# Magnetoresistance of metal films with low impurity concentrations in a parallel magnetic field

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The magnetoresistance $\sigma(H)$ of a thin film with diffuse electron scattering at the boundary and a low concentration of impurities is calculated for the case of a parallel magnetic field $H$. It is shown that a study of $\sigma(H)$ can be used to determine the mean free path $l$ for scattering by impurities.

## 1. INTRODUCTION

Interest has recently heightened in the study of the magnetoresistance of thin metal and semiconductor films.[1] This is due in large measure to the appearance of theoretical papers[2–6] which explain the anomaly in the magnitude and sign of the magnetoresistance of semiconductors by an effect of the magnetic field on the quantum corrections to the conductivity. This effect is particularly pronounced in films.

The anomalous magnetoresistance of films was calculated in Refs. 2 and 4; Ref. 2 considered the case in which the magnetic field is perpendicular to the film, and Ref. 4 considered the case of a parallel field. Both calculations apply under conditions in which the mean free path $l$ is much shorter than the film thickness $d$, i.e., the film is considered "dirty."

In many experiments amorphous metal films are used. In spite of the structural imperfections of these films, the mean free path in them is appreciable, and so the inequality $l \gg d$ may be satisfied. This is due to the relatively small size of the pseudopotential of an individual atom. This pseudopotential plays the role of the scattering center for electrons in amorphous systems. Therefore, the only effective process is diffuse scattering at the film surface.

Calculation of the anomalous magnetoresistance for $l \gg d$ is thus a pressing problem. Here the diffusion approximation, which has been used in the previous conductivity calculations, does not apply. Assuming the condition $L_H \gg \lambda$, where $L_H = (eH)^{-1/2}$ is the magnetic length and $\lambda$ is the electron wavelength on the Fermi surface ($\hbar = c = 1$ throughout), the problem at hand can be solved in the quasiclassical approximation. This yields equations of the kinetic type for the Green functions and cooperon, the latter being responsible for the leading quantum correction to the conductivity for $p_F l \gg 1$. These equations and their solutions are given in Sec. 3.

The formula for the correction to the conductivity in a magnetic field for $d \ll l$ can also be obtained (up to numerical coefficients) from simple qualitative considerations.

## 2. QUALITATIVE PICTURE

The main complication which arises in considering the magnetoresistance in a pure film in a parallel magnetic field is that the additional phase advance $e\int \mathbf{A}\cdot d\mathbf{r}$ arises not along the entire electron trajectory but only on individual segments on which the electron undergoes scattering by impurities. In fact, on a segment of the trajectory which crosses the film without collisions with impurities (see Fig. 1a) we have,[7] in the gauge $\mathbf{A} = (Hy,0,0)$,

$$e\int \mathbf{A}\,d\mathbf{r} = eH\int_{-1/2 d \operatorname{ctg}\alpha}^{1/2 d \operatorname{ctg}\alpha} y\,dx = eH \operatorname{ctg}\alpha \int_{-d/2}^{d/2} y\,dy = 0. \tag{1}$$

Therefore, in a film without impurities the phase advance on a closed trajectory, the quantity which determines the magnetoresistance, is independent of the length of a typical trajectory. If scattering by impurities occurs in the film, then even rare scattering events lead to growth of the phase advance as the length of the trajectory increases. Thus, if the time of motion along a trajectory is equal to $t$, while the time between collisions with impurities is $\tau$, then the phase advance is made up from the contributions of the individual scattering events, the number of which is $t/\tau$. This phase advance is given by

$$K(t) \equiv \langle e^{i\varphi(t)}\rangle = \prod_{i=1}^{t/\tau} \langle e^{i\varphi_i}\rangle. \tag{2}$$

The phases advance $\varphi_i$ depends on $H$ and the angle $\alpha$ (see Fig. 1b) between the velocity direction and the $x$ axis. For small angles $\alpha$ we have $\varphi_i \sim eHd^2/\alpha$, and the average in formula (2) means an average over angles.[1)] The minimum value of $\alpha$ is equal to $d/l$, where $l = v\tau$ is the mean free path for scattering by impurities. Therefore, the maximum value of $\varphi_i$ is of order $eHdl$. For $eHdl \ll 1$ we find

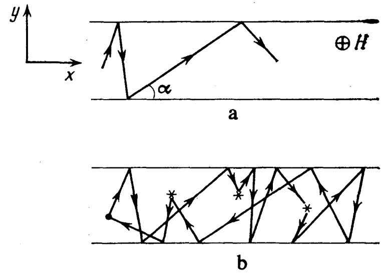


FIG. 1.

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$$K(t)=\exp\left[-\frac{t}{2\tau}\langle\varphi_i{}^2\rangle\right]=\exp\left[-\frac{t}{2\tau}\int_{d/l} d\alpha\left(\frac{eHd^2}{\alpha}\right)^2\right]$$

$$=\exp(-t/\tau_H), \tag{3}$$

$$1/\tau_H={}^1/_{16}(eHd)^2dv. \tag{4}$$

The coefficient $\frac{1}{16}$ is obtained as a result of a systematic evaluation [see formula (29)]. It is seen from (4) that in the weak-field region $1/\tau_H$ does not depend on $\tau$. In the case when $eHdl\gg1$, for angles not too close to grazing ($\alpha\gg eHd^2$) the phase increment is small, $\varphi_i\ll1$, while for $\alpha\lesssim eHd^2$ we have $\varphi_i\gtrsim1$. For the trajectories closest to grazing, on which $\varphi_i\gtrsim1$, each collision event, independent of $\alpha$, leads to phase relaxation. Therefore

$$K(t)=\exp\left[-\frac{t}{\tau}\int d\alpha f\left(\frac{eHd^2}{\alpha}\right)\right]=\exp(-t/\tau_H),$$
$$f(x)\sim\begin{cases}x^2, & x\ll1,\\ \text{const}, & x\gg1.\end{cases} \tag{5}$$

Rendering the integral in formula (5) dimensionless, we obtain for $eHdl\gg1$ [see also (31)]

$$1/\tau_H=eHd^2/3\tau. \tag{6}$$

The quantum correction to the conductivity can be evaluated with the formula[4]

$$\delta G(H)=-\frac{2De^2}{\pi}\int d^2q\left(Dq^2+\frac{1}{\tau_\varphi}+\frac{1}{\tau_H}\right)^{-1}, \tag{7}$$

where $D$ is the diffusion coefficient, $\tau_\varphi$ is the electron phase relaxation time, and the magnetoresistance is given by

$$\Delta G(H)=\delta G(H)-\delta G(0)=(e^2/2\pi^2)\ln(1+\tau_\varphi/\tau_H). \tag{8}$$

In principle, $\tau_H$ depends on $q$, and formulas (4) and (6) correspond to $q=0$. These formulas can be used if the longitudinal length scale $b$, which governs the dependence of $\tau_H$ on $q$, is greater than $1/q_*$, where $q_*^2=(\tau_H^{-1}+\tau_\varphi^{-1})/D$. In the case $dHdl\ll1$ this length scale is $b=l$, and the function (4) is valid for $L_\varphi\equiv(D_{\tau_\varphi})^{1/2}\gg l$. If $eHdl\gg1$, the corresponding scale is $b\sim d/eHd^2$, and formula (6) is valid for $L_\varphi\gg1/eHd$.

The dependence of $\tau_H$ on $q$ is important for

$$L_\varphi\ll1/eHd,\ l. \tag{9}$$

Under condition (9) the minimum grazing angle is $\alpha_{\min}\sim qd$, and $\varphi_i\ll1$ for all $\varphi_i$. As a result, by analogy with (3) we have

$$\frac{1}{\tau_H(q)}\sim\frac{1}{\tau}\frac{eHd^2}{qd}. \tag{10}$$

For typical values $q\sim q_*$, under condition (9) we have $\tau_\varphi\ll\tau_H(q_*)\equiv\tau_H$. Here the magnetoresistance is given by

$$\Delta G(H)=(e^2/2\pi^2)(\tau_\varphi/\tau_H), \tag{11}$$

where

$$\frac{1}{\tau_H}=\frac{\pi}{48\tau}(eHd^2)(eHdL_\varphi). \tag{12}$$

To determine the numerical coefficients in formulas (4), (6), and (12) we must carry out a systematic calculation, which is done in the next section.

## 3. EQUATION FOR THE COOPERON AND ITS SOLUTION

In the quasiclassical approximation and under the condition $l\gg d$, the diffusion equation[2] for the cooperon $C_v(\mathbf{rr}'t)$ can be replaced by an equation of the kinetic type:

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}-2ie\mathbf{vA}+\frac{1}{\tau_\varphi}+\frac{1}{\tau}\right)C_v(\mathbf{rr}'t)$$
$$-\int do_{v'}C_{v'}(\mathbf{rr}'t)w_{vv'}=\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'). \tag{13}$$

For the vector potential $\mathbf{A}$ we choose the Landau gauge $\mathbf{A}=(Hy,0,0)$. To evaluate the correction to the conductivity we need the quantity $C_v(\mathbf{rr})$, which is gauge invariant. The $y$ axis is taken perpendicular to the film, and $v$ is the electron velocity on the Fermi surface. As usual, we assume the condition $p_Fl\gg1$, which enables us to treat the quantum corrections in low orders of perturbation theory.

At the film boundaries $y=\pm d/2$ we apply the conditions of diffuse electron reflection:

$$vC_v(\mathbf{rr}'t)|_{\mathbf{vn}>0}=\frac{1}{\pi}\int_{\mathbf{nv}'<0} do_{v'}|\mathbf{nv}'|C_{v'}(\mathbf{rr}'t), \tag{14}$$

These conditions were introduced by Ovchinnikov[8] in a quasiclassical treatment of the problem of the critical point for superconducting thin films. In equation (14) $\mathbf{n}$ is the inward normal to the film surface. Equations (13) and (14) imply that $C_v(\mathbf{rr}'t)$ depends importantly only on $y$ and $y'$. After Fourier transformation on the variables $x$, $z$, $t$, we find

$$\left(-i\omega+\frac{1}{\tau_\varphi}+iv_xq_x+iv_zq_z+v_y\frac{\partial}{\partial y}-2iev_xHy+\frac{1}{\tau}\right)C_v(yy'\mathbf{q}\omega)$$
$$-\int do_{v'}C_{v'}(yy'\mathbf{q}\omega)w_{vv'}=\delta(y-y'). \tag{15}$$

For the case of point scatterers, the quantity $w=1/4\pi\tau$ can be taken out from under the integral sign for the integration over the Fermi surface.

In solving equation (15) we shall follow the method of Ovchinnikov.[8] Let us rewrite (15) in the form

$$C_v(yy'\mathbf{q}\omega)=C_v{}^0(yy'\mathbf{q}\omega)$$
$$+\frac{1}{4\pi\tau}\int do_{v'}\int_{-d/2}^{d/2}C_v{}^0(yy_1\mathbf{q}\omega)C_{v'}(y_1y'\mathbf{q}\omega)dy_1, \tag{16}$$

where $C_v^0$ satisfies equation (15) with $w=0$ and with the same boundary condition (14). The solution of this equation is of the form

$$C_v{}^0(yy'\mathbf{q}\omega)=\alpha(y')\exp\left[-\lambda\left(y+\frac{d}{2}\right)+\mu\left(y^2-\frac{d^2}{4}\right)\right]\theta(t)$$
$$+\beta(y')\exp\left[-\lambda\left(y-\frac{d}{2}\right)+\mu\left(y^2-\frac{d^2}{4}\right)\right]$$
$$\times\theta(-t)+\frac{1}{v|t|}\exp[-\lambda(y-y')$$
$$+\mu(y^2-y'^2)][\theta(t)\theta(y-y')+\theta(-t)\theta(y'-y)], \tag{17}$$

where

$$\lambda=\frac{1}{v_y}\left(iv_xq_x+iv_zq_z-i\omega+\frac{1}{\tau_\varphi}+\frac{1}{\tau}\right),\quad \mu=ieH\frac{v_x}{v_y},\tag{18}$$

$$\alpha(y')=\beta(-y')=\frac{D(d/2+y')+\Gamma(d)D(d/2-y')}{v[1-\Gamma^2(d)]},\tag{19}$$

$$\Gamma(d)=\frac{1}{\pi}\int_0^{2\pi}d\varphi\int_0^1 t\,dt\,e^{-\lambda d},\tag{20}$$

$$D\left(\frac{d}{2}\pm y'\right)=\frac{1}{\pi}\int_0^{2\pi}d\varphi\int_0^1 dt\exp\left[-\lambda\left(\frac{d}{2}\pm y'\right)\mp\mu\left(\frac{d^2}{4}-y'^2\right)\right],\tag{21}$$

$t=\cos\vartheta$, $\vartheta$ is the angle between the direction of $\mathbf{v}$ and the $y$ axis, and $\varphi$ is the angle in the plane of the film.

If $(q_x,q_z)d/\bar{t}\ll 1$, where $\bar{t}$ is the characteristic value of $\cos\vartheta$, then after integration over $\varphi$ we have

$$\Gamma(d)=2\int_0^1 t\,dt\exp\left(\frac{i\omega\tau-1}{lt}d\right)\left(1-\frac{1-t^2}{4t^2}q_\parallel^2d^2\right),\tag{22}$$

$$D\left(\frac{d}{2}\pm y'\right)=2\int_0^1 dt\exp\left[\frac{i\omega\tau-1}{lt}\left(\frac{d}{2}\pm y'\right)\right]\times\left\{J_0(p)\mp\frac{(1-t^2)^{1/2}}{t}\times q_x\left(\frac{d}{2}\pm y'\right)J_1(p)-\frac{1-t^2}{2t^2}\left(\frac{d}{2}\pm y'\right)^2\times[q_z^2J_0(p)+(q_x^2-q_z^2)J_1'(p)]\right\},\tag{23}$$

where $P=\frac{(1-t^2)^{1/2}}{t}eH\left(\frac{d^2}{4}-y'^2\right)$, $q_\parallel^2=q_x^2+q_z^2$, and $J_0(p)$ and $J_1(p)$ are Bessel functions. Since $d\ll l$, we have for $\omega\tau\ll 1$

$$\Gamma(d)\approx 1+\frac{2(i\omega\tau-1)d}{l}-\frac{q_\parallel^2d^2}{2}\ln\frac{l}{d}.\tag{24}$$

In the last terms the logarithmic part is due to grazing trajectories with $\cos\vartheta\gtrsim d/l$.

Let us now consider two different cases:

a) *The region* $L_\varphi, L_H^2/d\gg l$. In this case we find to leading order in the parameter $ld/L_H^2$ with the aid of (19), (23), and (24)

$$\alpha(y')=\beta(y')=\frac{\tau}{(1-i\omega\tau)d}\left[1-q_\parallel^2D\tau-{}^1/_8e^2H^2ld\left(\frac{d^2}{4}-y'^2\right)\right],\quad D=\frac{vd}{4}\ln\frac{l}{d}.\tag{25}$$

Since it is assumed that $L_H\gg d$, equation (16) can be averaged over $y$ and $y'$. In addition, let us average $C_v$ over the directions of the velocity vector. Introducing the notation

$$\overline{C_v}(\mathbf{q}\omega)=\frac{1}{4\pi d^2}\int do_v\iint_{-d/2}^{d/2}dy\,dy'\,C_v(yy'\mathbf{q}\omega)\tag{26}$$

and analogous notation for $\overline{C_v^0}$, we obtain from equation (16)

$$\bar{C}_v(\mathbf{q}\omega)=[[\overline{C_v^0}(\mathbf{q}\omega)]^{-1}-d/\tau]^{-1}.\tag{27}$$

Using (17) and (25) and interating over $y$, $y'$, and the velocity directions in accordance with (26), we find

$$\bar{C}_v(q)=\frac{1}{d}\left(\frac{1}{\tau_\varphi}+Dq^2+\frac{1}{\tau_H}\right)^{-1},\tag{28}$$

where

$$\frac{1}{\tau_H}=\frac{1}{\tau}\frac{ld^3}{16L_H^4}.\tag{29}$$

Angles $\bar{t}\sim d/l$ were important in obtaining this formula. The characteristic values $q\sim\bar{q}$ are found from the relation $D\bar{q}^2\sim 1/\tau_\varphi+1/\tau_H$. These conditions determine the validity region of our assumption $\bar{q}d/\bar{t}\ll 1$. This region is bounded by the inequality $L_\varphi\gg l$. Formula (25) is thus valid when the conditions $L_\varphi$, $L_H^2/d\gg l$ hold simultaneously.

b) *The region* $L_\varphi, l\gg L_H^2/d$. Discarding terms of order $eHd^2(L_H^2/ld)^2$, or smaller in evaluating integrals (19) and (20), we obtain

$$\alpha(y')=\beta(-y')=\frac{\tau}{(1-i\omega\tau)d}\left[1-D\tau q_\parallel^2-eH\left(\frac{d^2}{4}-y'^2\right)-4q_xy'\right].\tag{30}$$

To the same accuracy we have

$$1/\tau_H=d^2/3\tau L_H^2.\tag{31}$$

Estimating the characteristic values $\bar{t}$ and $\bar{q}$, we find that formula (31) is valid under the condition $L_\varphi, l\gg L_H^2/d$.

c) *The region* $l, L_H^2/d\gg L_\varphi$. In accordance with the qualitative picture in this region the characteristic values $\bar{q}$ and $\bar{t}$ satisfy the inequalities $\bar{q}d/\bar{t}\gg 1$ and $eHd^2/\bar{t}\ll 1$. The latter inequality permits an expansion in the magnetic field. Evaluation of the integrals (20), (21), and (26) gives

$$1/\tau_H=q_z^2d^3/12q^3L_H^4\tau,\tag{32}$$

i.e., $\tau_H$ turns out to be a function of the momentum $q$ with a sharp dependence on the angle between the directions of $\mathbf{q}$ and $\mathbf{H}$. The characteristic quantities for this case are the angle $\bar{t}\sim d/L_\varphi$ and $\bar{q}\sim 1/L_\varphi$. Formula (32) is therefore valid for $l, L_H^2/d\gg L_\varphi$.

## 4. QUANTUM CORRECTION TO THE CONDUCTIVITY

The formula for the correction in the case of a pure film has the same form as for the case $l\ll d$ considered in Refs. 1 and 4:

$$\Delta\sigma=-(2\sigma_0/\pi\nu)\underset{\sim}{C}(\mathbf{rr}),\tag{33}$$

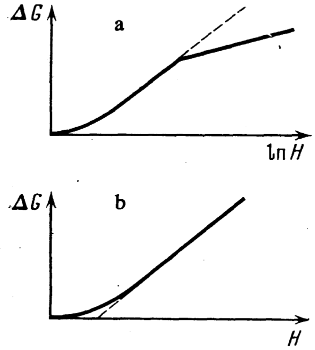


FIG. 2.

where

$$C(\mathbf{rr}) = \int \frac{d^2q}{(2\pi)^2} \bar{C}(\mathbf{q}) \tag{34}$$

is the cooperon for coincident arguments. In (33) the quantity $\sigma_0$ is the conductivity without the corrections and $\nu$ is the density of states at the Fermi level.

Now using formula (28) and the expressions found for $\tau_H$ in the various cases, we find that the magnetic-field dependent correction to the conductance of a square film is of the form

$$\Delta G_\square(H) = G_\square(H) - G_\square(0) = \frac{e^2}{2\pi^2\hbar} \ln\left(1 + \frac{ld^3}{16L_H^4} \frac{\tau_\varphi}{\tau}\right), \quad L_\varphi, \frac{L_H^2}{d} \gg l, \tag{35}$$

$$\Delta G_\square(H) = G_\square(H) - G_\square(0) = \frac{e^2}{2\pi^2\hbar} \ln\left(1 + \frac{d^2}{3L_H^2} \frac{\tau_\varphi}{\tau}\right), \quad L_\varphi, l \gg \frac{L_H^2}{d}, \tag{36}$$

$$\Delta G_\square(H) = G_\square(H) - G_\square(0) = \frac{e^2}{2\pi^2\hbar} \frac{\pi L_\varphi d^3}{48L_H^4} \frac{\tau_\varphi}{\tau}, \quad l, \frac{L_H^2}{d} \gg L_\varphi. \tag{37}$$

## 5. DISCUSSION OF RESULTS

Let us compare the resulting formulas (35)–(37) with the expression[4] for the correction to the conductance for a dirty film, $d \gg l$:

$$\Delta G_\square(H) = \frac{e^2}{2\pi^2\hbar} \ln\left(1 + \frac{d^2L_\varphi^2}{3L_H^4}\right). \tag{38}$$

For $L_\varphi$, $L_H^2/d \gg l$ formulas (35) and (38) differ only by a numerical coefficient within the logarithm. Furthermore, since the diffusion coefficient in the film is smaller than in the bulk by a factor of $d/l$, the magnetoresistance in a thin film turns out to be smaller.

For $L_\varphi, l \gg L_H^2/d$ the difference between (36) and (38) is more substantial, since the magnetic field enters the logarithm with a different power. Depending on the relationship between $L_\varphi$ and $l$, two different behaviors of $\Delta G_\square(H)$ are possible. If $L_\varphi \gg l$, then $\Delta G(H)$ becomes logarithmic at larger $H$, with the coefficient in front of the logarithm decreasing by a factor of two at $L_H^2 \sim ld$ (Fig. 2a). In the case $L_\varphi \ll l$ the quadratic dependence of $\Delta G$ on $H$ goes over to a linear dependence at $L_H^2 \sim L_\varphi d$ (Fig. 2b). The size of the magnetoresistance depends on the mean free time $\tau$ for scattering by impurities.

Thus, by studying the behavior of $\Delta G(H)$ one can determine $\tau$ in films with a low concentration of impurities.

We are grateful to B. L. Al'tshuler and A. G. Aronov for pointing out the special characteristics of pure films in a parallel field and to A. I. Larkin for a discussion.

[1)]As is usual in discussions of the magnetoresistance of thin films, we assume that $eHd^2 \ll 1$.

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