Novel type of oscillatory effects for hot photoelectrons

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A novel type of oscillatory effects in nonequilibrium photoelectrons is indicated. The cause of the oscillations is that the interelectron scattering decreases jumpwise the number of LO phonons emitted in the course of the relaxation of the photoelectrons when the density of the latter is increased. This should lead to a nonmonotonic dependence of the electron-gas temperature, obtained from the energy balance, on the pump intensity. Also investigated, as an auxiliary problem of independent interest, are the energy distribution and the balance equation of the electron temperature when the electrons are excited with light near the optical-phonon emission threshold. The results are used for a critical analysis of the energy balance equations used in a number of earlier studies.

INTRODUCTION

It is known that various photoelectron properties have oscillatory dependences on the exciting-light quantum energy h (see the review[1] and also Ref. 2). The cause of these oscillations is the following.

The value of $\hbar\nu$ determines the initial energy ε_0 of the photoelectron. Such a photoelectron emits a certain number s of optical phonons $\hbar\Omega_0$ (s is the integer part of $\varepsilon_0/\hbar\Omega_0$) and retains the energy $\tilde{\varepsilon}_0 = \varepsilon_0 - s\hbar\Omega_0$. The retained energy $\tilde{\varepsilon}_0$ is a periodic function of ε_0, and it is this which determines in the upshot all the oscillatory effects.

We wish to point out in this paper novel oscillatory phenomena that are due to a unique conjunction of scattering by optical phonons and electron-electron scattering.

We consider a situation[3] wherein almost all the photoelectrons accumulate in an energy region below the threshold for the emission of an optical phonon (passive region $\varepsilon < \hbar\omega_0$), where *ee* scattering establishes a Maxwellian distribution with electron temperature T_e and density n. The electrons above the threshold (active region $\varepsilon > \hbar\Omega_0$) relax by emitting $\hbar\Omega_0$ phonons and by scattering from the passive electrons. The distribution in the active region is not Maxwellian, so that a photoelectron becomes "thermalized" by *ee* scattering only after landing in the passive region. In the course of relaxation and thermalization the photoelectron imparts to the Maxwellian gas an energy $\tilde{\varepsilon}_0 = \varepsilon_0 - \tilde{s}\hbar\Omega_0$, where $\tilde{s}$ is the effective number of $\hbar\Omega_0$ phonons it emits. It will be shown that when account is taken of the *ee* scattering of active electrons by passive ones the value of $\tilde{s}$ is lower than the value s indicated above, and w is a steplike function of the density n. As a result the values of T_e and n determined from the energy and particle-number balance equations turn out to be nonmonotonic functions of the pump intensity.

In the calculation of the dependence of $\tilde{\varepsilon}_0$ on n we had to solve an auxiliary problem, that of the form of the distribution function when the optical pumping is near the production threshold of the optical-phonon $\varepsilon = \hbar\Omega_0$, i.e., when $|\varepsilon_0 - \hbar\Omega_0| \ll \hbar\Omega_0$. The results pertaining to this auxiliary problem (Secs. 4 and 5) are also of independent interest.

Knowledge of $\tilde{\varepsilon}_0$ enables us to write the energy-balance equation for the thermalized part of the distribution in the passive region. Comparison with the balance equations used in other papers has shown that in a number of cases the earlier balance equations were in error.

1. KINETIC EQUATION

The kinetic equation for the electron-energy distribution function $f(\varepsilon)$ under stationary excitation is

$$S_L + C_{ee} + G + R = 0. \tag{1}$$

Here $G(\varepsilon)$ is the term that describes generation of the electrons, and $R(\varepsilon)$ their recombination. The term S_L is responsible for the scattering by the lattice, due to acoustic and optic phonons: $S_L = S_A + S_O$. For A scattering we have

$$S_A(f|\varepsilon) = -\frac{1}{g(\varepsilon)}\frac{d}{d\varepsilon}[g(\varepsilon)J_A(\varepsilon)]; \tag{2}$$

here $g(\varepsilon)$ is the state density, and the flux along the energy axis, due to the A scattering, is

$$J_A(f|\varepsilon) = -\frac{\varepsilon}{\tilde{\tau}_A(\varepsilon)}\left[f(\varepsilon) + T_L\frac{df(\varepsilon)}{d\varepsilon}\right], \tag{3}$$

where $\tilde{\tau}_A(\varepsilon)$ is the energy relaxation time and T_L is the lattice temperature.

In the description of the O scattering we shall neglect processes having a probability proportional to $\exp(-\hbar\Omega_0/T_L)$, i.e., the absorption and stimulated emission. In this approximation we have

$$S_O(f|\varepsilon) = -\frac{f(\varepsilon)}{\tau_O(\varepsilon)} + B(f|\varepsilon) = -\frac{1}{g(\varepsilon)}\frac{d}{d\varepsilon}[g(\varepsilon)J_O(\varepsilon)],$$
$$B(f|\varepsilon) = \frac{g(\varepsilon+\hbar\Omega_0)}{g(\varepsilon)}\frac{f(\varepsilon+\hbar\Omega_0)}{\tau_O(\varepsilon+\hbar\Omega_0)}, \tag{4}$$

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where $\tau_0(\varepsilon)$ is the time of spontaneous emission of the $K\Omega_0$ phonon, and the flux along the energy axis is

$$g(\varepsilon)J_0(\varepsilon)=-\int_{\varepsilon}^{\varepsilon+\hbar\Omega_0}d\varepsilon' g(\varepsilon')\frac{f(\varepsilon')}{\tau_0(\varepsilon')}. \quad (5)$$

The term C_{ee} describes the *ee* scattering. In the Landau diffusion approximation we have

$$C_{ee}(f,f|\varepsilon)=-\frac{1}{g(\varepsilon)}\frac{d}{d\varepsilon}[g(\varepsilon)J_{ee}(\varepsilon)], \quad (6)$$

where the flux on the energy axis, due to *ee* scattering, is

$$J_{ee}(f|\varepsilon)=-\left[A_{ee}(f|\varepsilon)f(\varepsilon)+D_{ee}(f|\varepsilon)\frac{df(\varepsilon)}{d\varepsilon}\right]. \quad (7)$$

The designation C_{ee} emphasizes that this quantity is a bilinear functional of f; the dynamic friction coefficient A_{ee} and the energy-diffusion coefficient D_{ee} are linear functionals of f and are given according to Ref. 4 by

$$A_{ee}(f|\varepsilon)=\frac{\varepsilon}{\tau_{ee}(\varepsilon)}\frac{1}{n}\int_0^{\varepsilon}d\varepsilon' g(\varepsilon')f(\varepsilon'),$$

$$D_{ee}(f|\varepsilon)=\frac{2}{3}\frac{\varepsilon}{\tau_{ee}(\varepsilon)}\frac{1}{n} \quad (8)$$

$$\times\left[\int_0^{\varepsilon}d\varepsilon'\,\varepsilon' g(\varepsilon')f(\varepsilon')+\varepsilon^{1/2}\int_{\varepsilon}^{\infty}d\varepsilon'\frac{1}{\sqrt{\varepsilon'}}g(\varepsilon')f(\varepsilon')\right].$$

Here $\tau_{ee}(\varepsilon)$ is the *ee*-scattering time, and we shall find it useful to represent it in the form

$$\frac{1}{\tau_{ee}(\varepsilon)}=\frac{\Lambda_{ee}}{\bar\tau_{ee}}y^{-1/2},\quad y=\frac{\varepsilon}{\hbar\Omega_0}, \quad (9)$$

where $\Omega_{ee}(\varepsilon)$ is the Coulomb logarithm and depends on the form of $f(\varepsilon)$, while

$$\frac{1}{\bar\tau_{ee}}=n\frac{8\pi e^4 m}{\varkappa_0^2 p_0^3}.$$

In the last formula k_0 is the static dielectric constant of the lattice, m is the effective mass of the electron, and $p_0^2/2m=\hbar\Omega_0$.

We shall consider polarization *PO* interaction with longitudinal *LO* phonons. In this case

$$\frac{1}{\tau_{PO}(\varepsilon)}=\frac{1}{\bar\tau_{PO}}y^{-1/2}\operatorname{Arch}y^{1/2},\quad \frac{1}{\bar\tau_{PO}}=2\alpha\Omega_0, \quad (10)$$

where α is the Fröhlich coupling constant. At high energies ($y\gg1$),

$$\frac{1}{\tau_{PO}(\varepsilon)}=\frac{\Lambda_{PO}}{\bar\tau_{PO}}y^{-1/2},\quad \Lambda_{PO}=\ln 2y^{1/2}. \quad (11)$$

Near the threshold ($y-1\ll1$)

$$\frac{1}{\tau_{PO}(\varepsilon)}=\frac{1}{\bar\tau_{PO}}(y-1)^{1/2}. \quad (12)$$

We note that such a dependence on ε near the threshold is valid also for *DO* scattering by the deformation potential of the optical phonons.

Equation (1) is solved under the following assumptions.

1) Almost all the electrons are in the passive region $\varepsilon<\hbar\Omega_0$, and the distribution $f(\varepsilon)$ in this region is close to Maxwellian

$$f_{T_e}(\varepsilon)=n(mT_e/2\pi\hbar^2)^{-3/2}e^{-\varepsilon/T_e}. \quad (13)$$

Therefore in the lowest approximation the coefficients A_{ee} and D_{ee} can be calculated with the distribution (13); this yields

$$A_{ee}(\varepsilon)=\frac{\varepsilon}{\tau_{ee}(\varepsilon)}H(\varepsilon)=\frac{D_{ee}(\varepsilon)}{T_e}, \quad (14)$$

where

$$H(\varepsilon)=\frac{1}{n}\int_0^{\varepsilon}d\varepsilon' g(\varepsilon')f_{T_e}(\varepsilon'),\quad \Lambda_{ee}(\varepsilon)=\ln\frac{T_e}{\varepsilon\chi_{min}^2}, \quad (15)$$

where $\chi_{\min}$ is the minimum Coulomb-scattering angle.

2) The recombination and A scattering are slow compared with O scattering and *ee* scattering, so that the terms R and S_A can be left out of (1) in the lowest-order approximation. When this procedure is used, the parameters n and T_e should be initially regarded as given and should be later be determined from the energy and particle-number balance equations.

3) It is assumed that $T_e\ll\hbar\Omega_0$ and *ee* scattering by the active electrons can be neglected. In the active region we have $\varepsilon\gg T_e$, and we get therefore from (14) for the dynamic-friction and diffusion coefficients

$$A_{ee}(\varepsilon)=\varepsilon/\tau_{ee}(\varepsilon)=D_{ee}(\varepsilon)/T_e,\quad \varepsilon>\hbar\Omega_0. \quad (16)$$

4) The pump is assumed monochromatic, so that

$$G(\varepsilon)=[G_0/g(\varepsilon_0)]\delta(\varepsilon-\varepsilon_0), \quad (17)$$

where G_0 is the number of photoelectrons with energy ε_0 in 1 cm^3 and in 1 sec. It is assumed that $\varepsilon_0\gg\hbar\Omega_0$.

2. QUALITATIVE DESCRIPTION OF THE ENERGY DISTRIBUTION ABOVE THE THRESHOLD, AND THE NATURE OF THE OSCILLATIONS

We discuss the character of the distribution $f(\varepsilon)$ in the active region on the basis of qualitative considerations.

If *ee* scattering is neglected, $f(\varepsilon)$ is a sum of peaks[3] that have the same shape as the generation peak $G(\varepsilon)$ and are centered at the points

$$\varepsilon_k^0=\varepsilon_0-k\hbar\Omega_0,\quad k=0,\ 1,\ 2,\ \ldots,\ s-1, \quad (18)$$

where s is the maximum number of *LO* phonons that can be emitted by an electron of energy ε_0 as it cascades down over the levels ε_k^0; by assumption, $s\gg1$. It is assumed also that ε_0 is "nonresonant," i.e., it is not a multiple of integer $\hbar\Omega_0$; in other words,

$$\varepsilon_0-s\hbar\Omega_0\sim\hbar\Omega_0.$$

The number k of the electrons at the peak is 3

$$n_k=G_0\tau_{PO}(\varepsilon_k). \quad (19)$$

It can be seen from (10) that n_k decreases with decreasing energy.

We now take *ee* scattering into account, assuming initially that $T_e=0$. After staying a time $\tau_{PO}(\varepsilon)$ on the level ε, the electron can then "glide" downward in energy on ac-

count of dynamic friction by an amount

$$\delta\varepsilon_{ee}(\varepsilon)=A_{ee}(\varepsilon)\tau_{PO}(\varepsilon).$$

Using (9), (11), and (14) and neglecting the logarithmic dependences on the energy, we find that at $\varepsilon\gg\hbar\Omega_0$ the glide does not depend on ε, i.e.,

$$\delta\varepsilon_{ee}=\bar{\eta}\hbar\Omega_0,\quad \bar{\eta}=\frac{\Lambda_{ee}}{\Lambda_{PO}}\frac{\bar{\tau}_{PO}}{\bar{\tau}_{ee}}\equiv\frac{n}{n_c}.$$

Here n_c is the density at which, in the active region, the time of energy exchange in *ee* scattering becomes comparable with the time of energy relaxation on emission of *LO* phonons. We note that the parameter $\bar{\eta}$ differs somewhat from the parameter η used in Sec. 4, namely, $\eta=\bar{\eta}\Lambda_{PO}$. As a result of the glide, the distribution peaks are no longer at the points ε_k^0 (18), but at the points

$$\varepsilon_k=\varepsilon_k^0-(k+1)\delta\varepsilon_{ee}=\varepsilon_0-k\hbar\Omega_0-(k+1)\bar{\eta}\hbar\Omega_0.$$

The glide does not change the number of electrons in each peak, so that relation (19) is valid as before also in the presence of *ee* scattering.

So long as n is small and $\varepsilon_{s-1}>\hbar\Omega_0$, the number of distribution peaks in the active region and the number of *LO* phonons emitted by each photoelectron are each equal to s and independent of n. With increasing n, however, at a certain density n_1 determined from the condition

$$\varepsilon_{s-2}-\hbar\Omega_0=\hbar\Omega_0, \tag{20}$$

the last peak turns out to be at the threshold $\varepsilon=\hbar\Omega_0$. At $n>n_1$ the number of peaks and the number of emitted phonons will be each smaller by one. The critical density n_1 corresponds to the parameter value

$$\bar{\eta}_1=[\varepsilon_0-s\hbar\Omega_0]/(s-1)\hbar\Omega_0.$$

It can be seen from (20) that $\bar{\eta}_1\sim s^{-1}\ll 1$. One more peak vanishes at a density n_2 determined from the condition $\varepsilon_{s-3}-\hbar\Omega_0=\hbar\Omega_0$, i.e., at

$$\bar{\eta}_2=[\varepsilon_0-(s-1)\hbar\Omega_0]/(s-2)\hbar\Omega_0.$$

It is easy to verify that the interval between the critical densities is constant:

$$\Delta n=n_{k+1}-n_k=n_c\frac{\varepsilon_0}{\hbar\Omega_0}\frac{1}{s^2}\approx\frac{n_c}{y_0},\quad y_0=\frac{\varepsilon_0}{\hbar\Omega_0}.$$

The foregoing means that the number of emitted *LO* phonons decreases stepwise with increasing n, and it is this which explains the meaning of the oscillations referred to in the Introduction.

The oscillation smearing is due to the broadening of the peaks of the distribution $f(\varepsilon)$. We shall discuss here the broadening due to *ee* scattering. We consider first the limiting case $T_e=0$, when *ee* scattering produces no diffusion along the energy axis, and there is only dynamic friction—a systematic downward glide of the electron in energy. Nonetheless, the peaks broaden in this case, too. The point is that $\tau_{PO}(\varepsilon)$ is the *average* time of stay in the transition from the level ε to the next step $\varepsilon-\hbar\Omega_0$ of the cascade: some electrons stay longer, others less. Therefore some electrons glide more and others less. Then

$$\langle\delta\varepsilon_{ee}\rangle\sim\bar{\eta}\hbar\Omega_0,\quad\langle[\delta\varepsilon_{ee}-\langle\delta\varepsilon_{ee}\rangle]^2\rangle^{1/2}\sim\bar{\eta}\hbar\Omega_0,$$

where $\langle\ldots\rangle$ denotes averaging over electrons staying on the *LO*-phonon emission level ε. As a result, the peak at ε_0 is not only displaced by an amount $\bar{\eta}\hbar\Omega_0$, but also broadens by $\bar{\eta}\hbar\Omega_0$. The peak at ε_k^0 for $k\gg 1$ shifts by $(k+1)\bar{\eta}\hbar\Omega_0$ and broadens by $(k+1)^{1/2}\bar{\eta}\hbar\Omega_0$ (since the broadenings add up randomly).

Additional broadening of the peaks sets in at $T_e\neq 0$. The diffusion "length" after a stay lasting $\tau_{PO}(\varepsilon)$ at $\varepsilon\gg\hbar\Omega_0$ is

$$[D_{ee}(\varepsilon)\tau_{PO}(\varepsilon)]^{1/2}\sim(\bar{\eta}\theta)^{1/2}\hbar\Omega_0,\quad\theta=T_e/\hbar\Omega_0;$$

it does not depend on ε. The broadening of a peak with number $k\gg 1$ is a random sum of $(k+1)$ of such lengths, so that it amounts to $(k+1)^{1/2}(\bar{\eta}\theta)^{1/2}\hbar\Omega_0$.

Thus, finally, the peak numbered $k\gg 1$ is shifted downward on account of *ee* scattering by an amount $(k+1)\bar{\eta}\hbar\Omega_0$ and broadens by an amount of the order of $(k+1)^{1/2}(\bar{\eta}^2+\bar{\eta}\theta)^{1/2}\hbar\Omega_0$. We are interested in the region $\bar{\eta}\sim s^{-1}$, when the number of emitted *LO* phonon varies. It is easy to verify that in this region the displacement of the overwhelming majority of peaks greatly exceeds their broadening. This means at the same time that the widths of all the peaks are much smaller than $\hbar\Omega_0$, so that the oscillations due to the peak broadening are weakley smeared.

Everything stated above concerning the locations and widths of the peaks pertained, strictly speaking, only to peaks located far from the threshold, in the region $\varepsilon\gg\hbar\Omega_0$. The estimates of the width and of the glide are valid also for those few peaks that are located in the region $\varepsilon\sim\hbar\Omega_0$, since the width and the glide build up during the cascading and the main contribution is made to them by the steps for which $\varepsilon\gg\hbar\Omega_0$. As for Eq. (19) for the number of electrons in the peak, it is independent of the assumption $\varepsilon\gg\hbar\Omega_0$.

Another cause of the smearing of the oscillations might be a smearing of the threshold $\varepsilon=\hbar\Omega_0$ on account of *ee* scattering. The meaning of threshold smearing is the following. An electron located above the threshold can be dragged by dynamic friction and by diffusion below the threshold and can therefore not emit an *LO* phonon. On the other hand, an electron located below the threshold can be pushed out by diffusion above the threshold and emit an *LO* phonon. The probability of the phonon emission is given by the function $W(z)$, obtained in Sec. 4 [$z=(\varepsilon-\hbar\Omega_0)/\hbar\Omega_0$]. The smearing of the threshold is determined by the difference between the function $W(z)$ and the step function $\Theta(z)$. Using the characteristic scales given in Sec. 5 for the function W, it is easy to verify that at all combinations of the parameters $\bar{\eta}$ and θ these scales are smaller than the width of the last peak of the distribution $f(\varepsilon)$, so that the threshold smearing does not influence the oscillation smearing.

3. ENERGY DISTRIBUTION IN THE ACTIVE REGION FAR FROM THE THRESHOLD

It is clear from the results of Sec. 2 that in the density region $n\ll n_c$ corresponding to the onset of oscillations we can neglect the overlap of different peaks of the distribution $f(\varepsilon)$. Under these conditions the first peak of the distribution in the vicinity of ε_0 is obtained as the solution of Eq. (1) with $B=0$, which is the equation for diffusion with drift in the presence of a source G and a drain f/τ_{PO}; this solution must

be solved with the boundary conditions

$$f(+\infty)=f(-\infty)=0. \tag{21}$$

The peak near $\varepsilon_0-\hbar\Omega_0$ is obtained similarly, except that now $G=0$ and the role of the source is assumed by B of Eq. (4), in which $f(\varepsilon+\hbar\Omega_0)$ is now determined by the peak obtained near ε_0. All the succeeding peaks are obtained similarly; an exception is the last peak if it lands in the near-threshold region, where $|\varepsilon-\hbar\Omega_0|\ll\hbar\Omega_0$.

We transform to the dimensionless distribution

$$\varphi(z)=g(\varepsilon)\frac{f(\varepsilon)}{\tau_{PO}(\varepsilon)}\frac{\hbar\Omega_0}{G_0},\quad z=y_0-y,\quad y_0=\frac{\varepsilon_0}{\hbar\Omega_0}. \tag{22}$$

We note that if we integrate $\varphi(z)$ in the vicinity of one of the peaks and use (19) we obtain $\int dz\varphi(z)=1$. This means that the function $\varphi(z)$ is a superposition of the form factors of all the peaks. The variable z can be interpreted as the number of steps of the LO-phonon emission cascade.

Using for S_{PO} the representation (4) with the flux (5) and integrating the kinetic equation from ε to infinity with account taken of (17), we obtain the equation

$$\bar\eta(z)\left[\varphi(z)-\theta\frac{d}{dz}\varphi(z)\right]+\int_{z-1}^{z}dz'\varphi(z')=\Theta(z), \tag{23}$$

where

$$\bar\eta(z)=\varepsilon\tau_{PO}(\varepsilon)/\hbar\Omega_0\tau_{ee}(\varepsilon).$$

Most peaks lie in the region $\varepsilon\gg\hbar\Omega_0$, were $\bar\eta(z)$ can be replaced by the constant $\bar\eta$. It is important, however, that this substitution yields correct form factors also for the last few peaks located in the region $\varepsilon\sim\hbar\Omega_0$. This follows from the results of the qualitative treatment in Sec. 2.

Taking the Fourier transform

$$F(k)=\frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{+\infty}dz\,e^{ikz}\varphi(z),\quad \operatorname{Im}k>0$$

and transforming (23), we get

$$F(k)=-(2\pi)^{-1/2}D^{-1}(k),\quad D(k)\equiv ik\bar\eta(1+ik\theta)+(e^{ik}-1).$$

To solve Eq. (23) with the condition (21) at $\varepsilon=-\infty$ we must introduce in this equation a weak "drain," corresponding to the following inverse transform

$$\varphi(z)=\frac{i}{2\pi}\int_{+i0-\infty}^{+i0+\infty}dk\frac{e^{-ikz}}{D(k)}. \tag{24}$$

At $z<0$ the integration contour is closed in the half-plane $\operatorname{Im}k>0$, where there is a single pole at

$$\tilde k\approx i(1+\rho^{1/2})/2\theta,\quad \rho=1+4\theta/\bar\eta. \tag{25}$$

The high-energy wing of the first peak is therefore described near ε_0 by the function

$$\varphi(z)=\rho^{-1}\exp\left[-|z|(1+\rho^{1/2})/2\theta\right]. \tag{26}$$

This wing appears only in the presence of diffusion and is therefore absent at $T_e=0$.

At $z>0$ the integration contour is closed in the half-plane $\operatorname{Im}k<0$, so that

$$\varphi(z)=\frac{1}{1+\bar\eta}+\sum_{m=-\infty}^{+\infty}{}'c_m e^{-ik_m z},\quad c_m=i\left(\frac{dD}{dk}\right)^{-1}_{k=k_m}. \tag{27}$$

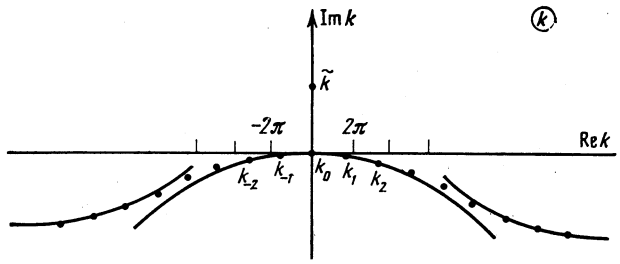

FIG. 1. Poles of the function $D(k)^{-1}$. The parabolic asymptote (28) and the logarithmic asymptote (29) are shown.

Here k_m are the zeros of $D(k)$, and the prime denotes that the term with $m=0$ corresponding to $k_m=0$ in the sum is written out separately.

The arrangement of the singularities k_m is shown in Fig. 1. At not too large numbers we have at $m\bar\eta\ll1$ and $m^2\bar\eta\theta\ll1$

$$k_m\approx2\pi m(1-\bar\eta)-i2\pi^2m^2\sigma^2,\quad \sigma^2=\bar\eta^2+2\bar\eta\theta. \tag{28}$$

The parameter σ is the total smearing of the peak (in units of $\hbar\Omega_0$) per step of the LO-phonon emission cascade—see Sec. 2.

For very large numbers, when $m\bar\eta\gg1$ and (or) $m^2\bar\eta\theta\gg1$ we get

$$k_m\approx2\pi m-i\ln\left[i2\pi m\bar\eta(1-i2\pi m\theta)\right],\quad \ln(1)=0. \tag{29}$$

A more lucid expression for $\varphi(z)$ at $z>0$ can be obtained in the region $z\gg1$; this corresponds to a large number of emitted phonons.

If the number of steps of the cascade is so large that the total broadening exceeds $\hbar\Omega_0$ and the peaks overlap strongly, i.e., $\sigma^2z\gg1$, only the terms with $m=\pm1$ are significant in the sum (27), so that

$$\varphi(z)=\frac{1}{1+\bar\eta}+2e^{-2\pi^2\sigma^2z}\cos\left[2\pi(1-\bar\eta)z\right]. \tag{30}$$

The principal term of this representation corresponds to that expression which can be obtained for $f(\varepsilon)$ by transforming in (4) at $\varepsilon\gg\hbar\Omega_0$ to the "diffusion" representation for the current

$$J_0(\varepsilon)=-\left[\hbar\Omega_0/\tau_{0.}(\varepsilon)\right]f(\varepsilon)$$

(only dynamic friction is present here, since LO-phonon absorption is not taken into account in (4)).

If, however, the number of cascade steps is not so large and the peaks do not overlap, i.e., $\sigma^2z\ll1$, the significant terms in (27) are those for which $m\sigma^2\ll1$. Noting that $c_m\approx1$ for these terms, using (28), and then extending the summation over m to infinity, we obtain at $z>0$

$$\varphi(z)=1+2\sum_{m=1}^{\infty}e^{-4\pi^2m^2t}\cos(2\pi mx)=\psi(x,t), \tag{31}$$

where

$$t=\sigma^2z/2,\quad x=(1-\bar\eta)z>0.$$

The sum over m can be transformed by noting that ψ satisfies the equation

$$(\partial/\partial t-\partial^2/\partial x^2)\psi(x,t)=0 \tag{32}$$

with the "initial" condition

$$\psi(x,0)=\sum_{k=-\infty}^{\infty}\delta(x-k). \tag{33}$$

At $t=0$ this ψ corresponds to the distribution $f(\varepsilon)$ obtained with allowance for the glide but without taking the peak broadening into account. The "development" with respect to t corresponds to accumulation of the broadening as the cascade progresses. Using the known Green's function of Eq. (32), we get

$$\varphi(z)=\sum_{k=1}^{\infty}\Phi(\sigma z^{1/4}|(1-\bar\eta)z-k)\approx\sum_{k=1}^{\infty}\Phi_0(\sigma\tilde k^{1/4}|z-\tilde k(1+\bar\eta)), \tag{34}$$

where Φ_0 is the normalized Gaussian distribution

$$\Phi_0(a|\xi)=(2\pi a^2)^{-1/2}\exp(-\xi^2/2a^2).$$

Returning to dimensional quantities, we see readily that (34) corresponds to the $f(\varepsilon)$ dealt with in Sec. 2.

The function (34) gives the correct form factors of all the peaks numbered $k\gg 1$; the total number of electrons in each of these peaks can be obtained from (19). An exception is the peak that lands on the threshold, a fact that can be seen already from (19), since $\tau_{PO}(\varepsilon)$ becomes infinite at the threshold. To find the form of this last peak we solve below, in Sec. 4, an auxiliary problem concerning the form of the distribution function in the case of near-threshold pumping, whose role in the case of interest to us is played by the arrival term $B(\varepsilon)$ (4) due to phonon emission by the electrons of the penultimate active-region peak that has not yet reached the threshold. The form factor and the number of electrons of this peak are determined from (34) and (19)

4. ELECTRON DISTRIBUTION FOR NEAR-THRESHOLD EXCITATION

We consider Eq. (1) for the case when electron generation by the light takes place near the phonon-emission threshold $\varepsilon=\hbar\Omega_0$, i.e., $|\varepsilon_0-\hbar\Omega_0|\ll\hbar\Omega_0$, in an energy band $\Delta\varepsilon_0\ll\hbar\Omega_0$ whose center is the point ε_0. The relation between $\Delta\varepsilon_0$ and the mismatch $\varepsilon_0-\hbar\Omega_0$, and also the sign of the mismatch, can be arbitrary. We introduce the pump form factor $\Phi(y-y_0)$, centered about $y_0=\varepsilon_0/\hbar\Omega_0$, having a width $\Delta y_0=\Delta\varepsilon_0/\hbar\Omega_0$ and normalized to

$$\int_{-\infty}^{+\infty}dy\,\Phi(y)=1.$$

The pump can then be represented in the form

$$G(\varepsilon)=(G_0/n_0)\Phi(y-(1+\delta)), \tag{35}$$

where G_0 is the number of electrons produced in 1 cm^3 per second and $\delta=(\varepsilon_0-\hbar\Omega_0)/\hbar\Omega_0$, while the characteristic density is

$$n_0=\hbar\Omega_0 g(\hbar\Omega_0)=p_0^3/(2\pi\hbar)^3.$$

Inasmuch as in the considered auxiliary problem electrons are present in the active region only near the threshold, we can use the expansion (12). Next, calculating from (16) A_{ee} and D_{ee} for the active electrons, we can put $\varepsilon=\hbar\Omega_0$. Near the threshold we have $B(\varepsilon)=0$, since there are no electrons in the region $\varepsilon\approx 2\hbar\Omega_0$.

Taking all the foregoing into account, we can represent Eq. (1) in the active region in the form

$$\eta\frac{d}{dy}\left[f(y)+\theta\frac{d}{dy}f(y)\right]-f(y)(y-1)^{1/2}=-p\Phi(y-(1+\delta)),\quad y>1. \tag{36}$$

We have introduced here the dimensionless pump $p=G_0\bar\tau_0/n_0$, as well as a parameter that determines the competition between the O scattering and the ee scattering, viz., $\eta=\Lambda_{ee}\tau_0/\bar\tau_{ee}$, where Λ_{ee} is calculated from (15) at $\varepsilon=\hbar\Omega_0$. In this notation, the equation in the passive region is

$$\frac{1}{y^{1/2}}\eta\frac{d}{dy}\left\{H(y)\left[f(y)+\theta\frac{d}{dy}f(y)\right]\right\}=-f(y+1)-p\Phi(y-(1+\delta)). \tag{37}$$

The boundary conditions imposed on the system (36) and (37) are

$$f(y)=0\quad(y\to\infty); \tag{38}$$

$$f(y-0)=f(y+0),\quad \frac{df}{dy}(y-0)=\frac{df}{dy}(y+0)\quad(y=1). \tag{39}$$

The absence of a boundary condition at $y=0$ leaves undetermined one of the four constants that arise in the integration of the system (36), (37). This constant is determined by specifying the density n.

The solution of (36) reduces to construction of the Green's function of the single-parameter equation

$$\left[\frac{d^2}{dx^2}+\lambda\frac{d}{dx}-x^{1/2}\right]g(x,x')=-\delta(x-x') \tag{40}$$

with the parameter $\lambda=(\eta^2/\theta^3)^{1/5}$. This can be easily verified by making the change of variables $y-1=\mu^2 x$, $\mu=(\eta\theta)^{1/5}$. We impose on the Green's function the boundary conditions $g(0,x)=0$, $g(\infty,x)=0$. The solution of (36) can then be written in the form

$$f(y)=f_1(y)+f_2(y)\quad(y>1), \tag{41}$$

$$f_1(y)=Au\left(\frac{y-1}{\mu^2}\right), \tag{42}$$

$$f_2(y)=\frac{p}{\mu^3}\int_1^{\infty}dy'\,\Phi(y'-(1+\delta))\,g\left(\frac{y-1}{\mu^2},\frac{y'-1}{\mu^2}\right), \tag{43}$$

where A is the integration constant and $u(x)$ is the solution of the homogeneous equation (40) with boundary conditions

$$u(0)=1,\quad u(\infty)=0.$$

Introducing also a second independent solution

$$v(x)=u(x)\int_0^{x}dx'\frac{e^{-\lambda x'}}{u^2(x')}, \tag{44}$$

that satisfies the boundary conditions

$$v(0)=0,\quad v'(0)=1.$$

we can express the Green's function explicitly:

$$g(x,x')=e^{\lambda x'}[\Theta(x-x')u(x)v(x')+\Theta(x'-x)u(x')v(x)]. \tag{45}$$

By the same token, the construction of the solution in the

active region reduces to construction of the function $u(x)$ which is investigated in the Appendix.

Substituting (41) in the right-hand side of (37) we obtain an equation for the distribution in the passive region. By assumption, this distribution differs little from Maxwellian, but this smallness has only an "integral" character. Thus in the matching region $y = 1$, where f_{T_e} is smallest, the true distribution can differ greatly from f_{T_e}. This is precisely why Eq. (37) must be solved in the passive region. Since we are interested in a solution only at $y \gg \theta$ we can assume that $H = 1$, after which (37) can be readily integrated. The result is

$$f(y)=Ce^{-y/\theta}-j+AK(y)+p\int_{\cdot}^{\infty}dy'\Phi(y'-(1+\delta))L(y,y'), \tag{46}$$

where C and j are the integration constants in the passive region. The second constant has the meaning of the (dimensionless) flux at the threshold:

$$j=(f+\theta df/dy)\,|_{y=1}.$$

The kernel L consists of two parts:

$$L(y,y')=\Theta(y'-1)L^+(y,y')+\Theta(1-y')L^-(y,y'). \tag{47}$$

The second term in (37) is connected with the pump $p\Phi$ in the right-hand side of (37). Then

$$L^-(y,y')=\frac{(y')^{1/2}}{\eta}e^{-y/\theta}[\Theta(y-y')(e^{y'/\theta}-1)+\Theta(y-y')(e^{y/\theta}-1)]. \tag{48}$$

This quantity is in essence the Green's function of Eq. (37) at $H = 1$. The first term in (47) is connected with the distribution f_2 (43) which is substituted in the right-hand side of (37); it is of the form

$$L^+(y,y')=\frac{1}{\mu}\int_0^{\infty}dxL^-(y,\mu^2x)\,g\left(x,\frac{y'-1}{\mu^2}\right). \tag{49}$$

The term AK in (46) is due to the distribution f_1 (42) substituted in the right-hand side of (37). In analogy with (49) we have

$$K(y)=\frac{1}{\mu}\int_0^{\infty}dxL^-(y,\mu^2x)\,u(x). \tag{50}$$

Using the properties of $u(x)$ whihc are described in the Appendix, it is easy to verify that at $y \gg \theta$ we have at all values of and

$$K(y)=\frac{b}{\eta}e^{-y/\theta}, \tag{51}$$

where

$$b=\int_0^{\infty}dx\,x^{1/2}e^{\lambda x}\,u(x). \tag{52}$$

In the region $y \gg \theta$ of interest to us the term AK in (46) can therefore be omitted, changing the meaning of the constant C. Since, by assumption, $f(\varepsilon)$ differs little from $f_{T_e}(\varepsilon)$, we can assume that C is determined by the density n, as in (13), and write ultimately for the distribution in the passive region

$$f(\varepsilon)=f_{T_e}(\varepsilon)-j+p\int_0^{\infty}dy'\Phi(y'-(1+\delta))L\left(\frac{\varepsilon}{\hbar\Omega_0},y'\right). \tag{53}$$

The constant j in this equation and the constant A in the distribution (41) in the active region are determined from the matching conditions (39); this yields

$$A=a\left[f_{T_e}(\varepsilon=\hbar\Omega_0)+p\int_0^{\infty}dy\Phi(y-(1+\delta))M(y)\right], \tag{54}$$

$$j=(1-a)\left[f_{T_e}(\varepsilon=\hbar\Omega_0)+p\int_0^{\infty}dy\Phi(y-(1+\delta))\bar{M}(y)\right], \tag{55}$$

where

$$\begin{aligned}M(y)&=\Theta(y-1)M^+(y)+\Theta(1-y)M^-(y),\\ \bar{M}(y)&=\Theta(y-1)a(a-1)^{-1}M^+(y)+\Theta(1-y)M^-(y),\\ M^-(y)&=\frac{1}{\eta}e^{-(1-y)/\theta},\\ M^+(y)&=\frac{1}{\eta}[e^{(y-1)/\theta}u(x)+e^{-1/\theta}(1-u(x))]\,|_{x=(y-1)/\mu^2}.\end{aligned} \tag{56}$$

and

$$a=\lambda|u'(0)|^{-1}. \tag{57}$$

We note that the distribution (53), obtained only for $\varepsilon \gg T_e$, can be assumed to be valid also at $\varepsilon \lesssim T_e$, for in this region the corrections to f_{T_e} are small. Further, it can be easily seen that in the solution of the kinetic equation we use only the fact that there are few electrons in the active region and that all are located near the threshold. The solution obtained holds therefore not only for the case of near-threshold pumping, but also for the case when the pumping goes into the passive region far from the threshold ($\delta < 0$, $|\delta| \sim 1$).

We proceed now to discuss the formulas obtained for the distribution $f(\varepsilon)$ and elucidate the meaning of the parameter λ. To this end we must understand the cause of the characteristic scale of the function $f_1(\varepsilon)$, which is given according to (A.5) and (A.9) by

$$\begin{aligned}&\Delta\varepsilon\sim\theta\hbar\Omega_0=T_e, \quad \lambda\gg1,\\ &\Delta\varepsilon\sim(\eta\theta)^{2/5}\hbar\Omega_0=\lambda T_e, \quad \lambda\ll1.\end{aligned} \tag{58}$$

We consider an electron produced at the threshold and ascertain how far it can penetrate into the active region by the diffusion due to ee scattering. The diffusion depth $\Delta\varepsilon$ is limited by phonon emission and by gliding into the passive region via dynamic friction. It can be estimated from the equation

$$(\Delta\varepsilon)^2/D_{ee}\sim\min\{\tau_0(\varepsilon+\Delta\varepsilon),\ \Delta\varepsilon/A_{ee}\}. \tag{59}$$

The left-hand side is the diffusion time, and the right contains the phonon emission time and the time of gliding via dynamic friction. The choice between the two possibilities in (59) depends on λ. If $\lambda \gg 1$, the diffusion is bounded by dynamic friction, but if $\lambda \ll 1$ it is bounded by phonon emission. The solution of Eq. (59) for the penetration depth $\Delta\varepsilon$ in these two cases is given by (58).

It can be seen from (53) that in the passive region the distribution function is distorted compared with f_{T_e} for two

reasons: the presence of pumping (the terms proportional to p) and the scattering by optical phonons (the terms remaining at $p = 0$).

Let us consider in greater detail the case when $p = 0$ (the pump is turned off or the electrons are heated by a static field E). The distribution in the active region is then f_1 (42) with $A = af_{T_e}$ ($\varepsilon = \hbar\Omega_0$), while the distribution in the passive region differs from f_{T_e} by the constant j, with $j = (1-a)f_{T_e}$ ($\varepsilon = \hbar\Omega_0$). At $\lambda \gg 1$, when the departure from the active region is determined by dynamic friction, we can use for $u(x)$ Eq. (A.6), and according to (A.12) we have $a \approx 1$. It is clear therefore that the distribution is close to $f_{T_e}(\varepsilon)$ for all ε. At $\lambda \ll 1$, when the departure is determined by emission of optical phonons, $a \approx 0$ and $j \approx f_{T_e}(\varepsilon = \hbar\Omega_0)$, so that the value $A = f(\varepsilon = \hbar\Omega_0)$ of the distribution function at the threshold decreases drastically compared with the Maxwellian distribution, while the distribution in the active region decreases at a penetration depth λT_e much smaller than T_e.

Turning now to the case $p \neq 0$, we show that the distribution $f(\varepsilon)$ can be calculated at $\lambda \ll 1$ and $\lambda \gg 1$ for different values of p and δ by using the results of the Appendix. The form of the distribution can vary greatly, depending on the sign of δ and on the values of p and λ.

The results of the present section were subsequently used to determine the form of the distribution function near the threshold and in the passive region for a pump (17) high in the band. To this end the pump form factor Φ contained in the equations of this section must be replaced by the form factor φ of the penultimate peak in the active region, shifted downward in energy by $\hbar\Omega_0$.

5. POWER LOSS IN THE CASE OF NEAR-THRESHOLD EXCITATION

The distribution (41) in the active region can be used to calculate the power transferred to the lattice via spontaneous emission of optical phonons in the case of near-threshold excitation

$$Q_0(f) = \int_{\hbar\Omega_0}^{\infty} d\varepsilon g(\varepsilon)\hbar\Omega_0 \frac{f(\varepsilon)}{\tau_0(\varepsilon)}. \tag{60}$$

Gathering together all the terms that are explicitly proportional to the pump, we can represent the calculation result in the form

$$Q_0 = Q_n + Q_G, \tag{61}$$

where

$$Q_n = Q_0\eta(1-a)f_{T_e}(\varepsilon = \hbar\Omega_0), \tag{62}$$

$$Q_G = pQ_0\int_0^{\infty} dy\Phi(y-(1+\delta))W(y-1). \tag{63}$$

We have introduced here the characteristic power loss $Q_0 = n_0\hbar\Omega_0/\bar{\tau}_0$ and the function

$$\begin{aligned} W(z) &= 1 - ae^{\lambda x}u(x)\,|_{x=z/\mu^2}, \quad z>0, \\ W(z) &= (1-a)e^{z/\theta}, \quad z<0. \end{aligned} \tag{64}$$

The term Q_n, which is proportional to the density n and does not depend explicitly on the pump, becomes simpler in limiting cases:

$$Q_n = \frac{\hbar\Omega_0}{\bar{\tau}_0} ne^{-\hbar\Omega_0/T_e}, \quad \lambda \gg 1; \tag{65}$$

$$Q_n = \frac{2}{\pi^{1/2}} \frac{\hbar\Omega_0}{\tau_{ee}(\varepsilon = T_e)} ne^{-\hbar\Omega_0/T_e}, \quad \lambda \ll 1. \tag{66}$$

The ee-scattering time drops out of Q_n at $\lambda \gg 1$, and the O-scattering time at $\lambda \ll 1$. The physical meaning of this is the following. The energy relaxation is due to the joint action of the ee and O scattering: the ee scattering pushes the electron out above the threshold, where it emits an optical phonon.[5] At $\lambda \gg 1$ the bottleneck is the second step, and at $\lambda \ll 1$ the first. The power loss (62) was obtained in Ref. 6. (The parameter B_2 introduced there is connected with the factor $1-a$ used by us.)

The function $W(z)$ with $z > 0$ is the probability that an electron produced above the threshold will manage to emit an optical phonon before it goes off into the passive region by the diffusion and the dynamic friction which are due to ee scattering. At $z < 0$, $W(z)$ is the probability that an electron produced below threshold will rise to the threshold by diffusion and then emit an $\hbar\Omega_0$ phonon. At the threshold, $W(0) = 1 - a$, so that a is the probability that an electron produced at the threshold will go into the passive region without emitting a phonon. It is easy to verify that $W(z)$ and $W'(z)$ are continuous at $z = 0$. A plot of $W(z)$ is shown in Fig. 2. The characteristic scale in z at $z < 0$ is θ for all λ. At $z > 0$ the characteristic scale depends on λ. It can be seen from (A.5) and (A.9) that it is equal to $\eta^{2/3} = \theta\lambda^{5/3} \gg \theta$ at $\lambda \gg 1$ and to $(\eta\theta)^{2/5} = \theta\lambda \ll \theta$ at $\lambda \ll 1$. In the limiting cases, using (A.5), (A.9), (A.10), and (A.11), we obtain: as $\lambda \to \infty$ (e.g., as $\theta \to 0$)

$$\begin{aligned} W(z) &= 1 - \exp\left\{-\frac{2}{3}\frac{z^{3/2}}{\eta}\right\}, \quad z>0, \\ W(z) &= 0, \quad z<0; \end{aligned} \tag{67}$$

and as $\lambda \to 0$ (e.g., as $\eta \to \infty$)

$$\begin{aligned} W(z) &= 1, \quad z>0, \\ W(z) &= e^{z/\theta}, \quad z<0. \end{aligned} \tag{68}$$

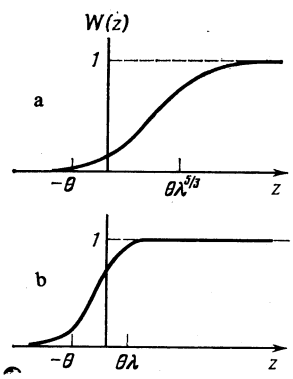

FIG. 2. Influence of ee scattering on the dependence of the probability of emitting an optical phonon (a—$\lambda \ll 1$, b—$\lambda \gg 1$).

The characteristic scales of the function $W(z)$ are easy to understand. We consider for this purpose the fate of an electron produced in the active region at a point ε near the threshold. This electron will emit an optical phonon if

$$\tau_0(\varepsilon) \ll (\varepsilon - \hbar\Omega_0)/A_{ee}, \quad (\varepsilon - \hbar\Omega_0)^2/D_{ee}. \tag{69}$$

On the right are written out the glide time and diffusion time from the point ε to the boundary of the active and passive region, while A_{ee} and D_{ee} were calculated at $\varepsilon = \hbar\Omega_0$. If either of the inequalities in (69) is reversed, the electron goes into the passive region by *ee* scattering. Substituting in (69) the emission time (12), we can verify that the parameter determines which of the inequalities in (69) is decisive.

If $\lambda \gg 1$ (low temperatures, high densities), the most dangerous is dynamic friction and it follows from (69) that

$$\varepsilon - \hbar\Omega_0 \gg \eta^{1/3}\hbar\Omega_0 = \theta\lambda^{5/3}\hbar\Omega_0 = \lambda^{5/3}T_e.$$

If, however, $\lambda \ll 1$ (high temperatures, low densities), diffusion dominates in the active region, and it follows then from (69) that

$$\varepsilon - \hbar\Omega_0 \gg (\eta\theta)^{2/3}\hbar\Omega_0 = \theta\lambda\hbar\Omega_0 = \lambda T_e.$$

This explains the scale of the function $W(z)$ at $z > 0$. At the same time, the limiting values (A.10) and (A.12) of the parameter a become clear. If dynamic friction predominates, $a = 1$ and $W(0) = 0$, i.e., an electron produced at the threshold certainly goes into the passive region without emitting a phonon. If, however, diffusion dominates, then $a = 0$ and $W(0) = 1$, i.e., the electron crosses many times the boundary between the regions and sooner or later emits a phonon.

We emphasize that the scales of the functions $f_1(y)$ and $W(z)$ (at $z > 0$) coincide only if $\lambda \ll 1$. In the case $\lambda \gg 1$ they are different, with the scale of $W(z)$ the larger.

We note that the power loss Q obtained in Ref. 6 at $\lambda \ll 1$ differs from Q obtained in Ref. 5, although at first glance the same formulation of the problem was used in both papers. The point is that in Ref. 6, just as in the present paper, the Landau diffusion approximation was used for C_{ee}. This means that, by assumption, the depth $\Delta\varepsilon$ of penetration into the active region is much larger than the characteristic energy $(\delta\varepsilon)_{ee} \sim \varepsilon\chi_{\min}^2$ transferred in *ee* scattering. In this approach, the distribution at the threshold $\varepsilon = \hbar\Omega_0$ is continuous. If, however, $(\delta\varepsilon)_{ee} \gg \Delta\varepsilon$, the diffusion approximation is not valid and $f(\varepsilon)$ has a discontinuity at the threshold. This is precisely the case considered in Ref. 5. If it is assumed that $\chi_{\min}$ is determined by the ratio of the *ee*-collision parameter at $\varepsilon = \hbar\Omega_0$ to the Debye length, then the energy transfer is

$$(\delta\varepsilon)_{ee} \sim \hbar\Omega_0 \frac{n}{n_D}, \quad n_D \equiv \frac{9}{32\pi}\left(\frac{\hbar\Omega_0}{e^2}\right)^3.$$

Comparing this transfer with $\Delta\varepsilon$ according to (58), we can verify that the diffusion approximation is applicable.

To conclude this section, we emphasize that the presence of a pump even below the threshold leads to an additional contribution to the power loss, a contribution that does not reduce to a change of the density n in Q_n.

6. POWER-LOSS OSCILLATIONS

We turn finally to our final problem, that of proving that upon excitation high into the band ($\varepsilon_0 \gg \hbar\Omega_0$) the power loss (60) due to spontaneous emission of optical phonons is a nonmonotonic function of the electron density.

Consider, for the sake of argument, the density region $n_1 \leqslant n < n_2$, when the active region far from the threshold contains $(s-1)$ peaks of the distribution $f(\varepsilon)$, with numbers $k = 0, 1, \ldots, (s-2)$. If resonance is present ($n \approx n_1$), the peak numbered $(s-1)$ lands near the threshold, and if there is no resonance this peak does not exist at all. Each of the peaks that are far from the threshold makes a contribution $G_0\hbar\Omega_0$ to Q, i.e., the total is $(s-1)G_0\hbar\Omega_0$. To calculate the contribution of the threshold region we use (61), taking Φ to mean, as indicated in Sec. 4, the form factor φ of the peak $(s-2)$, shifted by $\hbar\Omega_0$ downward in energy.

The peak $(s-2)$ is localized at the point $y_{s-2} = y_0 - s + 2 - (s-1)\bar{\eta}$. Writing $y_{s-2} = 2 + \delta$, where $\delta < 0$, we have $\delta = -(s-1)(\bar{\eta} - \bar{\eta}_1)$, so that $\delta = 0$ corresponds to $n = n_1$ and $\delta < 0$ corresponds to $n > n_1$. The form factor of the shifted peak is, according to (34),

$$\Phi_0(\sigma(s-2)^{1/2}|y-(1+\delta)).$$

Its width is of the order of $s^{1/2}(\bar{\eta}^2 + 2\bar{\eta}\theta)^{1/2}$ at $s \sim \bar{\eta}^{-1}$, i.e., of the order of $(\bar{\eta} + \theta)^{1/2}$. Going through all the possible relations between the parameters $\bar{\eta}$ and θ (and assuming that the parameters η and $\bar{\eta}$ are of the same order), we can show that the width $(\bar{\eta} + \theta)^{1/2}$ is always larger than the characteristic scales of the change of the function W contained in (63). In the integration we can therefore replace the function W by a step function, and obtain

$$Q_{PO} = (s-1)G_0\hbar\Omega_0 + Q_n + G_0\hbar\Omega_0 \int_1^\infty dy\,\Phi_0(\sigma(s-2)^{1/2}|y-(1+\delta)).$$

The last expression can be rewritten in the form

$$Q_{PO} = Q_n + G_0\hbar\Omega_0\tilde{s}, \quad \tilde{s} = \int_0^{y_0-1} dz\,\varphi(z), \tag{70}$$

where $\tilde{s}$ is the average number of the LO phonons emitted by each of the not-yet-thermalized photoelectrons, while Q_n is the power lost by the thermalized electrons. Obviously, Eq. (70) is valid not only in the considered interval $n_1 \leqslant n < n_2$, but at all densities.

From (24) we get at $y_0 \gg 1$

$$\tilde{s} = \frac{1}{2\pi}\int_{+i0-\infty}^{+i0-\infty} dk\,\frac{(e^{iky_0}-1)}{kD(k)}. \tag{71}$$

Calculating this integral for the limiting cases, we obtain the s that correspond to the distribution s(30) and (34). At $\sigma^2 y \gg 1$, when most peaks overlap strongly, we have

$$\tilde{s} = \frac{y_0}{1+\bar{\eta}} + \frac{1}{\pi}e^{-2\pi^2\sigma^2 y_0}\sin[2\pi(1-\bar{\eta})y_0]. \tag{72}$$

At $\sigma^2 y_0 \ll 1$, when the peaks do not overlap, we obtain the relation of interest to us

$$\tilde{s} = [y_0(1-\bar{\eta})] + \frac{1}{2}\operatorname{erfc}\left(\frac{1-\{y_0(1-\bar{\eta})\}}{(\sigma^2 y_0/2)^{1/2}}\right) - \frac{1}{2}\operatorname{erfc}\left(\frac{\{y_0(1-\bar{\eta})\}}{(\sigma^2 y_0/2)^{1/2}}\right). \tag{73}$$

The square and curly brackets in the last formula denote

respectively the integer and fractional parts of the number. The first term is obtained if the peak widths are neglected. It is a stepwise function of $\bar{\eta}-a$ "staircase." The remaining terms are responsible for the seamring of the stairs because of the finite widths of the peaks. The second term is connected with the high-energy tail (which actually does not exist) of the peak that landed in the passive region, and the third is connected with the low-energy tail of the last peak in the active region.

7. ENERGY BALANCE

The energy-balance equation for the determination of T_e is

$$G_0\tilde{\varepsilon}_0=Q_{PO}+Q_A, \tag{74}$$

where the two terms in the right-hand side correspond to energy relaxation by the optical and acoustic phonons. The term Q_A can be calculated by neglecting the deviation of the distribution from Maxwellian, i.e.,

$$Q_A=\int_0^\infty d\varepsilon g(\varepsilon) f_{T_e}(\varepsilon)(\varepsilon-\varepsilon^*)\frac{1}{\tilde{\tau}_A(\varepsilon)}, \tag{75}$$

where $\varepsilon^*=2T_L$ for *DA* scattering by a deformation potential and $\varepsilon^*=T_L$ for *PA* scattering by a piezoacoustic potential. We represent the term Q_{PO} in the form (61) and combine in (74) the terms proportional to the pump. The balance equation takes then the same form as for the thermalized part of the distribution in the passive region:

$$G_0\tilde{\varepsilon}_0=Q_n+Q_A, \tag{76}$$

where

$$\tilde{\varepsilon}_0=\varepsilon_0-\tilde{s}\hbar\Omega_0 \tag{77}$$

is the energy that each photoelectron brings into this region after emitting the optical phonons.

The oscillatory effect referred to above is connected with the nonmonotonic dependence of $\tilde{\varepsilon}_0$ on n in the case of excitation high into the band. This dependence is illustrated by the plot in Fig. 3, where the dimensionless quantities $\tilde{\varepsilon}_0/\hbar\Omega_0\equiv\tilde{y}_0=y-\tilde{s}$ and $n/n_c=\bar{\eta}$ are used. We assume for simplicity that $\theta=0$, so that $\sigma=\bar{\eta}$. At low densities, i.e., $\bar{\eta}\ll y_0^{-1/2}$, the plot is a smoothed "staircase" with step $\Delta\bar{\eta}=y_0^{-1}$ along the $\bar{\eta}$ axis and with step 1 along the $\tilde{y}_0$ axis. The staircase begins on the level $\tilde{y}_0=\{y_0\}$ (fractional part) at $\bar{\eta}=0$. The smearing of the first steps is $\delta\bar{\eta}\sim\bar{\eta}y_0^{-1/2}$. This part of the curve is described by Eq. (73). When $\bar{\eta}\sim y_0^{-1/2}$, the smearing $\delta\bar{\eta}$ becomes of the order of the step $\Delta\bar{\eta}$ and the staircase is completely smeared out. At higher densities, i.e., $y0^{-1/2}\ll\bar{\eta}\ll 1$, the plot approaches a smooth one corresponding to the first term of (72). The second term gives exponentially weak oscillations with a somewhat enlarged period.

In the region of the first steps, $\bar{\eta}\sim y_0^{-1}$, the jumps of the function $\tilde{y}_0$ are large, of the order of the function itself. It is therefore meaningless to speak of separating the monotonic part of the function y_0. The oscillations become relatively small at $\bar{\eta}\gg y_0^{-1}$. At such densities the monotonic part of the dependence of $\tilde{y}_0$ on $\bar{\eta}$ is

$$\tilde{y}_0=y_0\bar{\eta}/(1+\bar{\eta}). \tag{78}$$

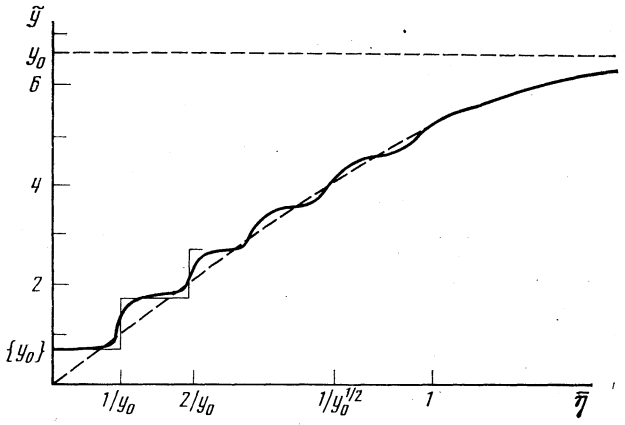

FIG. 3. Dependence of the energy delivered by the photoelectron to the thermalized part of the distribution (in units of $\hbar\Omega_0$) on the photoelectron density (in units of n_c).

It describes also the region of the remote steps of the staircase in the region when only exponentially weak oscillations remain. The relation (78) is valid also at $\bar{\eta}\gg 1$.

For excitation near the threshold we have

$$\tilde{\varepsilon}_0=\hbar\Omega_0[\delta+1-W(\delta)]. \tag{79}$$

If the pump is far from the threshold (outside of the region of variation of $W(z)$, where we can assume $W(z)=1$ at $z>0$ and $W(z)=0$ at $z<0$), we have $\tilde{\varepsilon}_0=\varepsilon_0$ for pumping below the threshold and $\tilde{\varepsilon}_0=\varepsilon_0-\hbar\Omega_0$ for pumping above the threshold, which is physically understandable.

Let us compare the balance energy (76) with the ones used in other papers. In Ref. 7 it was found from physical considerations that

$$\tilde{\varepsilon}_0=\varepsilon_0 n/(n+n_c), \tag{80}$$

which corresponds to (78). It is clear from the foregoing that (80) is a good approximation only in the region $n\gg n_c/s$, which corresponds, e.g., for GaAs at typical $s\sim 10$, to a density $n\gg 10^{17}$ cm^{-3}. It is assumed in Ref. 8 that

$$\tilde{\varepsilon}_0=(\varepsilon_0-s\hbar\Omega_0)+\varepsilon_0 n/(n+n_c). \tag{81}$$

It can be easily seen, however, that this relation is not an approximation of the true $\tilde{\varepsilon}_0$ in any region of the parameters. The point is the following. The second term is taken to mean the energy that the photoelectron transfers to the thermalized part of the distribution on account of *ee* scattering in the course of cascading with emission of *LO* phonons, while the first is taken as the energy that it contributes to this distribution after landing in the passive region. If, however, the *ee* scattering is significant, the electron glides in the active region and arrives in the passive region with an energy different from $\varepsilon_0-s\hbar\Omega_0$. This circumstance was likewise not taken into account in Ref. 3, where Eq. (35) corresponds to (81) at $n\ll n_c$.

In the balance equation used in Ref. 9, the energy $\tilde{\varepsilon}_0$ for the near-threshold excitation is taken to be (in our notation)

$$\begin{aligned}\tilde{\varepsilon}_0&=(\varepsilon_0-\hbar\Omega_0)+\left[1+\frac{\tau_{ee}(\varepsilon_0)}{\tau_{PO}(\varepsilon_0)}\right]^{-1},\\ \varepsilon_0&=\hbar\Omega_0\left[\delta+\left(1+\frac{\delta^{1/2}}{\eta}\right)^{-1}\right].\end{aligned} \tag{82}$$

This expression, obtained from qualitative considerations, does not agree with $\tilde{\varepsilon}_0$ (79) at any value of the parameters.

We turn now to the right-hand side of the balance equation (76). If only spontaneous emission of LO phonons is taken into account, the expression assumed for Q_n almost everywhere (see, e.g.,

$$Q_n = \int_{\hbar\Omega_0}^{\infty} d\varepsilon g(\varepsilon) \frac{\hbar\Omega_0}{\tau_0(\varepsilon)} f_{T_e}(\varepsilon). \tag{83}$$

It is presupposed in it that the distribution in the active region near the threshold, at $\varepsilon - \hbar\Omega_0 \sim T_e$, is Maxwellian. Yet, as shown in Ref. 6 and in Sec. 4, this is the case only at sufficiently high densities and low temperatures, if the following inequality holds:

$$\lambda = (\eta^2/\theta^3)^{1/3} \gg 1.$$

Transforming to dimensional quantities, we have

$$n \gg n_c (T_e/\hbar\Omega_0)^{3/2}.$$

For GaAs this means $n \gg 10^{15}$, 10^{16}, 10^{17} cm^{-3} respectively at $T_e = 5$, 20, 80 K. These conditions are satisfied far from always. For example, under the conditions typical of the experiment in Ref. 11, at $T_e = 20$ K and $n = 1.2 \times 10^{14}$ cm^{-3}, we have $\lambda \approx 0.5$. Thus, in many cases the energy balance equation used is in fact wrong.

8. EXPERIMENTAL MANIFESTATIONS OF THE OSCILLATORY AND THRESHOLD EFFECTS

The stepwise character of the dependence of $\tilde{\varepsilon}_0$ on n should lead, when solving the balance equation (76), to steplike singularities in the dependence of T_e on G_0, and these singularities might be experimentally observable. If the effective recombination time depends on T_e, the dependence of n on G_0 will also be steplike. It is even possible for hysteresis regions to appear, with non-single-valued dependence of T_e on G_0 in certain intervals of G_0.

Bearing in mind experiments in which the oscillations in question might be observable, we must discuss a few more circumstances.

The additional broadening of the peaks of the distribution $f(\varepsilon)$ and the ensuing smearing of the oscillations are the result of the finite dispersion of the LO phonons. Let us estimate this broadening $(\delta\varepsilon)_{LO}$. The dispersion law of LO phonons at low phonon momenta q is $\Omega_q = \Omega_0[1 - (qa)^2]$, where a is of the order of the lattice constant. Typical values of q are of the order of the electron momentum k, so that the characteristic broadening of the peak in one step of the cascade is $\hbar\Omega_0 k^2 a^2 \sim \hbar\Omega_0 \varepsilon/E$, where $E = \hbar^2/ma^2$. The total broadening can be estimated at $(\delta\varepsilon)_{LO} \sim s^{1/2}\hbar\Omega_0\varepsilon_0/E$. Thus, the smearing due to the dispersion of the LO phonon is quite small.

The exponential damping of the exciting light at the absorption depth leads to an energy release that is not uniform over the sample thickness. It follows hence that T_e will vary with the distance from the surface, and this will mask the oscillations, since the quantities usually measured are integral over the sample volume. This difficulty can be avoided by using the method of two-photon absorption, in which a sufficiently thin sample can be excited uniformly over its depth.

Additional difficulties are caused by the complicated structure of the hole band. The presence of two hole bands (light and heavy holes) leads, at a fixed photon energy $\hbar\nu$, to two values of the initial energy ε_0 corresponding to excitation of electrons from each band. The result should be two series of oscillations with different periods. In addition, owing to the corrugation of the hole bands both energies ε_0 will be smeared, and the smearing $\Delta\varepsilon_0$ will be larger the larger ε_0. It can be estimated from the formula

$$\frac{\Delta\varepsilon_0}{\varepsilon_0} = \frac{m_e/m_h}{1+m_e/m_h} \frac{\Delta m_h}{m_h},$$

where m_e and m_n are the masses of the electron and of the corresponding hole, while Δm_h is the variation of m_h along different directions of the hole momentum.

The variation of the mass of GaAs with direction is 25% for heavy holes and 2% for light ones. This yields for $\Delta\varepsilon/\varepsilon_0$ the values 0.03 and 0.01, respectively. Putting $s = 10$, we obtain for the ratio $\Delta\varepsilon_0/\hbar\Omega_0$ the values 0.3 and 0.1. The smearing of the heavy-hole peak is found to be quite appreciable.

Threshold singularities in the dependence of T_e on ε_0 were observed in Ref. 12. This dependence shows clearly how the *ee* scattering "smears" the threshold $\varepsilon = \hbar\Omega_0$.

To find T_e from Eq. (76) it is necessary to add to it the electron-number balance equation or to use some dependence of the density n on the pump G_0. By way of illustration we found the dependence of T_e on ε_0 for electrons in GaAs by using the relation

$$n = \bar{n}_0 (G_0/\bar{G}_0)^{\sigma}. \tag{84}$$

For the energy relaxation time in A scattering we have

$$\frac{1}{\bar{\tau}_A(\varepsilon)} = \frac{2ms^2}{\hbar\Omega_0}\left[\frac{1}{\bar{\tau}_{DA}} y^{1/2} + \frac{1}{\bar{\tau}_{PA}} y^{-1/2}\right], \tag{85}$$

where s is the speed of sound, and $\bar{\tau}_{DA}$ and $\bar{\tau}_{PA}$ are the characteristic times of the deformation and polarization scattering. Substituting (85) in (75) we obtain

$$Q_A = n\frac{2}{\pi^{1/2}}\frac{2ms^2}{\hbar\Omega_0}(T_e - T_L)\left[\frac{1}{\bar{\tau}_{DA}} y_L^{1/2} + \frac{1}{\bar{\tau}_{PA}} y_L^{-1/2}\right],$$

$$y_L = \frac{T_L}{\hbar\Omega_0}. \tag{86}$$

The pump was assumed monochromatic, i.e., we put $\Delta\varepsilon_0 = 0$. The calculation results for several values of the pump are shown in Fig. 4. It hardly makes sense to compare them with the experimental results, since the model employed does not take into account many factors that are possibly significant: the electron-energy relaxation on holes, and the presence of two electron-excitation points ε_0 due to the presence of two hole bands (thus, if electrons are produced at a threshold with $\varepsilon_0 = 36$ meV in transition from the heavy-hole band, electrons are simultaneously produced in transitions from the light-hole band at a threshold with $\varepsilon_0 = 20$ meV).

The following numerical values were used in the calculations and estimates for the parameters of GaAs:

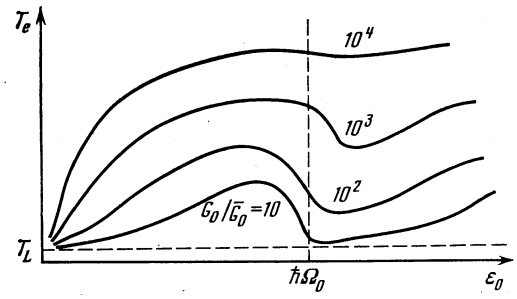

FIG. 4. Dependence of electron temperature T_e on the initial photoelectron energy e_0 for different ratios $G_0/\bar{G}_0 = 10, 10^2, 10^3, 10^4$. Lattice temperature $T_L = 4.2$ K.

energies

$\hbar\Omega_0=420$ K$=36$ meV, $ms^2=0{,}12$ K;

scattering times

$\bar{\tau}_{DA}=4{,}6$ psec $\bar{\tau}_{PA}=11$ psec $\bar{\tau}_{PO}=0{,}14$ psec

$\bar{\tau}_{ee}(n=n_0)=0{,}22$ psec

densities

$n_0=4{,}5\cdot 10^{17}$ cm^{-3}, $n_D=3\cdot 10^{18}$ cm^{-3}, $\bar{n}_0=2\cdot 10^{13}$ cm^{-3};

pumps

$n_0/\bar{\tau}_{PO}=3{,}2\cdot 10^{30}$ cm^{-3}/sec (at this G_0 we have $p=1$),

$\bar{G}_0=0{,}65\cdot 10^{20}$ cm^{-3}/sec (exponent $\sigma=0{,}7$).

We point out that at an absorption depth 1 μm and at a photon energy 1.53 eV the pump $\bar{G}_0$ corresponds to an absorbed flux 1.6 mW/cm^2.

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APPENDIX

To study the homogeneous equation (40) it is convenient to make the substitution

$$u(x)=e^{-\frac{1}{2}\lambda x}\varphi(x). \tag{A.1}$$

This yields the equation

$$d^2\varphi/dx^2-(x^{1/2}+\tfrac{1}{4}\lambda^2)\varphi=0. \tag{A.2}$$

At large $x\gg\lambda^4$ we can discard the term $\lambda^2\varphi$, after which the equation reduces to a Bessel equation whose solution is

$$\varphi(x)=x^{1/2}K_{2/5}(\tfrac{4}{5}x^{5/4})\equiv K(x). \tag{A3}$$

The solution $I_{2/5}$, as can be easily seen, does not satisfy the condition on $u(x)$ at $x=\infty$. At small $x\ll\lambda^4$ we have

$$\varphi(x)=e^{\pm\frac{1}{2}\lambda x}. \tag{A.4}$$

At $\lambda\ll 1$ the region $x\ll\lambda^4$ does not contribute to the necessary integrals with the function $u(x)$, so that we can put

$$u(x)=K(x)/K(0), \quad K(0)=\tfrac{1}{2}(\tfrac{5}{2})^{2/5}\Gamma(\tfrac{2}{5}). \tag{A.5}$$

Under these conditions

$$v(x)=I(x)/I'(0),$$
$$I(x)=x^{1/2}I_{2/5}(\tfrac{4}{5}x^{5/4}), \quad I'(0)=(\tfrac{5}{2})^{3/5}/\Gamma(\tfrac{2}{5}).$$

At $\lambda\gg 1$ the approximation (A.4), which corresponds to

$$u(x)=e^{-\lambda x}, \tag{A.6}$$

is in many cases insufficient, as will be shown subsequently. For such λ we can obtain a solution by the WKB method, viz.,

$$\varphi(x)=p(x)^{-1/2}\exp\left\{\pm\int_0^x dx'p(x')\right\},$$
$$p(x)=(x^{1/2}+\tfrac{1}{4}\lambda^2)^{1/2}. \tag{A.7}$$

This solution is valid at $x\gg\lambda^{-1}$. At $\gg\lambda^4$ such a solution with a minus sign in the exponential leads to an asymptotic expansion of the function $K(x)$ at $x\gg 1$. At $x\ll\lambda^4$ the solution (A.7) with a minus sign coincides with the decreasing exponential (A.4). It is clear therefore that at $\lambda\gg 1$ the function we need is of the form

$$u(x)=\left[\frac{p(0)}{p(x)}\right]^{1/2}\exp\left\{-\frac{1}{2}\lambda x-\int_0^x dx'p(x')\right\}. \tag{A.8}$$

Actually, at $\lambda\gg 1$ a sufficient approximation is obtained by expanding $p(x)$ in reciprocal powers of λ, namely

$$u(x)=\exp\left\{-\lambda x-\frac{2}{3}\frac{x^{3/2}}{\lambda}\right\}. \tag{A.9}$$

As for the function $v(x)$ at $\lambda\gg 1$, when it is used in the Green's function (45) to calculate the distribution $f(\varepsilon)$ it suffices to use the aprpoximation (A.4); then

$$v(x)=\frac{1}{\lambda}(1-e^{-\lambda x}).$$

With the aid of (A.5) we can obtain

$$a=\lambda\left(\frac{2}{5}\right)^{1/5}\frac{\Gamma(\tfrac{2}{5})}{\Gamma(\tfrac{3}{5})}, \quad \lambda\ll 1. \tag{A.10}$$

To find the corresponding expansion at large λ, it is convenient to use the relation

$$a=1+\frac{1}{u'(0)}\int_0^\infty dx x^{1/2}u(x), \tag{A.11}$$

which follows from the equation for the function $u(x)$. We can substitute in it $u(x)$, retaining only the first term of the expansion in (A.9); this yields

$$a=1-(\pi^{1/2}/2)\lambda^{-5/2}, \quad \lambda\gg 1. \tag{A.12}$$

We note that if we confine ourselves at $\lambda\gg 1$ to the approximation (A.6), the function $W(z)$ at $z>0$ turns out to be close to unity everywhere all the way to the threshold $z=0$ itself, patently contradicting the physical meaning. The integral (52) also diverges in this approximation.

[1]P. G. Harper, J. W. Hodby, and R. A. Stradling, Rep. Progr. Phys. **36**, 1 (1973).
[2]C. Weisbuch, Solid State Electronics **21**, 179 (1978).
[3]I. B. Levinson and B. N. Levinskiĭ, Zh. Eksp. Teor. Fiz. **71**, 300 (1976) [Sov. Phys. JETP **44**, 156 (1976)].
[4]V. L. Ginzburg and A. v. Gurevich, Usp. Fiz. Nauk **70**, 201 (1960) [Sov. Phys. Usp. **3**, 115 (1960)].

[5]I. B. Levinson and G. E. Mazhuolite, Zh. Eksp. Teor. Fiz. **50**, 1048 (1965) [Sov. Phys. JETP **23**, 697 (1966)].
[6]B. L. Gel'mont, R. I. Lyagushchenko, and I. N. Yassievich, Fiz. Tverd. Tela (Leningrad) **14**, 533 (1972) [Sov. Phys. Solid State **14**, 445 (1972)].
[7]J. Shah and R. C. Leite, Phys. Rev. Lett. **22**, 1394 (1969).
[8]J. Shah, Solid State Electronics **21**, 43 (1978).
[9]J. Shah, C. Liu, R. F. Leheny, and A. E. DiGiovanni, Solid State Commun. **18**, 487 (1976).
[10]S. Tanaka, T. Kuwata, T. Hokimoto, H. Kobayashi, and H. Saito, J. Phys. Soc. Jpn. **52**, 677 (1983).
[11]R. Ulbrich, Phys. Rev. **B8**, 5719 (1973).
[12]E. O. Goebel and O. Hildbrand, Phys. Stat. Sol. **B88**, 645 (1978).

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