Low-temperature properties of a tunnel junction with an amorphous layer

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The low-temperature properties acquired by a tunnel junction between two normal metals with an amorphous layer as a result of the presence of two-level systems (TLS) in the latter are considered. A set of equations that describe the tunneling in this situation is derived. It is shown that besides the direct influence of the TLS on the tunneling electrons an important role may be played both by the reaction of these electrons on the TLS (which leads to disequilibrium of the latter) and by the interaction of the TLS with the electrons of the "banks," which is not accompanied by tunneling (and contributes to the relaxation of the TLS). The contribution is calculated of the inelastic tunneling with participation of the TLS; this contribution leads to nonlinearity of the current-voltage characteristic of the junction, a measure of this nonlinearity being the quantity d^2I/dV^2. It is shown that at low temperatures a nonlinearity of this type can predominate. The possibility of separating in experiment the contribution of a TLS group with a specified energy E, due to the effect of resonant saturation of the TLS in the presence of an alternating electric field of low amplitude and of frequency $\omega = E/\hbar$, is considered. The junction tunnel current and voltage fluctuations due to the presence of TLS whose spectral density is proportional to ω^{-1} are investigated. The relaxation phenomena that occur when the external voltage is turned off are considered. It is shown that the dependence of the relaxation-phonon flux as well as of the junction voltage on the time acquires a nonexponential contribution proportional to t^{-1}.

The study of the low-temperature properties of glasses has lately attracted considerable interest. This is due, on the one hand, to advances in experimental techniques at infralow temperatures, and on the other to the success of the model of two-level systems (TLS) with almost constant density of states,[1,2] which made possible an explanation of the main details of the behavior of glasses at low temperatures. The traditional research methods are in this case experiments on the interaction of glasses with microwave radiation as well as of with ultrasound (absorption and echo).

It was recently observed that TLS are responsible for the unique properties of metallic glasses, in that interaction of the TLS with the conduction electrons leads to transitions between levels and acts as a source of relaxation.[3] The existence of this interaction gives rise to the interesting possibility of using the disequilibrium in the electron system as a tool for the investigation of TLS. One of the systems in which such a disequilibrium is realized is a tunnel junction. It is well known that tunnel spectroscopy yields unusually abundant information on the properties of the electron and phonon subsystems and constitutes a rather well developed and accurate experimental technique. It is therefore of interest, in our opinion, to assess the feasibility of using this technique to study the properties of TLS. An appropriate situation can be realized, in particular, in a tunnel junction with "banks" of ordinary crystalline metal and with an amorphous layer. Study of such a system is important also from the viewpoint that in real tunnel junctions the barrier (e.g., an exide layer) can have an amorphous structure, so that it is necessary to ascertain the extent to which TLS peculiar to the amorphous state can influence the observable characteristics of the junction. We note in this connection a recent experiment[4] in which generation of monochromatic phonons was observed in a Josephson junction under conditions of the nonstationary Josephson effect, with the phonon and Josephson frequencies equal. One of the possible mechanisms proposed in Ref. 4 to explain this effect involves the assumption that the oxide barrier is glasslike and the TLS make a contribution.

In this paper we consider, within the framework of the TLS model,[1,2] certain phenomena that can occur in a tunnel junction (made up of normal metals) with an amorphous layer. We obtain first a system of equations for the interaction of the TLS with the tunneling electrons. We discuss also the interaction, not accompanied by tunneling, between the TLS and the bank electrons. It is found that this interaction is an additional source of TLS relaxation. The main result of this paper is an expression for the tunnel current in the system under consideration. We shall show, in particular, that the presence of TLS leads to the appearance of an additional contribution to the tunnel current, due to inelastic tunneling wherein the transition of the electron from one band to the other is accompanied by excitation or deexcitation of TLS. Such a phenomenon was considered earlier for the case of molecular excitations in a barrier[5]; it was established that the corresponding contribution to the current is not ohmic and can be discerned by analyzing the d^2I/dV^2 characteristics. We shall investigate this effect in detail as applied to the TLS situation, which has a host of peculiarities, and shall show that its experimental observation can yield information on a number of characteristics of the TLS. Although in the usual situation d^2I/dV^2 as a function of V has no singularities, since there is no noticeable dependence of the TLS density of states on the energy, an interesting experimental possibility exists of separating the contribution of a group of TLS with a specified energy E. This possibility is due to the resonant saturation of the TLS in the presence of a small-amplitude alternating current of frequency $\omega = E/\hbar$; in this

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case d^2I/dV^2 acquires a singularity at $V=\hbar\omega/e$. A large number of phenomena can be caused by an important feature of the TLS such as a broad relaxation-time spectrum. In particular, this circumstance manifests itself substantially in the current-fluctuation spectrum, and leads to an ω^{-1} law for the spectral density.[6] We consider also relaxation effects that occur when the junction voltage is turned off, viz., the relaxation-phonon flux and the voltage relaxation, and shall show that the TLS gives rise to long-time nonexponential "tails" in the corresponding dependences.

1. EQUATION SYSTEM THAT DESCRIBES A TUNNEL JUNCTION WITH AN AMORPHOUS LAYER

We use the tunnel-Hamiltonian method which, while approximate, is known on the one hand to describe the tunnel effects with sufficient accuracy, and on the other to present a lucid physical picture of the phenomena. Within the framework of this approach we express the total Hamiltonian of the system as

$$H=\sum_{\mathbf{p}}(\varepsilon_{\mathbf{p}}+eV)a_{\mathbf{p}l}{}^{+}a_{\mathbf{p}l}+\sum_{\mathbf{p}}\varepsilon_{\mathbf{p}}a_{\mathbf{p}r}{}^{+}a_{\mathbf{p}r}+\sum_{i}E_i\hat{S}_z{}^{i}+H_T{}^{0}+H_T{}'+H_{is}+\tilde{H}. \quad (1)$$

Here a^{+} and a are the second-quantization electron operators, the subscripts l and r correspond to the left and right banks of the junction, V is the potential difference across the junction. The subscript i labels the TLS, which are described[1,3] by the spin operators $\hat{S}_z^i$ and $\hat{S}_x^i$.[1-3] Next, $E_i=[(\Delta_i)^2+(\Delta_{0i})^2]^{1/2}$, where Δ_i and Δ_{0i} are TLS parameters, the asymmetry of the two-well potential and the tunnel matrix element, respectively. The operator H_{is} describes the interaction of the TLS with the phonons, as well as the interaction, not accompanied by tunneling, with the bank electrons (diagonal in the indices $\alpha=l, r$). The operator

$$H_T{}^{0}=\sum_{\mathbf{pp}',\alpha\neq\alpha'}T_{\mathbf{pp}'\alpha\alpha'}a_{\mathbf{p}\alpha}{}^{+}a_{\mathbf{p}'\alpha'} \quad (2)$$

corresponds to the usual tunneling processes, and the operator H'_T to the tunneling with participation of the TLS. The operator $\tilde{H}$ includes the remaining parts of the total Hamiltonian (the phonon Hamiltonian, the electron-phonon interaction, etc.).

Since our calculation is an estimate, we shall not analyze further the influence of the disorder in the amorphous layer on the tunneling, and confine ourselves to an account of the TLS contribution. This simplification is legitimate since, on the one hand, the effects considered by us are connected just with the TLS and, on the other, Lifshitz and Kirpichenkov[7] have shown that in the absence of resonant scattering (i.e., when the barrier contains no impurities with a level equal to the Fermi level of the metal), the disorder itself does not cause substantial changes in the character of the tunneling, and leads only to a certain renormalization of the transmittance of the barrier.

The operator H'_T is described by the expression (see the Appendix)

$$H_T{}'=\sum_{i,\mathbf{pp}',\alpha\neq\alpha'}[v^{\perp}_{\mathbf{pp}'\alpha\alpha'}\hat{S}_x{}^{i}a_{\mathbf{p}\alpha}{}^{+}a_{\mathbf{p}'\alpha'}+v^{\parallel}_{\mathbf{pp}'\alpha\alpha'}\hat{S}_z{}^{i}a_{\mathbf{p}\alpha}{}^{+}a_{\mathbf{p}'\alpha'}+u_{\mathbf{pp}'\alpha\alpha'}\hat{J}a_{\mathbf{p}\alpha}{}^{+}a_{\mathbf{p}'\alpha'}], \quad (3)$$

where $\hat{J}$ is a unit operator in "spin" space, and the matrix elements u and v satisfy the relations

$$v^{\perp}_{\mathbf{pp}'rl}=\frac{\Delta_{0i}}{E_i}K_{\mathbf{pp}'},\quad v^{\parallel}_{\mathbf{pp}'rl}=\frac{\Delta_i}{E_i}K_{\mathbf{pp}'},$$
$$u_{\mathbf{pp}'rl}=u^{*}_{\mathbf{p}'\mathbf{p}lr}\sim-T_{\mathbf{pp}'}\frac{\varkappa d}{W_0}\left(\frac{V_{1\mathbf{q}}+V_{2\mathbf{q}}}{2}\right), \quad (4)$$
$$K_{\mathbf{pp}'}=K^{*}_{\mathbf{p}'\mathbf{p}}\sim-T_{\mathbf{pp}'}\frac{\varkappa d}{W_0}(V_{1\mathbf{q}}-V_{2\mathbf{q}}),$$

$$\varkappa=\frac{1}{\hbar}\{2m[U_0-(\varepsilon_{\mathbf{p}}-\varepsilon_{\perp})]\}^{1/2},$$
$$W_0=\frac{\hbar^2\varkappa^2}{2m},\quad \mathbf{q}=\frac{1}{\hbar}(\mathbf{p}-\mathbf{p}'). \quad (5)$$

Here $\varepsilon_{\perp}$ is the kinetic energy of the motion parallel to the barrier, V_{1q} and V_{2q} are the Fourier components of the TLS in the barrier and correspond to the location of the TLS in the particular well of the two-well potential (and normalized to the unit-cell volume), and d is the layer thickness.

We define the tunnel current as $I=-AdN_l/dt$, where $N_l=\Sigma_{\mathbf{p}}f_{\mathbf{p}l}$ and $f_{\mathbf{p}l}$ are respectively the density and the distribution function of the electrons on the left bank of the junction, and A is the junction area. To calculate the current it suffices therefore to find $f_{\mathbf{p}l}$. To obtain the corresponding kinetic equation, as well as equations for the spin components Si that describe the TLS, we use the correlation-uncoupling method, regarding the operator H'_T as a perturbation. This approach is justified because the perturbation contains an essential small quantity (due to the low transmittance of the barrier) and the lowest approximation is sufficient.

Following the indicated standard procedure we obtain for the mean value $f_{\mathbf{p}l}=\langle\hat{\rho}a_{\mathbf{p}l}{}^{+}a_{\mathbf{p}l}\rangle$ ($\hat{\rho}$ is the statistical operator of the system (cf. Ref. 8):

$$\left[\frac{df_{\mathbf{p}l}}{dt}\right]_s=\frac{1}{\hbar}\sum_{\mathbf{p}',i}|K|^2\frac{\pi}{8}\left(\frac{\Delta_{0i}}{E_i}\right)^2\times\{f_{\mathbf{p}'}(1-f_{\mathbf{p}})[\delta(\varepsilon_{\mathbf{p}}+eV-\varepsilon_{\mathbf{p}'}+E_i)n_i^{-}$$
$$+\delta(\varepsilon_{\mathbf{p}}+eV-\varepsilon_{\mathbf{p}'}-E_i)n_i^{+}]-f_{\mathbf{p}}(1-f_{\mathbf{p}'})[\delta(\varepsilon_{\mathbf{p}}+eV-\varepsilon_{\mathbf{p}'}+E_i)n_i^{+}$$
$$+\delta(\varepsilon_{\mathbf{p}}+eV-\varepsilon_{\mathbf{p}'}-E_i)n_i^{-}]\}+\frac{\pi}{\hbar}\sum_{\mathbf{p}',i}\left\{\delta(\varepsilon_{\mathbf{p}}+eV-\varepsilon_{\mathbf{p}'})(f_{\mathbf{p}'}-f_{\mathbf{p}})\right.$$
$$\times\left[\left|T+u+\frac{1}{2}\frac{\Delta_i}{E_i}K\right|^2n_i^{+}+\left|T+u-\frac{1}{2}\frac{\Delta_i}{E_i}K\right|^2n_i^{-}-|T|^2\right]\Bigg\}$$
$$-\frac{1}{\hbar}\sum_{\mathbf{p}',i}\left\{\langle S_x{}^{i}\rangle\left[|K|^2\frac{\pi}{2}\frac{\Delta_i\Delta_{0i}}{E_i^2}[\delta(\varepsilon_{\mathbf{p}}+eV-\varepsilon_{\mathbf{p}'}-E_i)-\delta(\varepsilon_{\mathbf{p}}+eV-\varepsilon_{\mathbf{p}'}+E_i)]\right.\right.$$
$$\left.\times(f_{\mathbf{p}'}+f_{\mathbf{p}}-2f_{\mathbf{p}}f_{\mathbf{p}'})-2i\pi\delta(\varepsilon_{\mathbf{p}}+eV-\varepsilon_{\mathbf{p}'})(f_{\mathbf{p}'}-f_{\mathbf{p}})2\frac{\Delta_{0i}}{E_i}\mathrm{Re}(Ku^{*})\right]$$
$$\left.+\langle S_y{}^{i}\rangle C\frac{\Delta_i\Delta_{0i}}{E_i^2}|K|^2(1-2f_{\mathbf{p}})\ln\left|1-\frac{E_i^2}{(\varepsilon_{\mathbf{p}'}-\varepsilon_F+eV)^2}\right|\right\},$$
$$C\sim1. \quad (6)$$

Here

$$n_i^- = {}^1/_2(1-2\langle S_z^i\rangle),\quad n_i^+ = {}^1/_2(1+2\langle S_z^i\rangle)$$

are the occupation numbers of the TLS levels; the notation [...] indicates that we have taken into account only that change of $f_{\mathbf{p}\,l}$ which is due to tunneling with participation of TLS; ε_F is the Fermi energy. The first sum in (6) has the form of the usual integral of the inelastic collisions that are accompanied by transitions to TLS. The second sum is the contribution of the processes not accompanied by transitions to the TLS, i.e., it is the increment to the usual "elastic" tunneling. It does not introduce a nonlinear dependence $I(V)$ and is small compared with the contribution of ordinary tunneling. Nonetheless, its allowance can be significant in the analysis of the tunnel-current fluctuations, which will be discussed below. Finally, the last sum in (6) contains the contribution of the spin components $\langle S_x\rangle$ and $\langle S_y\rangle$, which take into account the coherence of the TLS states. Effects connected with these components can be substantial only in the presence of a certain coherent excitation of the TLS. Since $\langle S_x\rangle$, $\langle S_y\rangle \propto \exp(iE_i t/\hbar)$, a contribution to the stationary current can appear here only in the next order in the corresponding perturbation.

Since the inelastic tunneling of interest to us depends substantially on the TLS states, the question can arise: To what degree can the reaction of the tunneling on these states, namely, the excitation of TLS by the tunneling electrons? To answer this question, we derive also a system of equations for the spin components with allowance for their interaction with the electrons. Proceeding as before, we obtain within the framework of the correlator-splitting method,

$$\frac{d\langle S_x^i\rangle}{dt} = -\frac{1}{\hbar}\left\{E_i\langle S_y^i\rangle - \sum_{\mathbf{pp}'}|K|^2 \times\left[\frac{\pi}{4}\frac{\Delta_i\Delta_{0i}}{E_i^2}Q - \frac{\pi}{2}\frac{\Delta_i^2}{E_i^2}P\langle S_x^i\rangle\right]\right\} - \{S_x^i\}_r, \tag{7a}$$

$$\frac{d\langle S_y^i\rangle}{dt} = -\frac{1}{\hbar}\left\{-E_i\langle S_x^i\rangle + \sum_{\mathbf{pp}'}|K|^2\left[\frac{\pi}{2}\frac{\Delta_i^2}{E_i^2}P\langle S_y^i\rangle + \pi\frac{\Delta_{0i}^2}{E_i^2}P_0\langle S_y^i\rangle\right]\right\} - \{S_y^i\}_r, \tag{7b}$$

$$\frac{d\langle S_z^i\rangle}{dt} = -\frac{1}{\hbar}\left\{\sum_{\mathbf{pp}'}|K|^2\left[\frac{\pi}{4}\frac{\Delta_{0i}^2}{E_i^2}Q - \frac{\pi}{2}\frac{\Delta_i\Delta_{0i}}{E_i^2}P\langle S_x^i\rangle\right]\right\} - \{S_z^i\}_r, \tag{7c}$$

$$\begin{aligned}Q = {} & n_i^+[\delta(\varepsilon_\mathbf{p}+eV-\varepsilon_{\mathbf{p}'}-E_i)f_{\mathbf{p}'}(1-f_\mathbf{p}) + \delta(\varepsilon_\mathbf{p}+eV-\varepsilon_{\mathbf{p}'}+E_i)f_\mathbf{p}(1-f_{\mathbf{p}'})] \\ & - n_i^-[\delta(\varepsilon_\mathbf{p}+eV-\varepsilon_{\mathbf{p}'}-E_i)f_\mathbf{p}(1-f_{\mathbf{p}'}) + \delta(\varepsilon_\mathbf{p}+eV-\varepsilon_{\mathbf{p}'}+E_i)f_{\mathbf{p}'}(1-f_\mathbf{p})],\end{aligned}$$

$$P = [\delta(\varepsilon_\mathbf{p}+eV-\varepsilon_{\mathbf{p}'}-E_i) + \delta(\varepsilon_\mathbf{p}+eV-\varepsilon_{\mathbf{p}'}+E_i)]\,(f_\mathbf{p}+f_{\mathbf{p}'}-2f_\mathbf{p}f_{\mathbf{p}'}),$$

$$P_0 = P(E_i=0).$$

The terms $\{S\}_r$ describe the TLS relaxation due to their interaction with the phonons and electrons of the banks, and also (in the case of S_x or S_y) with other TLS[9] (i.e., the contribution of the operator H_{is}). We note that in the derivation of (7) there appear, besides the terms proportional to δ functions, also contributions of the principal values of the integrals. It can be shown that the appearance of these contributions is equivalent to introducing into the Hamiltonian of the noninteracting TLS a term of the order of

$$(\alpha+\beta)\left(\frac{\Delta_i}{E_i}\hat{S}_z + \frac{\Delta_{0i}}{E}\hat{S}_x\right),$$

where

$$\alpha \sim \Delta_i\frac{|K|^2}{\varepsilon_F^2}\ln\left(\frac{E_i}{\varepsilon_F}\right),\quad |\beta| \sim \frac{|K|\,|T+u|}{\varepsilon_F},$$

i.e., to renormalization of the initial assummetry of the wells Δ_i by the amount $\alpha+\beta$. In the situation considered this renormalization can be regarded as small to the extent that the tunnel transmittance is small. At any rate, we shall assume below that this renormalization has already been taken into account in the initial Hamiltonian.

We discuss, finally, the question of the interaction, not accompanied by tunneling, of a TLS with the bank electrons in a dielectric. Such an interaction can serve as an additional source of relaxation, and an estimate of the corresponding contribution is thus necessary for the analysis of the processes connected with the disequilibrium of the spin system S^i. Several mechanisms of such an interaction can be indicated. The first—"contact"—is due to the fact that the electron wave function is not equal to zero at the location of the TLS. It is easily seen that an additional small factor $\sim |K|/\varepsilon_F$ appears here compared with the estimate of the interaction typical of metallic glasses. This mechanism can be described by equations of the type (7) by putting in them $V=0$.

The other—"long-range"—mechanisms are due to the fact that the TLS (which has the properties of an electric and elastic dipoles) produces in the medium an electric field and a strain field, both of which fall off with distance and reach the metallic banks. For an order-of-magnitude estimate of the contribution of the strain interaction we recognize that the strain ξ produced by a TLS at a distance r from it can be estimated at

$$\xi \sim 2a^3\lambda\left(\frac{\Delta_{0i}}{E_i}\langle S_x\rangle + \frac{\Delta_i}{E_i}\langle S_z\rangle\right)(\tilde{C}r^3)^{-1}. \tag{8}$$

Here $\tilde{C} = Ca^3$, where C is the elastic modulus, λ is the strain potential of the TLS, and a is the lattice constant. Neglecting for simplicity the difference between the elastic properties of the metal and the barrier, we write down the energy of the interaction of the TLS with the electron at a point $\mathbf{r}$ in the form $\Lambda(\mathbf{p})\xi(\mathbf{r})$, where $\Lambda(\mathbf{p})$ is the strain potential of the electrons and depends on the momentum $\mathbf{p}$. This interaction can be taken into account by introducing into the Hamiltonian the term

$$\sum_{kk'i} U_{kk'}a_k^+a_{k'}\left(\frac{\Delta_{0i}}{E}\hat{S}_x^i + \frac{\Delta_i}{E_i}\hat{S}_z^i\right),$$

$$U_{kk'} \sim \frac{\lambda a^3}{\tilde{C}}\int d^3r\,\frac{\Delta(k)}{r^3}\psi_k^*(\mathbf{r})\psi_{k'}(\mathbf{r}). \tag{9}$$

The integration in (9) is over the half-space occupied by the metal. We have taken into account here the fact that in the analysis of the processes not connected with tunneling an important role can be assumed by the deviation of the wave function k of the electron in the half-space from a plane wave (in the case of specular reflection from the barrier, the index k corresponds to $(\mathbf{p}_\parallel, |\mathbf{p}_\perp|)$, where $\mathbf{p}_\parallel$ and $\mathbf{p}_\perp$ are respectively the momentum components in the barrier plane and in the plane perpendicular to it). We assume in addition that the distance r_0 that separates the TLS from the bank is comparable with the barrier thickness, so that the conditions $pr \gg \hbar$, $|p-p'| \ll p$ are satisfied in (9).

We turn now to the contribution of the electric interaction. Inasmuch as in a metal the electric field is screened at distances $\sim\hbar/p_F$, its influence on the electron reduces to a certain modification of the surface energy barrier. The characteristic value of the increment to the potential inside the metal (an increment localized near the surface) is of the order of $\mathscr{E}_d(\hbar/p_F)$, where $\mathscr{E}_d$ is the field in the dielectric. Estimates show that the contribution of the interaction of the TLS with the electrons, due to such a surface potential, contains in comparison with the contribution of the strain interaction, at any rate, an extra small factor a/r_0. We note that at nonzero spin components $\langle S_x \rangle$ and $\langle S_y \rangle$ the TLS excites around itself likewise an alternating electric field having a frequency $E_i/\hbar$ and capable of penetrating into the metal to a large (skin) depth. In this case, however, an additional small factor appears, $(E_i/\hbar\sigma)^{1/2}$, where σ is the conductivity of the metal. We shall therefore neglect hereafter the contribution of the electric interaction of the TLS with the bank electrons.

On the basis of (9), the electron contribution to the TLS relaxation can be reduced to the form

$$\{S_x^i\}_r^e \sim \frac{1}{\tau_e^i E_i}\left[\int d\varepsilon\, d\varepsilon'\left(-\frac{\pi}{4}\frac{\Delta_i\Delta_{0i}}{E_i^2}\tilde{Q}+\frac{\pi}{2}\frac{\Delta_i^2}{E_i^2}\langle S_x^i\rangle\right)\right],$$
$$\{S_y^i\}_r^e \sim \frac{1}{\tau_e^i E_i}\left[\int d\varepsilon\, d\varepsilon'\left(\frac{\pi}{2}\frac{\Delta_i^2}{E_i^2}\tilde{P}\langle S_y^i\rangle+\pi\frac{\Delta_{0i}^2}{E_i^2}\tilde{P}_0\langle S_y^i\rangle\right)\right],$$
$$\{S_z^i\}_r^e \sim \frac{1}{\tau_e^i E_i}\left[\int d\varepsilon\, d\varepsilon'\left(\frac{\pi}{4}\frac{\Delta_{0i}^2}{E_i^2}\tilde{Q}-\frac{\pi}{2}\frac{\Delta_i\Delta_{0i}}{E_i^2}\tilde{P}\langle S_x^i\rangle\right)\right], \tag{10}$$

where

$$\tau_e^i \sim \frac{\Lambda^2\lambda^2a^3E_i}{C^2\varepsilon_F^2r_0^3\hbar},\quad \tilde{P},\tilde{Q},\tilde{P}_0=(P,Q,P_0)|_{V=0}.$$

Estimates show that at $r_0 \sim a$ the time τ_e is comparable in order of magnitude with the corresponding value for metallic glass: At $E_i \sim 10^{-16}$ erg and $r_0 \sim a$ we have $(\tau_e)^{-1} \sim 10^9$ sec^{-1}. At $E_i \lesssim 0.25\cdot 10^{-16}$ erg, therefore, even at $r_0 \sim 10a$, the contribution of the bank electrons to the TLS relaxation can exceed the phonon contribution, which is equal to

$$\tau_{ph}^{-1}|_{\Delta_i=\Delta_{0i},T=0} \sim \frac{\lambda^2E_i^2}{2\pi C\hbar^4s^3},$$

where s is the speed of sound.

We can now estimate the disequilibrium created in the spin system by the tunneling electrons. It cna be discerned, first, that in the stationary situation $\langle S_x \rangle, \langle S_y \rangle = 0$. Indeed, the vanishing of the right-hand side of (7c) ensures also the vanishing of the right-hand side of (7a), since the nonequilibrium increments to $\langle S_z \rangle$ enter both in like fashion—via the combinations Q and $\tilde{Q}$. At equilibrium we have

$$\langle S_z^i\rangle \equiv S_0 = {}^1/_2(1-2F_0(E_i)) = {}^1/_2\,\mathrm{th}\,(-E_i/2).$$

At $V \neq 0$ the degree of disequilibrium is defined by the ratio

$$\nu = E_i|K|^2\varepsilon_F^{-2}/\max\,(\hbar/\tau_e,\ \hbar/\tau_{ph}).$$

We note that we always have $\nu < 1$, since τ_e^{-1}, generally speaking, includes a contribution of the "contact" interaction with the electrons, of the order of $(E_i/\hbar)(K/\varepsilon_F)^2$. If $\nu \ll 1$, the TLS system can be regarded as at equilibrium when calculating the tunnel current. The nonequilibrium increment to $\langle S_z \rangle$ can be obtained by substituting in Q the relation $\langle S_z \rangle = S_0$. At $T=0$ it can be estimated at

$$(\delta S_z)_e \sim \nu\frac{(eV-E_i)}{E_i}\theta(eV-E_i), \tag{11}$$

where $\theta(x>0)=1$; $\theta(x<0)=0$.

2. CONTRIBUTION OF INELASTIC TUNNELING TO THE TUNNEL CURRENT

To calculate the contribution to the tunnel current by the inelastic tunneling we shall use Eq. (6), in which we substitute, in accord with the foregoing, $\langle S_x \rangle, \langle S_y \rangle = 0$. Integrating with respect to d^3p we have

$$I^1 \sim \frac{G}{e}\sum_i \sigma_i[\Phi(E_i,V)-\Phi(E_i,-V)]\left(\frac{\Delta_{0i}}{E_i}\right)^2, \tag{12}$$

where

$$\Phi=(E_i+eV)[\tilde{F}(-E_i-eV)n_i^+ + \tilde{F}(E_i+eV)n_i^-],$$
$$\tilde{F}(x)=[1-\exp(x/T)]^{-1}.$$

The quantity

$$\sigma_i \sim \pi a^2\left(\frac{\varkappa d}{W_0}\right)^2|V_{1q}-V_{2q}|^2$$

has the meaning of the cross section for electron scattering by the TLS potential[1)]; $G = dj_0/dV$, where j_0 is the tunnel-current density in the absence of the TLS. Transforming to the TLS distribution function in the parameter[10] $\rho = (\Delta_0/E)^2$, we have

$$I^1 \sim \frac{G}{e}A\,d\bar{\sigma}\int_0^\infty dE\int_0^1 d\rho\,\rho P(\rho)[\Phi(E,V)-\Phi(E,-V)], \tag{13}$$

where $\bar{\sigma}$ is the mean value and $P(\rho) = \bar{P}(1-\rho)^{-1/2}\rho^{-1}$. We assume that σ_i does not correlate with E_i and Δ_{0i}. To estimate this contribution, we compare it with the ohmic current $I_0 = GVA$ of the junction. At $d \sim 3\times10^{-7}$ cm, $\bar{\sigma} \sim 10^{-15}$ cm^2, $V \sim 3\times10^{-4}$ V, and $\bar{P} \sim 10^{33}$ erg$^{-1}\cdot$cm^{-3} (Ref. 11) we have $I^1/I_0 \sim 10^{-4}$.

It is important that the contribution I_1 is not ohmic. For its second derivative we obtain the expression

$$\frac{d^2I^1}{dV^2}=eGA\,d\bar{\sigma}\mathscr{F}(V);\quad \mathscr{F}=\int_0^1 d\rho\,\rho P(\rho)|_{E=eV},\quad eV \gg T, \tag{14}$$

$$\mathscr{F}=\int_0^\infty dE\int_0^1 d\rho\rho\frac{\partial^2\Phi}{\partial(eV)^2}\sim\frac{1}{T}\int_0^1 d\rho\,\rho P(\rho),\quad eV\ll T.$$

Thus, by studying the derivative d^2I/dV^2 we can estimate the TLS density. Moreover, it can be seen from (14) that if the distribution P (and hence the function $\mathscr{F}$) contains a certain dependence on E, it manifests itself directly in the dependence of this derivative on V, so that tunnel spectroscopy can be used to study the possible deviation of the TLS density of states from a constant.

We compare now the considered contribution to d^2I/dV^2 with the contributions of the other possible mechanisms of nonohmic origin. First of all, the inelastic tunneling can be due also to phonons. An estimate of the corresponding contribution to the current yields[2)]

$$I_{ph}\sim GVA\left(\frac{eV}{\varepsilon_F}\right)\left(\frac{eV}{\Theta}\right)^2\left(\frac{d}{a}\right)^2\left(\frac{eVd}{\hbar s}\right)^2,$$

where Θ is the Debye temperature. It can be seen that the $I(V)$ dependence turns out to be substantially stronger than in the case of the TLS, owing to the rapid decrease of the phase volume of the phonons that participate in the considered processes with decreasing V. The phonon contribution then becomes comparable with that of the TLS at eV on the order of several degrees. Another source of deviations from Ohm's law can be the energy dependence of the tunneling probability. As shown in Ref. 12, the contribution to the conductivity is determined in this case by the parameter $(eV\varkappa d/W_0)^2$ and is negligible at small V. Furthermore, a contribution of this type should not depend significantly on the temperature at eV $\lesssim T$, in contrast to the TLS contribution.

Finally, there exists in our opinion one more possibility of experimentally isolating the TLS contribution (12)–(14) to the current-voltage characteristic of the junction. This possibility is connected with a unique feature of the TLS—equalization of the populations of the levels in the case of resonant interaction with an alternating perturbation (sound or electric field) of rather small amplitude (saturation). Assume, thus, that besides the constant voltage V there is applied to the junction also an alternating signal $V_m\cos(\omega t)$ with frequency $\omega\sim eV/\hbar$. The direct interaction of this field with the TLS is described by introducing into the Hamiltonian the terms

$$\left(\mu_i\frac{eV_m}{d}\right)\cos(\omega t)\left[\frac{\Delta_{0i}}{E_i}\hat{S}_x+\frac{\Delta_i}{E_i}\hat{S}_z\right],$$

where μ_i is in the TLS dipole moment. Proceeding in standard fashion (cf. Ref. 11) we can show that the presence of this perturbation leads to the following expression for $\langle S_z\rangle$:

$$\langle S_z^i\rangle=S_0^i\left\{1+\frac{\tau_1\tau_2}{\hbar^2}\left(\mu\frac{V_m}{d}\frac{\Delta_{0i}}{E_i}\right)^2\times\left[\frac{4\omega^2}{(\omega^2-E_i^2/\hbar^2)^2\tau_2^2+4\omega^2}\right]\right\}^{-1}.\qquad(15)$$

Here τ_1 and τ_2 are respectively the relaxation times of the longitudinal $\langle S_z\rangle$ and of the transverse $\langle S_x\rangle$ and $\langle S_y\rangle$ spin components.[9] It can be readily seen from (15) that in the resonance case $\omega=E_i/\hbar$ the decrease of $\langle S_z\rangle$ begins at very small $V_m\sim V_m^0\equiv[(\mu/d\hbar)\ (\tau_1\tau_2)^{1/2}]^{-1}$; at $\mu\sim10^{-18}$ erg$^{1/2}\cdot$cm$^{3/2}$, $\tau_1\tau_2\sim10^{-14}$ sec^2, and $d\sim3\times10^{-7}$ cm we have $V_m^0\sim10^{-6}$ V. We note that so low an alternating voltage in the absence of TLS does not influence substantially the stationary tunnel current. Estimates show that the corresponding corrections to the ohmic part of the tunnel current contain the small quantity $(V_m/\hbar\omega)^2$, and the contribution nonlinear in V is $\sim V_m^2(\hbar\omega\Delta\varepsilon)^{-1}$, where $\Delta\varepsilon$ is the characteristic scale of the change of the density of states (cf. Ref. 13).[3)]

As a result, at $V_m>V_m^0$ the quantity $\langle S_z\rangle$ and hence the occupation numbers n^+ and n^- acquire as functions of E strong singularities at $E_i=\hbar\omega$, viz, $n^+(E_i=\hbar\omega)=n^-=1/2$. The width of these singularities is determined by the quantity $(\Delta E)_0\sim(\hbar/\tau_2)(V_m/V_m^0)$. Substituting the obtained distribution in (12), we can show that at $T=0$ the derivative d^2I/dV^2 vanishes at eV $=\hbar\omega$. Thus, in the absence of thermal broadening the presence of a weak alternating signal manifests itself for the value of this derivative in the form of a strong steep negative peak of width $\Delta V=(\Delta E)_0/e$ against the background of the smooth plot of (14); its "depth" is determined by the right-hand side of (14). It can be easily seen that at finite temperatures $T>(\Delta E)_0$ the peak broadens, $\Delta V\sim T$, and its amplitude decreases by a factor $T/(\Delta E)_0$. In this situation the peak amplitude can be reached by increasing the energy width ΔE of the TLS saturation band, say by broadening the spectrum of the saturating signal $\Delta\omega$. To preserve the saturation efficacy, this broadening should, naturally, be accompanied by an increase of the integral signal intensity $\overline{V_m^2}\sim V_m^{02}\Delta\omega\tau_2$.

It seems to us that observation of such a peak, suitably related to T, V_m, and V, could serve as experimental proof of the presence of TLS, and its study would allow the isolation of the contribution of the TLS against the background of the possible masking effects and obtain information on such parameters as $\bar{P}$, τ_2, and τ_1.

3. EFFECTS AT LARGE BIAS

We have discussed above the case when the junction bias is small, on the order of the TLS level spacing. It is known that the TLS model was originally suggested[1,2] by the notion that glass contains two-well potentials with random parameters. Besides the "ground" levels in the wells that make up the TLS there exist also excited ones. These, in particular, can lie above the barrier, so that the transition from one well to another does not call for tunneling.

Strictly speaking, as shown by Gurevich and Parshin,[14] at sufficiently high excitation energies, exceeding $E_0\sim(10$–$30)\cdot10^{-16}$ erg, the TLS model itself becomes invalid and a rigorous theoretical analysis of the phenomena connected with such energies is difficult. Recently, however, Parshin and Karpov[15] proposed an approach that constitutes, in essence, a generalization of the Anderson model and its extension to include higher energies. This approach permits, in particular, discussion of the properties of glasses at relatively high temperature 10–50 K, and the predictions of the theory are in qualitative agreement with the existing experimental data.

It seems to us that experiments on inelastic tunneling through an amorphous layer might yield additional information on the distribution of "soft" interatomic potentials, as well as on their structure (particularly on the location of the third level). Such data would be of great value for a comparison with the predictions of the theory[15] and for the determination of the parameters of the theory. Although in our analysis we used essentially the two-level model of Refs. 1 and 2, an expression such as (14) (in the $eV \gg T$ limit) can apparently be used also at sufficiently large $eV > E_0$, and the function $\mathscr{F}(V)$ describes in this case the total density of the excited states in the system. Indeed, Eq. (14) reflects the fact that the inelastic contribution to the tunnel current increases with increasing V both on account of "inclusion" of new systems, and s subsequently on account of processes with participation of the third and following excited levels. What is important is that in the latter case a contribution can be made also by systems with high and broad barriers (when the tunneling between the wells is weak and the relaxation time is long).

We wish to call attention here to one more effect that can manifest itself in tunnel junctions of sufficiently high resistance. Strictly speaking, for this effect the tunneling itself is a masking factor; we assume therefore initially that the layer is sufficiently thick and that we are dealing in fact with a capacitor whose dielectric contains TLS. In the presence of an electric field $\vec{\mathscr{E}}$ the interaction of the ith TLS with this field is described by the operator

$$2\mu_i\vec{\mathscr{E}}\left(\frac{\Delta_i}{E_i}\hat{S}_z^i+\frac{\Delta_{0i}}{E_i}\hat{S}_x^i\right).$$

If the field is static, the equilibrium level-population difference takes, when this interaction is taken into account, the form

$$n^+-n^-=2\langle S_z\rangle=\text{th}\left[\left(E_i+\mu_i\frac{\Delta_i}{E_i}\mathscr{E}\right)\Big/T^{-1}\right]. \qquad (16)$$

The field is then at equilibrium also with systems having sufficiently long relaxation times. We point out that corresponding to the distribution (16) is a certain dipole moment 2 $\mu_i\langle S_z^i\rangle$ containing a contribution that is not made to vanish by summation over i. If $\mu\mathscr{E} < T$, this total dipole moment is equal to

$$\overline{M}=\sum_i\mu_i^2\left(\frac{\Delta_i}{E_i}\right)^2\frac{\mathscr{E}}{T}\text{ch}^{-2}\left(\frac{E_i}{T}\right). \qquad (17)$$

Assume now that the field is turned off jumpwise at $t=0$. The distribution (16) begins to relax to the equilibrium value at $V=0$, and this relaxation is described by the law $\exp(-t/\tau_{1i})$. For the summary dipole moment (17) we obtain thus the expression

$$\overline{M}(t>0)$$
$$=d\mathscr{E}A\int_0^\infty dE\,\text{ch}^{-2}\left(\frac{E}{T}\right)\int d\rho\,(1-\rho)^{1/2}P(\rho)\mu^2\exp\left[-\frac{t}{\tau_1(\rho)}\right]. \qquad (18)$$

Since the presence of the layer dipole moment manifests itself in the form of a junction voltage $\widetilde{V}(t)=(4\pi/\varepsilon_0)\overline{M}(t)/A$ (ε_0 is the dielectric constant of the layer), a study of the $V(t)$ dependence makes it possible in principle to study the TLS relaxation processes all the way to systems with very weak tunneling. In this case $\widetilde{V}(t)$ is a measure of the number of TLS with $\tau_1 \gtrsim t$.

We have so far neglected in our arguments the presence of a tunnel current, i.e., of tunnel conductivity G. In the study of $\widetilde{V}(t)$ this neglect is justified only for times $t<t_0\sim\varepsilon_0/4\pi dG$. At $t>t_0$ the short-circuiting of the junction by the tunnel current becomes substantial, and the effect becomes unobservable.

4. FLUX ON RELAXATION PHONONS FROM THE LAYER

We have shown earlier that the tunneling electrons can excite TLS and produce a nonequilibrium distribution describable at $T=0$ by expression (11). The excited TLS relaxes after a time $\tau_1=\min(\tau_{\text{ph}},\tau_e)$ on account of emission of a phonon or else, if the TLS is close enough to the metallic bank, by interaction with the electrons. In the latter case, however, the energy is transferred to the phonon in final analysis also within electron-phonon relaxation times $\tau_{e-ph}\sim\Theta^2/E^3$, which we assume to be much shorter than τ_1. Thus, the presence of disequilibrium in the spin system manifests itself, in particular, in the appearance of a flux of relaxation phonons. In the stationary situation this flux cannot be isolated from the background of the flux due to the relaxation of the tunneling electrons. However, if the junction voltage is turned off abruptly enough,[4)] the flux of the phonons due to relaxation in the electron system terminates after a time[5)] $t\sim\tau_{e,\,\text{ph}}$, whereas the phonons due to the TLS relaxation will be emitted up to very long times connected with the slowly relaxing TLS. In this case we must also take into account, generally speaking, the relaxation of the TLS disequilibrium due to their "polarization" in the electric field, the polarization considered in the preceding section. It can be readily seen from (16) that the corresponding nonequilibrium (after turning off the field) part of $\langle S_z\rangle$ can be estimated for $\mu\mathscr{E}\ll T$ at

$$(\delta S_z^i)_{\mathscr{E}}\sim\left(\frac{\mu_i V}{dT}\frac{\Delta_i}{E_i}\right)^2 2\,\text{th}\left(\frac{E_i}{T}\right)\text{ch}^{-2}\left(\frac{E_i}{T}\right). \qquad (19)$$

For the energy flux due to relaxation phonons we have

$$\mathscr{P}_{ph}\approx\frac{1}{A}\sum_i E_i\frac{d(\delta S_z)}{dt}=d\int_0^\infty dEE(\delta S)\int_0^1 d\rho P(\rho)\frac{\rho}{\tau}e^{-t\rho/\tau},$$

$$\tau=\tau_1\rho, \qquad (20)$$

where $(\delta S)=(\delta S)_e+(\delta S)_{\mathscr{E}}$. Recognizing that the main contribution is made by TLS with $\rho\ll1$, we arrive at the estimate

$$\mathscr{P}_{ph}\sim\frac{d\overline{P}}{t}\int_0^\infty dEE(\delta S). \qquad (21)$$

Thus, this phonon flux falls off slowly with time like t^{-1}. Under conditions of sufficiently strong disequilibrium, when $(\delta S)\sim1/3$, $\overline{P}\sim10^{33}\,\text{erg}^{-1}\cdot\text{cm}^{-3}$, and $d\sim3\times10^{-7}$ cm, at typical values $E\sim10^{-16}$ erg, we have $\mathscr{P}_{ph}\sim(10^{-7}\,\text{W/cm}^2)\,10^{-6}\,\text{c}/t$.

5. FLUCTUATIONS INDUCED IN TUNNEL JUNCTIONS BY THE PRESENCE OF TLS

We have shown above that the presence of TLS in the dielectric layer of a tunnel junction influences a large number of effects, and the corresponding contributions to the physical quantities of interest to us (particularly to the tunnel current) are expressed in terms of the components $\langle S_z^i \rangle$ of the spins of the individual TLS, i.e., in terms of the occupation numbers of the levels. Thus, the fluctuations (due to transitions into TLS) of occupation numbers will lead ultimately to fluctuations of the observable physical parameters. It is important to note that, as indicated by Kogan and Nagaev,[6] the TLS can serve as a source of $1/f$ noise by virtue of the large scatter of the relaxation times.

We consider in the present section the fluctuations produced by the TLS in the tunnel current and voltage of the glass-layer tunnel junction considered by us. We shall assume that the transitions take place independently in the different TLS,[6)] and consequently the fluctuations of the occupation numbers n_i^- are statistically independent. With allowance for the foregoing, using the standard approach,[16] we obtain the following expression for the correlator of the fluctuations δn_i:

$$(\delta n_i^-, \delta n_j^-)_\omega = \frac{2}{\tau_1} \frac{n_0(1-n_0)}{\omega^2+1/\tau_1^2} \delta_{ij}, \quad n_0 = F_0(-E_i). \qquad (22)$$

We can now calculate the correlator $(I, I)_\omega$ of the junction-current fluctuations due to the TLS. The most interesting contribution to $(I, I)_\omega$ is due to elastic processes, since these are not accompanied by transitions, and slowly relaxing TLS also participate in them effectively. Turning to (6), we represent the corresponding part of the current in the form

$$I_e^1 \sim j_0 \sum_i (\sigma_i^+ n_i^+ + \sigma_i^- n_i^-), \qquad (23)$$

where j_0 is the current density in the absence of TLS, and

$$\sigma_i^+ \sim \frac{\pi a^2}{2} \frac{\varkappa d}{W_0} \mathrm{Re}\left\{ -T_q \left[V_{1q}\left(1+\frac{\Delta_i}{E_i}\right) + V_{2q}\left(1-\frac{\Delta_i}{E_i}\right)\right]\right\},$$

$$\sigma_i^- \sim \frac{\pi a^2}{2} \frac{\varkappa d}{W_0} \mathrm{Re}\left\{ -T_q \left[V_{1q}\left(1-\frac{\Delta_i}{E_i}\right) + V_{2q}\left(1+\frac{\Delta_i}{E_i}\right)\right]\right\}$$

have the meaning of the cross sections for elastic tunneling with TLS participation for the upper and lower TLS levels. Using (23) and (22) we obtain directly

$$(\delta I, \delta I)_\omega \sim j_0^2 A d \overline{(\sigma^+ - \sigma^-)^2} \int_0^\infty dE n_0(1-n_0) \int_0^1 d\rho \frac{P}{\rho} \frac{2\rho}{\tau} \frac{1}{\omega^2+\rho^2\tau^{-2}}$$

$$\sim \frac{I_0^2}{A} d \overline{(\sigma^+ - \sigma^-)^2} \overline{P} \frac{T}{\omega}, \quad \tau \equiv \tau_1 \rho. \qquad (24)$$

It can be seen from (24) that the spectral density of the fluctuations is proportional to ω^{-1}. At $I_0 \sim 10^{-1}$ A, $d \sim 3 \cdot 10^{-7}$ cm, $\sigma \sim 10^{-15}$ cm^2, $\overline{P} \sim 10^{33}$ erg^{-1}·cm^{-2}, $T \sim 1$ K and $A \sim 10^{-3}$ cm^2 we have $(\delta I, \delta I_\omega) \sim 10^{-18}$ A$^2/\omega$. If a resistor R whose fluctuations can be neglected is connected in the external circuit, the spectral density of the voltage fluctuations across the resistor become comparable with the spectral density of the usual Nyquist fluctuations at $R\,[\Omega]/\omega[\mathrm{sec}^{-1}] \sim 10^{-5}$.

Since the effect considered is proportional to I_0, it manifests itself most noticeably for junctions with sufficiently low resistance. We wish to point out, however, one more mechanism typical of junctions of very high resistance and leading to fluctuations of the voltage on the junction.

As already noted, the TLS contribute to the electric dipole moment of the layer; in the absence of tunneling and of an external circuit, this moment produces on the junction a potential difference

$$V \sim \frac{4\pi}{\varepsilon_0 A} \sum_i \mu_i \frac{\Delta_i}{E_i} \langle S_z^i \rangle.$$

Occupation-number fluctuations should lead thus to fluctuations of the voltage V. With allowance for (22), we obtain in analogy with (24) for the corresponding spectral density the estimate

$$(\delta V, \delta V)_\omega \sim \left(\frac{4\pi}{\varepsilon_0}\right)^2 \frac{d\mu^2}{A} \frac{\overline{P}T}{\omega}. \qquad (25)$$

In the case $A \sim 10^{-3}$ cm^2, $d \sim 10^{-6}$ cm, and $\overline{P} \sim 10^{33}$ erg^{-1}·cm^{-3} we have $(\delta V, \delta V)_\omega \sim 10^{-17}$ V$^2/\omega$. Clearly, the shunting action of the junction resistance R_j as well as of the external resistance R impose a limit on ω in (25): $\omega > d/[A \min(R_K, R)]$. On the other hand, the increase of R is connected with "turning on" an additional noise source. But if $RA\omega/d \gg 1$, the noise of R is limited by the shunting action of the junction capacitance. For the system parameters cited above, the noise spectral density determined from (25) exceeds the contribution of the Nyquist fluctuations in the case $R > 10^{11}\,[\Omega]/\omega\,[\mathrm{sec}^{-1}]$.

CONCLUSION

Thus, the presence of TLS in the insulating layer of a tunnel junction can lead to a large number of specific phenomena. The most direct influence on the properties of the junction is exerted by inelastic tunneling with participation of TLS, which contributes to the nonlinearity of its current-voltage characteristic (i.e., in fact to the second derivative d^2I/dV^2 of the tunnel current). We have shown that at sufficiently low temperatures this contribution (at any rate for the junctions considered here, with normal conductors) can exceed the contributions of the other nonlinearity mechanisms. The TLS can influence strongly also nonstationary properties of the system in question, primarily because they have such an important feature as the presence of a broad relaxation-time spectrum. In particular, the fluctuations of the current (more accurately, of the junction resistance) and of the voltage across the contact acquire contributions whose spectral density corresponds to an ω^{-1} dependence; these contributions increase with decreasing junction area. The indicated specific current fluctuations increase then with increasing density of the current through the junction. At small V they can exceed substantially the usual Nyquist noise. Finally, we have seen that when the external voltage is abruptly turned off the flux of the relaxation phonons from the junction acquires, owing to the presence of the TLS, a contribution that decreases with time nonexponentially, $\sim t^{-1}$. In junctions of sufficiently high resistance a similar nonexponential tail can be possessed also by the time dependence of the junction voltage.

Although the absolute values of the effects considered are small, since the typical TLS densities are not too high,[7)] the foregoing estimates show that they apparently are observable by contemporary experimental techniques, and their study can yield abundant information on the properties of TLS. On the other hand, it seems to us that allowance for the phenomena considered can be significant for the interpretation of the high-precision experiments typical of tunnel spectroscopy.

We have considered in this paper the properties of a normal-conductor tunnel junction. We purposely disregarded the case of superconducting junctions, which calls generally speaking for a special analysis. There are grounds for assuming that such a situation uncovers additional possibilities for the study of TLS in amorphous layers. Finally, a highly interesting situation can be realized, in our opinion, in a system whose properties are reminiscent of a tunnel junction, namely in a point contact of two metals,[17] in the vicinity of which TLS are located. (Since such a contact is usually made by "punching" through the insulating layer, amorphous properties of the conductor structure near the contact can be expected.) In particular, since the number of TLS is in this case insignificant, and the current densities are high, noticeable current fluctuations due to the TLS can be expected.

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APPENDIX

Derivation of the expression for the operator of tunneling with participation of TLS

Since, as already noted, ordinary impurities (in the absence of resonant scattering) do not influence the tunneling significantly,[7] we use a model in which the insulating layer is described by a rectangular potential barrier; let its boundaries correspond to $x_1 = 0$ and $x_2 = d$. In accord with the tunnel-Hamiltonian method (see, e.g., Ref. 18) the matrix element of the tunneling is defined by the expression

$$T_{lr} = -\frac{\hbar^2}{2m}\left(\psi_l^* \frac{d\psi_r}{dx} - \psi_r \frac{d\psi_l^*}{dx}\right)\Big|_{x=x_B}. \quad (A.1)$$

Here ψ_l and ψ_r are the solutions of the Schrödinger equation in the barrier region $x_1 \leqslant x_B \leqslant x_2$, which are matched to the exact solutions for the respective regions $x < x_1$ and $x > x_2$. In the absence of the perturbation due to the TLS we have

$$\psi_l{}^0 = Ae^{-i\mathbf{k}_\perp \mathbf{r} - \varkappa x}, \quad \psi_r{}^0 = Be^{i\mathbf{k}_\perp \mathbf{r} + \varkappa x},$$

$$\varkappa = \frac{1}{\hbar}\{2m[U_0 - (\varepsilon - \varepsilon_\perp)]\}^{1/2};$$

here U_0 is the barrier height, while $\varepsilon_\perp$ and $\hbar\mathbf{k}_\perp$ are the kinetic energy and the momentum of the electron in a plane parallel to the barrier.

In the presence of TLS in the barrier, we should take into account, when determining ψ_l and ψ_r, also the potential $V(r)$ of the interaction of the tunneling electron with the TLS. We note that mechanism of such an interaction can generally speaking be different. Namely, since the TLS is located in the insulating layer at a sufficient distance from the metallic bank, one can expect, besides the short-range contribution typical of metallic glasses (and due to the usual scattering pseudopotential,[3)] also the appearance of a contribution of an electric dipole moment $\boldsymbol{\mu}$ of the TLS; this contribution is long-range and amounts, if the distance r_0 from the TLS exceed the lattice constant, to $e(\mu \mathbf{r}_0)/r_0^3$. Assume that the potential V is small enough to permit the use of perturbation theory (in the case $\varkappa\tilde{r} > 1$, where $\tilde{r}$ is the characteristic scale of the potential, this corresponds to the requirement $|V| \ll (\hbar^2/m\tilde{r}^2)\varkappa\tilde{r}$). In this case the Schrödinger equation inside the barrier takes in first-order perturbation theory the form

$$\Delta\psi_{l,r}^{(1)} - (U_0 - \varepsilon)\psi_{l,r}^{(1)} = \frac{2mV}{\hbar^2}\psi_{l,r}^0. \quad (A.2)$$

The solution of (A.2) is described in the general case by the expression

$$\psi^{(1)}(x, y, z) = -\frac{m}{2\pi\hbar^2}\int \frac{d^3r'}{R}\psi^{(0)}V(x', y', z')e^{-\varkappa R},$$
$$R^2 = (x - x')^2 + (y - y')^2 + (z - z')^2. \quad (A.3)$$

We assume next that $\tilde{r}^2 \gg d/2\varkappa$, $\varkappa \gg k_\perp$.[8)] In this case the solutions take the much simpler form:

$$\psi_{l,r}^1 \approx \mp \frac{m}{\hbar^2\varkappa}\psi_{l,r}^0 \int_0^x V\,dx' + C, \quad (A.4)$$

where C is determined from the condition that they match the exact solution at $x = 0$ and $x = d$. Since the corresponding matching is ensured by the zeroth-approximation functions, we put $\psi_l^1(x_1) = 0$, $\psi_r^1(x_2) = 0$, whence

$$\psi_l{}^1 \approx -\frac{m}{\hbar^2\varkappa}\psi_l{}^0 \int_0^x V\,dx',$$

$$\psi_r{}^1 \approx \frac{m}{\hbar^2\varkappa}\psi_r{}^0 \left(\int_0^x V\,dx' - \int_0^d V\,dx\right). \quad (A.5)$$

If we now substitute the obtained functions ψ_l and ψ_r in (A.1), we get besides the contribution of the usual tunneling T_{lr}^0, due to the zeroth-approximation functions, the increment

$$T_{lr}{}^1 = -\frac{m}{\hbar^2\varkappa}T_{lr}{}^0 \int_0^d V\,dx \quad (A.6)$$

(in the derivation of (A.6) we took it into account that it is not consistent to differentiate the terms $\int_0^x V dx'$ in the approximation considered).

It must next be recognized that the potential V is different for the TLS states that correspond to its location in different wells of the two-well potential. Introducing the corresponding Fourier components V_{1q} and V_{2q} and using a procedure similar to that employed to describe the interaction of TLS with electrons in metallic glasses,[3] we obtain ultimately the contribution of interest to us to the tunnel Hamiltonian.

As for the quantity $(V_{1q} - V_{2q})$, it can be assumed that, just as for metallic glasses, it is of the order of 1 eV. We note that for the long-range component, at typical values $\mu \sim 10^{-18}$ erg$^{1/2}$·cm$^{3/2}$, the value of $(V_{1q} - V_{2q})$ is also of the order of 1 eV, and when account is taken of the long-range action the corresponding contribution to the interaction cross section may turn out to be predominant.

1)Such an estimate is obtained for a short-range potential; if the main contribution to the interaction is due to the dipole electric moment of the TLS (see the Appendix), then (cf. Ref. 5) $\sigma_i \sim (4\pi m e^2 \mu^2/\varepsilon_0^2 \hbar^2 W_0) \ln (d/a)$.

2)The factor $(eVd/\hbar s)$ stems from the circumstance that in inelastic tunneling only the interaction with the phonons in the layer itself is significant; at small V the wavelength of these phonons turns out to be larger than d, and this leads to a corresponding decrease of the probability of the process.

3)We call attention to the fact that notwithstanding the presence of an alternating voltage, the terms proportional to $\langle S_x \rangle$ and $\langle S_y \rangle$ need not be taken into account in the expression (6) for the current. Although the terms proportional to $\langle S_x^i \rangle$ and $(\mu_i V_m) \cos \omega t$ can contain stationary contributions, their sum over i vanishes since $\Sigma_i \mu_i = 0$.

4)It is readily understood that the lower bound of the turning-off time is estimated at $t_0 = \min (\varepsilon_0/4\pi dG, \varepsilon_0 AR/4\pi d)$, where R is the resistance of the external circuit.

5)We do not discuss here the question of the subsequent phonon propagation, which is determined by the geometry of the experiment.

6)This assumption is justified, since the TLS relaxation is caused by phonons (or electrons) for which the interaction with the TLS in the layer is only a weak perturbation, so that the equilibrium in the phonon and electron systems is established independently of the TLS.

7)There are definite grounds for assuming that in surface layers, particularly near interfaces, the TLS density can exceed by an order of magnitude the corresponding value in the bulk.

8)The last inequality is obviously satisfied, since the main contribution to the tunneling is made by particles with small $k_\perp$. As for the first inequality [in the derivation of which we have put $x' = d/2$ in (A.3)], on the other hand, one can expect in a dielectric quite large values of $\tilde{r}$ (since there is no screening). On the other hand, it can be easily seen that violation of this inequality leads only to appearance of an additional coefficient $4\varkappa\tilde{r}^2/d < 1$ in the final estimates.

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