Effect of impurities on absorption in semiconductors and on generation by semiconductor lasers

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The effect of impurity scattering of electrons on the operation of semiconductor lasers and on absorption in semiconductors is considered. It is shown that the limiting field of a neodymium laser is increased by $(\tau_{ph}/\tau_{im})^{1/2}$ times (τ_{ph} and τ_{im} are the times of electron scattering by phonons and impurities) compared with pure lasers, and that under certain conditions the dependence of the field on the pump current has a maximum. Combined scattering leads to a broadening $\sim(1\tau_{im}\tau_{ph})^{1/2}$ of the absorption curve. It is found that impurity scattering alone does not broaden the absorption line.

INTRODUCTION

A kinetic theory of semiconductor lasers, developed in Refs. 1 and 2, takes correct account of the microscopic character of the electromagnetic-field radiation and of the change of the distribution function and of the electron spectrum in a strong field. The most important prediction of the theory is that a limiting field exists for a single-mode semiconductor laser. We recall that it follows from the customarily employed rate equations[3] (linear laser theory) that one mode increases without limit, contradicting the experimental data.

Let us elucidate the physical meaning of this phenomenon. The elementary act of the interaction of a semiconductor with a field is emission (or absorption) of a photon with transition of the electron from the conduction to the valence band. The fundamental circumstances is that photon emission is possible only in the presence of simultaneous electron scattering (e.g., with emission or absorption of a phonon), which upsets the coherence of the electron interaction with the field. In the contrary case there is no radiation, since the electron will execute periodic transitions between the bands, at a frequency $\lambda = \mathbf{d}\cdot\mathbf{E}_0$, where $\mathbf{d}$ is the dipole moment of the interband transition and $\mathbf{E}_0$ is the field amplitude. Thus, the photon emission rate Q should depend on the reciprocal coherence-loss time (e.g., the phonon-emission time τ_{ph}), i.e., $Q \propto 1/\tau_{ph}$.

Equating the emission rate N/τ_0 to the rate of departure of N photons from the cavity (τ_0 is the photon lifetime in the cavity), we can find the limiting field[1,2]

$$\lambda_0 = \beta/2\tau_{ph},$$

where β is a dimensionless parameter defined below; $\beta \ll \omega_{ph}\tau_{ph}$; $\omega_{ph} = 2p_0 s \gg 1/\tau_{ph}$, p_0 is the Fermi momentum, and s is the speed of sound. The field λ_0 corresponds to the maximum value of the reciprocal electron-phonon relaxation time $1/\tau_{ph}$ reached at a definite pump value.

With further increase of the pump current the one-mode lasing power increases. The limiting-field effect (i.e., the saturation of the watt-ampere characteristic of the laser) was apparently observed in a number of studies (see, e.g., Refs. 3 and 4) and is the possible cause of multimode lasing. The expression given above for the limiting field is valid only for pure semiconductors.

It is natural to expect scattering by impurities also to lead to coherence loss. Indeed, it is known from experiment that doping exerts a substantial influence on the interband absorption and on the parameters of the laser.

This paper deals with the action of impurity scattering of electrons on absorption in semiconductors and on the operation of the laser. It is shown that in the presence of combined electron-phonon and impurity scattering and at $\tau_{im} \ll \tau_{ph}$, where τ_{im} is the impurity-scattering time, the limiting field becomes equal to

$$\lambda_{0i} = \beta(2\tau_{ph}\tau_{im})^{-1/2},$$

i.e., it increases by a factor $(2\tau_{ph}/\tau_{im})^{1/2}$. In addition, at small the dependence of the field on the pump becomes nonmonotonic, viz., it has a maximum.

Combined scattering broadens also the absorption curve, but to a lesser degree than is customarily assumed. It turned out that pure impurity scattering does not broaden the absorption line. For this reason, any field is strong.

The influence of impurity scattering on the limiting field was considered earlier[5] for the case of weak doping ($\tau_{im} \gg \tau_{ph}$), so that the corrections naturally turned out to be small.

§1. GENERAL EQUATIONS

We consider a straight-band semiconductor with equal electron and electron masses, in a field

$$\mathbf{E}(t) = \mathbf{E}_0 \cos \Omega t$$

having a frequency Ω somewhat higher than the band gap E_g. It is known that the field causes interband electron transitions that lead to absorption (or emission).

1. 1. Absorption

Defining the absorption Q as the time average of the interband current $\mathbf{J}$ and of the field $\mathbf{E}$ (divided by $\hbar\Omega$), we readily obtain[2,6]

$$Q = \frac{\overline{\mathbf{JE}}}{2\Omega} = -i\sum_{\mathbf{p}}\lambda[\rho(\mathbf{p}) - \rho^*(\mathbf{p})], \quad \hbar = c = 1. \tag{1}$$

Here $\rho(\mathbf{p}) = \langle a_p{}^+ b_p{}^+ \rangle$ is a nondiagonal density matrix that describes electron transitions between the valence and conduction bands; a_p^+ and b_p^+ are the electron and hole cre-

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ation operators. We have neglected in (1) the photon wave vector; this is legitimate in the low-Q regime ($\lambda \ll \omega_{ph}$),[1] which is assumed in the present paper.

We note that here Q is the number of photons absorbed (emitted) per unit time.

For inverted population $Q<0$, so that we obtain the equation for the field in the laser by equating to $-Q$ the number N of photons that leave the cavity, i.e.,

$$-Q=N/\tau_0. \tag{2}$$

1.2. Equation for the density matrix

The equation for the density matrix $\rho(\mathbf{p})$ can be obtained by the Bogolyubov method[6,7]:

$$\left(i\frac{\partial}{\partial t}+2\xi_p\right)\rho(\mathbf{p})=\lambda(2f_p-1)+i\left(\frac{\partial\rho}{\partial t}\right)_{st}. \tag{3}$$

Here $\xi_p=\rho^2/2m-\mu$, $\mu=(\Omega-E_g)/2$, $f_p=\langle a_p{}^+ a_p\rangle$ is the distribution function of the electrons in the conduction band; $(\partial\rho/\partial t)_{st}$ describes the changes of ρ on account of dissipative scattering processes. We consider a model in which the electrons are assumed to interact with the phonons and the impurities. In this case we have

$$\left(\frac{\partial\rho}{\partial t}\right)_{st}=\left(\frac{\partial\rho}{\partial t}\right)_{ph}+\left(\frac{\partial\rho}{\partial t}\right)_{im}. \tag{4}$$

The electron-phonon contribution can be represented in the form

$$\left(\frac{\partial\rho}{\partial t}\right)_{ph}=-\gamma_{ph}\rho(\mathbf{p})+\sum_{\mathbf{p}'}K(\mathbf{p},\mathbf{p}')\rho(\mathbf{p}'), \tag{5}$$

$$\gamma_{ph}=\sum_{\mathbf{p}'}K(\mathbf{p},\mathbf{p}'),$$

$$K(\mathbf{p},\mathbf{p}')=2\pi g^2(\mathbf{p}-\mathbf{p}')[(1-f_{\mathbf{p}'})\delta(\xi_{\mathbf{p}}+\xi_{\mathbf{p}'}+\omega_{\mathbf{p}-\mathbf{p}'})$$

$$+f_{\mathbf{p}'}\delta(\xi_{\mathbf{p}}+\xi_{\mathbf{p}'}-\omega_{\mathbf{p}-\mathbf{p}'})], \tag{6}$$

where $g^2(\mathbf{p})$ is the matrix element of the electron-phonon scattering, $\omega_q=qs$ is the phonon frequency, and the number N_{ph} of the phonons is customarily assumed to be zero.

It was assumed in the derivation that the field is weak (an exact criterion will be given below) and that the electron spectrum was renormalized.

Impurity scattering leads to the following change of the matrix ρ:

$$\left(\frac{\partial\rho}{\partial t}\right)_{im}=-\gamma_{im}\rho(\mathbf{p})+2\pi\sum_{\mathbf{q}}|V_{\mathbf{q}}|^2\rho(\mathbf{p}-q)\delta(\xi_{\mathbf{p}}+\xi_{\mathbf{p}-\mathbf{q}}), \tag{7}$$

$$\gamma_{im}=2\pi\sum_{\mathbf{q}}|V_{\mathbf{q}}|^2\delta(\xi_{\mathbf{p}}+\xi_{\mathbf{p}-\mathbf{q}}), \tag{8}$$

where V_q is the matrix element of the impurity scattering. We note that (7) and (8) can be obtained from (5) and (6) by putting $\omega_q=0$ and letting $g^2\to v_q^2$.

1.3. Equation for electron distribution function

Equation (3) must be supplemented with an equation for the distribution function, which can be obtained by a similar method:

$$\frac{\partial f_{\mathbf{p}}}{\partial t}=-i\lambda(\rho-\rho^*)+\left(\frac{\partial f}{\partial t}\right)_{st}, \tag{9}$$

$$\left(\frac{\partial f}{\partial t}\right)_{st}=\left(\frac{\partial f}{\partial t}\right)_{ph}+\left(\frac{\partial f}{\partial t}\right)_{im}. \tag{10}$$

The collision integrals, which describe the collisions of the electrons with the phonons and with the impurities, are here of the form

$$\left(\frac{\partial f}{\partial t}\right)_{ph}=-\sum_{\mathbf{p}'}g^2(\mathbf{p}-\mathbf{p}')[f_{\mathbf{p}}(1-f_{\mathbf{p}'})\delta(\xi_{\mathbf{p}}-\xi_{\mathbf{p}'}+\omega_{\mathbf{p}-\mathbf{p}'})$$

$$-(1-f_{\mathbf{p}})f_{\mathbf{p}'}\delta(\xi_{\mathbf{p}}-\xi_{\mathbf{p}'}-\omega_{\mathbf{p}-\mathbf{p}'})], \tag{11}$$

$$\left(\frac{\partial f}{\partial t}\right)_{im}=-\sum_{\mathbf{p}'}|V(\mathbf{p}-\mathbf{p}')|^2[f_{\mathbf{p}}-f_{\mathbf{p}'}]\delta(\xi_{\mathbf{p}}-\xi_{\mathbf{p}'}). \tag{12}$$

§2. DENSITY MATRIX AND FIELD ABSORPTION

2.1 Electron scattering and field absorption

It is known that stationary absorption is possible only if the electron undergoes scattering (e.g., by phonons or impurities) that upsets the coherence of the interaction with the field. In fact, putting in (3) $(\partial\rho/\partial t)_{st}=0$, we find that

$$\rho=\rho^*=\lambda(2f-1)/2\xi,$$

and hence $Q\equiv 0$.

It turns out that the character of the absorption depends, generally speaking, on the scattering mechanism. We consider therefore in succession absorption in electron-phonon, impurity, and combined interaction.

2.2. Electron-phonon scattering

We obtain the stationary matrix ρ from Eq. (3). The integral term in (5) is usually neglected. The remaining term describes then damping of the matrix, with a time γ_{ph}^{-1}.

In this approximation the matrix $\rho(\xi)$ takes the form

$$\rho(\xi)=\frac{\lambda(2f-1)}{2\xi+i\gamma_{ph}}=\lambda(2f-1)\frac{2\xi-i\gamma_{ph}}{4\xi^2+\gamma_{ph}{}^2} \tag{13}$$

and the absorption is accordingly equal to

$$Q=\alpha(\Omega)N,\quad \alpha(\Omega)=\frac{2\alpha_0}{\pi}\int_{-\infty}^{\infty}\frac{d\xi[1-2f(\xi)]\gamma_{ph}}{4\xi^2+\gamma_{ph}{}^2}, \tag{14}$$

where $\alpha(\Omega)$ and α_0 are the semiconductor absorption coefficient in a nonequilibrium and in the ground (i.e., in a very weak field at $T=0$) states.

It is easy to verify that if we let $\gamma_{ph}\to 0$ and assume slow variation of $f(\xi)$, we get for α the usual perturbation-theory value that is independent of the scattering mechanism: $\alpha=\alpha_0(1-2f(\xi=0))$.

Let us estimate the order of magnitude of the integral term discarded in (5). Recognizing that at $\gamma_{ph}/\omega_{ph}\ll 1$ the function $\rho(\xi)$ differs from zero near $\xi=0$ in a narrow energy region $\sim\gamma_{ph}$, we find

$$\sum_{\mathbf{p}}K(\mathbf{p},\mathbf{p}')\rho(p')\sim\rho(0)\frac{\gamma_{ph}{}^2}{\omega_{ph}},$$

i.e., at $\gamma_{ph}/\omega_{ph}\ll 1$ it is small compared with the term $\gamma_{ph}\rho$ that describes the damping.

2.3. Impurity scattering

Results of a different kind are obtained for impurity scattering. The fundamental difference from electron-phonon scattering is the impossibility of neglecting in $(\partial p/\partial t)_{im}$ the ρ terms with arrival for owing to the inelasticity of the scattering ($\omega_q = 0$) the arrival makes a contribution comparable with the departure.

In the isotropic case ρ depends only on ξ, so that we obtain from (7)

$$\left(\frac{\partial \rho}{\partial t}\right)_{im} = -\gamma_{im}[\rho(\xi)-\rho(-\xi)], \tag{15}$$

i.e., the even part of the matrix $\rho(\xi)$ is not subject to damping on account of impurity scattering. If we confine ourselves only to impurity scattering, the stationary equation for ρ takes the form

$$(\xi+i\gamma_{im})\rho(\xi) = \lambda(2f-1)+i\gamma_{im}\rho(-\xi). \tag{16}$$

Assuming for simplicity a weak dependence of γ_{im} and of f on ξ, we get from (16)

$$\rho(\xi) = \lambda(2f-1)\,(\xi-i2\gamma_{im})/\xi^2. \tag{17}$$

It can be ssen from (17) that $\rho(\xi)$ tends to infinity at $\xi = 0$ and differs greatly from ρ as described by (13).

Substitution of the imaginary part of $\rho(\xi)$ in expression (1) for the absorption would lead to divergence of the integral at $\xi = 0$. The reason is that the imaginary (even) part of $\rho(\xi)$ is not subject to damping. Thus, despite the widespread intuitive notion, pure impurity scattering does not broaden the absorption line.

To avoid divergences in (17) we must forego the perturbation theory in terms of the field, which was used to derive Eqs. (3), (5), and (7). The criterion of validity of (5) for electron-phonon interaction is the condition[2]

$$\lambda \ll \omega_{ph}.$$

Since $\omega_q \equiv 0$ for impurity elastic scattering, perturbation theory is not valid in any arbitrarily weak field. It can be shown, by using the quasiparticle approach,[1] that an exact treatment leads to replacement of ξ^2 in the denominator of (17) by $\xi^2 + \lambda^2$.

2.4. Combined scattering

We take into account in (3) both types of scattering:

$$[2\xi+i(\gamma_{ph}+\gamma_{im})]\rho(\xi) = \lambda(2f-1)+i\gamma_{im}\rho(-\xi). \tag{18}$$

This yields for $\rho(\xi)$

$$\rho(\xi) = \lambda(2f-1)\,\frac{2\xi-i(\gamma_{ph}+2\gamma_{im})}{4\xi^2+\gamma_{ph}(\gamma_{ph}+2\gamma_{im})}. \tag{19}$$

The "width" of the function $\rho(\xi)$ is

$$\gamma = \gamma_{ph}(1+2\gamma_{im}/\gamma_{ph})^{1/2}$$

or for $\gamma_{\text{im}} \gg \gamma_{\text{ph}}$

$$\gamma \approx (2\gamma_{im}\gamma_{ph})^{1/2};$$

it increases like $\gamma_{\text{im}}^{1/2}$ with impurity density, i.e., impurity scattering leads to broadening only when combined with electron-phonon scattering.

The question arises: how large can the broadening γ become as $\gamma_{\text{im}} \to \infty$? It can be shown that γ has an upper limit $\sim\omega_{\text{ph}}$. Indeed, at this value of γ the width of the function $\rho(\xi)$ becomes $\sim\omega_{\text{ph}}$, and we can no longer neglect the integral term (arrival) in $(\partial\rho/\partial t)_{\text{ph}}$. If we do take it into account we obtain the aforementioned result.

Substituting (19) in (1), we get the absorption coefficient

$$\alpha(\Omega) = \alpha_0 \frac{2}{\pi}\int_{-\infty}^{\infty} \frac{dx\,\bar\gamma\,(1-2f(x))}{x^2+\bar\gamma\tilde\gamma_{ph}}, \tag{20}$$

where

$$\bar\gamma = \frac{\gamma_{ph}+2\gamma_{im}}{2\omega_{ph}}, \quad \tilde\gamma_{ph} = \gamma_{ph}/2\omega_{ph}, \quad x = \xi/\omega_{ph}.$$

We note the substantial difference between (20) and (14) for a pure semiconductor. In the latter, the area under the function $\tilde\gamma/(x^2+\tilde\gamma^2)$, which describes the shape of the interaction region, does not depend on $\tilde\gamma$ and remains constant. On the contrary, in (20) the area under the function $\bar\gamma/(x^2+\bar\gamma\tilde\gamma)$ depends on γ_{im} and γ_{ph}:

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{dx\,\bar\gamma}{x^2+\bar\gamma\tilde\gamma_{ph}} = \left(\frac{\bar\gamma}{\tilde\gamma_{ph}}\right)^{1/2} = \left(1+\frac{2\gamma_{im}}{\gamma_{ph}}\right)^{1/2}. \tag{21}$$

This means that doping increases the absorption coefficient. We shall show that this circumstance influences strongly the laser operation.

§3. GENERATION IN A DOPED SEMICONDUCTOR LASER

3.1. System of equations for laser

In the stationary case the laser is described by Eq. (2), which takes on substitution of Q the form

$$\frac{\bar\gamma}{\bar\tau_0} = \int_{-\infty}^{\infty}\frac{dx\,\bar\gamma[2f(x)-1]}{x^2+\bar\gamma\tilde\gamma_{ph}}, \quad \bar\tau_0 = \tau_0\alpha_0\bar\gamma, \tag{22}$$

and by the kinetic equation for f, obtained from (9) with account taken of (19):

$$\left(\frac{\partial f}{\partial t}\right) = -\frac{\bar\lambda^2(2f-1)}{x^2+\bar\gamma\tilde\gamma_{ph}} + \left(\frac{\partial f}{\partial t}\right)_{ph} + \left(\frac{\partial f}{\partial t}\right)_{im} - \frac{f}{\tau_r} + J_p(1-f), \tag{23}$$

where $\bar\lambda^2 = \lambda^2\bar\gamma\tau_{\text{ph}}/\omega_{\text{ph}}\tau_r$ is the recombination time, and J_p is the pump source. At $\gamma_{\text{im}} = 0$ the system (22) and (23) goes over into the equations investigated in Ref. 2.

Impurity scattering "broadens" the region of the field interaction with the electrons $[\tilde\gamma_{\text{ph}} \to (\tilde\gamma_{\text{ph}}\bar\gamma)^{1/2}]$. This result, confirms the qualitative arguments advanced in a number of papers, favoring "the homogenizing action of the doping." At the same time, it is much less than expected, viz., we obtain $(\gamma_{\text{im}}\gamma_{\text{ph}})^{1/2}$ in lieu of γ_{im}.

3.2 Threshold conditions

At the lasing threshold ($\lambda = 0$) Eq. (22), with account taken of the smoothness of the threshold distribution function $f^{(0)}(\xi)$, leads to the condition

$$\beta[2f^{(0)}(0)-1]\left(1+\frac{2\gamma_{im}}{\gamma_{ph}}\right)^{1/2} = 1, \quad \beta = \pi\tau_0\alpha_0. \tag{24}$$

If (24) is compared with the condition for a pure semiconductor

$$\beta[2f^{(0)}(0)-1] = 1, \quad \beta > 1,$$

it can be seen that doping leads to another factor $(1 + 2\gamma_{\text{im}}/$

$\gamma_{ph})^{1/2}$, which generally speaking lowers the threshold. Indeed, β can now be less than unity ($\beta \sim \gamma_{ph}/\gamma_{im} \ll 1$), or at a given β we need a smaller value of $f^{(0)}(0)$ and a correspondingly smaller threshold current.

The physical reason is the following. Impurity scattering increases the interaction-coherence-loss probability more strongly ($\propto \gamma_{im}$) than broadening of the transition function ($\propto \gamma_{im}^{1/2}$). The latter is connected with the absence of damping of the imaginary part of the density matrix on account of pure impurity scattering.

3.3 Electron distribution

The kinetic equation (23) for a pure semiconductor was investigated in detail in Ref. 2. It was shown that at large pump currents and accordingly in strong fields the distribution differs substantially from that at the threshold $f^{(0)}$ and behaves as follows. Near the energy with $x = 0$ there appears a dip ("hole burning" due to saturation) of width $\tilde{\gamma}_{ph}$, and the right-hand limit ($x < 0$) shifts towards positive energies. A hump appears thus in the distribution at $x > 0$, and its area

$$\tilde{a} = \int_0^1 f(x)\,dx$$

increases with increasing pump current.

The situation is similar in combined scattering, only the width $\tilde{\gamma}_{ph}$ must be replaced by $\tilde{\gamma} = (\bar{\gamma}\tilde{\gamma}_{ph})^{1/2}$. At sufficiently strong currents, when $\tilde{a} \gg \tilde{\gamma}$, Eq. (23) can be simplified in the energy region $|x| < \tilde{\gamma}$ and an analytic solution can be obtained. Indeed, it is permissible under these conditions to neglect in $(\partial f/\partial t)_{ph}$ the second term (departure), which is of the order of $\tilde{\gamma}$. In addition we leave out the terms that describe recombination (since $\tau_{ph} \ll \tau_r$) and the pump that acts in the region of large x. The last terms can easily be calculated exactly, but they do not change the qualitative picture.

As a result we arrive at an equation for $f(x)$:

$$[1-f(x)]\tilde{a} = \frac{\bar{\lambda}^2[2f(x)-1]}{x^2+\tilde{\gamma}_{ph}\bar{\gamma}}, \tag{25}$$

the solution of which is

$$f(x) = \frac{1}{2}\left(1 + \frac{x^2+\bar{\gamma}\tilde{\gamma}_{ph}}{x^2+\bar{\gamma}\tilde{\gamma}_{ph}+2\bar{\lambda}^2/\tilde{a}}\right). \tag{26}$$

$f(x)$ goes over at $\gamma_{im} = 0$ into the expression obtained in Ref. 2. It can be easily seen that $f(x)$ at $x = 0$ a dip whose width is $(\bar{\gamma}\tilde{\gamma}_{ph} + 2\bar{\lambda}^2/\tilde{a})$. In a weak field, when

$$2\bar{\lambda}^2/\tilde{a} \ll \bar{\gamma}\tilde{\gamma}_{ph} \quad \text{or} \quad \lambda^2 \ll \gamma_{ph}/\tau_{ph}, \tag{27}$$

the dip width turns out to equal $(\bar{\gamma}\tilde{\gamma}_{ph})^{1/2}$, and to increase with increasing γ_{im} as $(\gamma_{im}\gamma_{ph})^{1/2}$ when $\gamma_{im} \gg \gamma_{ph}$.

It must be emphasized that the criterion (27) does not contain the impurity densities.

If the field becomes strong, the width of the dip begins to depend on the field and is equal to

$$(\bar{\lambda}^2/\tilde{a})^{1/2} = \lambda(\tau_{ph}/2\tau_{im}+\tau_{ph})^{1/2}.$$

Thus, impurity scattering increases the width of the dip in a strong field by a factor $(\tau_{ph}\tau_{im})^{1/2}$ compared with the width ($\sim\lambda$) in a pure semiconductor.

3.4. Field generated by a semiconductor laser

Let us find the laser-generated field as a function of $\tilde{a}$ and of other parameters. Substituting f from (26) in Eq. (22) for the field, we obtain the sought dependence

$$\lambda^2 = \frac{\tilde{a}\beta^2}{2}\,\frac{\gamma_{ph}+2\gamma_{im}}{\tau_{ph}}\left(1 - \frac{\tilde{\gamma}_{ph}}{\bar{\gamma}\beta^2}\right). \tag{28}$$

At $\gamma_{im} = 0$ we get from (28) an expression first obtained for λ_0 in Refs. 1 and 2 by another method:

$$\lambda = \tilde{a}(\beta^2-1)^{1/2}/2\tau_{ph}. \tag{29}$$

If $\beta \gg 1$ and $\tilde{a} = 1$, the field reaches the limiting value cited in the Introduction:

$$\lambda_0 = \beta/2\tau_{ph}. \tag{30}$$

We obtain the limiting field in a doped semiconductor for the case $\beta \gg 1$ and $\tau_{im} \ll \tau_{ph}$ from the general expression (28):

$$\lambda_{0i} = \beta/(2\tau_{im}\tau_{ph})^{1/2}. \tag{31}$$

Thus, doping increases the limiting field $\lambda_{0i} \propto 1/\sqrt{\tau_{im}}$ (and correspondingly the power $P_0 \propto 1/\tau_{im}$).

This important result explains why a stronger field is obtained in one mode in doped semiconductors. It is confirmed also in studies devoted exclusively to this question. It was observed in Ref. 8, for example, that the one-mode regime exists right up to $J/J_{th} = 2$ in lasers doped to a density 10^{19} cm^{-3}. In weakly doped lasers ($7 \cdot 10^{17}$ cm^{-3}), at the same time, a multimode regime is observed at an insignificant excess over the threshold current J_{th}.

A curious dependence of λ^2 on $\tilde{a}$ (i.e., on the pump current) is predicted by Eq. (28) at $\beta < 1$ and $\tau_{im} \ll \tau_{ph}$:

$$\lambda^2 = \frac{\tilde{a}\beta^2}{\tau_{ph}\tau_{im}}\left(1 - \frac{\tilde{a}\tau_{im}}{\beta^2\tau_{ph}}\right).$$

It is easily seen that when the condition

$$\beta < \frac{2\tau_{im}}{\tau_{ph}} \ll 1, \quad \tilde{a} < 1 \tag{32}$$

is satisfied the $\lambda^2(\tilde{a})$ dependence becomes nonmonotonic and reaches a maximum at

$$\tilde{a}_{max} = \frac{\beta^2\tau_{ph}}{2\tau_{im}}, \quad \lambda^2(\tilde{a}_{max}) = \frac{\beta^2}{8\tau_{im}^2}.$$

The physical reason is that a growth of $\gamma_{ph} = \tilde{a}/\tau_{ph}$ means an effective increase of the threshold current (see above). At $\tilde{a} > \tilde{a}_{max}$ the influence of γ_{ph} becomes predominant and the field decreases with increasing $\tilde{a}$. This behavior was observed in experiment many times. It is usually attributed to overheating as the threshold current increases. There are, however, experiments (see, e.g., Ref. 4) that seem to demonstrate realization of the described mechanism in pure form, since there is no overheating in them and the nonmonoticity is observed at small β.

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