Magnon interaction and relaxation in yttrium iron garnet, a twenty-sublattice ferromagnet

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A theory is constructed for the interaction of magnons in yttrium iron garnet (YIG), a classical object of experimental research in magnetism, in the temperature range from 0 to 300 K. Neutron-scattering data are used to refine the values of the exchange integrals in YIG and to obtain the corresponding magnon spectrum, which consists of twenty branches. It is shown that in the energy range $T < 260$ K only magnons of the lower branch are excited; the spectrum of these "ferromagnons" is quadratic in the wave vector only up to 40 K and becomes linear in the region $\omega_{\mathbf{k}} > 40$ K. The amplitude of the four-magnon exchange interaction is determined, and the temperature correction to the frequency is evaluated. This temperature correction is positive, in contrast to the case of a simple cubic ferromagnet with nearest-neighbor interactions. The exchange relaxation rate is calculated for normal and umklapp processes. It is shown that the magnetic dipole interaction is important only for the ferromagnons; the amplitude of this interaction and the corresponding relaxation rate are determined. Three-magnon scattering processes are allowed only for wave vectors larger than a certain k_l; at $k = k_l$ there is a discontinuity in the wave-vector dependence of the damping. A calculation is given for the nonvanishing contribution to the relaxation at $\mathbf{k} = 0$ on account of scattering processes involving optical magnons; this contribution is due to the local uniaxial anistropy. The relative role of each of the investigated relaxation mechanisms is discussed, and the correspondence of the present results with the experimental data is examined.

INTRODUCTION

The unique properties of yttrium iron garnet (YIG-$Y_3Fe_5O_{12}$) lend it a special status among the magnetic dielectrics. Kittel[1] has remarked that the role of this crystal in the physics of magnetism is analogous to that of the fruit fly in genetics. In the first place, it has the narrowest known ferromagnetic-resonance line and the smallest spin-wave damping. Second, with 80 atoms in the unit cell, YIG crystals can be grown so perfectly that the damping of sound in them is smaller than in quartz. Third, the high Curie temperature of YIG, $T_C \approx 560$ K, permits its use in technical devices and enables one to conduct experiments at room temperature. Unfortunately, many important properties of this magnet are still not understood. This is primarily because of the complex crystal structure of YIG: its unit cell contains four formula units of $Y_3Fe_2^{3+}Fe_3^{3+}O_{12}^{2-}$, with the magnetic ions Fe^{3+} occupying two inequivalent positions with regard to the character of its immediate O^{2-} environment—octahedral (a) and tetrahedral (d). There are 20 magnetic ions in all ($8a + 12d$) and, accordingly, 20 magnon branches in the energy range from 0 to 1000 K. The fundamental characteristics of a ferrite are the magnon frequency $\omega_j(\mathbf{k})$ and relaxation time $\gamma_j^{-1}(\mathbf{k})$ (j is the number of the branch and $\mathbf{k}$ is the wave vector). The magnon spectrum in YIG is well enough understood. The frequencies of the homogeneous ($\mathbf{k} = 0$) oscillations are known,[2] and, in addition, approximate expressions have been obtained[3] for the frequencies $\omega_j(\mathbf{k})$ and eigenvectors over the entire Brillouin zone. As regards the magnon relaxation, the damping of the ferromagnons—oscillations of the lower branch $\omega_1(\mathbf{k})$—has been studied[4] in some detail in the range from $k = 0$ to $k = 10^6$ cm^{-1}. These data were interpreted using the theory for magnon relaxation in ferromagnets having a quadratic dispersion relation. It has been pointed out that the wave-vector dependent part of the ferromagnon damping $\Delta(\mathbf{k}) = \gamma_1(\mathbf{k}) - \gamma_1(0)$ is due mainly to the three-magnon dipole-dipole and four-magnon exchange interactions with other ferromagnons.[4] It should be pointed out that the spectrum of the ferromagnons is quadratic, $\omega_1(\mathbf{k}) = \omega_{ex}(ak)^2$, only up to energies of 40 K; after this, as we have shown,[3] the spectrum $\omega_1(\mathbf{k})$ becomes almost linear: $\omega_1(\mathbf{k}) \approx -\Delta + \omega_l(ak)$. Consequently, for $T > 40$ K the theory of the ferromagnon relaxation in YIG should differ substantially from the corresponding theory for ferromagnets. Furthermore, at $T > 260$ K other types of magnons are excited, and the scattering by these magnons must also be taken into account.

The present study deals with the interactions of magnons in YIG in thermodynamic equilibrium at temperatures up to 300 K. We consider the exchange and magnetic-dipole terms in the YIG Hamiltonian and also a term due to the local uniaxial crystallographic anisotropy, find the corresponding amplitudes of the three- and four-magnon processes, and calculate the relaxation rate and the correction to the magnon energy due to these interactions.

In Sec. 1 we study the magnon exchange interaction. We evaluate the temperature correction to the ferromagnon frequency to first order in the interaction. This correction is positive, in contrast to the case of ferromagnets, and is proportional to $(T/T_C)^{5/2}$ at temperatures up to 150 K, in agreement with experiment. The exchange-relaxation rate of the magnons is found as a function of the wave vector and temperature. In the region $T \lesssim 250$ K this rate agrees with the

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familiar expression for ferromagnets [see (9.2.62) in Ref. 5]. At higher temperatures, at which the main contribution to the exchange damping is from the magnons of the linear part of the spectrum, the temperature dependence of the damping becomes stronger.

In Sec. 2 we study the magnetic dipole interaction of magnons. It is shown that this interaction is substantial only for the ferromagnons, since the variable magnetization component, which is due to optical modes, is practically absent. The amplitude of the three-magnon magnetic dipole interaction is well approximated over the entire Brillouin zone by the familiar long-wavelength asymptotic expression for the amplitude of this process in a ferromagnet. However, the magnetic dipole relaxation of the long-wavelength ferromagnons in YIG differs substantially from the corresponding relaxation in ferromagnets, since it is due to processes of coalescence with ferromagnons near the boundary of the Brillouin zone. The conservation laws admit these processes for k larger than a certain value k_l [see (2.9) below]; at $k = k_l$ the processes of coalescence with the ferromagnons are turned on over the entire linear region of the spectrum, leading to a jump in the wave-vector dependence of the relaxation rate at $k = k_l$. In the region $k < 3k_l$ the relaxation rate is nearly constant, and it is only for $k > 3k_l$ that the damping is linear in k, as it is in a ferromagnet. In spite of the fact that the amplitude of the magnetic dipole interaction is independent of the direction of the wave vectors with respect to the crystallographic axes, the damping of the long-wavelength ferromagnons is anisotropic because of the nonspherical shape of the Brillouin zone.

As we know, the contributions to the ferromagnon relaxation from the dipole-dipole and exchange interactions go to zero as $\mathbf{k}\to 0$, but experiment reveals the presence of a nonzero damping $\gamma_1(0)$ even in very pure samples.[5] This damping depends linearly on temperature in the range 150–350 K, and so it is natural to assume that it is due to three-wave processes. Kasuya and Le Craw, who first detected the damping $\gamma_1(0)$ in 1960, assumed that the relaxation of the long-wavelength ferromagnons is due to their scattering by a phonon and a ferromagnon with $k\approx 4\cdot 10^6\ \mathrm{cm}^{-1}$, the interaction being caused by modulation of the local uniaxial anisotropy constant in the field of the phonon.[6] From that time on the relaxation of ferromagnons with $\mathbf{k}\to 0$ and other poorly understood properties of the damping of ferromagnons with $\mathbf{k}\neq 0$ (e.g., its anisotropy) were commonly attributed to "Kasuya–Le Craw processes." However, an accurate analysis of this mechanism shows that the estimate of the damping $\gamma_1(0)$ in Ref. 6 is at least an order of magnitude too large.

In Sec. 3 we show that the main relaxation mechanism for long-wavelength ferromagnons is their coalescence with "optical" magnons having a gap in their spectrum. The gaps of the nineteen optical magnons in YIG exceed 200 K, whereas the frequency of the ferromagnons at $\mathbf{k}\to 0$ is of the order of 1 K. Therefore, the three-magnon processes involving long-wavelength ferromagnons are allowed near points of intersection or tangency of the optical branches of the spectrum. We show that these processes are due to the uniaxial crystallographic anisotropy of the Fe^{3+} ions in octahedral positions (a) calculate the corresponding contribution of the relaxation of ferromagnons with $\mathbf{k}\to 0$.

In the Conclusion we compare the contributions to the ferromagnon relaxation in the various wave-vector and temperature regions and discuss the relationship of our results with experiment.

1. EXCHANGE INTERACTION OF MAGNONS

1.1. Spin Hamiltonian and the magnon spectra. It is well known that the exchange interaction, which governs the magnetic order of the material, is the strongest. In YIG the predominant interaction is the *a–d* exchange; for nearest neighbors $J_{ad}\approx 40$ K. This interaction leads to an antiparallel orientation of the spins of the *a* and *d* ions; the *a–a* and *d–d* exchange interactions are weaker than the *a–d* exchange. The exchange interaction with the remaining coordination spheres will be neglected. As a result, the Heisenberg Hamiltonian H_{ex} is written in the form

$$H_{ex}=-2\sum_n\Big\{J_{aa}\sum_{i,j=1,\,j>i}^{8}\mathbf{S}_i(\mathbf{R}_{in})\times\sum_{\mathbf{d}_{ij}}\mathbf{S}_j(\mathbf{R}_{in}+\mathbf{d}_{ij})+J_{ad}\sum_{i=1}^{8}\sum_{j=9}^{20}\mathbf{S}_i(\mathbf{R}_{in})\times\sum_{\mathbf{d}_{ij}}\mathbf{S}_j(\mathbf{R}_{in}+\mathbf{d}_{ij})+J_{dd}\sum_{i,j=9,\,j>i}^{20}\mathbf{S}_i(\mathbf{R}_{in})\sum_{\mathbf{d}_{ij}}\mathbf{S}_j(\mathbf{R}_{in}+\mathbf{d}_{ij})\Big\}. \tag{1.1}$$

Here n numbers the primitive cell, i and j number the sublattices ($i = 1, \dots, 8$ number the a ions, $i = 9, \dots, 20$ number the d ions), $\mathbf{S}_i(\mathbf{R}_{in})$ are the spin and coordinate of an ion of the ith sublattice in the nth cell, and $\mathbf{d}_{ij}$ is the distance to the nearest neighbor in the jth sublattice. For the exchange integrals we take the values

$$J_{ad}=-40.0\pm0.2\ \text{K},\quad J_{dd}=-13.4\pm0.2\ \text{K},\quad J_{aa}=-3.8\pm0.4\ \text{K}, \tag{1.2}$$

which differ somewhat from the values adopted in our previous paper [Eq. (1.13) of Ref. 3]. We shall discuss the question of refining the values of the exchange integrals for YIG later on in this paper.

The magnon spectra YIG in have been studied by Harris[7] and by the present authors.[3] Converting from the spin exchange Hamiltonian to boson operators with the aid of a Holstein-Primakoff transformation and then going over to the k representation by the formula

$$a_j(\mathbf{k})=\frac{1}{N^{1/2}}\sum_n\exp(-i\mathbf{k}\mathbf{R}_{jn})\,a_{jn} \tag{1.3}$$

(N is the number of cells in the crystal), we obtain the quadratic part of the Hamiltonian in the form

$$H^{(2)}=\sum_{\mathbf{k}}H_{\mathbf{k}}^{(2)},$$

$$H_{\mathbf{k}}^{(2)}=\sum_{i,j=1}^{8}A_{ij}(\mathbf{k})\,a_{i\mathbf{k}}^{+}a_{j\mathbf{k}}+\sum_{i,j=9}^{20}D_{ij}(\mathbf{k})\,a_{i\mathbf{k}}^{+}a_{j\mathbf{k}}+\sum_{i=1}^{8}\sum_{j=9}^{20}[B_{ij}(\mathbf{k})\,a_{i\mathbf{k}}^{+}a_{j-\mathbf{k}}^{+}+\text{H.c.}],$$

$$\frac{1}{8}\sum_{i,j} A_{ij}(\mathbf{k})=A_{\mathbf{k}}, \quad \frac{1}{12}\sum_{i,j} D_{ij}(\mathbf{k})=D_{\mathbf{k}},$$

$$\frac{1}{(8\cdot 12)^{1/2}}\sum_{i,j} B_{ij}(\mathbf{k})=B_{\mathbf{k}}. \tag{1.4}$$

Detailed expressions for the matrices A, B, and D are given by Harris.[7] After a linear u-v transformation $H_{\mathbf{k}}^{(2)}$ assumes the diagonal form

$$H_{\mathbf{k}}^{(2)}=\sum_{j=1}^{20} \omega_{j\mathbf{k}} b_{j\mathbf{k}}^{+} b_{j\mathbf{k}}. \tag{1.5}$$

We have proposed an efficient means of approximately diagonalizing Hamiltonian (1.4) by converting to a quasinormal basis in which the diagonal elements of the Hamiltonian matrix are close to the eigenfrequencies of the magnons, while the off-diagonal terms are small.[3] In this approximation the eigenvector and frequency of the ferromagnetic mode are

$$b_{1\mathbf{k}}=\frac{u_{\mathbf{k}}}{\sqrt{12}}\sum_{i=9}^{20} a_{i\mathbf{k}}+\frac{v_{\mathbf{k}}}{\sqrt{8}}\sum_{i=1}^{8} a_{i-\mathbf{k}}^{+},$$

$$\omega_{1\mathbf{k}}={}^{1}/_{2}[(C_{\mathbf{k}}^{2}-4|B_{\mathbf{k}}|^{2})^{1/2}-A_{\mathbf{k}}+D_{\mathbf{k}}], \quad C_{\mathbf{k}}=A_{\mathbf{k}}+D_{\mathbf{k}}, \tag{1.6}$$

$$\begin{pmatrix} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{pmatrix}=\frac{(C_{\mathbf{k}}+2|B_{\mathbf{k}}|)^{1/2}\pm(C_{\mathbf{k}}-2|B_{\mathbf{k}}|)^{1/2}}{2(C_{\mathbf{k}}^{2}-4|B_{\mathbf{k}}|^{2})^{1/4}}.$$

In the long-wavelength limit this leads to the well-known formula[2]

$$\omega_{1\mathbf{k}}=\omega_{ex}(ak)^{2}, \quad \omega_{ex}={}^{5}/_{16}(8J_{aa}+3J_{dd}-5J_{ad}), \tag{1.7}$$

where a is the lattice constant.

1.2. Determination of the exchange integrals in YIG. In the first stage of this study[3] we chose the value $J_{ad}=-(35+3)$ K on the basis of an analysis of the indirect experimental data of Ref. 8. We then took the value $\omega_{ex}=41$ K obtained from measurements[9] of the frequency $\omega_{1\mathbf{k}}$ of the ferromagnetic mode in the region of small k. The missing information was obtained from comparison of the experimental $M(T)$ curve in the region 4–300 K with the $M(T)$ curve calculated in the noninteracting-spin-wave approximation. We thus obtained the following "admissible" values for the exchange integrals: $J_{ad}=-(35\pm 3)$ K, $J_{dd}=-(16\pm 3)K$, $J_{aa}=(0$–$3)$ K. More definite values of J can of course be obtained by comparing our analytical expressions with direct measurements of the magnon frequencies over the entire Brillouin zone. Plant[10] has reported low-temperature neutron-diffraction measurements of the frequencies $\omega_1(\mathbf{k})$ and $\omega_{a1}(\mathbf{k})$ of the ferromagnetic and antiferromagnetic modes, respectively, and the frequency $\omega_x(\mathbf{k})$ of another optical magnon mode in YIG with an undetermined pattern of oscillation. From the antiferromagnetic gap $\omega_{a1}(0)=10J_{ad}$, Plant found $J_{ad}=-39.8$ K. Comparison of the theory (1.6) with experiment for the ferromagnetic mode $\omega_1(\mathbf{k})$ in the directions $\mathbf{k}\|[110]$ and [100], with $\mathbf{k}$ varying from 0 to the Brillouin zone boundary, we obtained the value of the combination of exchange integrals in (1.7) as $\omega_{ex}=(40.0\pm 1.0)$ K and found $J_{aa}=-(4\pm 1)$ K. Using this set of J values to calculate the gaps of all the optical modes, we became convinced that the role of $\omega_x(\mathbf{k})$ could be played by only one of the triad of zone-center-degenerate modes ω_{d8j}, $j=1$, 1, 3. By comparing $\omega_{d8j}(0)=20(J_{dd}-J_{ad})$ with the experimental value $\omega_x(0)=530$ K, we find $J_{dd}=-(13.4\pm 0.2)$ K, and we then get the refined value $J_{aa}=-(3.8\pm 0.2)$ K.

1.3. The magnon exchange interaction Hamiltonian can be obtained by expanding (1.1) up to terms of fourth order and transforming to the normal operators b, b^{+}:

$$H^{(4)}=\frac{1}{2}\sum_{12,34,i\leq j,l\leq m} T_{12,43}^{ijlm} b_{i1}^{+} b_{j2}^{+} b_{l3} b_{m4}\Delta(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3-\mathbf{k}_4). \tag{1.8}$$

We shall henceforth be concerned with the interactions of ferromagnons with one another and with low-lying optical magnons. In the quasinormal approximation it follows from (1.6) that

$$T_{12,34}^{11\,11}=T_{12,34}+{}^{1}/_{4}(\tilde{T}_{12,34}+\tilde{T}_{21,34}+\tilde{T}_{12,43}+\tilde{T}_{21,43}),$$

$$T_{12,34}=\frac{C_{12,34}}{16NS_0}(\omega_1+\omega_2+\omega_3+\omega_4-\omega_{13}-\omega_{14}-\omega_{23}-\omega_{24}),$$

$$C_{12,34}={}^{1}/_{6}(2u_1u_2u_3u_4-3v_1v_2v_3v_4), \tag{1.9}$$

$$\tilde{T}_{12,34}=-\frac{B_{13}}{24NS_0}\left[2\sqrt{6}u_1v_2u_3v_4-2u_1u_2u_3u_4\frac{v_{13}}{u_{13}}-3v_1v_2v_3v_4\frac{u_{13}}{v_{13}}\right].$$

Here S_0 is the ion spin, $\omega_1\equiv\omega_1(\mathbf{k})$, $u_1\equiv u(\mathbf{k}_1)$, $\omega_{13}\equiv\omega_1(\mathbf{k}_{13})\equiv\omega_1(\mathbf{k}_1-\mathbf{k}_3)$, $B_{13}\equiv B(\mathbf{k}_1-\mathbf{k}_3)$, etc. In the limit of small $\mathbf{k}$ the expression for $T_{12,34}^{11,11}$ goes over to the familiar expression for the interaction of magnons in a ferromagnet in the continuum approximation. In the long-wavelength limit $T_{12,34}\propto k^2$, while $\tilde{T}_{12,34}\propto k^4$ and can be neglected. At the zone boundary $\tilde{T}_{12,34}$ is approximately 1/2 of $T_{12,34}$, and for purposes of estimation one can assume $T_{12,34}^{11,11}=T_{12,34}$. The form of the expression for $T_{12,34}$ is reminiscent of the familiar formula for ferromagnets[11] with spin $\overline{S}=4S_0$ and dispersion law $\omega_{\mathbf{k}}$. It should, however, be kept in mind that $C_{12,34}$ falls off from 1 (at $\mathbf{k}_i\to 0$) to 0.5 (when all the $\mathbf{k}_i$ lie on the zone boundary) and that the form of $\omega_{\mathbf{k}}$ for ferromagnons in YIG is considerably different from that of $\omega_{\mathbf{k}}$ in ferromagnets.

1.4. Temperature dependence of the ferromagnon frequency. In the spin-wave approximation the temperature correction to the ferromagnon frequency is given by the formula

$$\Delta\omega_{\mathbf{k}}=\sum_j \Delta\omega_{\mathbf{k}}^{(j)}, \quad \Delta\omega_{\mathbf{k}}^{(1)}=2\sum_{\mathbf{k}'} T_{\mathbf{k}\mathbf{k}'}^{11} n_{\mathbf{k}'}^{(1)},$$

$$\Delta\omega_{\mathbf{k}}^{(j)}={}^{1}/_{2}\sum_{\mathbf{k}'} T_{\mathbf{k}\mathbf{k}'}^{1j} n_{\mathbf{k}'}^{(j)}, \quad j\neq 1, \tag{1.10}$$

$$T_{\mathbf{k}\mathbf{k}'}^{1j}\equiv T_{\mathbf{k}\mathbf{k}',\mathbf{k}\mathbf{k}'}^{1j,1j},$$

in which $n_{\mathbf{k}}^{(j)}$ are the equilibrium occupation numbers for the magnons of branch j. For $T<300$ K in YIG only the ferromagnons ($j=1$) are excited, and for determining $\Delta\omega_{\mathbf{k}}(T)$ it is sufficient to take only $T_{\mathbf{k}\mathbf{k}'}$ into account. The expression for $\Delta\omega_{\mathbf{k}}(T)$ even in this case will be awkward, and we shall therefore discuss only the long-wavelength limit ($k<k_0$, ω_{k_0}

$= 40$ K) and allow for the fact that $\omega_{\mathbf{k}}$ for ferromagnons in YIG is practically independent of the direction of $\mathbf{k}$. Then

$$\Delta\omega_{\mathbf{k}}=2\sum_{\mathbf{k}'} T^{11}_{\mathbf{k}\mathbf{k}'}n_{\mathbf{k}'}=\frac{a^3}{2\pi^2}\int_0^{k_B}\langle T^{11}_{\mathbf{k}\mathbf{k}'}\rangle n_{\mathbf{k}'}k'^2\,dk'. \qquad (1.11)$$

Here $\langle T^{11}_{\mathbf{k}\mathbf{k}'}\rangle$ is expresion (1.9) for $T^{11}_{\mathbf{k}\mathbf{k}'}$ averaged over the angles of the wave vector $\mathbf{k}'$, and k_B is the average wave vector on the boundary of the Brillouin zone.

At low temperatures ($T < 40$ K) the k' integration in (1.11) is taken over the long-wavelength region ($k' < k_0, \omega_{k_0} = 40$ K), in which the dispersion relation simplifies to the quadratic form in (1.7). Here the expression for $\langle T^{11}_{\mathbf{k}\mathbf{k}'}\rangle$ also simplifies substantially:

$$\langle T^{11}_{\mathbf{k}\mathbf{k}'}\rangle\approx 0.32\,|J_{ad}|\,(a^4k^2k'^2/N\bar{S}). \qquad (1.12)$$

It is seen that even in the long-wavelength limit the matrix element $\langle T^{11}_{\mathbf{k}\mathbf{k}'}\rangle$ differs not only in magnitude but also in sign from the corresponding expression for a simple cubic ferromagnetic with a nearest-neighbor interaction

$$\langle T_{\mathbf{k}\mathbf{k}'}\rangle = -0.04J\,(a^4k^2k'^2/N\bar{S}). \qquad (1.13)$$

In the continuum limit for ferromagnets, in which case $\omega_{\mathbf{k}} \propto k^2$, one has $\langle T_{\mathbf{k}\mathbf{k}'}\rangle = 0$. This quantity therefore depends on the specific type of crystal structure and functional form of $J_{nn'}$. The possibility of a positive $\langle T^{11}_{\mathbf{k}\mathbf{k}'}\rangle$ in two-sublattice ferromagnets was pointed out in Ref. 12. It should also be noted that in evaluating $\langle T^{11}_{\mathbf{k}\mathbf{k}'}\rangle$ in YIG one cannot neglect in (1.9) the terms $\widetilde{T}$, which contribute substantially to the terms of order $k^2k'^2$ in the expansion.

Substituting the expression for $\langle T^{11}_{\mathbf{k}\mathbf{k}'}\rangle$ into the integral (1.11), we obtain the temperature correction to the frequency: $\Delta\omega_{\mathbf{k}}(T) = \Delta\omega_{ex}(T)(ak)^2$. The function $\Delta\omega_{ex}(T)$ is plotted in Fig. 1. The dashed curve shows this function as evaluated in the low-temperature limit with the aid of (1.11) and (1.7):

$$\frac{\Delta\omega_{ex}}{\omega_{ex}} = 0.44\left(\frac{T}{10|J_{ad}|}\right)^{5/2} \approx 1.0\left(\frac{T}{T_c}\right)^{5/2}. \qquad (1.14)$$

Strictly speaking, this expression is valid only at temperatures in the range $T \lesssim 10$ K, at which only long-wavelength magnons with the quadratic spectrum are excited. It is seen from Fig. 1, however, that expression (1.14) can be used with 10% accuracy up to $T = 150$ K and with 30% accuracy up to $T = 250$ K; this situation is explained by the following circumstances: First, in the higher-energy region $ak' \gtrsim 1$, where the magnon dispersion law ω_k, becomes linear, there is a slowing of the growth of the matrix element $\langle T^{11}_{\mathbf{k}\mathbf{k}'}\rangle$. Second, in this region one has $\omega_{\mathbf{k}'} < \omega_{ex}(ak')^2$, and so the occupation number $n_{\mathbf{k}}$, is greater than that on the quadratic spectrum for the same $\mathbf{k}'$. As a result, the product $\langle T^{11}_{\mathbf{k}\mathbf{k}'}\rangle n_{\mathbf{k}'}$ describing the temperature correction to the frequency turns out to be close to its long-wavelength limit at values $ak' \lesssim 3$. The temperature dependence of $\omega_{ex}(T)$ obtained here agrees with experiment both in the sign and magnitude of the effect for temperatures up to 150–200 K.

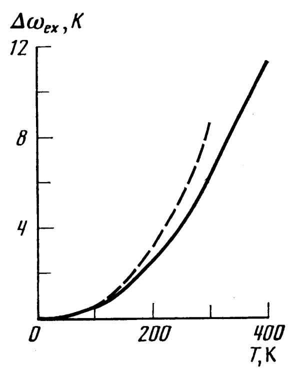

FIG. 1. Temperature dependence of the exchange frequency $\Delta\omega_{ex} = \omega_{ex}(T) - \omega_{ex}(0)$ in YIG. The dashed curve is the low-temperature expansion for $\Delta\omega_{ex}$ in a ferromagnet with a spin corresponding to the spin of the unit cell of YIG.

1.5. Exchange relaxation of ferromagnons. The relaxation rate of magnons of the ith branch in four-magnon scattering is given by the well-known expression

$$\gamma_{\mathbf{k}}{}^i=2\pi\sum_{1,2,3,\mathbf{b}}\ \sum_{j,l,m}|T^{ijlm}_{\mathbf{k}\,1,23}|\,\{n_1{}^j(n_2{}^l+n_3{}^m+1)-n_2{}^ln_3{}^m\}\times\delta(\omega_k{}^i+\omega_1{}^j-\omega_2{}^l-\omega_3{}^m)\times\Delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3), \qquad (1.15)$$

where $\mathbf{b}$ are reciprocal lattice vectors, $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$ lie in the first Brillouin zone, $\Delta(\mathbf{k}) = 0$ for $\mathbf{k} = 0$ and $\Delta(0) = 1$, and i, j, l, and m number the magnon branches.

The relaxation of the low-frequency ferromagnons $\omega_{\mathbf{k}} \lesssim 1$ K, corresponding to $ak \lesssim 0.1 \ll 1$, can currently be studied in experiment, and for this reason it is of greatest interest to evaluate the damping of these magnons. At moderate temperatures $T \lesssim 200$ K one can neglect the scattering by optical magnons, which have an activation energy $\gtrsim 260$ K, i.e., one can set $i = j = l = m = 1$ in (1.15). In the long-wavelength limit one can neglect the contribution $\widetilde{T}_{\mathbf{k}\,1,23}$ (which is quadratic in ak) in expression (1.9) for the matrix element $T^{1111}_{\mathbf{k}1,23}$. It can also be seen that when the conservation laws are taken into account, $C_{\mathbf{k}\,1,23}$ varies by less than 15% over the entire Brillouin zone. As a result, we obtain for $T^{1111}_{\mathbf{k}1,23}$ the following approximate expression:

$$T^{1111}_{\mathbf{k}1,23}=-\frac{1}{8NS_0}(\mathbf{v}_2+\mathbf{v}_3)\mathbf{k}, \qquad (1.16)$$

where $\mathbf{v}_{\mathbf{k}} = \partial\omega_{\mathbf{k}}/\partial\mathbf{k}$ is the group velocity. This expression is valid for arbitrary $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$.

At temperatures which are not too high the contribution of umklapp processes and the finiteness of the Brillouin zone are not important, and the dispersion law $\omega_{\mathbf{k}}$ is spherically symmetric. This permits us to perform the integration over angles in (1.15) and to reduce the five-dimensional integral to a double integral. Changing to the dimensionless variable $\varepsilon_{\mathbf{k}} = \omega_{\mathbf{k}}/10|J_{ad}|$, we obtain

$$\gamma_{\mathbf{k}}=\frac{32}{3\pi^3}\frac{(ak)^2\omega_{\mathbf{k}}}{A^2\bar{S}^2}F\left(\frac{T}{10|J_{ad}|},\varepsilon_{\mathbf{k}}\right), \qquad (1.17)$$

$$F(\tau,\varepsilon)=\frac{1}{\tau}\iint\limits_{\varepsilon_2+\varepsilon_3>\varepsilon}^{\varepsilon_2+\varepsilon_3<\varepsilon_m}\left[\frac{(2\varepsilon_2+1)^2-1}{(2\varepsilon_2+1)^2}+\frac{(2\varepsilon_3+1)^2-1}{(2\varepsilon_3+1)^2}\right.$$

$$+\frac{8\varepsilon_2\varepsilon_3}{(2\varepsilon_2+1)(2\varepsilon_3+1)}\Bigg]\frac{(2\varepsilon_2+2\varepsilon_3+1)(2\varepsilon_2+1)(2\varepsilon_3+1)}{(e^{\varepsilon_2/\tau}-1)(e^{\varepsilon_3/\tau}-1)(1-e^{-(\varepsilon_2+\varepsilon_3)/\tau})}d\varepsilon_2\,d\varepsilon_3,$$
$$\varepsilon_m=0.88. \qquad (1.18)$$

Here we have used an approximation for $\varepsilon_{\mathbf{k}}$ that follows from (1.6) with our choice of exchange integrals:

$$\varepsilon_{\mathbf{k}}=[(1+Aq^2)^{1/2}-1]/2, \quad \mathbf{q}=(a\mathbf{k})/8,$$
$$A=40(1-J_{dd}/2J_{ad}-2J_{aa}/J_{ad})\approx 25.7 \text{ K}.$$

This approximation is valid to within $\lesssim 20\%$ over the entire Brillouin zone. At temperatures below 40 K, where magnons are excited in the quadratic part of the spectrum, the damping $\gamma_{\mathbf{k}}$ coincides with the damping (evaluated in Refs. 11 and 13) of magnons in a ferromagnet with a dispersion law $\omega_{ex}(ak)^2$ and a unit-cell spin $\overline{S}$. In this case

$$F=16\left(\frac{T}{10|J_{ad}|}\right)^2\left[\ln^2\frac{T}{\omega_{\mathbf{k}}}-\frac{10}{3}\ln\frac{T}{\omega_{\mathbf{k}}}-0{,}3\right]. \qquad (1.19)$$

However, by analyzing expression (1.18) one is readily convinced that the angle-averaged square of the matrix element—the expression in braces—differs little from its long-wavelength asymptotic behavior at energies all the way up to ≈ 200 K, i.e., $\varepsilon_{\mathbf{k}}\approx 0.5$. Therefore, expression (1.19) can be used for the damping of ferromagnons in YIG at temperatures up to $T\lesssim 200$ K (see Fig. 2). For the temperature range $200\lesssim T\lesssim 350$ K one can obtain from (1.18) another approximate expression which corresponds to integration only over the linear part of the dispersion law:

$$F\approx 32(T/10|J_{ad}|)^4. \qquad (1.20)$$

1.6. The contribution of umklapp processes to the exchange relaxation of long-wavelength ferromagnons. The energy of the magnons which take part in these processes is approximately $\omega_B/2\approx 150$ K (ω_B is the ferromagnon frequency on the boundary of the Brillouin zone). Therefore, starting at around this temperature, umklapp processes can be important. Integration and a summation over the twelve reciprocal lattice vectors of the [110] type with allowance for the conservation laws yield

$$\gamma_{\mathbf{k}}^u=2.4\cdot 10^{-4}\,\omega_{\mathbf{k}}(ak)^2\frac{10|J_{ad}|/T}{(e^{\omega_B/2T}-1)^2(1-e^{-\omega_B/T})}, \qquad (1.21)$$

where ω_B is the ferromagnon frequency at the Brillouin zone boundary in the [110] direction. At T = 300 K the umklapp contribution (1.21) amounts to $\gamma_{\mathbf{k}}^u = 8.5\cdot 10^{-5}\omega_{\mathbf{k}}(ak)^2$, which is considerably smaller than the damping (1.20) in

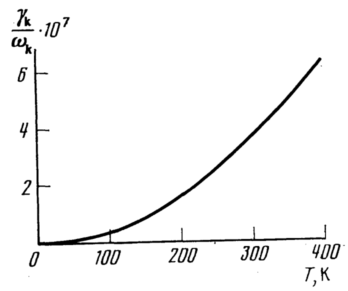

FIG. 2. Temperature dependence of the exchange relaxation rate of ferromagnons in YIG for $k = 10^5$ cm^{-1}.

normal processes: $\gamma_{\mathbf{k}}^n = 6.2\cdot 10^{-4}\omega_{\mathbf{k}}(ak)^2$. At temperatures above 300 K one has $\gamma^u \propto T^2$, while $\gamma^n \propto T^4$, and so the relation $\gamma_{\mathbf{k}}^u \ll \gamma_{\mathbf{k}}^n$ remains valid. The smallness of $\gamma_{\mathbf{k}}^u$ is due to the smallness of the phase volume in which umklapp processes are allowed.

2. MAGNETIC DIPOLE INTERACTION AND RELAXATION OF FERROMAGNONS

2.1. The Hamiltonian for the magnetic dipole-dipole interaction for an infinite volume can be written[5]

$$H_m=2\pi\frac{(g\mu_B)^2}{v}\sum_{\mathbf{k}}\left|\mathbf{n}\sum_{i=1}^{20}\mathbf{s}_{i\mathbf{k}}\right|^2, \qquad (2.1)$$

where $\mathbf{n}=\mathbf{k}/k$, s_i is the deviation of the ith spin from equilibrium, g is a factor ≈ 2, v is the volume of the primitive cell, and the Fourier transformation is defined in (1.3). This expression implies, in particular, the familiar long-wavelength (in all $\mathbf{k}$, $\mathbf{k}_1$, and $\mathbf{k}_2$) approximation for the Hamiltonian of the three-magnon interaction involving only ferromagnons (FMs):

$$V_{\mathbf{k}\mathbf{k}_1,\mathbf{k}_2}=V_{\mathbf{k}}+V_{\mathbf{k}_1}, \quad V_{\mathbf{k}}=-\frac{\omega_m}{4(2S_0)^{1/2}}\sin(2\theta_{\mathbf{k}})\exp(i\varphi_{\mathbf{k}}),$$
$$H_{m,\mathrm{FM}}^{(3)}={}^1/_2\sum_{\mathbf{k}\mathbf{k}_1\mathbf{k}_2}V_{\mathbf{k}1,2}b_{\mathbf{k}}^+b_1^+b_2\Delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2)+\text{H.c.} \qquad (2.2)$$

We have shown that this expression for $H_{m,\mathrm{FM}}^{(3)}$ works with good accuracy (10–15%) even for the case in which only one of the wave vectors, say $\mathbf{k}$, is small ($ka\lesssim 1$), while the others, $\mathbf{k}_2$ and $\mathbf{k}_3$, are completely arbitrary.[14]

2.2. The magnetic-dipolar damping of ferromagnons, as we know, is due to processes of decay and coalescence:

$$\omega_{\mathbf{k}}=\omega_{\mathbf{k}_1}+\omega_{\mathbf{k}_2}, \quad \mathbf{k}=\mathbf{k}_1+\mathbf{k}_2, \qquad (2.3)$$

$$\omega_{\mathbf{k}}+\omega_{\mathbf{k}_1}=\omega_{\mathbf{k}_2}, \quad \mathbf{k}+\mathbf{k}_1=\mathbf{k}_2. \qquad (2.4)$$

For magnons with small $\mathbf{k}$ only the coalescence processes (2.4) are allowed by the conservation laws. The contribution to the damping from these processes is given by the well-known formula

$$\gamma_{m\mathbf{k}}=\pi\sum_{\mathbf{k}_1,\mathbf{k}_2}|V_{\mathbf{k}1,2}|^2(n_1-n_2)\,\delta(\omega_{\mathbf{k}}+\omega_1-\omega_2)\,\Delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2) \qquad (2.5)$$

with the matrix element $V_{\mathbf{k}\,1,2}$ from (2.2) substituted in. In evaluating the integral in (2.5) one can assume that the ferromagnon dispersion relation consists of a quadratic and a linear part:

$$\omega_{\mathbf{k}}=\omega_0+\omega_{ex}(ak)^2, \quad k\leqslant k_0,$$
$$\omega_{\mathbf{k}}=-\Delta+\omega_l(ak), \quad k\geqslant k_0, \qquad (2.6)$$

where ω_0 is the gap in the ferromagnon spectrum (usually $\omega_0\lesssim 1$ K). The values of k_0 and Δ are determined from the continuity conditions for $\omega_{\mathbf{k}}$ and for the group velocity $\mathbf{v}_{\mathbf{k}}=\partial\omega_{\mathbf{k}}/\partial\mathbf{k}$:

$$k_0a=\omega_l/2\omega_{ex}, \quad \Delta=\omega_l^2/4\omega_{ex}-\omega_0. \qquad (2.7)$$

One finds that for $\omega_l = 2\omega_{ex}$ the values of $\omega_{\mathbf{k}}$ evaluated with formulas (2.6) differ from the ferromagnon dispersion relation (1.6) by no more than 10%. We shall therefore use

$$k_0a=1,\quad \Delta=\omega_{ex}=40\text{ K},\quad \omega_l=80\text{ K}. \tag{2.8}$$

For $T<40$ K the main contribution to $\gamma_{m\,\mathbf{k}}$ comes from the integration over the quadratic part of the spectrum and, consequently, the expression for $\gamma_{m\,\mathbf{k}}$ in YIG is the same as the familiar expression for ferromagnets (see, e.g., formula (9.2.51) of Ref. 5). For $T>$ 40 K the integration over the quadratic part of the spectrum can be done using the Rayleigh-Jeans approximation $n_{\mathbf{k}}=T/\omega_{\mathbf{k}}$. The corresponding contribution to the magnon damping is then

$$\gamma_{ms}(\mathbf{k})=\frac{1}{3}\frac{k}{k_l}\frac{\omega_m^2T}{16\pi\bar{S}\omega_l^2}\left(1-\frac{k_l^2}{k^2}\right)$$
$$\times\Big\{\Big(1+\frac{17}{2}\sin^2\theta_{\mathbf{k}}-\frac{35}{4}\sin^4\theta_{\mathbf{k}}\Big)$$
$$+\frac{k_l^2}{k^2}\Big[\cos^2\theta_{\mathbf{k}}\Big(1+\frac{11}{2}\sin^2\theta_{\mathbf{k}}\Big)-2\frac{k_l^2}{k^2}\Big(\cos^2\theta_{\mathbf{k}}(1-4\sin^2\theta_{\mathbf{k}})$$
$$+\frac{3}{8}\sin^4\theta_{\mathbf{k}}\Big)\Big]\Big\},\quad k\geqslant k_l. \tag{2.9}$$

On the linear part of the spectrum one must take the Planck distribution for $n_{\mathbf{k}}$. The corresponding contribution to the damping is

$$\gamma_{ml}(\mathbf{k})=\frac{k_l}{k}\frac{\omega_m^2T}{16\pi\bar{S}\omega_l^2}\cdot1{,}5\frac{\omega_B}{\omega_l}F(T)\Big\{\sin^2 2\theta_{\mathbf{k}}-\Big(1-\frac{k_l^2}{k^2}\Big)$$
$$\times\Big[\frac{1}{8}(7\sin^2 2\theta_{\mathbf{k}}-\sin^4\theta_{\mathbf{k}})$$
$$-\frac{k_l^2}{k^2}\Big(1-\frac{5}{4}\sin^2 2\theta_{\mathbf{k}}-\frac{5}{8}\sin^4\theta_{\mathbf{k}}\Big)\Big]\Big\},\quad k\geqslant k_l. \tag{2.10}$$

The total three-magnon damping in coalescence processes is equal to the sum of (2.9) and (2.10). In these formulas $k_la=\omega_0/\omega_l,\omega_k\approx\omega_0$, and $F(T)$ is a dimensionless function of temperature given by the integral

$$F(T)=\frac{1}{1{,}5\omega_BT^2}\int_{\Delta}^{\omega_B}\frac{(\omega+\Delta)^2e^{\omega/T}}{(e^{\omega/T}-1)^2}d\omega,$$
$$F(T)\to1\quad\text{as}\quad T\to\infty. \tag{2.11}$$

The values of this function for $\omega_B=350$ K and $\Delta=40$ K are given in Ref. 14.

It can be seen from (2.9) that γ_{ms} is a linear function of temperature. It follows from (2.10) that the deviation of γ_{ml} from a linear temperature dependence is determined by the factor $F(T)$, which is chosen such that $F\to1$ when the temperature exceeds the maximum ferromagnon energy $\omega_B\approx350$ K. The factor $F(T)$ is a slowly varying function of T: in particular, as T varies by a factor of three over the actual range from 100 to 300 K, F varies by only 30%. This means that the temperature dependence of γ_{ml} for $T>100$ K can also be assumed to be approximately linear.

Such an assumption is more accurate than it would seem at first glance. In fact, for $T>100$ K it is generally necessary to take the temperature dependence of the magnetic dipole interaction matrix element into account. This question has not been studied in detail, but it is most natural to assert that the function $V_{1,23}(T)$ is obtained by replacing $\bar{S}$ by $\bar{S}(T)$, since the magnetic dipole interaction matrix element of long-wavelength spin waves should depend on the average magnetic moment $g\mu_B\bar{S}(T)$. This is taken into account in formula (2.10) for γ_{ml} by replacing $F(T)$ by $\tilde{F}(T)=F(T)\bar{S}(T)/\bar{S}(0)$. As is shown in Ref. 14, this function is even more slowly varying than $F(T)$ in the temperature range 100–300 K.

Let us now consider γ_{ms} and γ_{ml} as functions of the magnitude and direction of the wave vector. We notice first of all that for $k<k_l=\omega_0/\omega_la$ the coalescence process is forbidden by the conservation laws (2.4) because the ferromagnon group velocity is bounded by the value $v_{\max}=a\omega_l$. Accordingly, $\gamma_{ms}=\gamma_{ml}=0$ if $k<k_l$. For $\omega_0=2\pi\cdot17$ GHz ($h\omega_0\approx1$ K we have $k_l\approx10^5$ cm^{-1}. For $k>k_l$ the damping coefficients γ_{ms} and γ_{ml} are given by formulas (2.9) and (2.10). At $k=k_l$, γ_{ml} has the finite value

$$\Delta\gamma_m(k_l)\equiv\gamma_{ml}(k_l)=\frac{\omega_m^2T}{16\pi\bar{S}\omega_l^2}\frac{\omega_B}{\omega_l}F(T)\sin^2 2\theta_{\mathbf{k}}. \tag{2.12}$$

The value of $\Delta\gamma_m(k_l)$ is maximum at $\theta_{\mathbf{k}}=\pi/4$, and at $T=300$ K and $\omega_B=350\,K$ it is equal to $7.2\cdot10^6$ sec^{-1}. The occurrence of this jump can be easily understood by analyzing the conservation laws (2.4) with spectrum (2.6): The coalescence processes (2.4), which are forbidden for $k<k_l$, become allowed at $k=k_l$ simultaneously with all the magnons belonging to the linear part of the spectrum and having wave vectors $\mathbf{k}_1||\mathbf{k}$. Since the distance k_B from the center to the boundary of the Brillouin zone depends on the direction of the wave vector, the size of the jump $\Delta\gamma_m(\mathbf{k}_l)$ will depend on the direction of $\mathbf{k}_l$ with respect to the crystallographic axes. If we neglect the weak dependence of $F(T)$ on $\omega_B(\mathbf{k}_B)$, then $\Delta\gamma_m(\mathbf{k}_l)$ will be proportional to the zone-boundary magnon frequency ω_B in the direction of $\mathbf{k}$ (for a fixed angle $\theta_{\mathbf{k}}$ with respect to the direction of the magnetization $\mathbf{M}$). The crystallographic anisotropy of the jump is rather large: $\Delta\gamma_m(\langle100\rangle)/\Delta\gamma_m(\langle110\rangle)\approx1.5$.

It must be said that this simple geometric picture for the occurrence and anisotropy of the jump $\Delta\gamma_m$ results from the idealization (2.6) of the ferromagnet dispersion relation. In actuality the group velocity $v_{\mathbf{k}}$ depends, though slightly, on $\mathbf{k}$ at $k>k_0$: $\Delta v_{\mathbf{k}}/v_{\mathbf{k}}\sim0.1$. This leads to a smearing of the jump $\Delta\gamma_m(\mathbf{k}_l)$ over the interval $\Delta k/k_l\sim0.1$ and to some decrease in the crystallographic anisotropy.

As we see from (2.9) and (2.10), for $k\gg k_l$ the damping γ_{ml} decreases as k^{-1}, while γ_{ms} increases linearly with k. Asymptotically for $k\gg k_l$, formula (2.9) describes the familiar function $\gamma_m(k,\theta_{\mathbf{k}})$ [see, e.g., (9.2.51) of Ref. 5]. The damping γ_m is a universal function of the dimensionless wave vector $x=k/k_l$, with the magnon frequency entering only in the expression for k_l: $ak_l=\omega_0/\omega_l$. In the region $k\sim k_l$ the function $\gamma_m(x)$ is shown in Fig. 3 for $\theta_{\mathbf{k}}=\pi/2$ and $\theta_{\mathbf{k}}=\pi/4$.

We note that for $k<k_l$ there is a nonzero contribution to the ferromagnon relaxation from four-magnon scattering processes due to the magnetic dipole interaction:

$$\gamma_{4m}\approx\frac{1}{2(8\pi)^3}\frac{\omega_m^2T^2}{4\bar{S}^2\omega_{ex}^3}. \tag{2.13}$$

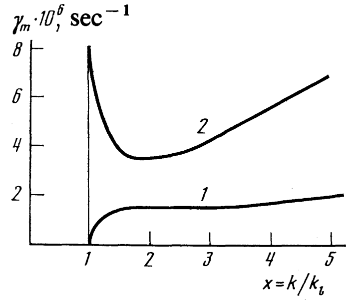

FIG. 3. Wave-vector dependence of the magnetic-dipolar relaxation rate of ferromagnons in YIG for the two directions $\theta_{\mathbf{k}}=\pi/2$(1) and $\theta_{\mathbf{k}}=\pi/4$ (2); $x=k/k_l (k_l=\omega_0/\omega_l a)$.

Here we have taken into account both the direct contribution to the four-magnon scattering amplitude from the magnetic dipole interaction and also the nonvanishing (at $\mathbf{k}\to 0$) exchange scattering which arises on account of the contribution of the magnetic dipole interaction to the magnon dispersion relation. At room temperature we have $\gamma_{4m}\approx 9\cdot 10^2$ sec^{-1}.

3. FERROMAGNON RELAXATION AT k→0

The ferromagnon damping $\gamma_{\mathbf{k}\to 0}$ is due to the local uniaxial anisotropy of the a ions, which gives rise to a coalescence of the ferromagnons with magnons of the optical branches of the spectrum:

$$\omega_0+\omega_{j\mathbf{k}}=\omega_{j'\mathbf{k}},\qquad j,\ j'\geqslant 2. \tag{3.1}$$

The symmetry of the nearest- neighbor environment of an a ion is lower than cubic, and the anisotropy energy of such an ion contains a term quadratic in S:

$$H_a^{(a)}=DS_\xi^2+{}^1/_6A'^2(S_x^4+S_y^4+S_z^4). \tag{3.2}$$

Here ξ is the three-fold trigonal axis, the coefficient D for YIG is[7] of the order of 0.3 K, while $A'^2\approx 0.03$ K. The term DS_ξ^2 is usually not taken into account, since its contribution to the interaction after summation over equivalent a positions goes to zero for ferromagnons with $\mathbf{k}\to 0$, which are homogeneous oscillations of the magnetic moment. For ferromagnons with $\mathbf{k}\neq 0$ the contribution proportional to D is suppressed by a factor $(ak)^2$ and, as a rule, is small compared to the contribution in A'^2. For the interaction involving optical magnons, however, there is no small long-wavelength factor $(ak)^2$, and the interaction DS_ξ^2 gives the main contribution to the amplitude of the processes of interest (3.1). The Hamiltonian of the a-ion local uniaxial anisotropy responsible for these processes has symmetry group O_h^{10} and is given by

$$H_a^{(a)}=\sum_n H_{n,a},$$

$$\begin{aligned}H_{n,a}={}^1/_3D[&(S_{1x}+S_{1y}+S_{1z})^2+(S_{2z}+S_{2y}-S_{2x})^2\\&+(S_{3z}+S_{3x}-S_{3y})^2+(S_{4z}-S_{4x}-S_{4y})^2+(S_{5x}+S_{5y}+S_{5z})^2\\&+(S_{6z}+S_{6y}-S_{6x})^2\\&+(S_{7z}-S_{7y}+S_{7x})^2+(S_{8x}+S_{8y}-S_{8z})^2].\end{aligned} \tag{3.3}$$

Here x, y, and z are the crystallographic axes (the edges of the cubic unit cell), and S_i with $i=1,\ldots,8$, number the a ions of the nth primitive cell.

Since only the a ions have uniaxial anisotropy, after changing in (3.3) to the variables b^+, b which diagonalize the quadratic Hamiltonian, we obtain the matrix elements for the interaction of ferromagnons with the optical modes of types a and a, d only. The lowest-lying of these is the a, d triad of modes ω_{d9j}, $j=1,2,3$, which is degenerate at $\mathbf{k}=0$: $\omega_{d9j}(0)=\Omega\approx 290$ K. The excitation energy of the remaining modes of types a and a, d is substantially higher, and we shall not consider them. Because of the degeneracy, the eigenvectors b_{d9j} are rather complicated nonanalytic functions of the wave vector: at $\mathbf{k}\to 0$ they depend on the direction: $b_{d9j}(\mathbf{k})=b_{d9j}(\mathbf{n})$, where $\mathbf{n}=\mathbf{k}/k$. The corresponding formulas are found in our preprint.[3] Omitting the awkward manipulations, we shall immediately give the expressions for the matrix elements of the Hamiltonian describing the interaction of $\mathbf{k}=0$ ferromagnons with optical magnons of the lower a, d triad:

$$H^{(3)}_{\mathrm{FM},a-d}=\sum_{\substack{\mathbf{k},j_1,j_2=1,2,3\\ j_1\neq j_2}}\Big[V\begin{pmatrix}d9j_1 & 1, & d9j_2\\ \mathbf{k} & 0, & \mathbf{k}\end{pmatrix}\times b^+_{d9j_1}(\mathbf{k})\,b_{10}{}^+\,b_{d9j_2}(\mathbf{k})+\mathrm{H.c.}\Big] \tag{3.4}$$

$$V\begin{pmatrix}d9j_1 & 1, & d9j_2\\ \mathbf{k} & 0, & \mathbf{k}\end{pmatrix}=Dv^2(S_0/2)^{1/2}\,W_{j_1,j_2}(\mathbf{n}),\qquad \mathbf{n}=\mathbf{k}/k,$$

$$\left.\begin{matrix}|W_{12}(\mathbf{n})|^2\\ |W_{13}(\mathbf{n})|^2\end{matrix}\right\}=\frac{1}{2}\Big\{1+4n_z^4-3n_z^2$$
$$\pm\frac{3(4n_z^2-1)\,[n_z^2(1-n_z^2)^2-n_x^2n_y^2(1+n_z^2)]}{(f_4^2(\mathbf{n})-f_6(\mathbf{n}))^{1/2}}\Big\},$$

$$|W_{23}(\mathbf{n})|^2=9\,\frac{n_y^6(n_x^2-n_z^2)^2+n_x^6(n_z^2-n_y^2)^2}{f_4^2(\mathbf{n})-f_6(\mathbf{n})}, \tag{3.5}$$

$$f_4(\mathbf{n})=3(n_x^2n_y^2+n_x^2n_z^2+n_y^2n_z^2),\qquad f_6(\mathbf{n})=27n_x^2n_y^2n_z^2.$$

Here v is a coefficient of the u, v transformation for this a, d triad from the irreducible to the quasinormal basis. With the values (1.2) for the exchange integrals we have $v\approx 0.47$ when $\mathbf{M}||[001]$.

To evaluate the damping

$$\gamma_1(0)=4\pi\sum_{\mathbf{k}j_1j_2}\left|V\begin{pmatrix}d9j_1 & 1, & d9j_2\\ \mathbf{k} & 0, & \mathbf{k}\end{pmatrix}\right|^2(n_{\mathbf{k}}^{d9j_1}-n_{\mathbf{k}}^{d9j_2})\times\delta(\omega_0+\omega_{d9j_1}(\mathbf{k})-\omega_{d9j_2}(\mathbf{k})) \tag{3.6}$$

it is necessary to know the dispersion relation $\omega_{d9j}(\mathbf{k})$ at least for small $\mathbf{k}$ with $ak\lesssim 1$. An analysis with the aid of perturbation theory in $\mathbf{k}$ (see Sec. 6 of Ref. 3) yields the following expressions (valid for $ak\lesssim 1$) for the frequencies of the a, d triad:

$$\Delta\omega_{32}(\mathbf{q})=\omega_{d9,3}(\mathbf{q})-\omega_{d9,2}(\mathbf{q})$$
$$=-2Zq^2[f_4^2(\mathbf{n})-f_6(\mathbf{n})]^{1/2}+Rq^4[f_4^2(\mathbf{n})-f_6(\mathbf{n})]^{1/4},$$

$$\Delta\omega_{31}(\mathbf{q})=\Delta\omega_{21}(\mathbf{q})=Bq^2,\qquad \mathbf{q}=a\mathbf{k}/8, \tag{3.7}$$

where

$$Z\approx 6\ \mathrm{K},\qquad R\approx 1200\ \mathrm{K},\qquad B\approx 370\ \mathrm{K}. \tag{3.8}$$

The coefficient of the q^2 term in $\Delta\omega_{32}$ turns out to be anomalously small, and this term can be neglected in comparison with the q^4 term:

$$\Delta\omega_{32}(\mathbf{q}) = Rq^4[f_4^2(\mathbf{n}) - f_6(\mathbf{n})]^{1/4}. \tag{3.9}$$

The damping (3.6) is given by the sum of three terms: $\gamma_1(0) = \gamma_{32}(0) + \gamma_{31}(0) + \gamma_{21}(0)$, corresponding to the coalescence processes

$$\omega_0 = \Delta\omega_{32}(\mathbf{q}), \quad \omega_0 = \Delta\omega_{31}(\mathbf{q}), \quad \omega_0 = \Delta\omega_{21}(\mathbf{q}). \tag{3.10}$$

For the frequencies of the a, d triad we obtain from (3.8) and (3.7)

$$\begin{gathered} \gamma_{32}(0) \approx v^4 \frac{D^2 e^{\Omega/T}}{T(e^{\Omega/T}-1)^2} \left(\frac{\omega_0}{R}\right)^{3/4} \frac{4\bar{S}}{\pi^2} C, \\ \gamma_{31}(0) + \gamma_{21}(0) \approx v^4 \frac{D^2 e^{\Omega/T}}{T(e^{\Omega/T}-1)^2} \left(\frac{\omega_0}{B}\right)^{1/2} \frac{128\bar{S}}{5\pi}. \end{gathered} \tag{3.11}$$

Here Ω is the gap of the a, d triad, and the constant C is given by

$$C = \int do \frac{|W_{23}(\mathbf{n})|^2}{[f_4^2(\mathbf{n}) - f_6(\mathbf{n})]^{3/16}} \approx 6.8. \tag{3.12}$$

If we take for Ω the value of the gap of the a, d triad at zero temperature ($\Omega \approx 290$ K), then the damping $\gamma_1(0)$ evaluated by formulas (3.11) turns out to be $0.36 \cdot 10^6\ \mathrm{sec}^{-1}$. This is several times smaller than the experimentally observed (at room temperature) value of $\gamma_1(0)$. However, at $T = 300$ K it is necessary to take into account the temperature dependence of the gap of the a, d triad. Certain experimental facts (see Ref. 10) indicate that the gaps of the optical modes in YIG behave under changes in temperature like the average magnetization of the sample. With allowance for this circumstance the gap of the a, d triad at $T = 300$ K becomes $\Omega \approx 200$ K, and accordingly, the damping (3.11) is

$$\gamma_1(0) \approx 0.9 \cdot 10^6\ \mathrm{sec}^{-1}; \tag{3.13}$$

in fair agreement with the experimental data (see Conclusion below). The damping $\gamma_1(0)$ as a function of T with allowance for the temperature dependence of the gap Ω is shown in Fig. 4.

We note that processes involving coalescence with optical magnons at the corners of the Brillouin zone give a nonzero contribution to the damping of ferromagnons with $\mathbf{k} \to 0$. However, simples estimates show that the volume of $\mathbf{k}$ space in which these processes are allowed is small, and the contribution from these processes is substantially smaller than the contributions which we have calculated.

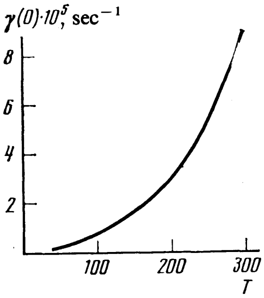

FIG. 4. Temperature dependence of the relaxation rate $\gamma(0)$ at $k \to 0$ due to the interaction with optical magnons.

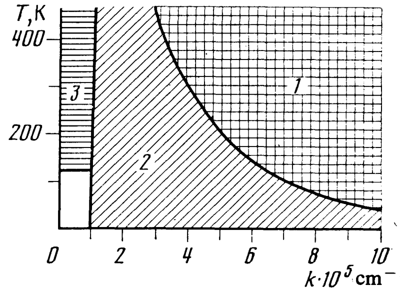

FIG. 5. Diagram of the relative contributions of various relaxation processes in YIG: 1) the region in which exchange relaxation is dominant [see Sec. 1], 2) the region of magnetic dipolar relaxation [see Sec. 2], 3) the region of relaxation involving optical magnons [Sec. 3], 4) the region in which the leading contribution is from relaxation involving defects ($\omega_0 = 1$ K).

CONCLUSION

Each of the elementary ferromagnon relaxation processes considered here—exchange scattering, the magnetic dipole interaction, and relaxation involving optical magnons—is dominant in a certain parameter region. Figure 5 shows the regions of temperature T and wave vector k in which the various elementary processes give the leading contribution. For long-wavelength magnons with $k < k_l$ the magnetic dipole interaction is forbidden and the amplitude of the exchange interaction is very small, so that in the region $k < k_l (k_l = \omega_0/a\omega_l)$ the leading contribution to the relaxation is that due to scattering by optical magnons; this scattering was considered in Sec. 3 [see (3.12)]. At low temperatures $T \lesssim 150$ K the damping $\gamma(0)$ falls off exponentially. In this temperature region the leading intrinsic relaxation process is four-magnon magnetic-dipole scattering (2.13), but, as a rule, under real conditions the ferromagnon damping at $T \lesssim 120$ K and $k < k_l$ is due to defects.[5] The contributions from the exchange and three-magnon magnetic-dipolar damping, as can be seen from (1.17) and (2.9), are comparable for $T \propto k^{-2}$ in the low-temperature region. At higher temperatures this dependence becomes smoother and asymptotically approaches $T \propto k^{-2/3}$ [see (1.20)].

There have been many experimental measurements of the ferromagnon relaxation in YIG; the most detailed study, as far as we know, was done by Anisimov and Gurevich.[4,5] These experiments, which pertain to the region $k \sim 10^5$ cm^{-1}, demonstrated good agreement of the k-dependent part of the damping with the theory for a ferromagnet. These experiments also detected the k-independent part of the damping in the region $k > 10^5$ cm^{-1}; this part was found to be $2.7 \cdot 10^6\ \mathrm{sec}^{-1}$ at $T = 300$ K and varied linearly with the temperature. Our theory implies that the k-independent part of the damping for such wave vectors ($k > k_l$) is the sum of two terms. One of these terms is the damping due to sattering by optical magnons, viz. $\gamma(0) \approx 0.9 \cdot 10^6\ \mathrm{sec}^{-1}$ [see (3.11)], and the other is the magnetic-dipolar damping on the plateau (see Fig. 3), equal to $1.5 \cdot 10^6\ \mathrm{sec}^{-1}$. The total relaxation rate is $2.4 \cdot 10^6\ \mathrm{sec}^{-1}$ at $T = 300$ K and agrees well with experiment both in magnitude and in temperature dependence in the 150–350 K region. When k/k_l is changed from 1.5 to 1, the magnetic dipolar relaxation is turned off, and the damping decreases sharply to $\gamma(0)$.

It must be said that in the experiments on parametric excitation of magnons by the method of parallel pumping,[4] our predicted "dip" in the damping for $k \leqslant k_l$ was not detected because magnons with $k < k_l$ and $\theta_{\mathbf{k}} = \pi/2$ were not excited even though the resonance conditons $\omega_p/2 = \omega(k,\pi/2)$ were satisfied. Instead, magnons with $k > k_l$ and $\theta_{\mathbf{k}} < \pi/2$ were excited. We believe that this is explained by the presence of elastic scattering of magnons (two-magnon processes) by defects and the boundaries of the sample. Nevertheless, this dip has been observed in experiments on the relaxation of magnons excited under conditions of kinetic instability.[15] In this case magnons were excited close to the bottom of the spectrum, with $k < k_l$ and $\theta_{\mathbf{k}} = 0$. The elastic scattering could not remove the magnons from this region and was therefore unimportant. The experimentally determined relaxation rate was $5 \cdot 10^5\ \mathrm{sec}^{-1}$ at a magnon frequency $\omega_{\mathbf{k}} = 2\pi \cdot 2 \cdot 10^6\ \mathrm{sec}^{-1}$. This value is smaller by a factor of three than the magnetic dipolar damping on the plateau, but it is larger than the rate $\gamma(0) = 1.7 \cdot 10^5\ \mathrm{sec}^{-1}$ for scattering by optical magnons. The discrepancy between the theoretical value of $\gamma(0)$ and the experimentally measured damping may be due,[15] on the one hand, to experimental error in determining $\gamma(0)$ as a result of the strong fluctuations in the emission from the magnons and, on the other hand, to insufficiently accurate knowledge of the local uniaxial anisotropy constant and to relaxation processes involving impurities.

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