Deep inelastic scattering by a polarized target in quantum chromodynamics

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The basis for fermion and boson twist-3 operators in quantum chromodynamics is constructed for deep inelastic scattering of electrons by a polarized target. In the calculations of the matrix elements for the operators, the quarks and gluons can be considered to lie on the mass shell, so that the helicity-amplitude technique can be used. A theoretical description of the process within the framework of the parton model calls for introducing a correlation density matrix expressed in terms of the product of wave functions with different numbers of partons. Evolution equations for the correlation density matrix are derived in the nonsinglet and singlet cases. The anomalous-dimensionalities matrix is found and its rank is shown to increase with the angular-momentum number.

1. INTRODUCTION

The study of deep inelastic processes involving polarized particles is of considerable interest as one of the methods of verifying quantum chromodynamics, especially in connection with the possibility of separating the contributions of the higher-twist operators in the expansion of the product of currents on the light cone.[1,2] The total probabilities of the processes, connected with the lowest twist-2 operator, can be described in the leading logarithmic approximation (LLA) in terms of the usual parton model [3,4]. It suffices here to introduce the inclusive probabilities $D_h^j(x)$ of observing partons in a hadron and $D_j^h(x)$ of observing hadrons in a parton (x is the fraction of the energy of the particle in question relative to the initial energy); these probabilities have also a dependence on $\ln(-q^2)$ (q is the momentum transfer) and this dependence is given by the evolution equations.[3,4] On the other hand, when considering quantities connected with higher-twist operators, such a description becomes impossible, since interference exists here between states with different numbers of partons. In particular, the structure functions of the deep inelastic scattering of a polarized electron by a polarized proton can be expressed in terms the matrix elements of twist-3 operators between the hadron states whose numbers of partons differ by unity. It becomes necessary then to introduce certain three-particle operators which are generalizations of the usual inclusive probabilities.[5–7] The present paper is devoted to consideration of this process in chromodynamics. In our earlier note[7] are given results for the nonsinglet (in flavor in the t channel) part of the amplitude. Here we take into account also the singlet contribution connected with the possible pure gluon states in the t channel. In the derivation of the evolution equations that determine the dependence of the quantities considered on $\ln(-q^2)$ we use a helicity-amplitude technique that is simpler and more effective than the one previously employed. The use of this technique is made possible by a felicitous choice of an independent set of operators that become intermixed in the course of the evolution. For the operators we need, the partons can be regarded as being on the mass shell. A similar criterion for the choice of an independent system of operators was proposed in Ref. 8 for the problem of finding the power-law corrections to deep inelastic scattering of electrons by an unpolarized target. For our calculation method, however, another system of operators, which differs also from that used in Ref. 9, is found to be more convenient.

The differential cross section for deep inelastic scattering of an electron by a nucleon with momentum p is determined by the imaginary part $\pi W_{\mu\nu}$ of the amplitude $T_{\mu\nu}$ of scattering a virtual γ quantum with momentum q through a zero angle. If the target is polarized, the tensor $W_{\mu\nu}$ has an antisymmetric increment[1]:

$$W_{\mu\nu}{}^A=\frac{1}{2}(W_{\mu\nu}-W_{\nu\mu})=\frac{1}{\pi}\operatorname{Im}T_{\mu\nu}{}^A$$
$$=-\frac{2m_h}{pq}i\varepsilon_{\mu\nu\lambda\sigma}q^\lambda[a^\sigma g_1(x,Q^2)+a_\perp{}^\sigma g_2(x,Q^2)],\tag{1}$$

$$a_\perp=a-aqp/pq,$$

where a^σ is the four-dimensional nucleon-polarization vector and coincides in the rest system with the spin direction. The structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$ depend on the Bjorken variable $x=Q^2/2pq$, where $Q^2=-q^2$. For the tensor $T_{\mu\nu}^A$ in the leading logarithmic approximation, the use of the Wilson operator expansion for the T-product of electromagnetic currents $j_\mu(x)$ and $j_\nu(0)$ leads to the equation[1,2]

$$T_{\mu\nu}{}^A=-i\varepsilon_{\mu\nu\rho\sigma}q^\rho\frac{1}{Q^2}\sum_{n=0}^{\infty}[1+(-1)^n]\left(\frac{2}{Q^2}\right)^n q_{\mu_1}\dots q_{\mu_n}$$
$$\times\left[\langle h|R_{1\sigma\mu_1\dots\mu_n}|h\rangle+\frac{2n}{n+1}\langle h|R_{2\sigma\mu_1\dots\mu_n}|h\rangle\right].\tag{1a}$$

The operators R_1 and R_2 have respectively twist 2 and 3

$$R_{1\sigma\mu_1\dots\mu_n}=i^n\mathop{\mathrm{S}}_{\sigma\mu_1\dots\mu_n}\bar\psi\gamma_5\gamma_\sigma D_{\mu_1}\dots D_{\mu_n}\psi,$$

$$R_{2\sigma\mu_1\dots\mu_n}=i^n\mathop{\mathrm{A}}_{\sigma\mu_1}\mathop{\mathrm{S}}_{\mu_1\dots\mu_n}\bar\psi\gamma_5\gamma_\sigma D_{\mu_1}\dots D_{\mu_n}\psi.\tag{1b}$$

The system of intermixing twist-3 operators includes also

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the following operators for the nonsinglet channel[2,7] (we have here a full analogy with the Abelian theory,[6] the difference being that color matrices are taken into account):

$$R_{3\sigma\mu_1\ldots\mu_n}=i^{n-1}m \underset{\mu_1\ldots\mu_n}{\mathrm{S}} \underset{\sigma\mu_1}{\mathrm{A}} \bar{\psi}\gamma_5\gamma_\sigma\gamma_{\mu_1}D_{\mu_2}\ldots D_{\mu_n}\psi,$$

$$R_{4l\sigma\mu_1\ldots\mu_n}=i^{n-1} \underset{\mu_1\ldots\mu_n}{\mathrm{S}} \bar{\psi}\gamma_5\gamma_{\mu_1}D_{\mu_2}\ldots D_{\mu_l}gF_{\sigma\mu_{l+1}}D_{\mu_{l+2}}\ldots D_{\mu_n}\psi, \quad (1c)$$

$$R_{5l\sigma\mu_1\ldots\mu_n}=i^{n-2} \underset{\mu_1\ldots\mu_n}{\mathrm{S}} \bar{\psi}\gamma_{\mu_1}D_{\mu_2}\ldots D_{\mu_l}g\tilde{F}_{\sigma\mu_{l+1}}D_{\mu_{l+2}}\ldots D_{\mu_n}\psi,$$

where

$$D_\mu=\partial_\mu+igt^aA_\mu^a, \quad F_{\mu\nu}=t^a(\partial_\mu A_\nu^a-\partial_\nu A_\mu^a-gf^{abc}A_\mu^bA_\nu^c), \quad (1d)$$

$$\tilde{F}_{\mu\nu}={}^1/_2\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma},$$

m and g are respectively the bare quark mass and the coupling constant. The symbols S and A denote symmetrization or antisymmetrization with respect to the indices under them, followed by subtraction of the traces over the symmetrized indices.

2. MATRIX ELEMENTS OF TWIST 2 AND 3 OPERATORS

We begin with consideration of a nonsinglet channel. We introduce by definition the functions $E(\beta)$, $A(\beta)$,, ... , a linear combination of which can be used to express the matrix elements of the operators R_1–R_5 referred to above:

$$\langle h|\bar{\psi}\gamma_5\frac{\hat{q}'}{pq}(i\partial.)^n\psi|h\rangle=\frac{aq'}{pq}\int d\beta\,\beta^{n-1}E(\beta)=\frac{aq'}{pq}E_n,$$

$$\langle h|\bar{\psi}\gamma_5\gamma_\sigma^\perp(i\partial.)^n\psi|h\rangle=a_\sigma^\perp\int d\beta\,\beta^{n-1}A(\beta)=a_\sigma^\perp A_n,$$

$$\left\langle h|\bar{\psi}\gamma_5\frac{\hat{q}'}{pq}(i\partial_\sigma^\perp)(i\partial.)^{n-1}\psi|h\right\rangle=a_\sigma^\perp\int d\beta\,\beta^{n-1}B(\beta)=a_\sigma^\perp B_n,$$

$$\langle h|\bar{\psi}m\gamma_5\gamma_\sigma^\perp\frac{\hat{q}'}{pq}(i\partial.)^{n-1}\psi|h\rangle=a_\sigma^\perp\int d\beta\,\beta^{n-1}C(\beta)=a_\sigma^\perp C_n,$$

$$\left\langle h|\bar{\psi}\gamma_5\frac{\hat{q}'}{pq}(i\partial.)^{l-1}g\hat{A}_\perp\gamma_\sigma^\perp(i\partial.)^{n-l}\psi|h\right\rangle$$

$$=a_\sigma^\perp\int d\beta_1\,d\beta_2\,\beta_1^{l-1}\,\beta_2^{n-l}\,D_1(\beta_1,\beta_2)=a_\sigma^\perp(D_1)_n{}^l,$$

$$\langle h|\bar{\psi}\gamma_5\frac{q'}{pq}(i\partial.)^{l-1}\gamma_\sigma^\perp g\hat{A}_\perp(i\partial.)^{n-l}\psi|h\rangle$$

$$=a_\sigma^\perp\int d\beta_1\,d\beta_2\beta_1^{l-1}\,\beta_2^{n-l}\,D_2(\beta_1,\beta_2)=a_\sigma^\perp(D_2)_n{}^l.$$

Here $|h\rangle$ is the state of a hadron with momentum p and 4-vector polarization a_σ ($a^2=-1, pa=0$); q' is a light-like vector:

$$q'=q+xp, \quad x=\frac{-q^2}{2pq}, \quad q'^2=0; \quad (2a)$$

the symbol $\perp$ means projection of a vector on a plane perpendicular to p and q', while a dot denotes convolution of the corresponding tensor with the vector $q'_\mu/(pq')$. The quark fields ψ and $\bar{\psi}$ are transformed in accord with the fundamental representation of the color group; m is the current mass of the quark (different in general for different flavors); $A_\mu=A_\mu^a t^2$, where t^a are color matrices. We use for the gluon field the axial gauge

$$q_\mu' A_\mu^a=0. \quad (3)$$

The cross section for deep inelastic ep scattering contains, if the initial particles are polarized, besides $W_1(x)$ and $W_2(x)$ also two additional structure functions $g_1(x)$ and $g_2(x)$, which satisfy the following expressions in terms of the distributions introduced in (1) (see Refs. 6 and 7):

$$g_1(x)=-\sum_q\frac{e_q^2}{2x}(E_q(x)-E_q(-x)),$$

$$g_1(x)+g_2(x)=-\sum_q\frac{e_q^2}{2x}(A_q(x)-A_q(-x)). \quad (4)$$

The subscript q, which was left out of Eqs. (1) for simplicity, labels the quark flavors ($q=u,d,s\ldots$), and e_q denotes the charge of the corresponding quark (in units of electron charge). The fact that Eqs. (4) contain only combinations that are even in x is due to the positive charge parity of the two-photon state in the t-channel. For the same reason, the evolution equations will contain only a charge-even combination of the functions D_1 and D_2:

$$D(\beta_1,\beta_2)={}^1/_2[D_1(\beta_1,\beta_2)+D_2(\beta_2,\beta_1)]. \quad (5)$$

We note that the matrix elements B, D_1, and D_2 cannot be written in a gauge-invariant form; only the differences $-(D_1)_n^{l\prime}, (D_2)_n^l-(D_2)_n^{l\prime},(D_1)_n^l+(D_2)_n^l-2B_n$ are invariant. We shall find it convenient to define the quantity

$$Y(\beta_1,\beta_2)=(\beta_1-\beta_2)D(\beta_1,\beta_2);$$

$$Y_n{}^l=\int d\beta_1\,d\beta_2\,\beta_1^{l-1}\,\beta_2^{n-l-1}\,Y(\beta_1,\beta_2)=D_n^{l+1}-D_n{}^l, \quad (6)$$

whose moments y_n^l are connected with the matrix elements of gauge-invariant operators (see Ref. 6).

The functions introduced in (2) are connected by two relations[6]:

$$A(\beta)=B(\beta)+C(\beta)-\int d\beta_1 D(\beta_1,\beta), \quad (7)$$

$$E(\beta)=A(\beta)-\beta\frac{d}{d\beta}B(\beta)+\beta\int d\beta_1\frac{D(\beta_1,\beta)-D(\beta,\beta_1)}{\beta_1-\beta}. \quad (8)$$

The first is the consequence of the equations of motion for the fields $\psi(x)$. The second follows from the relativistic invariance: both sides of (8) are expressed in terms of matrix elements of different components of one and the same twist-2 operator. Eliminating $B(\beta)$ from (7) and (8) and using (6) we obtain the following connection between the gauge-invariant quantities introduced above:

$$\left(1-\beta\frac{d}{d\beta}\right)A(\beta)=E(\beta)-\beta\frac{d}{d\beta}C(\beta)$$

$$+\beta\int\frac{d\beta_1}{\beta_1-\beta}\left[\frac{\partial}{\partial\beta}Y(\beta_1,\beta)+\frac{\partial}{\partial\beta_1}Y(\beta,\beta_1)\right], \quad (9)$$

from which we see that the three functions $E(\beta)$, $C(\beta)$ and $Y(\beta_1,\beta)$ can be chosen to be independent. When solving the differential equation (9) for $A(\beta)$, the integration constant must be chosen such that $A(\beta)=0$ at $|\beta|>1$. If $A(\beta)$ and $C(\beta)$ are assumed to be continuous at the point $\beta=0$ (this assumption is confirmed by the explicit form the evolution equations), we obtain from (9) the equality

$$\int_{-1}^{1} d\beta \frac{A(\beta)-E(\beta)}{\beta}=0. \tag{10}$$

It is equivalent, when (4) is taken into account, to the Cottingham sum rule[10]

$$\int_{0}^{1} dx g_2(x)=0,$$

which reflects the fact that one of the operators in the expansion (namely the one whose matrix element reduces to the difference A_n-E_n) is absent at $n=0$. (We note that in the scalar theory[5] the Cottingham sum rule was violated, for there $A(\beta)$ had a discontinuity at $\beta=0$; this was attributed to the presence of a subtraction term in the dispersion relation for g_1+g_2 with respect to $2pq$.)

If the mass m of the current quarks is negligibly small ($C\ll 1$) and the gluon-containing wave-function component is not large ($Y\ll 1$), we obtain from (9) and (4) an approximate connection between the structure functions g_1 and g_2:

$$g_1(x)+g_2(x)=\int_{x}^{1} \frac{d\beta}{\beta} g_1(\beta), \tag{11}$$

which corresponds to allowance for only twist-2 operators in the operator expansion.

In terms of Feynman diagrams, the quantities $E(\beta)$, $A(\beta)$, $B(\beta)$, and $C(\beta)$ are represented by vertex parts (Fig. 1a) with the integration over the fraction β of the quark energy removed:

$$\begin{gathered} E(\beta)=\left\langle \beta\frac{\hat{q}'}{pq'}\right\rangle, \quad A(\beta)=-\langle\beta\gamma_5\hat{a}_\perp\rangle, \\ B(\beta)=-\left\langle \frac{(ak_\perp)}{pq'}\gamma_5\hat{q}'\right\rangle, \\ C(\beta)=-\left\langle \frac{m}{pq'}\gamma_5\hat{a}_\perp\hat{q}'\right\rangle, \end{gathered} \tag{12}$$

where the brackets $\langle O\rangle$ denote convolution of the vertex O with the block in Fig. 1a at a fixed value $\beta=kq'/pq'$. Corresponding to exactly the same functions is the three-particle diagram, Fig. 1b, with removed integration over β_1 and β_2:

$$\begin{gathered} D_1(\beta_1,\beta_2)=-\left\langle \frac{g}{pq'}\gamma_5\hat{q}'\hat{A}^\perp\hat{a}_\perp\right\rangle, \\ D_2(\beta_1,\beta_2)=-\left\langle \frac{g}{pq'}\gamma_5\hat{q}'\hat{a}_\perp\hat{A}^\perp\right\rangle. \end{gathered} \tag{13}$$

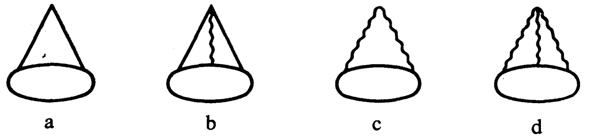


FIG. 1.

We proceed to consider the singlet channel. In this case account must be taken also of operators made up of gluon fields. There exists only one gluon twist-2 operator with the quantum numbers we need (Fig. 1c):

$$\mathscr{E}_{\sigma\mu_1\ldots\mu_n}=\underset{\sigma\mu_1\ldots\mu_n}{\mathrm{S}} \operatorname{Tr} G_{\rho\mu_1}(iD_{\mu_2})\ldots(iD_{\mu_n})\tilde{G}_{\rho\sigma}, \tag{14}$$

where

$$G_{\rho\mu}=t^a G_{\rho\mu}{}^a, \quad \tilde{G}_{\rho\sigma}={}^1/{}_2\varepsilon_{\rho\sigma\lambda\eta}G_{\lambda\eta}$$

are the gluon-field tensors; Tr stands for the trace over the color indices, and S for symmetrization with respect to the Lorentz indices under it (with the traces separated). In analogy with (1) we can introduce a function $\mathscr{E}(\beta)$:

$$\begin{aligned} \langle h|\mathscr{E}_{\underbrace{\cdots}_{n+1}}|h\rangle &= \frac{1}{2}\langle h|G^a_{\rho\cdot}(i\partial_\cdot)^n\tilde{G}^a_{\rho\cdot}|h\rangle \\ &= \frac{aq'}{pq'}\int d\beta\beta^{n-1}\mathscr{E}(\beta)=\frac{aq'}{pq'}\mathscr{E}_n. \end{aligned} \tag{15}$$

It follows from the identity of the gluons that $\mathscr{E}_n=0$ at odd n, so that the physical results can include only an odd combination of $\mathscr{E}(\beta)-\mathscr{E}(-\beta)$. We shall assume that

$$\mathscr{E}(\beta)=-\mathscr{E}(-\beta). \tag{16}$$

In analogy with (12), the function $\mathscr{E}(\beta)$ is represented by a gluon vertex without integration over β:

$$\mathscr{E}(\beta)=\frac{pq'}{aq'}\langle i\varepsilon_{\alpha\beta}{}^\perp\delta_{ab}\rangle, \quad \varepsilon_{\alpha\beta}{}^\perp=\frac{1}{pq'}\varepsilon_{\alpha\beta\gamma\delta}q_\gamma{}'p_\delta. \tag{17}$$

Turning now to twist-3 operators made up of gluon fields, we note first that in the non-Abelian theory there are three-gluon operators that have positive charge parity:

$$\begin{gathered} R^l_{6\sigma\underbrace{\cdots}_{n}}=if_{abc}G^a_{\rho\cdot}(i\partial_\cdot)^{l-2}igG^b_{\sigma\cdot}(i\partial_\cdot)^{n-l-1}(-i)\tilde{G}^c_{\rho\cdot}, \\ R^l_{7\sigma\underbrace{\cdots}_{n}}=if_{abc}iG^a_{\rho\cdot}(i\partial_\cdot)^{l-2}(-ig)\tilde{G}^b_{\sigma\cdot}(i\partial_\cdot)^{n-l-1}G^c_{\rho\cdot}, \\ 2\leqslant l\leqslant n-1 \end{gathered} \tag{18}$$

(we have written out only the tensor components that we shall need hereafter). These operators are analogous to the operators (1c) R_4 and R_5 for a nonsinglet channel. In addition there exist two-gluon twist-3 operators [of type (14)]; their matrix elements can be expressed with the aid of a formula of the type (12) and (17) in terms of the vertex function $\langle O\rangle$, where the current O is a linear combination of two tensor structures:

$$\begin{gathered} ia_\sigma{}^\perp(\varepsilon_{\sigma\alpha}{}^\perp k_\beta-\varepsilon_{\sigma\beta}{}^\perp k_\alpha); \\ ia_\sigma{}^\perp(\varepsilon_{\sigma\alpha}{}^\perp k_\beta{}^\perp-\varepsilon_{\sigma\beta}{}^\perp k_\alpha{}^\perp)=-i\varepsilon_{\alpha\beta}{}^\perp(k^\perp a^\perp). \end{gathered} \tag{19}$$

We shall presently see, however, that two-gluon operators cannot be considered. The point is that, in analogy with Eqs. (7) and (8) for a nonsinglet channel, there are two relations that connect the matrix elements of two- and three-particle vertices in a singlet channel. One of them is the consequence of the equation of motion for the gluon field (Maxwell's equation):

$$D_\rho G_{\rho\mu}{}^a = g\bar{\psi}\gamma_\mu t^a \psi, \tag{20}$$

just as Eq. (7) follows from another (Dirac) equation of motion. The second relation is an analog of Eq. (8) and follows from the connection, due to relativistic invariance, between the matrix elements of the different components of the tensor (14):

$$\langle h | \mathscr{E}^{\perp}_{\underbrace{\sigma\ldots}_{n}} | h \rangle = \frac{1}{n+1} \frac{a_\sigma{}^\perp (pq')}{(aq')} \langle h | \mathscr{E}_{\underbrace{\ldots}_{n+1}} | h \rangle. \tag{21}$$

The left-hand side of (21) is expressed in terms of the aforementioned two vertices (19). Thus, two-gluon vertices can be eliminated with the aid of the relations indicated above and can be reduced to the three-gluon contributions (18) and to the matrix elements of a twist-2 operator.

We note that by virtue of the Bose statistics of the gluons not all operators (18) are independent. Namely, accurate to quantities that are total derivatives and make no contributions to the matrix elements between hadron states with equal momenta, the following relations hold:

$$R^l_{6\sigma\ldots} = (-1)^{n+1} R^{n-l+1}_{6\sigma\ldots}, \quad R^l_{7\sigma\ldots} = (-1)^n R^{n-l+1}_{7\sigma\ldots},$$

$$R^l_{6\sigma\ldots} = \sum_{r=2}^{l} R^r_{7\sigma\ldots} (-1)^r [C^{r-2}_{l-2} - (-1)^n C^{r-2}_{n-l-1}],$$

$$C_r{}^s = \frac{r!}{s!(r-s)!}. \tag{22}$$

The last equation of (22) can be obtained with the aid of the identity

$$\varepsilon_{\rho\delta}{}^\perp \delta_{\sigma\tau}{}^\perp = \varepsilon_{\rho\sigma}{}^\perp \delta_{\tau\delta}{}^\perp + \varepsilon_{\sigma\delta}{}^\perp \delta_{\rho\tau}{}^\perp.$$

We introduce in analogy with (13) the function

$$H(\beta_1, \beta_2) = -a_\sigma{}^\perp \langle f_{abc} (\varepsilon_{\sigma\gamma}{}^\perp \delta_{\alpha\beta}{}^\perp - \varepsilon_{\sigma\alpha}{}^\perp \delta_{\beta\gamma}{}^\perp - \varepsilon_{\sigma\beta}{}^\perp \delta_{\gamma\alpha}{}^\perp) \rangle, \tag{23}$$

which corresponds to diagram d of Fig. 1. Since the gluons are identical, this function has the symmetry properties

$$H(\beta_1, \beta_2) = -H(\beta_2, \beta_1) = H(-\beta_1, -\beta_2) = -H(-\beta_2, -\beta_1). \tag{24}$$

The matrix elements R_6 and R_7 (18) are expressed in terms of this function as follows:

$$\langle h | R^l_{\underbrace{6\sigma\ldots}_{n}} | h \rangle = \frac{3}{2} a_\sigma{}^\perp \int d^3\beta\, \delta\left(\sum \beta\right) \times \beta_1 (\beta_2^{l-1} \beta_3^{n-l} + \beta_3^{l-1} \beta_2^{n-l}) H(\beta_1, \beta_3),$$

$$\langle h | R^l_{\underbrace{7\sigma\ldots}_{n}} | h \rangle = \frac{3}{2} a_\sigma{}^\perp \int d^3\beta\, \delta\left(\sum \beta\right) \times \beta_1 (\beta_2^{l-1} \beta_3^{n-l} - \beta_3^{l-1} \beta_2^{n-l}) H(\beta_1, \beta_3). \tag{25}$$

The evolution equations discussed below (Sec. 4) will connect, in a singlet channel, the function $H(\beta_1,\beta_2)$ (23) with the previously introduced function $D(\beta_1,\beta_2)$ (5). It is convenient here to introduce in lieu of $H(\beta_1,\beta_2)$, in analogy with (6), a function $L(\beta_1,\beta_2)$ defined by

$$L(\beta_1, \beta_2) = \beta_1 \beta_2 (\beta_1 - \beta_2) H(\beta_1, \beta_2). \tag{26}$$

3. PARTON REPRESENTATIONS OF STRUCTURE FUNCTIONS

To derive the relations that express the structure functions in terms of the quark-number densities we must transform in the matrix elements of type (1) to the interaction representation, expanding the fields ψ, $\bar{\psi}$, and A in the free-particle creation and annihilation operators.[5,12] Neglecting the quark mass, we choose as the wave functions for the free quark and antiquark spinors that correspond to states with a definite helicity $1/2\lambda = \pm 1/2$:

$$\hat{k} u^{(\lambda)}(k) = 0, \quad \gamma_5 u^{(\lambda)}(k) = -\lambda u^{(\lambda)}(k);$$

$$\hat{k} v^{(\lambda)}(-k) = 0, \quad \gamma_5 v^{(\lambda)}(-k) = \lambda v^{(\lambda)}(-k), \quad k^2 = 0. \tag{27}$$

It is convenient, as will be seen later on, to use the following normalization of the spinors u and v:

$$\bar{u}^{(\lambda_1)}(k) \gamma_\mu u^{(\lambda_2)}(k) = \bar{v}^{(\lambda_1)}(-k) \gamma_\mu v^{(\lambda_2)}(-k) = 2\beta k_\mu \delta_{\lambda_1\lambda_2},$$

$$\beta = kq'/pq'. \tag{28}$$

When calculating the matrix elements of the currents E, B, and C (12), the quarks with momentum k (Fig. 1a) can be regarded as being on the mass shell. Indeed, the numerator of each of the quark propagators can be written in the form

$$\hat{k} = \hat{\bar{k}} + \frac{k^2}{2pq'} \hat{q}', \quad \bar{k}^2 = 0; \tag{29}$$

the term proportional to $\hat{q}'$ can be discarded since $q'^2 = 0$, whereas the remainder is a projector on physical states with helicities $1/2\lambda = \pm 1/2$:

$$\hat{\bar{k}} = \frac{1}{\beta} \sum_{\lambda=\pm 1} [\theta(\beta) u^{(\lambda)}(\bar{k}) \bar{u}^{(\lambda)}(\bar{k}) + \theta(-\beta) v^{(\lambda)}(-\bar{k}) \bar{v}^{(\lambda)}(-\bar{k})]. \tag{30}$$

For the $E(\beta)$ vertex, for example, we obtain then

$$-\frac{(aq')}{(pq')\beta} E(\beta) = \theta(\beta) [q^+(\beta) - q^-(\beta)] + \theta(-\beta) [\bar{q}^+(-\beta) + \bar{q}^-(-\beta)], \tag{31}$$

where $q^\pm(\beta)$ and $\bar{q}^\pm(-\beta)$ are the number densities of the quarks and antiquarks respectively with helicities $\pm 1/2$ in the hadron. From this, taking (4) into account, we obtain the known parton representation for g_1.[13]

To obtain the corresponding representation for the function g_2 it is necessary, according to (4), to consider in similar fashion the vertex A (12). Here, however, a complication arises because the quarks of Fig. 1a can no longer be regarded as located on the mass shell. Indeed, the second term of the right-hand side of (29) does not vanish in this case, so that the numerators of the quark propagators cannot be written in the form of projectors on physical states. (This circumstance was not taken into account in Ref. 5, and Eq. (38) of that reference is not a parton representation.) This difficulty can be circumvented by using the following identity for γ matrices

$$\gamma_5\gamma_\sigma^\perp=\gamma_5\frac{\hat{q}'k_{\perp\sigma}}{kq'}+m\gamma_5\gamma_\sigma^\perp\frac{\hat{q}'}{kq'}-\frac{(\hat{k}-m)\gamma_5\hat{q}'\gamma_\sigma^\perp}{2kq'}-\frac{\gamma_5\hat{q}'\gamma_\sigma^\perp(\hat{k}-m)}{2kq'}, \quad (32)$$

which is equivalent to relation (7). The factor $(\hat{k}-m)$ in the last two terms cancels the denominator of one of the neighboring quark propagators for the current $O=\gamma_5\gamma_\sigma^\perp$ on Fig. 1a, and the nearest quark-gluon vertex "is drawn" into the two-quark current vertex O, forming thereby a local three-particle vertex. It is significant that in the right-hand sides of (32) and (7) both the two-particle vertices B and C and the three-particle ones $D_{1,2}$ contain the factor $\hat{q}'$, owing to which the second term of (29) can be omitted and the quarks on Figs. 1a and 1b can be regarded as real. It is easy to verify that the last conclusion is valid also for the gluon on Fig. 1b. Indeed, let us write the numerator of the gluon propagator in the axial gauge (3) in the form

$$\Delta_{\alpha\beta}(k)=-\delta_{\alpha\beta}+\frac{k_\alpha q_\beta'+k_\beta q_\alpha'}{kq'}=\Delta_{\alpha\beta}(\tilde{k})+\frac{q_\alpha' q_\beta'}{(kq')^2}k^2, \quad \tilde{k}=k-\frac{k^2}{2kq'}q', \quad \tilde{k}^2=0. \quad (33)$$

Just as in the analogous expansion (29) for a quark, the last term proportional to $q_\alpha' q_\beta'$ in (33) drops out on convolution with the vertices (13), and the remainder is a projector on the physical states of the gluon:

$$\Delta_{\alpha\beta}(\tilde{k})=\frac{1}{\beta^2}\sum_{\lambda=\pm1} e_\alpha^{(\lambda)}(\tilde{k})e_\beta^{(\lambda)*}(\tilde{k}). \quad (34)$$

Here $e_\alpha^{(\lambda)}$ are the polarization vectors of a gluon in states with definite helicity $\lambda=\pm1$ and satisfy the equations

$$\tilde{k}_\alpha e_\alpha^{(\lambda)}(\tilde{k})=q_\alpha' e_\alpha^{(\lambda)}(\tilde{k})=0, \quad \frac{i}{kq'}\varepsilon_{\alpha\beta\gamma\delta}q_\gamma'\tilde{k}_\delta e_\beta^{(\lambda)}(\tilde{k})=\lambda e_\alpha^{(\lambda)}(\tilde{k}) \quad (35)$$

and the normalization condition

$$e_\alpha^{(\lambda)}(\tilde{k})e_\alpha^{(\lambda')}(\tilde{k})=-\beta^2\delta_{\lambda\lambda'}. \quad (36)$$

Thus, after using Eqs. (32) we can assume the partons on Figs. 1a and 1b to be on the mass shell; $A(\beta)$ is expressed with the aid of (7) in terms the other functions (12) and (13). The quantity $B(\beta)$ has a parton representation similar to (31):

$$-B(\beta)=2\int d^2k_\perp(ak_\perp)\{\theta(\beta)[q^+(\beta,k_\perp)-q^-(\beta,k_\perp)]+\theta(-\beta)[\bar{q}^+(-\beta,k_\perp)-\bar{q}^-(-\beta,k_\perp)]\} \quad (37)$$

except that the last expression contains the parton distribution densities not only over the longitudinal but also over the transverse component of the momentum. (We note that the following correlation is possible here

$$q^\lambda(\beta,k_\perp)\sim\lambda(ak_\perp)f(k_\perp^2,\beta),$$

so that $B(\beta)$ does not vanish.) The remaining functions C and $D_{1,2}$, however, cannot be similarly expressed in terms of the density of the number of partons; it is necessary to introduce more general quantities, viz., parton correlators that are not diagonal in the parton states. Thus, if the two-quark correlator $(\bar{q}q)^{\lambda_1\lambda_2}(x)$ is defined by the relation

$$\langle h|\bar{\psi}_\alpha(0)\psi_\beta(0)|h\rangle=\int dx\,\bar{u}_\alpha^{(-\lambda_1)}(-xp)u_\beta^{(\lambda_2)}(xp)(\bar{q}q)^{\lambda_1\lambda_2}(x), \quad (38)$$

the quark or antiquark number density is given (apart from a factor) by its value at $\lambda_1=-\lambda_2$, whereas the function $C(x)$ can be expressed in terms of this correlator at $\lambda_1=\lambda_2$:

$$C(x)=-\sum_\lambda(\bar{q}q)^{\lambda\lambda}(x)\bar{u}^{(-\lambda)}(xp)m\frac{\gamma_5\hat{a}^\perp\hat{q}'}{pq'}u^{(\lambda)}(xp). \quad (39)$$

For the functions $D_{1,2}$ it is necessary to introduce a three-particle correlator N that connects the states of a hadron with a different number of partons:

$$\langle h|\bar{\psi}_{\alpha_1}^{a_1}(0)A_{\alpha_3}^{a_3}(0)\psi_{\alpha_2}^{a_2}(0)|h\rangle=\int dx_1\,dx_2\,dx_3\,\delta\left(\sum x_i\right)\bar{u}_{\alpha_1}^{(-\lambda_1)}(-x_1p)\times u_{\alpha_2}^{(\lambda_2)}(x_2p)e_{\alpha_3}^{(\lambda_3)}(xp)N_{a_1a_2a_3}^{\lambda_1\lambda_2\lambda_3}(x_1,x_2,x_3) \quad (40)$$

(a_i are the color indices). The arguments x_i of the correlator N, which are connected by the condition $\Sigma x_i=0$, can be of either sign, and accordingly Eq. (40) covers six different kinematic regions. Depending on the sign of x_i, the corresponding particle is in an initial ($x_i>0$) or final ($x_i<0$) state, and in the latter case the correlator corresponds to creation of an antiparticle with momentum $-x_ip$ and helicity-1/$2\lambda_i$. In each of the six regions one can write for N an integral representation in terms of a product of parton wave functions with unity difference between the number of the partons. The relations for the functions D_1 and D_2 are

$$D_1(\beta_1,\beta_2)=-a_\sigma\sum_\lambda gt_{a_1a_2}^{a_3}\cdot4\lambda e_\sigma^{(\lambda)}\times((\beta_1-\beta_2)p)N_{a_1a_2a_3}^{-\lambda\lambda\lambda}(-\beta_1,\beta_2,\beta_1-\beta_2),$$
$$D_2(\beta_1,\beta_2)=-a_\sigma\sum_\lambda gt_{a_1a_2}^{a_3}\cdot4\lambda e_\sigma^{(-\lambda)}\times((\beta_1-\beta_2)p)N_{a_1a_2a_3}^{-\lambda\lambda-\lambda}(-\beta_1,\beta_2,\beta_1-\beta_2). \quad (41)$$

Equations (41), (39) and (37) yield via (4) and (7) a parton-like representation for $g_1(x)+g_2(x)$.

An expression similar to (41) can be obtained also for the three-gluon function H (23) by introducing the three-gluon correlator

$$\langle h|A_{\alpha_1}^{a_1}(0)A_{\alpha_2}^{a_2}(0)A_{\alpha_3}^{a_3}(0)|h\rangle=\int dx_1\,dx_2\,dx_3\,\delta\left(\sum x_i\right)e_{\alpha_1}^{(\lambda_1)}(x_1p)\times e_{\alpha_2}^{(\lambda_2)}(x_2p)e_{\alpha_3}^{(\lambda_3)}(x_3p)M_{a_1a_2a_3}^{\lambda_1\lambda_2\lambda_3}(x_1,x_2,x_3). \quad (42)$$

As for the two-gluon function $\mathscr{G}$ (17) that corresponds to twist 2, it has the usual parton representation of type (31) in terms of the number of gluons, which we shall not write out here.

4. EVOLUTION EQUATIONS

To find the dependence of the structure functions on $\ln(-q^2)$ we must construct evolution equations. The general method of obtaining these equations is known[3,4] and was described in detail in the preceding papers.[5,6]

We differentiate expressions of the type (13) and (17) with respect to the logarithm of the ultraviolet cutoff parameter Λ which limits the integrals over the virtual transverse momenta. The dependence on Λ systems from two sources. First, the ultraviolet divergences are contained inside the blocks of Fig. 1. If all the tails of the blocks correspond to real particles, such divergences are singled out on account of the renormalization invariance in the form of a factor $\Pi_i z_i^{1/2}$, where z_i is the renormalization constant of the Green's function of the ith particle (the block includes the two-particle Green's functions of the quarks and gluons interacting directly with the external current). The case of virtual tails, in which the analysis of the renormalizations in the axial gauge (3) becomes considerably more complicated,[6] can be disregarded if (7) is used to exclude the two-particle vertex $A(\beta)$. A similar procedure can be applied to the two-gluon vertex (19); the analog of identity (32) is here the relation

$$k_\alpha \Delta_{\alpha\beta}(k) = \frac{k^2}{kq'} q_\beta'. \tag{43}$$

The second source of the dependence on Λ in the vertex functions of Fig. 1 is connected with the logarithmic divergence of the integrals over the transverse momenta of those particles that interact directly with the external current. To find this dependence, it is necessary to expand in part the blocks of Fig. 1 after separating in explicit form the loop diagram that leads to this dependence. Figure 2 shows such a separation for the case when the vertex contains initially an operator with a quark-antiquark pair. The shaded blocks denote the total gauge-invariant sum of the Feynman diagrams for the corresponding processes in the Born approximation (see Fig. 3). We note that the diagram 2d is responsible for the mixing of the quark and strictly gluon operators. A similar group of diagrams is shown in Fig. 4 for the case when the initial operator is strictly gluon.

We emphasize once more that the partons corresponding to the inner lines of Figs. 2 and 4 can be assumed, in the operator basis chosen by us, to be on the mass shell. This allows us to find, as an intermediate stage of the calculations, all the two-particle scattering amplitudes, corresponding to the processes in Fig. 3, for physical particles. However, the equations of motion (7) and (20) [or their corrolaries (32) and (43)], with the aid of which we succeeded in excluding the "poor" operators, make it necessary to include in the calculations the last terms of the numerators of the virtual-particle propagators (29) and (33). After the denominator is cancelled out, these terms lead to the already mentioned effect of "drawing" one chromodynamic vertex into another. In Fig. 3, slashes are drawn through the lies in which only the "drawing" effect operates. We note that it is possible, with equal accuracy, to set these lines in correspondence with a total propagator, since the contribution of the first terms in Eqs. (29) and (33) can be shown to vanish in the indicated diagrams after averaging over the directions in the $\mathbf{k}_\perp$ plane.

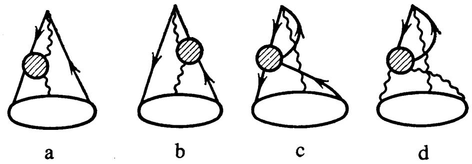


FIG. 2.

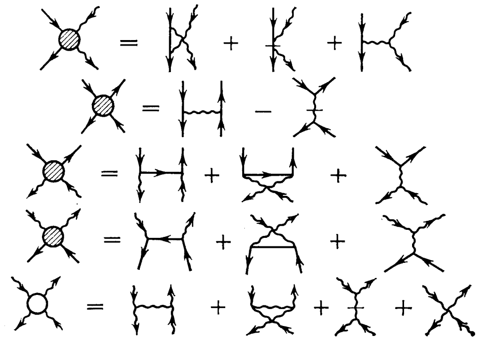


FIG. 3.

The contribution of the diagrams of Fig. 3 can therefore be constructed in full accord with the Feynman rules, thus ensuring gauge invariance of the calculation results.

To find the four-point amplitudes shown in Fig. 3 it is convenient to use the states of particles with definite helicity, which were introduced in the preceding section. This method is similar to that used in Ref. 4 to study the evolution equations for quantities connected with twist 2, but there it sufficed to use amplitudes without transfer of momentum and color to the t-channel. Here, however, we must calculate these amplitudes for the more general case. We note that the results enable us to write, without great difficulty, the evolution equations for operators with higher twist (4 and more), since it can be shown that in this case only pair interactions are significant in the LLA, but the investigated current vertices must contain a large number of external lines. We write down explicit expressions for the spinors $u^{(\lambda)}$ and $v^{(\lambda)}$ and for the vectors $e^{(\lambda)}$ satisfying Eqs. (27) and (35):

$$u^{(\lambda)}(k) = \frac{1}{2pq'} \hat{k}\hat{q}' u^{(\lambda)}(p), \quad v^{(\lambda)}(-k) = u^{(-\lambda)}(k),$$
$$e_\rho^{(\lambda)}(k) = \beta e_\rho^{(\lambda)}(p) - \frac{q_\rho'}{pq'}(k_\perp e^{(\lambda)}(p)), \quad e^{(\lambda)*}(k) = e^{(-\lambda)}(-k). \tag{44}$$

Here $u^{(\lambda)}(p)$ and $e^{(\lambda)}(p)$ are solutions of the following equations:

$$\gamma_5 u^{(\lambda)}(p) = -\lambda u^{(\lambda)}(p), \quad \hat{p} u^{(\lambda)}(p) = 0;$$
$$i\varepsilon_{\rho\tau} e_\tau^{(\lambda)}(p) = \lambda e_\rho^{(\lambda)}(p), \quad p_\rho e_\rho^{(\lambda)}(p) = q_\rho' e_\rho^{(\lambda)}(p) = 0; \tag{45}$$

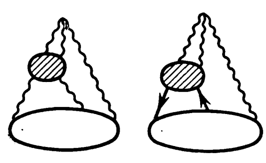
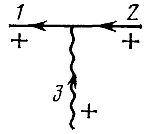


FIG. 4.

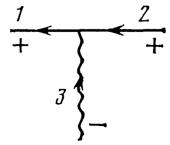
with normalization conditions compatible with (38) and (36),

$$\bar{u}^{(\lambda)}(p)\gamma_\mu u^{(\lambda')}(p)=2p_\mu\delta_{\lambda\lambda'},\quad e^{(\lambda)*}(p)e^{(\lambda')}(p)=-\delta_{\lambda\lambda'},\quad e^{(\lambda)*}(p)=-e^{(-\lambda)}(p). \tag{46}$$

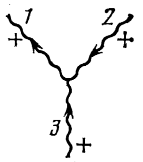
These equations yield the following expressions for three-particle vertices of interaction of particles with definite helicities (the helicity is marked + or − on the diagrams):



$$=t^a\cdot 2\beta_1(Ke^{(+)}(p)),$$

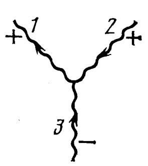


$$=t^a\cdot 2\beta_2(Ke^{(-)}(p)),$$



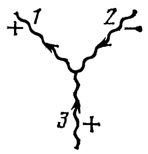
$$=-if_{a_1a_2a_3}\cdot 2\beta_1^2(Ke^{(+)}(p)),$$

$$K=\beta_3k_{1\perp}-\beta_1k_{3\perp};$$



$$=-if_{a_1a_2a_3}\cdot 2\beta_2^2(Ke^{(-)}(p)),$$

$$K=\beta_3k_{2\perp}-\beta_2k_{3\perp};$$



$$=-if_{a_1a_2a_3}\cdot 2\beta_3^2(Ke^{(-)}(p)),$$

$$K=\beta_1k_{2\perp}-\beta_2k_{1\perp};\qquad k_1=k_2+k_3.$$

We supplement them with the following rules:

1) helicity is conserved along a fermion line;

2) the sign of the helicity for any gluon line can be reversed if the arrow is simultaneously reversed;

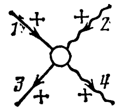
3) if the helicities of all three particles reverse sign, the form of the vertex remains unchanged, but $e^{(+)}(p)$ is replaced by $e^{(-)}p$ and vice versa;

4) a three-gluon vertex is zero if the helicities of all particles remain the same when all three arrows are rotated inward or outward;

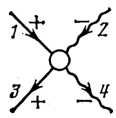
5) a factor $1/\beta k^2$ should correspond to each internal quark line and a factor $1/\beta^2k^2$ to each gluon line.

By applying these rules it is easy to find the four-point vertices (Fig. 3). A significant simplifying factor in their calculation is the averaging over the directions of the vector $\mathbf{k}_\perp$. It is important here that the transverse momenta of the particles are of different order of magnitude in the logarithmic kinematics considered by us: they are small for the two lower particles of the four-point diagram and large and approximately equal for the two upper ones ($|\mathbf{k}_{1\perp}|=|\mathbf{k}_{2\perp}|\gg|\mathbf{k}_{3\perp}|\sim|\mathbf{k}_{4\perp}|$). The averaging results in a convolution of the two vectors $e^{(\lambda)}(p)$, which is eliminated with the aid of the orthogonality relation (46).

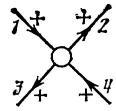
We present explicit expressions for the four-point diagrams in the case when the color structure in the t-channel is a triplet and an octet:



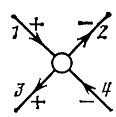
$$=\beta_3\beta_4\left[-2\left(C_F-\frac{C_V}{2}\right)\beta_1\frac{k_2^2}{k_{14}^2}+C_V\frac{\beta_1\beta_4k_2^2-\beta_2^2k_1^2}{\beta_{13}k_{13}^2}\right],$$



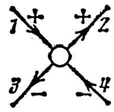
$$=\beta_3\beta_4\left[-2\left(C_F-\frac{C_V}{2}\right)\beta_{14}\frac{k_{1\perp}^2}{k_{14}^2}+2C_F\frac{\beta_1\beta_2}{\beta_1+\beta_2}+C_V\frac{\beta_4\beta_{14}k_{1\perp}^2-s\alpha_1\beta_1\beta_2(\beta_2+\beta_4)}{\beta_{13}k_{13}^2}\right]$$



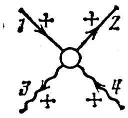
$$=\beta_3\beta_4\left[2\left(C_F-\frac{C_V}{2}\right)\frac{(\beta_2+\beta_3)k_{1\perp}^2+2s\alpha_1\beta_1\beta_2}{\beta_{13}k_{13}^2}-2n_f\frac{\beta_1\beta_2}{\beta_{12}^2}\right],$$



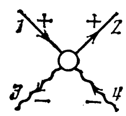
$$=2\left(C_F-\frac{C_V}{2}\right)\beta_3\beta_4\frac{(\beta_1+\beta_2)k_{1\perp}^2+2s\alpha_1\beta_1\beta_2}{\beta_{13}k_{13}^2},$$



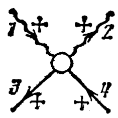
$$=-2n_f\frac{\beta_1\beta_2\beta_3\beta_4}{\beta_{12}^2},$$



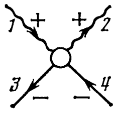
$$=\frac{n_f}{2}\beta_3\beta_4\left[\frac{s\alpha_1\beta_1\beta_2}{k_{13}^2}+\frac{k_{1\perp}^2(\beta_1+\beta_4)}{(k_1+k_4)^2}+2\frac{\beta_1\beta_2(\beta_3+\beta_4)}{\beta_{12}^2}\right],$$



$$=-\frac{n_f}{2}\beta_3\beta_4\left[\frac{s\alpha_1\beta_1\beta_2}{(k_1+k_4)^2}+\frac{\beta_{13}k_{1\perp}^2}{k_{13}^2}-2\frac{\beta_1\beta_2(\beta_3+\beta_4)}{\beta_{12}^2}\right],$$

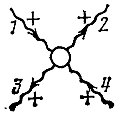


$$=-C_V\beta_3\beta_4\left[\frac{(\beta_2+\beta_3)k_{1\perp}^2+s\alpha_1\beta_1\beta_2}{k_{13}^2}-\frac{(\beta_1+\beta_4)k_{1\perp}^2}{(k_1+k_4)^2}-2\frac{\beta_1\beta_2(\beta_1+\beta_2)}{\beta_{12}^2}\right],$$

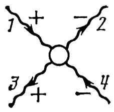


$$=-C_V\beta_3\beta_4\left[\frac{\beta_{31}k_{1\perp}^2}{k_{31}^2}+\frac{\beta_{13}k_{1\perp}^2+s\alpha_1\beta_1\beta_2}{(k_1+k_4)^2}\right.$$

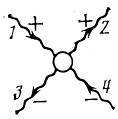
$$-2\frac{\beta_1\beta_2(\beta_1+\beta_2)}{\beta_{12}^2}\Big],$$



$$=\frac{C_V}{2}\beta_3\beta_4\Big[\frac{1}{\beta_{13}k_{13}^2}[k_{1\perp}^2(2\beta_3\beta_4(\beta_2+\beta_3)-\beta_1\beta_2\beta_{13})$$
$$+s\alpha_1\beta_1\beta_2(\beta_1+\beta_3)(\beta_2+\beta_4)]$$
$$+2\frac{(\beta_1+\beta_4)^2k_{1\perp}^2}{(k_1+k_4)^2}+2\beta_1\beta_2\frac{(\beta_1+\beta_2)(\beta_3+\beta_4)}{\beta_{12}^2}+3\beta_1\beta_2\Big],$$

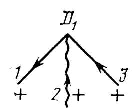


$$=\frac{C_V}{2}\beta_3\beta_4\Big[\frac{1}{\beta_{13}k_{13}^2}[k_{1\perp}^2(2\beta_1^2\beta_4+2\beta_2^2\beta_3+\beta_1\beta_2\beta_{13})$$
$$+s\alpha_1\beta_1\beta_2(\beta_1+\beta_3)$$
$$\times(\beta_2+\beta_4)]+\frac{(\beta_1+\beta_4)^{-1}}{(k_1+k_4)^2}[k_{1\perp}^2(2\beta_1^2\beta_3+2\beta_2^2\beta_4-\beta_1\beta_2(\beta_1+\beta_4))$$
$$-s\alpha_1\beta_1\beta_2\beta_{14}\beta_{23}]\Big],$$

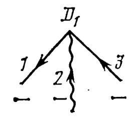


$$=\frac{C_V}{2}\beta_3\beta_4\Big[-\frac{(\beta_1+\beta_4)^{-1}}{(k_1+k_4)^2}$$
$$[k_{1\perp}^2(2\beta_3\beta_4\beta_{24}-\beta_1\beta_2(\beta_1+\beta_4))$$
$$+s\alpha_1\beta_1\beta_2\beta_{14}\beta_{23}]-2\beta_{13}^2\frac{k_{1\perp}^2}{k_{13}^2}+2\beta_1\beta_2\frac{(\beta_1+\beta_2)(\beta_3+\beta_4)}{\beta_{12}^2}-3\beta_1\beta_2\Big].$$

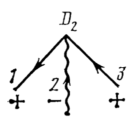
In these expressions $k_{ij}=k_i-k_j$, $\beta_{ij}=\beta_i-\beta_j$, $k_i=\alpha_i q'+\beta_i p+k_{i\perp}$, $s=2pq'$, $C_F=(N^2-1)/(2N)$, $C_V=N$, $N=3$ is the number of colors, and n_f is the number of flavors. To derive the evolution equations it is necessary also to calculate the matrix elements of the three-particle vertices (13) and (24) between the helicity states of the partons. With the aid of (44)–(46) we get the following results:



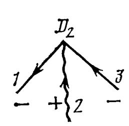
$$=-2\beta_1\beta_2\beta_3(ae^{(+)}(p)),$$



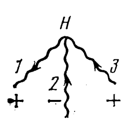
$$=2\beta_1\beta_2\beta_3(ae^{(-)}(p));$$



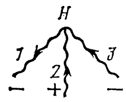
$$=-2\beta_1\beta_2\beta_3(ae^{(-)}(p)),$$



$$=2\beta_1\beta_2\beta_3(ae^{(+)}(p));$$



$$=-2\beta_1\beta_2\beta_3(ae^{(-)}(p)),$$



$$=2\beta_1\beta_2\beta_3(ae^{(+)}(p)).$$

The formulas presented enable us to write the equations for the three-particle functions $D(\beta_1,\beta_2)$ and $H(\beta_1,\beta_2)$. In the calculation of the logarithmic integrals it is convenient to use relations of the form

$$\int\frac{d^4k\,k_\perp^2}{k^2(k-k_1)^2(k-k_2)^2}=-i\pi^2\ln\frac{\Lambda^2}{k^2_{max}}\int d\beta J^0_{111}(\beta,\beta-\beta_1,\beta-\beta_2),$$

etc., where $k^2_{\max}=\max(k_1^2,k_2^2)$; some relations for the functions $\mathbf{J}^m_{n_i\ldots n_k}(\beta_i,\ldots,\beta_k)$ are contained in Refs. 5 and 6. The resultant equations are quite cumbersome; they can be simplified somewhat by changing from the functions D and H to the functions Y and L in accord with Eqs. (6) and (26), and are given here in the latter form:

$$\frac{\partial Y(\beta_1,\beta_2)}{\partial\xi}=-4C_FC(\beta_2)(\beta_1-\beta_2)J_{21}^0(\beta_1,\beta_1-\beta_2)$$
$$+C_FY(\beta_1,\beta_2)[3-2l(\beta_1)-2l(\beta_2)]-2C_VY(\beta_1,\beta_2)l(\beta_1-\beta_2)$$
$$-2\left(C_F-\frac{C_V}{2}\right)\Big[\int dx\,Y(x,\beta_2)J^0_{211}(\beta_1,\beta_1-\beta_2,\beta_1-\beta_2+x)$$
$$+\int dx\,Y(\beta_1,x)J_{12}^0(\beta_2,x-\beta_1+\beta_2)$$
$$+\int dx\,Y(\beta_1-x,\beta_2-x)\frac{1}{x}((x-\beta_1+\beta_2)$$
$$\times J^0_{111}(\beta_1,\beta_2,x)+2\beta_2J_{11}^0(\beta_2,x))\Big]$$
$$-C_V\int dx\,Y(x,\beta_2)\Big[\beta_1(\beta_1-\beta_2)$$
$$\times J^2_{211}(\beta_1-\beta_2,\beta_1,\beta_1-x)$$
$$+\frac{1}{\beta_1-x}((\beta_1+\beta_2-x)J^0_{111}(\beta_1,\beta_1-\beta_2,\beta_1-x)$$
$$+2\beta_1J_{11}^0(\beta_1,\beta_1-x))\Big]$$
$$+C_V\int dx\,Y(\beta_1,x)\frac{1}{x-\beta_2}[\beta_1J^0_{111}(\beta_2,\beta_2-\beta_1,\beta_2-x)$$
$$+2xJ_{11}^0(\beta_2,\beta_2-x)$$
$$+(x-\beta_2)J_{12}^0(\beta_2-\beta_1,\beta_2-x)]+\frac{2n_f\beta_1\beta_2}{(\beta_1-\beta_2)^2}J_{11}^0(\beta_1,\beta_2)$$
$$\times\int dx[Y(x,x+\beta_1-\beta_2)$$
$$+Y(x+\beta_1-\beta_2,x)]-\frac{1}{2}n_f\int dx\,L(x,\beta_2-\beta_1)$$
$$\times[\beta_1\beta_2J^3_{221}(\beta_1,\beta_2,\beta_2-x)-J^1_{221}(\beta_1,\beta_2,\beta_1+x)];\qquad(47)$$

$$\frac{\partial L(\beta_1,\beta_2)}{\partial\xi}=L(\beta_1,\beta_2)\Big[-\frac{2}{3}n_f+\frac{11}{3}C_V-2C_V(l(\beta_1)$$
$$+l(\beta_2)+l(\beta_1-\beta_2))\Big]$$
$$+C_V\int dx\,L(\beta_1-x,\beta_2-x)\frac{\beta_2}{x}J_{12}^0(\beta_2,x)$$

$$+C_V\int dx\,L(\beta_1,x)\,[J^0_{221}(\beta_2,\beta_2-\beta_1,\beta_2-\beta_1+x)$$
$$-\beta_2^2J^2_{221}(\beta_2,\beta_2-\beta_1,\beta_2-x)$$
$$+2x^2J^2_{211}(\beta_2,\beta_2-\beta_1,\beta_2-x)+\frac{1}{\beta_2-x}\,(\beta_2J_{12}{}^0(\beta_2,\beta_2-x)$$
$$+(x-\beta_1)J_{11}{}^0(\beta_2-\beta_1,\beta_2-x))\Big]$$
$$-C_V\int dx[Y(x,x-\beta_2)+Y(-x,-x+\beta_2)]$$
$$\times\Big[(\beta_2-x)J_{11}{}^0(\beta_1,\beta_1-\beta_2+x)-\frac{1}{\beta_2-x}((\beta_1-x)^2J_{11}{}^0(\beta_1,\beta_1-x)$$
$$-(\beta_1-\beta_2)^2J_{11}{}^0(\beta_1,\beta_1-\beta_2))-2\beta_2^{-2}(\beta_1-\beta_2)^2$$
$$\times(2\beta_1+\beta_2)J_{11}{}^0(\beta_1,\beta_1-\beta_2)\Big]$$
$$+C_V\int dx\frac{\beta_1}{x}J_{12}{}^0(\beta_1,x)\cdot L(\beta_1-x,\beta_2-x)+(\beta_1\to\beta_2). \qquad (48)$$

In these relations

$$\xi=\frac{1}{b}\ln\left(1+b\frac{g^2}{16\pi^2}\ln\frac{\Lambda^2}{m^2}\right),\quad b=\frac{11}{3}C_V-\frac{2}{3}n_f; \qquad (49)$$

$$l(\beta)=\int_0^{|\beta|}\frac{dx}{x};$$

$$J^m_{jkl}(a,b,c)=\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{dy\,y^m}{(ay-1+i\varepsilon)^j(by-1+i\varepsilon)^k(cy-1+i\varepsilon)^l}$$

(the permutation $\beta_1\leftrightarrow\beta_2$ in (48) applies to the entire right-hand side). The infrared divergence at the lower limit of the integral $l(\beta)$ is actually cancelled by a similar divergence in other terms of the equations. Equations (47) and (48) were written for a singlet channel; in the case of a nonsinglet channel only the first of the equations remains, and furthermore its last term containing L drops out.

The right-hand side of (47) includes a term that contains $C(\beta_1)$ and takes into account the nonzero mass of the quark. To find this term it is necessary to calculate in the diagrams of Fig. 5 the contribution linear in mass. It suffices for this purpose to replace the numerator of the quark propagator in the loop by the quark mass. It is likewise easy to obtain an equation for the function $C(\beta_1)$ itself, which has a "ladder" form similar to the case of twist 2

$$\frac{\partial C(\beta)}{\partial\xi}=4C_F\Big[-C(\beta)l(\beta)+\theta(\beta)\int_\beta^1 dx\frac{\beta C(x)}{x(x-\beta)}$$
$$-\theta(-\beta)\int_{-1}^{\beta}dx\frac{\beta C(\beta)}{x(x-\beta)}\Big]. \qquad (50)$$

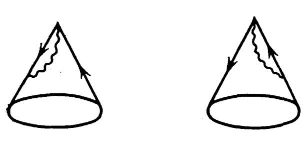


FIG. 5.

We present also equations for the functions E and $\mathscr{E}$ which are connected with the twist 2 (see Ref. 4):

$$\frac{\partial E(\beta)}{\partial\xi}=2C_F\Big[-E(\beta)\int_0^\beta dx\frac{\beta^2+x^2}{\beta^2(\beta-x)}$$
$$+\int_\beta^1 dx\,E(x)\frac{\beta^2+x^2}{x^2(x-\beta)}\Big]$$
$$-n_f\int_\beta^1 dx\,\mathscr{E}(x)\frac{x-2\beta}{x^2},$$
$$\frac{\partial\mathscr{E}(\beta)}{\partial\xi}=-\mathscr{E}(\beta)\int_0^\beta\frac{dx}{\beta}\Big[n_f\frac{x^2+(\beta-x)^2}{\beta^2}$$
$$+2C_Vx(\beta-x)\left(\frac{1}{x^2}+\frac{1}{\beta^2}\frac{1}{(\beta-x)^2}\right)\Big]$$
$$+4C_V\int_\beta^1 dx\,\mathscr{E}(x)\frac{\beta x+2(x-\beta)^2}{x^2(x-\beta)}-2C_F\int_\beta^1 dx\,E(x)\frac{2x-\beta}{x^2}. \qquad (51)$$

(Equations (51) pertain to the case $\beta>0$; at $\beta<0$ $\int_\beta^1 dx$ must be replaced in them, as well as in (50), by $\int_{-1}^{\beta}dx$.) Together with (9) and (4), Eqs. (47)–(51) determine in principle the dependence of the structure functions g_1 and g_2 on $\ln(\Lambda^2/m^2)=\ln(Q^2/m^2)$.

Taking in the evolution equations (47) and (48) the Laplace transforms with respect to the variables ξ and transforming from the functions A, Y, L, and C to their moments:

$$L_n{}^l=\int d\beta_1\,d\beta_2\,\beta_1^{l-1}\beta_2^{n-l-1}L(\beta_1,\beta_2),\quad A_n=\int d\beta\,\beta^nA(\beta),$$
$$Y_n{}^l=\int d\beta_1\,d\beta_2\,\beta_1{}^l\beta_2{}^{n-l}Y(\beta_1,\beta_2),\quad C_n=\int d\beta\,\beta^nC(\beta) \qquad (52)$$

[see also Eqs. (1), (6), and (15)], the equations become algebraic:

$$(n+1)A_n=E_n+nC_n+\sum_{k=1}^{n-1}(n-k)Y_n{}^k, \qquad (53)$$
$$\nu Y_n{}^l=-\frac{8C_F}{l(l+1)(l+2)}C_n+Y_n{}^l\Big\{3C_F+2\left(C_F-\frac{C_V}{2}\right)$$
$$\times\Big[\frac{2(-1)^l}{l(l+1)(l+2)}$$
$$-\frac{(-1)^{n-l}}{n-l+1}+\frac{1}{n}-S_l-S_{n-l}\Big]+C_V\Big[\frac{2}{l(l+2)}$$
$$-\frac{n+2}{(l+1)(n-l+1)}-2(S_l+S_{n-l})\Big]\Big\}$$
$$+\sum_{k=l+1}^{n-1}Y_n{}^k\Big\{2\left(C_F-\frac{C_V}{2}\right)(-1)^k$$
$$\times\Big[-\frac{(-1)^n}{n-l+1}C^{k-l}_{n-l-1}+(-1)^l\frac{C^k_{n-1}}{C^l_{n-1}}\frac{n+k-l}{n(k-l)}\Big]$$
$$+C_V\frac{(n-k)(n-k+1)}{(n-l)(n-l+1)(k-l)}\Big\}+\sum_{k=1}^{l-1}Y_n{}^k\Big\{2\left(C_F-\frac{C_V}{2}\right)(-1)^k$$

$$\times\left[\frac{2C_l^k}{l(l+1)(l+2)}+(-1)^l\frac{C_{n-1}^{k-1}}{C_{n-1}^{l-1}}\frac{n+l-k}{n(l-k)}\right]$$

$$+C_V\frac{(k+1)(k+2)}{(l+1)(l+2)(l-k)}\Bigg\}-2n_f\frac{1+(-1)^n}{n+1}\sum_{k=1}^{n-1}Y_n^k$$

$$\times(-1)^{l+k}\frac{C_{n-2}^{k-1}}{C_n^l}+\frac{1}{2}n_f\sum_{k=1}^{n-2}L_{n-1}^k\sum_{m=k+1}^{n-1}(-1)^m$$

$$\times\left[\frac{2(m-k)C_{n-l-1}^{n-m-1}}{m(m+1)(m+2)}+(-1)^n\frac{(m-k)}{m+2}C_l^{n-m-1}\right]; \quad (54)$$

$$\nu L_n^l=L_n^l\left[-\frac{1}{3}n_f+C_V\left(\frac{11}{6}-S_{l-1}-S_{l+1}-S_{l+3}\right)\right]$$

$$+C_V\Bigg\{\sum_{k=1}^{l-1}L_n^k\frac{(-1)^{l-k}}{n-k+1}\frac{C_{l-1}^{k-1}}{C_{n-k}^{n-l+1}}$$

$$+\sum_{k=1}^{l}L_n^k(-1)^k\frac{6C_{l+1}^{k+1}}{l(l+1)(l+2)(l+3)}$$

$$+\sum_{k=1}^{l-1}L_n^k\frac{(k+2)(k+3)}{(l+2)(l+3)(l-k)}\Bigg\}+C_V(1-(-1)^n)$$

$$\times\Bigg\{\sum_{k=1}^{n-l}Y_{n+1}^k\frac{(-1)^k}{l}C_{n-l-1}^{k-1}$$

$$-\sum_{k=l+2}^{n}Y_{n+1}^k(-1)^k\frac{2C_{n-l-1}^{k-l-2}}{l(l+1)(l+2)}$$

$$+\sum_{k=1}Y_{n+1}^k(-1)^k\frac{2C_{n-1}}{l(l+2)}\left[\frac{n}{(l+1)(n-k+1)}-\frac{6}{l+3}\right]\Bigg\}$$

$$+(l\leftrightarrow n-l); \quad (55)$$

$$\nu C_n=-4C_FC_nS_{n+1};\quad S_l=\sum_{k=1}^{l}\frac{1}{k}. \quad (56)$$

For the sake of completeness we present also the equations for the matrix elements of the twist-2 operators 4:

$$\nu E_n=C_FE_n\left(3+\frac{2}{n(n+1)}-4S_n\right)-n_f\frac{n-1}{n(n+1)}\mathscr{E}_n; \quad (57)$$

$$\nu\mathscr{E}_n=2\mathscr{E}_n\left[-\frac{1}{3}n_f+C_V\left(\frac{11}{6}+\frac{4}{n(n+1)}-2S_n\right)\right]$$

$$-2C_F\frac{n+2}{n(n+1)}E_n. \quad (58)$$

The notation in (54)–(57) is

$$S_n=\sum_{k=1}^{n}\frac{1}{k},\quad C_V=N,\quad C_F=\frac{N^2-1}{2N},$$

where N is the rank of the gauge group ($N=3$ for quantum chromodynamics). In the case of a nonsinglet channel the evolution equations coincide with (54) and (57) with $n_f=0$.

The number of equations in the system (53), (54) increases with increasing n, and with it also the degree of the secular equation that determines the anomalous dimensionalities ν_i. At $n=2$ Eq. (53) takes the form

$$\nu Y_2^1=-{}^4/_3C_FC_2+Y_2^1\left({}^1/_3C_F-3C_V-\frac{2}{3}n_f\right), \quad (59)$$

from which we have an anomalous dimensionality $\nu=-137/12$, $n_f=4$, which coincides with that obtained in Ref. 2. At $n=4$ we have

$$\nu Y_4^1=-\frac{1}{5}n_f(Y_4^1-2Y_4^2+Y_4^3)-\frac{17}{120}n_fL_3^1+(\nu Y_4^1)_{NS},$$

$$\nu Y_4^2=\frac{2}{15}n_f(Y_4^1-2Y_4^2+Y_4^3)-\frac{7}{120}n_fL_3^1+(\nu Y_4^2)_{NS},$$

$$\nu Y_4^3=-\frac{1}{5}n_f(Y_4^1-2Y_4^2+Y_4^3)+\frac{1}{40}n_fL_3^1+(\nu Y_4^3)_{NS},$$

$$\nu L_3^1=L_3^1\left(-\frac{2}{3}n_f-\frac{307}{60}C_V\right)-\frac{37}{30}C_VY_4^3$$

$$-\frac{7}{20}C_VY_4^2+\frac{23}{30}C_VY_4^1,$$

where NS denotes the contribution of the nonsinglet channel[7]:

$$(\nu Y_4^3)_{NS}=-\left(\frac{37}{60}C_F+\frac{25}{6}C_V\right)Y_4^3-\left(\frac{23}{10}C_F-\frac{7}{4}C_V\right)Y_4^2$$

$$+\frac{3}{10}C_FY_4^1-\frac{1}{15}C_FC_4,$$

$$(\nu Y_4^2)_{NS}=-\left(\frac{1}{6}C_F-\frac{5}{12}C_V\right)Y_4^3-\left(3C_F+\frac{41}{12}C_V\right)Y_4^2$$

$$-\left(\frac{7}{6}C_F-\frac{13}{12}C_V\right)Y_4^1-\frac{1}{6}C_FC_4, \quad (60)$$

$$(\nu Y_4^1)_{NS}=\left(C_F-\frac{5}{12}C_V\right)Y_4^3-\left(\frac{7}{2}C_F-\frac{9}{4}C_V\right)Y_4^2$$

$$-\left(\frac{7}{4}C_F+\frac{37}{12}C_V\right)Y_4^1-\frac{2}{3}C_FC_4.$$

The dependence of the structure functions $g_1(x)$ and $g_2(x)$ on ξ is given in the case of a singlet channel by the equations

$$g_1^n(\xi)=\sum_{i=1}^{2}C_ie^{\nu_i\xi},$$

$$g_1^n(\xi)+g_2^n(\xi)=\frac{1}{n+1}\sum_{i=1}^{2}C_ie^{\nu_i\xi}+\sum_{i=3}^{2n+2}C_ie^{\nu_i\xi}, \quad (61)$$

where ν_i are the anomalous dimensionalities of the twist-2 ($i=1,2$) and of the twist-3 ($3\leqslant i\leqslant 2n+2$) operators; we have neglected the contribution of the operator C, whose matrix element is proportional to the current-quark mass. To determine the coefficients C_i from experiment one must measure the moments $g_{1,2}^n$ at $2n+2$ values of the momentum transfers. It would be of interest to obtain these coefficients by rigorous theory using the quantum chromodynamics sum rules or from nonrelativistic quark models (cf. Ref. 2).

If experimental data are available for the $2n+2$ moments at different values of Q^2:$Q_1^2,\dots,Q_m^2$, $m>2n+2$, Eq. (61) can serve as a self-consistency check.

We note that the formalism developed above can be used to obtain equations similar to (47) and (48) above for twist-4 matrix operators.

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[1)]The importance of taking them into account was emphasized also in Ref. 11.

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