Polarization phenomena due to ion drift in plasmas

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The ordering of the angular momenta of atoms and ions in low-pressure plasmas is predicted and investigated theoretically. It is due to the anisotropy in the distribution of relative collision velocities produced by ion drift. The analysis is based on the theory of anisotropic collisional relaxation of polarization moments of the density matrix, which generalizes the D'yakonov-Perel' theory [Sov. Phys. JETP **20**, 997 and **21**, 227 (1965)] to the case of collisions with a preferred direction of relative velocity. It is shown that, when anisotropic relaxation is present, collisions lead not only to the destruction of the polarization moments and intramultiplet mixing, but also to the creation of even polarization moments and to the mutual transformation of polarization moments of different rank. These general results of the theory are used together with a numerical solution of the equations derived by the impact-parameter method to examine the ^2P doublets of ions and neutrals in low-temperature plasmas. The rate constants for the appearance of alignment under the influence of ion drift and for the depolarization and intramultiplet mixing during anisotropic collisions between atoms and ions are reported. The results show that, in low-pressure plasmas, the drift mechanism of alignment and the associated linear polarization of light are the dominant effects for ion lines, and compete successfully with other alignment-inducing mechanisms (fast electrons and light beams) in the case of neutral-atom lines. The orientation of the atomic (ionic) angular momenta and the associated circular polarization of light under the combined effect of ion drift and internal magnetic field of the plasma is predicted.

INTRODUCTION

Studies of the polarization properties of spectral lines yield valuable information on elementary plasma processes and may be useful in plasma diagnostics. The phenomenon of partial linear polarization of spectral lines emitted by plasmas was first discovered in the upper atmosphere of the sun[1] and subsequently in low-temperature plasmas under laboratory conditions (Refs. 2–4). It is explained by the alignment of the atomic angular momenta during excitation by fast-electron[2] or light[3,4] beams. A review of astrophysical and laboratory studies of alignment processes is given in Ref. 5.

The use of anisotropic light beams and fast-electron beams in low-temperature plasmas is not the only way of ordering the angular momenta of atoms and ions. One of the alternatives is the recently proposed[6] method whereby alignment is produced by the slow (as compared with electron velocities) drift of ions with energies insufficient for the excitation of atoms. The present paper is devoted to an examination of this new mechanism for the alignment of atomic and ionic states in low-pressure plasmas.

Our approach is founded on the theory of anisotropic collisional relaxation of atomic states.[7–10] It has enabled us not only to establish the general conditions for the appearance of alignment under the influence of ion drift, but also to determine the quantitative characteristics of this phenomenon as functions of the plasma parameters and the properties of aligned atoms or ions. Because the corresponding rate constants are quite high, the new mechanism will not only successfully compete with other alignment mechanisms but, under certain definite conditions, will be the main cause of alignment signals from low-pressure gas discharges.

ANISOTROPIC COLLISIONAL RELAXATION OF POLARIZATION MOMENTS

The method of polarization moments[14–17] that was developed in the theory of the nucleus and of gamma-emission,[11–13] and was successfully transplanted to atomic theory, is one of the most convenient and rigorous ways of describing the ordering of the angular momenta of an ensemble of atoms (or ions). The polarization moments arise when the density matrix $\rho_{MM_1}^{JJ_1}$ of an ensemble is expanded over the irreducible representations of the rotation group, and are determined by the formula

$$\rho_Q{}^K(JJ_1)=\sum_{MM_1}(-1)^{J_1-M_1}\begin{bmatrix} J & J_1 & K \\ M & -M_1 & Q\end{bmatrix}\rho_{MM_1}^{JJ_1}. \tag{1}$$

The brackets in this expression are the Clebsch-Gordan coefficients and J and M are the internal angular momentum of the atom and its z-component, respectively. The polarization moments of $\rho_Q^K(JJ)$ of different rank K that are diagonal in J determine the population n_J of a given J state and the different types of ordering of the angular momenta of the atoms in this state. We then have

$$n_J=(2J+1)^{1/2}\rho_0{}^0(JJ). \tag{2}$$

The three quantities $\rho_Q^1(JJ)$ corresponding to $Q=0, \pm 1$ are the spherical components of the so-called orientation vector and determine the circular polarization of light emitted from level J; the five quantities $\rho_Q^2(JJ)$ corresponding to $Q=0, \pm 1, \pm 2$ are the spherical components of a tensor of rank two—the so-called alignment tensor—and determine the linear polarization of the emitted light. Polarization moments of higher rank (with $K \geqslant 3$) do not appear directly in

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dipole radiation, but can be observed for radiation of higher multipolarity.[18]

Suppose that an ensemble of atoms undergoes collisions with foreign particles and that the distribution of the relative collision velocities is axially symmetric relative to some particular direction which we shall take as the z axis. The effect of the collisions on the state of the ensemble then corresponds to the symmetry group $C_{\infty v}$ and is described by the relaxation matrix R which splits into blocks belonging to different irreducible representations of this group. The contribution of collisions to the change in the polarization moments of the ensemble is described by the following set of equations:

$$\dot{\rho}_Q{}^K(JJ_1)=\sum_{K_1J_2J_3} R_Q^{KK_1}(JJ_1;J_2J_3)\rho_Q{}^{K_1}(J_2J_3). \tag{3}$$

The dot represents differentiation with respect to time, which is due to collisional processes. It follows from the properties of the group $C_{\infty v}$ that the matrix R is diagonal in Q but not in K: collisions with anisotropic velocity distribution (with a preferred direction) will mix the components of polarization moments of different rank K but the same Q. For the longitudinal components of the polarization moments ($Q=0$) there is an additional selection rule: the elements of the matrix $R_0^{KK_1}$ are nonzero only when K and K_1 have the same parity. This means that the longitudinal polarization moments will mix in anisotropic collisions in which the parity of the rank of each of them is conserved.

The mixing of polarization moments of different rank is the essential difference between anisotropic collisional relaxation and the usually examined isotropic relaxation[14–17] that occurs under the conditions of completely random collisions. Mixing in the course of anisotropic collisions gives rise to interesting physical effects that are fundamentally impossible in the case of isotropic collisions. They include the collisional transformation of linear polarization to circular polarization, predicted in Refs. 19 and 20 and observed in Ref. 21. This transformation corresponds to the collisional mixing of the polarization moments $\rho^2_{\pm1}$ and $\rho^1_{\pm1}$. Among the other cases of mixing of polarization moments, the most interesting is the common collisional relaxation of the longitudinal components of even polarization moments. It gives rise to alignment and to the dependence of the rates of collisional intermultiplet mixing on the alignment of multiplet sublevels.

ALIGNMENT DURING INTRAMULTIPLET MIXING IN ANISOTROPIC COLLISIONS

We now establish the general condition under which anisotropic collisions may lead to the longitudinal alignment $\rho_0^2(JJ)$ from atomic level populations n_J. It follows from (2) and (3) that

$$\dot{n}_J=(2J+1)^{1/2}\sum_{J_1J_2K} R_0{}^{0K}(JJ;J_1J_2)\rho_0{}^K(J_1J_2). \tag{4}$$

Let us first consider the situation where slow collisions do not mix states with different J. We then have $\dot{n}_J=0$ on the left, whereas only terms with $J_1=J_2=J$, i.e., terms of the form $R_0^{0K}(JJ;JJ)\rho_0^K(JJ)$, remain on the right. The right-hand side should then vanish identically for arbitrary instantaneous values of $\rho_0^K(JJ)$. It follows that, in this case, $R_0^{0K}(JJ;JJ)=0$ for all K. When the symmetry of the relaxation matrix is taken into account, all quantities of the form $R_0^{K0}(JJ;JJ)$ are then also zero. Moreover, it is clear from (3) that the rate of alignment of $\rho_0^2(JJ)$ in level J from its population n_J is $R_0^{20}(JJ;JJ)n_J/(2J+1)^{1/2}$. It follows that collisional alignment is not possible in the absence of mixing of different J states. The derivatives $\dot{n}_J$ in (4) and the matrix elements $R_0^{0K}(JJ;JJ)=R_0^{K0}(JJ;JJ)$ (with even K) are then different from zero, so that collisional longitudinal alignment from the populations n_J becomes possible.

The condition for mixing of different J states is well satisfied for narrow fine-structure multiplets in which the splitting ΔE is appreciably smaller than the kinetic energy of the colliding particles and, as a rule, satisfies the additional condition

$$\Delta E/\hbar \ll v/R, \tag{5}$$

which ensures the breaking of the coupling between **L** and **S** during the collision time,[22–25] where v is the relative velocity and R is a characteristic distance related to the cross section σ for the process by $R\sim(\sigma/\pi)^{1/2}$. This breaking of the coupling means that only the orientation of the vector **L** changes during the collision; the vector **S** "does not manage to react" and retains its orientation. The only part of the density matrix that is then found to relax is that connected with the quantization of the orbital angular momentum **L**. Let this part be denoted by $\rho_{MM_1}^{LL}$ and its polarization moments by $\rho_q^{\varkappa}(L)$. By analogy with (3), the equations describing anisotropic relaxation of the latter are

$$\dot{\rho}_q{}^{\varkappa}(L)=\sum_{\varkappa_1} R_q^{\varkappa\varkappa_1}(L)\rho_q{}^{\varkappa_1}(L). \tag{6}$$

The velocity v is assumed to be not too high, so that it satisfies condition (5), but does not lead to collisional "flipover" of atoms from levels in a given **L**, **S** multiplet to levels in other multiplets. The total population of each multiplet is then conserved and, by virtue of what was said above in connection with (4), all the elements of the matrix $R_q^{\varkappa\varkappa_1}(L)$ for which at least one of the indices $(\varkappa,\varkappa_1)$ is zero are all zero.

If we now apply the algebraic methods of the theory of angular momenta[11] and use the expansion of the polarization moments $\rho_Q^K(JJ_1J_2J_3)$ and their spin analogs, we can derive from (6) the following expression for the anisotropic collisional relaxation matrix for the polarization moments of a given **L**, **S** multiplet, $R_Q^{KK_1}(JJ_1;J_2J_3)$, in terms of the matrix $R_q^{\varkappa\varkappa_1}(L)$ describing the analogous relaxation of orbital polarization moments $\rho_q^{\varkappa}(L)$:

$$\begin{aligned} R_Q^{KK_1}(JJ_1;\ J_2J_3) &= \sum_{\varkappa\varkappa_1 lpq} (-1)^{J^*}(2l+1)[(2\varkappa+1)(2\varkappa_1+1)(2K+1) \\ &\quad \times (2K_1+1)(2J+1)(2J_1+1)(2J_2+1)(2J_3+1)]^{1/2} \\ &\times \begin{Bmatrix} S & S & l \\ L & L & \varkappa \\ J & J_1 & K \end{Bmatrix} \begin{Bmatrix} S & S & l \\ L & L & \varkappa_1 \\ J_2 & J_3 & K_1 \end{Bmatrix} \begin{Bmatrix} \varkappa_1 & \varkappa & p \\ K & K_1 & l \end{Bmatrix} \begin{bmatrix} \varkappa_1 & \varkappa & p \\ q & -q & 0 \end{bmatrix} \begin{bmatrix} K_1 & K & p \\ Q & -Q & 0 \end{bmatrix} \\ &\times R_q^{\varkappa\varkappa_1}(L), \end{aligned} \tag{7}$$

where $J^* = J_1 + J_3 - J - J_2 + K + K_1 - l + p + Q + q$. The problem thus reduces to the determination of the matrix $R_q^{\varkappa\varkappa_1}(L)$ describing anisotropic collisional relaxation of purely orbital polarization moments of the ensemble of atoms. Let us now consider the matrix $\tilde{R}_q^{\varkappa\varkappa_1}(L,v)$ in the limiting case where all collisions occur strictly along the z axis with the same relative velocity $\mathbf{v}$. In the real situation of a nonmonokinetic velocity distribution $(f\theta,v)$, where θ is the angle between the relative velocity vector and the z axis, the collisional relaxation of purely orbital polarization moments is then determined by the matrix

$$R_q^{\varkappa\varkappa_1} = \sum_{q_1} \int D_{qq_1}^{\varkappa}(\varphi, \theta, \psi) D_{qq_1}^{\varkappa_1 *}(\varphi, \theta, \psi) f(\theta, v) \tilde{R}_{q_1}^{\varkappa\varkappa_1}(v) v^2\, dv \sin\theta\, d\theta\, d\varphi\, d\psi, \quad (8)$$

where D is the Wigner function. Let S denote the scattering matrix that determines the mixing of the purely orbital magnetic sublevels of the atom (ion) in collisions with relative velocity $\mathbf{v}$ parallel to the z axis. The matrix describing the monokinetic relaxation of orbital polarization moments is then given by

$$\tilde{R}_q^{\varkappa\varkappa_1}(v) = 2\pi N v \sum_{MM_1M_2M_3} (-1)^{M_1-M_3} \begin{bmatrix} L & L & \varkappa \\ M & -M_1 & q \end{bmatrix} \times \begin{bmatrix} L & L & \varkappa_1 \\ M_2 & -M_3 & q \end{bmatrix} \int_0^\infty b\, db [S_{MM_1} S_{M_2M_3}^* - \delta_{MM_1}\delta_{M_2M_3}], \quad (9)$$

where N is the flux density of incident particles and the integral is evaluated with respect to the impact parameter b (see Ref. 9).

The theoretical determination of anisotropic collisional relaxation of the polarization moments of an ensemble of atoms in the course of collisions with heavy particles is thus seen to reduce to the evaluation of the S-matrix that determines the collisional reorientation of the angular momentum $\mathbf{L}$, the evaluation of the matrix $\tilde{R}_q^{\varkappa\varkappa_1}$ in (9), the determination of the matrix $R_q^{\varkappa\varkappa_1}$, and, finally, the determination of the complete anisotropic collisional relaxation matrix $R_Q^{KK_1}$ with the aid of (7). The elements of the last matrix are contained in (3) which describes the appearance, disappearance, and mutual transformation of different types of ordering of the angular momenta of atoms in the course of anisotropic collisions. Depending on the specific form of the relative velocity distribution function $f(\theta,v)$, the theory advanced here will cover a wide range of situations, ranging from extremely anisotropic collisions in narrow colliding beams to the totally random (isotropic) distributions of collision directions. As an example, we consider in the next section the effect of ion drift in gas-discharge plasmas on the ordering of the angular momenta of atoms and ions.

COLLISIONAL ALIGNMENT OF ATOMS AND IONS IN A GAS DISCHARGE

According to Ref. 26, the velocities v_a of neutral atoms in a low-pressure discharge have the isotropic Maxwellian distribution, whereas the ion velocities have the Maxwellian distribution with a superimposed drift velocity that we shall denote by $\mathbf{v}_0 = v_0\mathbf{e}_2$. The distribution of the relative velocities of atoms and ions is then described by the function

$$f(\theta, v) = (\gamma/\pi)^{3/2} \exp[-\gamma(v^2 - 2vv_0\cos\theta + v_0^2)]. \quad (10)$$

The quantity γ is determined by the masses of atoms and ions (m_a and m_i) and their effective temperatures T_a and T_i:

$$\gamma = m_a m_i / 2k(m_i T_i + m_a T_a). \quad (11)$$

The anisotropy of the velocity distribution is characterized by the parameter

$$\lambda = v_0\sqrt{\gamma}. \quad (12)$$

Substituting (10) in (8), and using the expansion of a plane wave in terms of spherical functions, we obtain

$$R_q^{\varkappa\varkappa_1}(L) = \frac{4\gamma^{3/2}}{\pi^{1/2}} \exp(-\gamma v_0^2) \times \sum_{q_1, l} (-1)^{q+q_1} \begin{bmatrix} \varkappa & \varkappa_1 & l \\ -q_1 & q_1 & 0 \end{bmatrix} \begin{bmatrix} \varkappa & \varkappa_1 & l \\ -q & q & 0 \end{bmatrix} \times \int dv \tilde{R}_{q_1}^{\varkappa\varkappa_1}(v) \exp(-\gamma v^2) \left(\frac{v}{2\gamma v_0}\right)^l \left(\frac{1}{v}\frac{d}{dv}\right)^l \frac{\operatorname{sh} 2\gamma v_0 v}{2\gamma v_0 v}. \quad (13)$$

It is clear from (10) that excited neutral atoms in the gas discharge undergo anisotropic collisions with ions moving preferentially in the direction of their drift, whereas excited ions find themselves in a stream of neutral atoms moving relative to the ions in the direction opposite to the ion-drift velocity. It follows that all the phenomena associated with anisotropic collisional relaxation of polarization moments that were described in the last section arise as a result of the drift of ions in the discharge, both in the neutral excited atoms and in the drifting ions themselves. The entire difference reduces to the fact that the matrices $\tilde{R}_q^{\varkappa\varkappa_1}$ describing collisional reorientation of the orbital angular momenta of the electrons are different for neutrals and for ions.

We have carried out calculations of the S-matrix and the corresponding matrix $\tilde{R}_q^{\varkappa\varkappa_1}$ in (9) for both atomic and ionic P-states ($L = 1$). The calculations were performed by numerical integration of the equations obtained by the impact parameter method in the approximation of rectilinear trajectories,[27] using the program described in Ref. 19. In the interaction between the colliding pair of particles, we have taken into account the leading term in the multipole expansion that leads to the splitting of the M_L levels of the atom or ion under investigation and, as a consequence, to the collisional reorientation of its electron orbital angular momentum $\mathbf{L}$. When we consider the orientation of the angular momentum $\mathbf{L}$ of a neutral atom in the P-state in a collision with an atom of charge Ze, this splitting is

$$\Delta C_a / R^3 = -3ZQ_a e^2 / 2R^3, \quad (14)$$

whereas the splitting of the M_L levels of the ion in the P-state during the interaction with unexcited neutral atoms is

$$\Delta C_i / R^6 = (9ZQ_i\alpha_a e^2 + \Delta)/R^6, \quad (15)$$

where Q_a and Q_i are the quadrupole moments of the electron shell of the atom and ion, α_a is the dipole polarizability of the unexcited netural atom, and Δ is the difference between the dispersion interaction constants of the excited ion and the unexcited atom for $M_L = \pm 1$ and for $M_L = 0$.

Computer calculations have enabled us to find the following numerical values of the nonzero elements of the matrix $\widetilde{R}_q^{\varkappa\varkappa_1}$ for $L = 1$. In the case of the reorientation of the angular momentum **L** of a neutral atom in collisions with ions,

$$\widetilde{R}_0^{11}=-6.56s_i,\ \widetilde{R}_{\pm1}^{11}=-2.41s_i,\ \widetilde{R}_0^{22}=-1.64s_i,$$
$$\widetilde{R}_{\pm1}^{22}=-3.49s_i,\ \widetilde{R}_{\pm2}^{22}=-3.56s_i,\ \widetilde{R}_{\pm1}^{21}=\widetilde{R}_{\pm1}^{12}=\pm0.39is_i. \tag{16a}$$

whereas for the reorientation of the angular momentum **L** of an excited ion in collisions with neutral atoms,

$$\widetilde{R}_0^{11}=-4.61s_a,\ \widetilde{R}_{\pm1}^{11}=-1.97s_a,\ \widetilde{R}_0^{22}=-1.45s_a,$$
$$\widetilde{R}_{\pm1}^{22}=-2.94s_a,\ \widetilde{R}_{\pm2}^{22}=-2.55s_a,\ \widetilde{R}_{\pm1}^{21}=\widetilde{R}_{\pm1}^{12}=\mp0.96is_a.$$

The quantities

$$s_i=3N_i|ZQ_a|e^2/2\hbar,\quad s_a=N_a v^{3/5}|(9ZQ_ie^2\alpha_a+\Delta)/\hbar|^{2/5} \tag{17}$$

have the dimensions of s^{-1} and depend on the particular electronic properties of the colliding particles; N_1 and N_a are the concentrations of ions and neutrals.

Substituting for the elements of the matrix R from (16) into (13), and using (7) and (3), we can calculate all the characteristics of anisotropic collisional relaxation of narrow ${}^{2S+1}P$ multiplets of ions and atoms in a gas discharge for arbitrary electron spin S. We shall confine our attention to $S=\frac{1}{2}$, i.e., to the ${}^2P$ doublets. Let us now consider the $Q=0$ block in (3). This describes the simultaneous collisional relaxation of the doublet level populations $n_{1/2}$ and $n_{3/2}$ with arbitrary alignment of the $J=\frac{3}{2}$ level, which is characterized by

$$a_{3/2}=\langle 3J_z^2-J^2\rangle/J(2J-1)\,|_{J=3/2}=2\rho_0^2(3/2\ 3/2).$$

This yields the following set of equations:

$$\dot{n}_{1/2}=N(-\langle v\sigma_n\rangle n_{1/2}+{}^1/_2\langle v\sigma_n\rangle n_{3/2}+{}^1/_2\langle v\sigma_b\rangle a_{3/2}),$$
$$\dot{n}_{3/2}=N(\langle v\sigma_n\rangle n_{1/2}-{}^1/_2\langle v\sigma_n\rangle n_{3/2}-{}^1/_2\langle v\sigma_b\rangle a_{3/2}), \tag{18}$$
$$\dot{a}_{3/2}=N(\langle v\sigma_b\rangle n_{1/2}-{}^1/_2\langle v\sigma_b\rangle n_{3/2}-\langle v\sigma_g\rangle a_{3/2}),$$

where N represents the concentration of perturbing particles (N_i or N_a). We have also introduced the rate constants for the following collisional processes: $\langle v\sigma_n\rangle$ determines the direct transfer of populations between the $\frac{1}{2}$ and $\frac{3}{2}$ levels, $\langle v\sigma_b\rangle$ describes the creation of the (positive) longitudinal alignment $a_{3/2}$ from the population $n_{1/2}$ of the $\frac{1}{2}$ level and the simultaneous creation of the (negative) alignment $a_{3/2}$ from the population $n_{3/2}$ of the $\frac{3}{2}$ level, and $\langle v\sigma_g\rangle$ describes collisional depolarization (removal) of alignment $a_{3/2}$. Moreover, it is clear from (18) that the quantity $\langle v\sigma_b\rangle$ also describes the influence of the alignment of the $\frac{3}{2}$ state on the resultant rates of collisional transitions between the doublet $\frac{1}{2}$ and $\frac{3}{2}$ levels. Thus, the number of particles executing the $\frac{1}{2}\to\frac{3}{2}$ and $\frac{3}{2}\to\frac{1}{2}$ transitions per unit time is respectively given by

$$\Delta n_{3/2}=N(\langle v\sigma_n\rangle n_{1/2}-{}^1/_2\langle v\sigma_b\rangle a_{3/2}),$$
$$\Delta n_{1/2}={}^1/_2N(\langle v\sigma_n\rangle n_{3/2}+\langle v\sigma_b\rangle a_{3/2}), \tag{19}$$

whereas the following alignment is produced per unit time from the populations $n_{1/2}$ and $n_{3/2}$:

$$\Delta a_{3/2}=(N/2)(2n_{1/2}-n_{3/2})\langle v\sigma_b\rangle. \tag{20}$$

It is clear from (20) that the rate of collisional alignment is proportional to the deviation of the populations of the $\frac{1}{2}$ and $\frac{3}{2}$ levels from their statistical weights ($\Delta a_{3/2}=0$ for $n_{1/2}:n_{3/2}=1:2$). It is also clear from (19) that the rate of collisional transfer of atoms (ions) between the levels of the multiplet depends on the alignment of their angular momenta. This phenomenon was recently predicted for colliding beams[28] in which the rates of collisional $\frac{1}{2}\to\frac{3}{2}$ and $\frac{3}{2}\to\frac{1}{2}$ transitions may change by a considerable factor depending on the alignment of the $\frac{3}{2}$ state. It is clear from (19) that this effect occurs in the gas-discharge plasma as well, but it is then reduced because of the low degree of anisotropy of the velocity distribution. In addition to collisional alignment, the dependence of the rate of collisional flipover ("intramultiplet mixing") on the alignment is a characteristic feature of anisotropic collisional relaxation. Both effects are absent for isotropic velocity distributions.

For clarity, we also note the connection between the above rate constants and the elements of the R-matrix in (3):

$$N\langle v\sigma_n\rangle=\sqrt{2}R_0^{00}(1/2\ 1/2;\ 3/2\ 3/2)=-R_0^{00}(1/2\ 1/2;\ 1/2\ 1/2)$$
$$=-2R_0^{00}(3/2\ 3/2;\ 3/2\ 3/2)=\sqrt{2}R_0^{00}(3/2\ 3/2;\ 1/2\ 1/2),$$
$$N\langle v\sigma_b\rangle=\sqrt{2}R_0^{20}(3/2\ 3/2;\ 1/2\ 1/2)=\sqrt{2}R_0^{02}(1/2\ 1/2;\ 3/2\ 3/2)$$
$$=-2R_0^{20}(3/2\ 3/2;\ 3/2\ 3/2)=-2R_0^{02}(3/2\ 3/2;\ 3/2\ 3/2), \tag{21}$$
$$N\langle v\sigma_g\rangle=-R_0^{22}(3/2\ 3/2;\ 3/2\ 3/2).$$

RATE CONSTANTS FOR ANISOTROPIC COLLISIONAL RELAXATION OF THE ${}^2P$ STATES OF ATOMS AND IONS UNDER THE INFLUENCE OF ION DRIFT

We shall now consider quantitative calculations of the rate constants $\langle v\sigma_n\rangle$, $\langle v\sigma_b\rangle$ and $\langle v\sigma_g\rangle$ that we have carried out for the relative velocity distribution (10) which takes into account ion drift in the gas-discharge plasma.

Figure 1 shows the rate constants for different values of the anisotropy parameter λ, given by (12), in the case of reorientation of the angular momenta of atoms in the ${}^2P$ state under the influence of collision with ions (case a) and for the reorientation of the angular momenta of ions in ${}^2P$ states during collisions with unexcited atoms of an inert buffer gas (case b). When $\lambda=0$, there is no ion drift, the alignment constants $\langle v\sigma_b\rangle$ are zero, and the other two rate constants

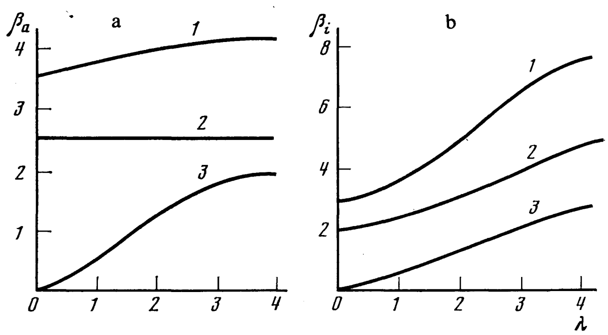

FIG. 1. Rate constants $\langle v\sigma_g\rangle$ (1), $\langle v\sigma_n\rangle$ (2) and $\langle v\sigma_b\rangle$ (3) as functions of the anisotropy parameter λ of the relative velocity distribution of atoms and ions: a—alignment of the excited atom in the ${}^2P$ state during collisions with ions; b—alignment of the excited ion in the ${}^2P$ state during collisions with neutral atoms. The rate constants are expressed in units of β_a and β_i given by (22), respectively.

TABLE I. Rate constants (in units of 10^{-6} cm^{3}s^{-1}) for the n^2P doublets of Li and Na atoms during collisions with singly-charged ions. The calculations were performed for the anisotropy parameter $\lambda = 1$.

State of atom	Li			Na		
	$\langle v\sigma_n\rangle$	$\langle v\sigma_b\rangle$	$\langle v\sigma_g\rangle$	$\langle v\sigma_n\rangle$	$\langle v\sigma_b\rangle$	$\langle v\sigma_g\rangle$
2^2P	0,26	0,06	0,39	–	–	–
3^2P	1,56	0,34	2,34	0,36	0,08	0,54
4^2P	5,29	1,18	7,94	2,00	0,44	3,00
5^2P	13,3	2,97	20,0	6,39	1,42	9,58
6^2P	28,2	6,28	42,4	15,5	3,45	23,3
7^2P	52,9	11,8	79,5	32,0	7,12	48,1
8^2P	91,0	20,2	137	59,0	13,1	88,6
9^2P	147	32,6	220	100	22,3	150
10^2P	224	49,9	337	159	35,5	239

assume values characteristic for isotropic collisional relaxation. In the limiting isotropic case ($\lambda \to \infty$), all three rate constants tend to finite values describing the situation in parallel colliding beams.

The ordinate axis in Fig. 1 gives the values of

$$\beta_a = 3|ZQ_a|e^2/2\hbar, \quad \beta_i = |(9ZQ_i\alpha_a e^2 + \Delta)/\hbar|^{2/5}\gamma^{-3/10}. \tag{22}$$

They depend on the particular characteristics of the ^2P doublet of the neutral atom or ion under investigation, and can be calculated, for example, by the quantum-defect method, using reference tables.[29,30] We have carried out such numerical calculations of the rate constants for the first ^2P doublets of Li and Na and a number of excited ^2P doublets of Mg^+ for $\lambda = 1$, i.e., for a drift velocity comparable with the thermal velocity of the ions. In the case of neutral atoms (Table I), the interaction law (14) shows that the rate constants should be independent of temperature. In the case of ions, the rate constants for the interaction law (15) are proportional to $\gamma^{-3/10}$; the data of Table II refer to the situation where $T_i \approx T_a = 650$ K, which is close to the conditions prevailing in a typical experiment.

It is clear from Tables I and II that the rate constants in the case of collisions between an excited neutral atom and ions are of the order of 10^{-6} cm^{3}s^{-1}, i.e., they are higher by a factor of about 100 as compared with the analogous rate constants in the case of collisions between an excited ion and neutral atoms (10^{-8} cm^{3}s^{-1}). However, the neutral-atom concentration in the low-pressure plasma ($N_a \sim 10^{13}$–10^{15} cm^{-3}) exceeds the ion concentration by three to five orders of magnitude ($N_i \sim 10^{10}$ cm^{-3}). The ion-drift alignment mechanism for the angular momentum should therefore have a stronger effect in the case of excited ions than excited neutral atoms. Experiments[31] have in fact shown that alignment signals associated with ions lines are appreciably stronger than those for neutral atoms in low-pressure plasma.

CALCULATION OF THE DEGREE OF ALIGNMENT OF THE $^2P_{3/2}$ STATE WITH ALLOWANCE FOR ION DRIFT

To find the alignment $a_{3/2}$ of the $^2P_{3/2}$ level in the discharge, we must first introduce into (18) the terms representing radiative damping (Γ_0) of the given ^2P doublet and the rates of population of its sublevels, $F_{1/2}$ and $F_{3/2}$. Assuming that the population of the doublet levels under the influence of electrons occurs without alignment in the $\frac{3}{2}$ level, we obtain

$$dn_{1/2}/dt = N(-\langle v\sigma_n\rangle n_{1/2} + {}^1/_2\langle v\sigma_n\rangle n_{3/2} + {}^1/_2\langle v\sigma_b\rangle a_{3/2}) - \Gamma_0 n_{1/2} + F_{1/2},$$
$$dn_{3/2}/dt = N(\langle v\sigma_n\rangle n_{1/2} - {}^1/_2\langle v\sigma_n\rangle n_{3/2} - {}^1/_2\langle v\sigma_b\rangle a_{3/2}) - \Gamma_0 n_{3/2} + F_{3/2},$$
$$da_{3/2}/dt = N(\langle v\sigma_b\rangle n_{1/2} - {}^1/_2\langle v\sigma_b\rangle n_{3/2} - \langle v\sigma_g\rangle a_{3/2}) - \Gamma_0 a_{3/2}. \tag{23}$$

This set of equations was solved for a steady-state discharge (when all the time derivatives on the left-hand sides are zero)

TABLE II. Rate constants (in units of 10^{-8} cm^{3}s^{-1}) for the n^2P doublets of Mg^+ during collisions with atoms of heavy inert gases. Anisotropy parameter $\lambda = 1$, temperatures $T = T_i = T_a = 650$ K.

State of ion Mg^+	Inert gas	$\langle v\sigma_n\rangle$	$\langle v\sigma_b\rangle$	$\langle v\sigma_g\rangle$
6^2P	Ar	2,18	0,46	3,32
	Kr	2,39	0,50	3,64
	Xe	2,85	0,49	4,36
7^2P	Ar	2,90	0,61	4,43
	Kr	3,18	0,67	4,86
	Xe	3,78	0,79	5,75
8^2P	Ar	3,65	0,77	5,57
	Kr	4,02	0,85	6,14
	Xe	4,94	1,04	7,53
9^2P	Ar	4,49	0,94	6,85
	Kr	4,96	1,04	7,57
	Xe	5,89	1,24	8,99
10^2P	Ar	5,41	1,14	8,25
	Kr	5,94	1,25	9,07
	Xe	7,07	1,49	10,7

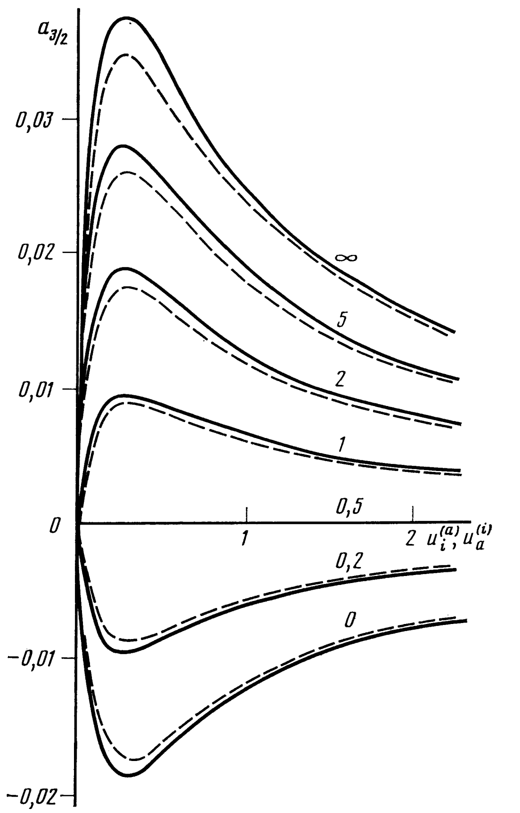

FIG. 2. Ion-drift induced alignment of the $J = \frac{3}{2}$ levels of the ^2P doublets of atoms (solid lines) and ions (dashed lines) as a function of the parameters $u_i^{(a)}$ and $u_a^{(i)}$, given by (24), for different values of $x = F_{1/2}/F_{3/2}$ (shown against the curves) corresponding to $J = \frac{1}{2}$ and $J = \frac{3}{2}$. The anisotropy parameter of the relative velocity distribution of the atoms and ions is assumed to be $\lambda = 1$.

with anisotropy parameter $\lambda = 1$ and different values of the ratio x of the level population efficiency ($x = F_{1/2}/F_{3/2}$). The alignment $a_{3/2}$ of the $^2P_{3/2}$ levels of neutral excited atoms and excited ions is shown in Fig. 2 for different values of the dimensionless parameters $u_i^{(a)}$ (neutrals) and $u_a^{(i)}$ (ions), where

$$u_i^{(a)} = N_i\beta_a/\Gamma_0, \quad u_a^{(i)} = N_a\beta_i/\Gamma_0, \tag{24}$$

for different values of x. In accordance with (19), the alignment $a_{3/2}$ is positive for $x > 0.5$ and negative for $x < 0.5$. It vanishes altogether for $x = 0.5$. The last case is, however, not typical of low-pressure plasmas where, as a rule, the relative population of the multiplet components is appreciably different from the statistical weights.[32] The dependence of the alignment of neutral atoms on $u_i^{(a)}$ and of ions on $u_a^{(i)}$ is very similar. The alignment $a_{3/2}$ first increases with increasing $u_i^{(a)}$ (or $u_a^{(i)}$), but then reaches a maximum near $u_i^{(a)} \approx 0.3$ and $u_a^{(i)} \approx 0.3$. It then decreases monotonically with increasing $u_i^{(a)}$ (or $u_a^{(i)}$). This behavior of the alignment signal is due to the fact that the depolarization rate constant $\langle v\sigma_g \rangle$ exceeds the alignment rate constant $\langle v\sigma_b \rangle$ for both neutrals and ions (Fig. 1). It follows that, for each ^2P doublet of a particular atom (or the corresponding ion), the alignment signal has an optimum ion density N_i (or, correspondingly, neutral-atom density N_a) in the discharge plasma. These optimum densities are determined by the condition that the values of the parameters $u_i^{(a)}$ and $u_a^{(i)}$ should lie in the region near the maxima of the alignment curves in Fig. 2.

Tables III and IV list our calculations of the alignment signals for the n^2P doublets of neutral Li and Na atoms and Mg^+ ions. These results not only clearly demonstrate the existence of optimum ion and neutral-atom densities (for the alignment of neutral atoms and ions, respectively), but can also be seen to show the variation in the alignment signal with the degree of excitation of the atom or ion and the principal quantum number n of the n^2P doublet. As n increases, the numerators of the fractions in (24) are found to increase because of the monotonic rise in the quadrupole moments of the atoms (ions) and the anisotropy of the dispersion interaction constants. The rates of radiative damping in the denominators decrease simultaneously with increasing n. Hence, as the principal quantum number n increases, the parameters $u_i^{(a)}$ and $u_a^{(i)}$ are found to increase rapidly and very rapidly passthrough the region corresponding to the maximum alignment signal. The consequence of this is that the ion-drift induced alignment of the neutral atoms, and of the drifting ions themselves, is selective in the degree of excitation of atoms and ions: there is an optimum value of n for which the maximum alignment is induced in the n^2P doublets. It is clear from Tables III and IV that the only group of states that is appreciably aligned is that for which the values of n^2P are close to the optimum

The estimated degree of linear polarization of light

$$(I_x - I_z)/(I_x + I_z) = 3a_{3/2}/(4n_{3/2} - a_{3/2}), \tag{25}$$

TABLE III. Alignment $a_{3/2}$ (in %) for the n^2P states of Li and Na atoms due to the drift of singly-charged ions as a function of the ion density N_i in units of 10^9 cm^{-3}. Anisotropy parameter $\lambda = 1$, efficiency of population of doublet levels $x = 1$.

State of atom	Li			Na		
	$N_i = 1$	$N_i = 10$	$N_i = 100$	$N_i = 1$	$N_i = 10$	$N_i = 100$
2^2P	≪0,1	≪0,1	<0,1	–	–	–
3^2P	≪0,1	≪0,1	0,11	≪0,1	≪0,1	<0,1
4^2P	≪0,1	<0,1	0,72	≪0,1	<0,1	0,1
5^2P	<0,1	0,43	0,92	≪0,1	0,10	0,72
6^2P	0,20	0,81	0,56	<0,1	0,55	0,82
7^2P	0,42	0,92	0,24	0,37	0,92	0,38
8^2P	0,63	0,77	0,11	0,65	0,76	0,11
9^2P	0,90	0,43	<0,1	0,91	0,41	<0,1
10^2P	0,93	0,27	≪0,1	0,88	0,21	≪0,1

TABLE IV. Alignment $a_{3/2}$ (in %) of the n^2P states of Mg^+ ions induced by ion drift in an inert gas. Calculations were performed for different pressures (in mm Hg) of the inert gas and anisotropy parameter $\lambda = 1$; population efficiency ratio $x = 1$, temperatures $T = T_i = T_a = 650$ K.

State of ion Mg^+	Pressure of inert gas	Ar	Kr	Xe
6^2P	0,001	0,13	0,13	0,16
	0,01	0,78	0,80	0,82
	0,1	0,51	0,46	0,42
7^2P	0,001	0,27	0,32	0,35
	0,01	0,87	0,87	0,86
	0,1	0,27	0,25	0,21
8^2P	0,001	0,52	0,53	0,60
	0,01	0,76	0,73	0,68
	0,1	0,10	<0,1	<0,1
9^2P	0,001	0,72	0,60	0,86
	0,01	0,57	0,67	0,48
	0,1	<0,1	<0,1	<0,1
10^2P	0,001	0,80	0,80	0,87
	0,01	0,40	0,37	0,32
	0,1	<0,1	<0,1	<0,1

emitted as a result of the $^2P_{3/2} - {}^2S_{1/2}$, transition is found to agree to within an order of magnitude with the measured polarization of spectral lines emitted by atoms in gas discharges,[33] which is a qualitative confirmation of the efficacy of alignment by ion drift.

Finally, we note that circularly polarized light may appear in plasma emission. The alignment induced by ion drift

$$\rho_0{}^2 = \langle 3J_z{}^2 - J^2 \rangle / 2J(2J-1) \tag{26}$$

is quantized in the direction of the z axis (local ion drift direction), which is at an angle to the tube axis in the wall region of the discharge because of ambipolar diffusion. The magnetic field due to the discharge current has a component perpendicular to the local z axis in this kind of region. The magnetic field transforms the alignment ρ_0^2 into an alignment of the form $\rho_{\pm 1}^2$ which, because of the mechanism described in the second section of the present paper,[19,20] is in turn transformed by ion drift into orientation of the form $\rho_{\pm 1}^1$. Although the orientation is small (for $\lambda = 1$ it is of the order of $\sim 10^{-4}$), the associated partial circular polarization of the emitted light can be detected by modern techniques and can be used as a way of examining the distribution of the magnetic field in plasmas.

It is important to note that all the effects associated with the ordering of the angular momenta in plasmas are found to increase rapidly with the ion-velocity anisotropy parameter λ. Our estimates were made for the moderately anisotropic case ($\lambda = 1$) and must be replaced by higher values for higher λ. The orientation $\rho_{\pm 1}^1$ and the associated circular polarization of light increase particularly rapidly with increasing λ. The corresponding calculations are readily performed with the aid of (13) by analogy with the preliminary studies of these effects made in Ref. 6.

All the predictions of our theory are amenable to experimental verification. Hence, composite studies of alignment and orientation signals recorded for atomic and ionic lines can yield valuable information about the properties of low-temperature plasmas (anisotropy in ion velocity distribution, nonuniform population of multiplet sublevels, and magnetic-field strength) in macroscopically small volumes of low-pressure plasma, and in elementary processes occurring during slow collisions between atoms and ions.

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