\mathbf{k}_1\Omega_1(\mathbf{k}_1))\Big[\frac{\partial\,\mathrm{Re}\,\varepsilon_i(\mathbf{k},\omega(\mathbf{k}))}{\partial\omega(\mathbf{k})}\Big]^{-1}\times v_\beta(\mathbf{k}_2,\Omega'(\mathbf{k})-\Omega_1'(\mathbf{k}_1)|\mathbf{k},\Omega'(\mathbf{k}))I(\mathbf{k}_1)\times\Big[1+\varepsilon_i(\mathbf{k}-\mathbf{k}_1,\Omega(\mathbf{k})-\Omega_1(\mathbf{k}_1)+q\omega_0)+\sum_{m=-\infty}^{\infty}J_m^2(a)\varepsilon_e(\mathbf{k}_2,\Omega(\mathbf{k})-\Omega_1(\mathbf{k}_1)+(q-m)\omega_0)\Big]^{-1}\tag{4.5}$$

(Σ'' denotes summation over v, p, q, r from $-\infty$ to $+\infty$ with the condition $v+r=p+q$). We have in (4.4) and (4.5) introduced the following notation:

$$v_\alpha(\mathbf{k},\Omega(\mathbf{k})|\mathbf{k}_1,\Omega_1(\mathbf{k}_1))=V_\alpha(\mathbf{k},\Omega(\mathbf{k})|\mathbf{k}_1,\Omega_1(\mathbf{k}_1))+V_\alpha(\mathbf{k},\Omega(\mathbf{k})|\mathbf{k}-\mathbf{k}_1,\Omega(\mathbf{k})-\Omega_1(\mathbf{k}_1)),$$

$$w_\alpha(\mathbf{k},\Omega(\mathbf{k})|\mathbf{k}_1,\Omega_1(\mathbf{k}_1)|\mathbf{k},\Omega(\mathbf{k})+(v-q)\omega_0)=W_\alpha(\mathbf{k},\Omega(\mathbf{k})|\mathbf{k}_1,\Omega_1(\mathbf{k}_1)|\mathbf{k},\Omega(\mathbf{k})+(v-q)\omega_0)+W_\alpha(\mathbf{k},\Omega(\mathbf{k})|\mathbf{k}_1,\Omega_1(\mathbf{k}_1)|-\mathbf{k}_1,\Omega_1(-\mathbf{k}_1)-(v-q)\omega_0),$$

$$\Omega(\mathbf{k})=\omega(\mathbf{k})-v\omega_0,\quad \Omega_1(\mathbf{k}_1)=\omega_1(\mathbf{k}_1)-p\omega_0,\quad \Omega_2(\mathbf{k}_2)=\omega_2(\mathbf{k}_2)-q\omega_0,$$

$$\Omega'(\mathbf{k})=\omega(\mathbf{k})-v_1\omega_0,\quad \Omega_1'(\mathbf{k}_1)=\omega_1(\mathbf{k}_1)-p_1\omega_0,\quad \Omega_2'(\mathbf{k}_2)=\omega_2(\mathbf{k}_2)-q_1\omega_0,$$

$$B^{(e)}_{v,p,q,r}=J_v(a)e^{-iv\delta}J_p(a_1)e^{ip\delta_1}J_q(a)e^{iq\delta}J_r(a_1)e^{-ir\delta_1},\quad B^{(i)}_{v,p,q,r}=\delta_{v0}\delta_{p0}\delta_{q0}\delta_{r0}.$$

Equation (4.4) differs from the equation for the intensity $I(\mathbf{k})$ in the usual weak turbulence theory (see, e.g., Ref. 6) by the factors $A^{(e)}$ and $B^{(e)}$ and the presence of shifts in the frequencies ω, ω_1, and ω_2 by amounts which are multiples of the pump wave field frequency. Pursuing the analogy with the usual kinetic equation for waves we shall say that the third and fourth terms on the right-hand side of Eq. (4.4) describe decay processes involving the waves $(\mathbf{k},\Omega(\mathbf{k}))$, $(\mathbf{k}_1,\Omega_1(\mathbf{k}_1))$, $(\mathbf{k}_2,\Omega_2(\mathbf{k}_2))$. The nonlinear growth rate $\Gamma(\mathbf{k})$ describes induced scattering processes of waves by free particles (the first term in (4.5)) and by the polarization clouds of virtual waves $(\mathbf{k}_2,\Omega(\mathbf{k})-\Omega_1(\mathbf{k}_1)+q\omega_0)$ (second term in (4.5)).

One should note that it follows from (2.23) that the intensity $I(\mathbf{k})$ of the oscillations given by Eq. (4.3) is equal to the intensity of the oscillations in the laboratory system of coordinates:

$$I(\mathbf{k},t)=|\xi(\mathbf{k},t)|^2=|\varphi(\mathbf{k},t)|^2.\tag{4.6}$$

In the case when

$$|\mathrm{Re}\,\varepsilon_e(\mathbf{k},\omega)|\sim1\gg\{|\mathrm{Im}\,\varepsilon_e(\mathbf{k},\omega)|,\ |\varepsilon_i(\mathbf{k},\omega)|\},\tag{4.7}$$

arising, for instance, when we study the electron cyclotron parametric kinetic instability of a plasma with hot ions and cold electrons ($T_i > T_e$),[4] one must for the construction of an equation for the intensity $I(\mathbf{k})$ of the oscillations start from the nonlinear equation for $\psi(\mathbf{k},\omega)$. We define in that case the intensity of the oscillations in the form

$$\langle\psi(\mathbf{k},\ \omega)\psi^*(\mathbf{k}',\ \omega')\rangle=I(\mathbf{k})\delta(\omega-\omega(\mathbf{k}))\delta(\omega-\omega')\delta(\mathbf{k}-\mathbf{k}'), \quad (4.8)$$

where $\omega(\mathbf{k})$ is the solution of the equation $1+\mathrm{Re}\,\varepsilon_e(\mathbf{k},\omega)=0$. The equation for $I(\mathbf{k})$ as defined by (4.8) can be obtained from (4.4), (4.5) by substituting everywhere the indexes $i\rightleftarrows e$ and taking the summation indexes, apart from the indexes of the Bessel functions, with the opposite sign.

5. THE EQUATIONS OF THE QUASILINEAR APPROXIMATION

In the present section we consider the equations of the quasilinear approximation when there are parametric instabilities presence in a uniform plasma both in the presence and in the absence of an external magnetic field.

The equations of the quasilinear theory for the averaged distribution function F_α of particles of the species α when there is a pump wave present were considered in Ref. 1 in the variables $F_\alpha^{(n)}$ and $\varphi^{(n)}$ which are the expansion coefficients of expansions of the form

$$F_\alpha=\sum_{n=-\infty}^{\infty}F_\alpha^{(n)}e^{-in\omega_0t}$$

of the function F_α and the potential $\varphi(\mathbf{r},t)$ of the oscillations. The use of the Fourier transformation in the moving system of coordinates in the cases (2.18) and (4.7) significantly simplifies the derivation and the final form of the kinetic equation for F_α. We obtain the equation for the particle distribution function, averaged over the initial phases of the oscillations, of a uniform plasma when there is no external magnetic field by averaging the Vlasov equation:

$$\frac{\partial F_\alpha}{\partial t}+\frac{e_\alpha}{m_\alpha}\left(\mathbf{E}_0(t)\frac{\partial F_\alpha}{\partial\mathbf{v}}-\left\langle\nabla\varphi\frac{\partial f_\alpha}{\partial\mathbf{v}}\right\rangle\right)=0\quad(\alpha=i,e). \quad (5.1)$$

In a system of coordinates moving with the velocity

$$\mathbf{u}_\alpha(t)=\frac{e_\alpha}{m_\alpha}\int^t\mathbf{E}_0(\tau)\,d\tau,$$

in which $\mathbf{v}=\mathbf{v}_\alpha+\mathbf{u}_\alpha(t)$ (5.1) takes the form

$$\frac{\partial F_\alpha(\mathbf{v}_\alpha,t)}{\partial t}-\frac{e_\alpha}{m_\alpha}\left\langle\nabla\varphi\frac{\partial f_\alpha}{\partial\mathbf{v}}\right\rangle=0. \quad (5.2)$$

We denote the Fourier transform of the function $f_\alpha(\mathbf{v},\mathbf{r},t)$ $(\alpha=i,e)$ obtained in the moving system of coordinates, by $g_\alpha(\mathbf{v}_\alpha,\mathbf{k},\omega)$; it is connected with the Fourier transform of $f_\alpha(\mathbf{v},\mathbf{r},t)$ in the laboratory system of coordinates, $f_\alpha(\mathbf{v},\mathbf{k},\omega)$, through Eq. (2.14). From the Fourier transform of the linearized equation for the perturbation $f_\alpha(\mathbf{v},\mathbf{r},t)$:

$$\frac{\partial f_\alpha}{\partial t}+\mathbf{v}\frac{\partial f_\alpha}{\partial\mathbf{r}}+\frac{e_\alpha}{m_\alpha}\mathbf{E}_0(t)\frac{\partial f_\alpha}{\partial\mathbf{v}}-\frac{e_\alpha}{m_\alpha}\nabla\varphi(\mathbf{r},t)\frac{\partial F_\alpha}{\partial\mathbf{v}}=0$$

it follows that

$$g_i(\mathbf{v}_i,\mathbf{k},\omega)=-\frac{e_i}{m_i}\xi(\mathbf{k},\omega)(\omega-\mathbf{k}\mathbf{v}_i)^{-1}\mathbf{k}\frac{\partial F_i(\mathbf{v}_i)}{\partial\mathbf{v}_i}, \quad (5.3)$$

$$g_e(\mathbf{v}_e,\mathbf{k},\omega)=\frac{e}{m_e}\psi(\mathbf{k},\omega)(\omega-\mathbf{k}\mathbf{v}_e)^{-1}\mathbf{k}\frac{\partial F_e(\mathbf{v}_e)}{\partial\mathbf{v}_e}. \quad (5.4)$$

We now restrict our considerations to the case (2.18) in which the quantity $\xi(\mathbf{k},\omega)$ is determined by Eq. (2.20). Using (5.3), (2.20) in Eq. (5.2) we get the following equation for F_i:

$$\frac{\partial F_i(\mathbf{v}_i,t)}{\partial t}=\pi\frac{e_i^2}{m_i^2}\int d\mathbf{k}\left(\mathbf{k}\frac{\partial}{\partial\mathbf{v}_i}\right)\mathbf{k}\frac{\partial F_i}{\partial\mathbf{v}_i}I(\mathbf{k})\delta(\omega(\mathbf{k})-\mathbf{k}\mathbf{v}_i). \quad (5.5)$$

This equation is the same in form as the quasilinear kinetic equation when there is no pump wave. The difference is connected with the fact that in (5.5) the velocity $\mathbf{v}_1=\mathbf{v}-\mathbf{u}_i(t)$ is used instead of the velocity $\mathbf{v}$.

Using the relation (2.15) between $\xi(\mathbf{k},\omega)$ and $\psi(\mathbf{k},\omega)$ we get in the case (2.18) also the following equation for F_e:

$$\frac{\partial F_e(\mathbf{v}_e,t)}{\partial t}=\pi\frac{e^2}{m_e^2}\int d\mathbf{k}\left(\mathbf{k}\frac{\partial}{\partial\mathbf{v}_e}\right)\sum_{m,n=-\infty}^{\infty}J_m(a)J_n(a)$$
$$\times\cos[(m-n)(\omega_0t+\delta)]\left(\mathbf{k}\frac{\partial F_e}{\partial\mathbf{v}_e}\right)I(\mathbf{k},t)\delta(\omega(\mathbf{k})-m\omega_0-\mathbf{k}\mathbf{v}_e), \quad (5.6)$$

in which the principal value of the integral vanishes because the integrand is odd in $\mathbf{k}$.

We easily get in the usual way[6] from (5.5) the H-theorem and the H-like theorem. We can also prove these theorems for Eq. (5.6) in the asymptotic limit $\omega_0t\gg1$. We integrate Eq. (5.6) over the time:

$$F_e(\mathbf{v}_e,t)=F_e(\mathbf{v}_e,0)+\pi\frac{e^2}{m_e^2}\int d\mathbf{k}\left(\mathbf{k}\frac{\partial}{\partial\mathbf{v}_e}\right)\times\sum_{m=-\infty}^{\infty}J_m^2(a)\delta(\omega(\mathbf{k})-m\omega_0-\mathbf{k}\mathbf{v}_e)$$
$$\times\int_0^t d\tau I(\mathbf{k},\tau)\left(\mathbf{k}\frac{\partial F_e(\mathbf{v}_e,\tau)}{\partial\mathbf{v}_e}\right)+\pi\frac{e^2}{m_e^2}\int d\mathbf{k}\left(\mathbf{k}\frac{\partial}{\partial\mathbf{v}_e}\right)\times\sum_{m,n=-\infty}^{\infty}J_m(a)J_n(a)$$
$$\times\delta(\omega(\mathbf{k})-m\omega_0-\mathbf{k}\mathbf{v}_e)\int_0^t d\tau\cos[(m-n)(\omega_0\tau+\delta)]I(\mathbf{k},\tau)\times\left(\mathbf{k}\frac{\partial F_e(\mathbf{v}_e,\tau)}{\partial\mathbf{v}_e}\right). \quad (5.7)$$

Substituting F_e in the form

$$F_e=\bar{F}_e+\tilde{F}_e, \quad (5.8)$$

where $\tilde{F}_e$ is the part of the distribution function F_e oscillating with a frequency which is a multiple of ω_0 and $\bar{F}_e$ its nonoscillating part, we find from (5.7) that

$$\bar{F}_e(\mathbf{v}_e,t)=\bar{F}_{e0}(\mathbf{v}_e,t)[1+O(\gamma/\omega_0,(\omega_0t)^{-1})],$$
$$\tilde{F}_e\sim\bar{F}_{e0}(\mathbf{v}_e,t)O(\gamma/\omega_0,(\omega_0t)^{-1}),$$
$$\bar{F}_{e0}(\mathbf{v}_e,t)=\bar{F}_e(\mathbf{v}_e,0)+\frac{\pi e^2}{m_e^2}\int d\mathbf{k}\left(\mathbf{k}\frac{\partial}{\partial\mathbf{v}_e}\right)\sum_{m=-\infty}^{\infty}J_m^2(a)$$
$$\times\delta(\omega(\mathbf{k})-m\omega_0-\mathbf{k}\mathbf{v}_e)\int_0^t d\tau I(\mathbf{k},\tau)\left(\mathbf{k}\frac{\partial\bar{F}_{e0}(\mathbf{v}_e,\tau)}{\partial\mathbf{v}_e}\right),$$
$$2\gamma(\mathbf{k})=|d\ln I(\mathbf{k},t)/dt|. \quad (5.9)$$

It follows from (5.9) that $\bar{F}_{e0}$ satisfies the equation

$$\frac{d\bar{F}_{e0}}{dt}=\pi\frac{e^2}{m_e^2}\int d\mathbf{k}\left(\mathbf{k}\frac{\partial}{\partial \mathbf{v}_e}\right)\sum_{m=-\infty}^{\infty}J_m^2(a)\left(\mathbf{k}\frac{\partial \bar{F}_{e0}}{\partial \mathbf{v}_e}\right)I(\mathbf{k},t)$$

$$\delta(\omega(\mathbf{k})-m\omega_0-\mathbf{k}\mathbf{v}_e). \tag{5.10}$$

We multiply (5.6) by F_e, integrate over velocity space and time and, using (5.9), we get

$$\sigma\equiv\int F_e^2(\mathbf{v}_e,t)\,d\mathbf{v}_e=\int F_e^2(\mathbf{v}_e,0)\,d\mathbf{v}_e-\frac{\pi e^2}{m_e^2}\int d\mathbf{v}_e\int d\mathbf{k}\sum_{m=-\infty}^{\infty}J_m^2(a)$$

$$\times\delta(\omega(\mathbf{k})-m\omega_0-\mathbf{k}\mathbf{v}_e)\int_0^t d\tau\, I(\mathbf{k},\tau)\left(\mathbf{k}\frac{\partial \bar{F}_{e0}}{\partial \mathbf{v}_e}\right)^2$$

$$\times[1+O(\gamma/\omega_0,(\omega_0 t)^{-1})]. \tag{5.11}$$

All terms in (5.11) are positive; hence it follows that σ decreases with time. However, the fact σ is positive requires that as $\omega_0 t\to\infty$

$$I(\mathbf{k},t)\left(\mathbf{k}\frac{\partial \bar{F}_{e0}}{\partial \mathbf{v}_e}\right)\Big|_{\omega(\mathbf{k})=m\omega_0+\mathbf{k}\mathbf{v}_e}\to 0. \tag{5.12}$$

Condition (5.12) is the analog of the well known condition of quasilinear relaxation.[6] We note that one can obtain the result (5.12) in the usual way (see, e.g., Ref. 6) if one starts from Eq. (5.10).

We define the entropy density s in the form

$$s=-\left\langle\int d\mathbf{v}_e F_e \ln F_e\right\rangle_{\Delta t}, \tag{5.13}$$

where the brackets $\langle\dots\rangle_{\Delta t}$ indicate averaging over a time interval $\Delta t\gg\omega_0^{-1}$ and we get

$$\frac{ds}{dt}=\frac{\pi e^2}{m_e^2}\int d\mathbf{k}\int d\mathbf{v}_e\frac{1}{\bar{F}_{e0}}\left(\mathbf{k}\frac{\partial \bar{F}_{e0}}{\partial \mathbf{v}_e}\right)^2$$

$$\times\sum_{m=-\infty}^{\infty}J_m^2(a)I(\mathbf{k},t)\delta(\omega(\mathbf{k})-m\omega_0-\mathbf{k}\mathbf{v}_e)$$

$$+O(\gamma/\omega_0,(\omega_0 t)^{-1})>0, \tag{5.14}$$

i.e., an increase in the entropy density of the resonance electrons over time intervals larger than the period of the pump wave as a result of the quasilinear relaxation. If we define the entropy density s' in the form

$$s'=-\left\langle\int d\mathbf{v}_e \bar{F}_{e0}\ln \bar{F}_{e0}\right\rangle, \tag{5.13'}$$

one can, starting from Eq. (5.10), obtain an expression for the rate of change of the entropy ds'/dt which will differ from (5.14) by terms of order $O(\gamma/\omega_0, (\omega_0 t)^{-1}$.

When there is a constant uniform magnetic field present in the system of coordinates moving with the velocity $\mathbf{u}_\alpha(t)$ the equation

$$\frac{\partial F_\alpha}{\partial t}+\frac{e_\alpha}{m_\alpha}\left(\mathbf{E}_0(t)+\frac{1}{c}[\mathbf{v}\mathbf{B}_0]\right)\frac{\partial F_\alpha}{\partial \mathbf{v}}-\frac{e_\alpha}{m_\alpha}\left\langle\nabla\varphi\frac{\partial f_\alpha}{\partial \mathbf{v}}\right\rangle=0$$

for the averaged part F_α of the distribution function of the plasma particles takes the form (5.2). We assume that $F_\alpha(\mathbf{v}_\alpha)$ is independent of the azimuthal angle χ_α in the space of the velocities $\mathbf{v}_\alpha=\mathbf{v}-\mathbf{u}_\alpha(t)$.

From the linearized Vlasov equation for that part of the distribution function f_α which oscillates with the frequencies of the eigenoscillations,

$$\frac{\partial f_\alpha}{\partial t}+\mathbf{v}\frac{\partial f_\alpha}{\partial \mathbf{r}}+\frac{e_\alpha}{m_\alpha}\left(\mathbf{E}_0(t)+\frac{1}{c}[\mathbf{v}\times\mathbf{B}_0]\right)\frac{\partial f_\alpha}{\partial \mathbf{v}}-\frac{e_\alpha}{m_\alpha}\nabla\varphi\frac{\partial F_\alpha}{\partial \mathbf{v}}=0,$$

we can get the following expression for $g_\alpha(\mathbf{v}_\alpha,\mathbf{k},\omega)$:

$$g_\alpha(\mathbf{v}_\alpha,\mathbf{k},\omega)$$

$$=-\frac{e_\alpha}{m_\alpha}\zeta(\mathbf{k},\omega)\sum_{n=-\infty}^{\infty}\frac{\exp[i\mu_\alpha\sin(\chi_\alpha-\theta)-in(\chi_\alpha-\theta)]}{(\omega-n\omega_{c\alpha}-k_\parallel v_{\alpha\parallel})}$$

$$\times J_n(\mu_\alpha)\left[\frac{n\omega_{c\alpha}}{v_{\alpha\perp}}\frac{\partial F_\alpha(\mathbf{v}_\alpha)}{\partial v_{\alpha\perp}}+k_\parallel\frac{\partial F_\alpha(\mathbf{v}_\alpha)}{\partial v_{\alpha\parallel}}\right], \tag{5.15}$$

in which $\zeta(\mathbf{k},\omega)\equiv\xi(\mathbf{k},\omega)$ for $\alpha=i$ and $\zeta(\mathbf{k},\omega)\equiv\psi(\mathbf{k},\omega)$ for $\alpha=e$,

$$\mathbf{v}_\alpha=(v_{\alpha\perp},\chi_\alpha,v_{\alpha\parallel}),\quad \mathbf{k}=(k_\perp,\theta,k_\parallel),\quad v_{\alpha\parallel}\|k_\parallel\|B_0,$$

$$\mu_\alpha=v_{\alpha\perp}k_\perp/\omega_{c\alpha}.$$

We restrict our considerations to the case (2.18); in that case, using (5.15), (2.20), (4.3) in Eq. (5.2) we get a quasilinear equation for F_i in the form

$$\frac{\partial F_i}{\partial t}=\pi\frac{e_i^2}{m_i^2}\int d\mathbf{k}\sum_{n=-\infty}^{\infty}\left(\frac{n\omega_{ci}}{v_{i\perp}}\frac{\partial}{\partial v_{i\perp}}+k_\parallel\frac{\partial}{\partial v_{i\parallel}}\right)$$

$$\times\left(\frac{n\omega_{ci}}{v_{i\perp}}\frac{\partial F_i}{\partial v_{i\perp}}+k_\parallel\frac{\partial F_i}{\partial v_{i\parallel}}\right)$$

$$\times J_n^2(\mu_i)I(\mathbf{k})\delta(\omega(k)-n\omega_{ci}-k_\parallel v_{i\parallel}), \tag{5.16}$$

which, like (5.5) differs from the quasilinear equation when there is no pump wave through the use of the variables $\mathbf{v}_i=\mathbf{v}-\mathbf{u}_i(t)$ instead of $\mathbf{v}$.

Using the relation (2.15) between $\xi(\mathbf{k},\omega)$ and $\psi(\mathbf{k},\omega)$ we get in the case (2.18) also the following equation for the function $F_e(\mathbf{v}_e,t)$:

$$\frac{\partial F_e(\mathbf{v}_e,t)}{\partial t}$$

$$=\pi\frac{e^2}{m_e^2}\int d\mathbf{k}\sum_{m,p,n=-\infty}^{\infty}J_m(a)J_p(a)\cos[(m-p)(\delta+\omega_0 t)]$$

$$\left(\frac{n\omega_{ce}}{v_{e\perp}}\frac{\partial}{\partial v_{e\perp}}+k_\parallel\frac{\partial}{\partial v_{e\parallel}}\right)$$

$$\times\left(\frac{n\omega_{ce}}{v_{e\perp}}\frac{\partial F_e}{\partial v_{e\perp}}+k_\parallel\frac{\partial F_e}{\partial v_{e\parallel}}\right)J_n^2(\mu_e)I(\mathbf{k},t)\delta$$

$$\times(\omega(\mathbf{k})-m\omega_0-n\omega_{ce}-k_\parallel v_{e\parallel}). \tag{5.17}$$

We integrate Eq. (5.17) over the time. Writing $F_e(\mathbf{v}_e,t)$ as the sum (5.8) we find that

$$\bar{F}_e(\mathbf{v}_e,t)=\bar{F}_{e0}(\mathbf{v}_e,t)\,(1+O(\gamma/\omega_0,(\omega_0 t)^{-1})],$$

$$\tilde{F}_e(\mathbf{v}_e,t)\sim\bar{F}_{e0}(\mathbf{v}_e,t)\,O(\gamma/\omega_0,(\omega_0 t)^{-1}),$$

while

$$\bar{F}_{e0}(\mathbf{v}_e,t)$$

$$=F_e(\mathbf{v}_e,0)+\pi\frac{e^2}{m_e^2}\int d\mathbf{k}\sum_{m,n=-\infty}^{\infty}J_m^2(a)\left(\frac{n\omega_{ce}}{v_{e\perp}}\frac{\partial}{\partial v_{e\perp}}+k_\parallel\frac{\partial}{\partial v_{e\parallel}}\right)$$

$$\times \int_0^t \left(\frac{n\omega_{ce}}{v_{e\perp}} \frac{\partial \bar{F}_{e0}(\mathbf{v}_e, \tau)}{\partial v_{e\perp}} + k_\parallel \frac{\partial \bar{F}_{e0}(\mathbf{v}_e, \tau)}{\partial v_{e\parallel}} \right) I(\mathbf{k}, \tau) J_n^2(\mu_e) \quad (5.18)$$

$$\times \delta(\omega(\mathbf{k}) - m\omega_0 - n\omega_{ce} - k_\parallel v_{e\parallel})\, d\tau.$$

We multiply (5.17) by F_e and integrate over the velocity space and the time. Using (5.8) and (5.18) we find that

$$\int F_e^2(\mathbf{v}_e, t)\, d\mathbf{v}_e = \int F_e^2(\mathbf{v}_e, 0)\, d\mathbf{v}_e - \frac{\pi e^2}{m_e^2} \int d\mathbf{v}_e \int d\mathbf{k} \sum_{m,n=-\infty}^{\infty} J_m^2(a) J_n^2(\mu_e) \times \delta(\omega(\mathbf{k}) - m\omega_0 - n\omega_{ce} - k_\parallel v_{e\parallel}) \times \int_0^t d\tau\, I(\mathbf{k}, \tau) \left(\frac{n\omega_{ce}}{v_{e\perp}} \frac{\partial \bar{F}_{e0}}{\partial v_{e\perp}} + k_\parallel \frac{\partial \bar{F}_{e0}}{\partial v_{e\parallel}} \right)^2 \times [1 + O(\gamma/\omega_0, (\omega_0 t)^{-1})], \quad (5.19)$$

whence follows the quasilinear relaxation condition

$$I(\mathbf{k}, t) \left| \left(\frac{n\omega_{ce}}{v_{e\perp}} \frac{\partial \bar{F}_{e0}}{\partial v_{e\perp}} + k_\parallel \frac{\partial \bar{F}_{e0}}{\partial v_{e\parallel}} \right) \right|_{\omega(\mathbf{k}) = m\omega_0 + n\omega_{ce} + k_\parallel v_{e\parallel}} \to 0 \quad (5.20)$$

as $\omega_0 t \to \infty$.

One can obtain this condition also in the usual way[6] starting from the equation for the function $\bar{F}_{e0}(\mathbf{v}_e, t)$ which, as follows from (5.8), has the form

$$\frac{\partial \bar{F}_{e0}}{\partial t} = \pi \frac{e^2}{m_e^2} \int d\mathbf{k} \sum_{m,n=+\infty}^{\infty} J_m^2(a) \left(\frac{n\omega_{ce}}{v_{e\perp}} \frac{\partial}{\partial v_{e\perp}} + k_\parallel \frac{\partial}{\partial v_{e\parallel}} \right) \times \left(\frac{n\omega_{ce}}{v_{e\perp}} \frac{\partial \bar{F}_{e0}}{\partial v_{e\perp}} + k_\parallel \frac{\partial \bar{F}_{e0}}{\partial v_{e\parallel}} \right) I(\mathbf{k}, t) J_n^2(\mu_e) \delta \times (\omega(\mathbf{k}) - m\omega_0 - n\omega_{ce} - k_\parallel v_{e\parallel}). \quad (5.21)$$

In concluding this section we note that the quasilinear relaxation process leads to an increase in the entropy density, given by Eq. (5.13). For time intervals $\Delta t \gg \omega_0^{-1}$ we find that

$$\frac{ds}{dt} = \pi \frac{e^2}{m_e^2} \int d\mathbf{k} \int d\mathbf{v}_e \frac{1}{\bar{F}_{e0}} \times \sum_{m,n=-\infty}^{\infty} J_m^2(a) J_n^2(\mu_e) \left(\frac{n\omega_{ce}}{v_{e\perp}} \frac{\partial \bar{F}_{e0}}{\partial v_{e\perp}} + k_\parallel \frac{\partial \bar{F}_{e0}}{\partial v_{e\parallel}} \right)^2 \times I(\mathbf{k}, t) \delta(\omega(\mathbf{k}) - m\omega_0 - n\omega_{ce} - k_\parallel v_{e\parallel}) + O(\gamma/\omega_0, (\omega_0 t)^{-1}) > 0. \quad (5.22)$$

One can also reach the same conclusion starting from Eq. (5.21) and defining the entropy density s' in the form (5.13').

In the usual way we can for Eq. (5.16) obtain a H-theorem and an H-like theorem which in form are the same as the corresponding theorems when there is no pump wave.

6. CONCLUSION

Above we showed that for kinetic type parametric instabilities caused by the build-up of oscillations by resonance particles one can when there is a small parameter $|\gamma(\mathbf{k})/\omega(\mathbf{k})|$ construct a generalized weak turbulence theory for the general case of finite displacements of particles in the pump wave field. In the limiting case of small displacements $k\xi_E \ll 1$ the expressions obtained go over into well known expressions.

As an example we apply the results obtained above to a study of the nonlinear stage of the ion cyclotron parametric kinetic instability. Such instabilities are excited in the hf heating of a plasma using fast magneto-sonic or ion cyclotron (Alfvén) waves with a frequency $\omega_0 \sim \omega_{ci}$ (see Ref. 7). The appearance of the instability may be caused either by the oscillations of the electrons relative to the ions under the action of the electric field of these waves or by the oscillations of the ions of different kinds relative to one another.[8] These instabilities can particularly easily be excited at the periphery of a plasma filament where the plasma temperature is low and the amplitudes of the hf fields used to heat the plasma are large so that the relative velocity of the oscillations may exceed the thermal ion velocity.

The development of parametric turbulence will lead to an undesirable heating of the periphery of the plasma and, possibly, to the bombardment of the walls by fast ions and the entry of impurities into the discharge. In small devices one often uses for plasma heating a large amplitude hf field. In that case ion cyclotron parametric instabilities are excited also inside the plasma filament leading to a fast turbulent plasma heating. In experiments[9] one has observed an anomalously fast heating of the hydrogen in the plasma of the stellarator "Uragan-2" after a time appreciably shorter (by a factor 10 to 10^3) than the time for the exchange of energy between the hydrogen and deuterium ions during cyclotron resonance for a small group ($\lesssim 10\%$) of deuterium ions. The study of ion cyclotron parametric turbulence is thus of great interest for the problem of hf plasma heating.

The frequency and growth rate of the instability for the case when the relative velocity of the electrons and ions $\mathbf{u}$ in the pump wave field is less than or of the order of the ion thermal velocity v_{Ti} and the pump wave frequency $\omega_0 \sim \omega_{ci}$ are given by the estimates[3]

$$\omega(k) \approx n\omega_{ci}\left(1 + \frac{1}{k_\perp \rho_i}\right), \quad \gamma \sim n\omega_{ci} \frac{T_i}{T_e} \frac{1}{k_\perp \rho_i}, \quad k_\perp \rho_i \sim \frac{v_{Ti}}{u} \gg 1, \quad (6.1)$$

which are the same as the estimates for the frequency and the growth rate for the short-wavelength beam ion cyclotron instability for a plasma with a transverse current.[10]

As in the weak turbulence theory obtained in the present paper the expressions for the matrix elements V_α and W_α differ from the corresponding matrix elements of the usual kinetic equation for the waves only through the shifts in the frequencies ω, ω_1, and ω_2 by amounts which are multiples of ω_0 and the quantity $a \sim 1$ for the estimates (6.1), it turns out to be possible to use the results of the analysis of the nonlinear stage of the short-wavelength beam instability of a plasma with a transverse current given in Ref. 10. From this analysis it follows that also for the parametric instability considered, as for the beam instability, the nonlinear stage will be determined by the induced scattering of ion cyclotron waves by free ions and that the level W of the energy density of the oscillations in the saturation state when the nonlinear damping rate becomes, as to order of magnitude, equal to the quantity (6.1), will be equal to

$$\frac{W}{n_i T_i} \sim \frac{T_i}{T_e}\left(\frac{u}{v_{Ti}}\right)^4 \frac{1}{\ln (v_{Ti}/u)}. \tag{6.2}$$

When one takes this process into account there occurs then at the same time a nonlinear amplification of unstable ion cyclotron waves (ICW) with the smallest allowable frequency (6.1) $\omega(k) \approx n\omega_{ci} \approx m\omega_0$. The restriction of the growth of these oscillations can occur as the result of the nonlinear broadening of the cyclotron resonance at the level[10] $W \sim (u/v_{Ti})^4 n_i T_i$ exceeding (6.2). In that case the nonlinear damping rate $\Gamma(\mathbf{k})$ of the oscillations with higher frequencies becomes larger than the linear growth rate which leads to the nonlinear suppression of these oscillations. The same effect of the suppression of the high-frequency part of the spectrum of the unstable ICW must also occur for a beam instability.,

On the basis of the quasilinear Eq. (5.17) for F_e one can see that Cherenkov absorption and emission of ion cyclotron oscillations by resonance electrons leads to the heating of the electron component with a rate

$$n_e dT_{e\parallel}/dt \sim \gamma W. \tag{6.3}$$

With the same speed the increase of the transverse ion temperature due to stochastic shocks from turbulent pulsations of the electric field of the ICW will take place.

The estimates given on the basis of the equation for the intensity $I(\mathbf{k})$, (4.8), under the condition (4.7), similar to (4.4), show that for electron cyclotron turbulence[4] the presence of energy transfer along the spectrum, caused by the induced scattering of electron cyclotron waves by free electrons, is also characteristic. As for the ion cyclotron instability, this leads to the suppression of the high-frequency part of the spectrum of the electron cyclotron oscillations; the unstable oscillations with the smallest frequency are amplified as a result of this process and their saturation occurs due to the effect of the broadening of the cyclotron resonances at a level $W \sim (u/v_{Te})^4 n_e T_e$. The same effect is also characteristic for electron cyclotron turbulence of a plasma with a transverse current.

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Translated by D. ter Haar