Acceleration of electrons to ultrarelativistic energies by shock waves and synchrotron radiation of these electrons

A. A. Galeev

Institute of Cosmic Studies, Academy of Sciences of the USSR
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The Čerenkov interaction of electrons with the natural oscillations of a plasma excited by ions reflected from the fronts of strong shock waves is examined as a mechanism for the acceleration of electrons to ultrarelativistic energies by strong shock waves. The energy spectrum of the accelerated electrons at ultrarelativistic energies is a power-law spectrum with a universal exponent $\gamma_e = 2$. The angular distribution of the electrons is highly anisotropic, since they are accelerated primarily along magnetic lines of force. The spectrum of the synchrotron radiation of the accelerated electrons is calculated, with allowance for the synchrotron energy loss and the tendency toward a less anisotropic electron angular distribution.

1. INTRODUCTION

The acceleration of charged particles by collisionless shock waves is usually attributed to a Fermi acceleration of the first kind. This mechanism operates when the scattering of particles by plasma inhomogeneities ahead of and behind the shock front makes it possible for some of the particles to be reflected repeatedly from these inhomogeneities. Since the inhomogeneities ahead of the shock front are carried off by the plasma flow more rapidly than are inhomogeneities behind the front, there is effectively a motion of inhomogeneities in opposite directions, and a particle acquires energy by bouncing between these inhomogeneities. This mechanism has been under discussion for a long time now in connection with the observation of bursts of high-energy particles at the fronts of interplanetary shock waves (Axford's review[1], has an exhaustive bibliography). Interest in this mechanism increased dramatically when it was learned[2,3] that the energy spectrum of the particles accelerated in this manner is a universal one and almost exactly the same as the observed spectrum of high-energy particles of cosmic origin (the so-called cosmic rays[4]). A crucial question here is the nature of the inhomogeneities which scatter the particles. Although we do not yet have a self-consistent theory for the appearance of inhomogeneities, analysis of the experimental data on interplanetary shock waves and shock waves in the space environment of the earth shows that these inhomogeneities are apparently low-frequency MHD waves[5] excited by a firehose or ion cyclotron instability of ion streams emerging forward from the vicinity of a shock front.[6] Since the emission of ions from the front region is particularly likely when shock waves are propagating nearly along magnetic lines of force, the Fermi acceleration mechanism described above is characteristic of so-called quasiparallel shock waves, which are propagating at an angle $\lesssim 45°$ from the magnetic field. A quasilinear theory[7] for the scattering and acceleration of ions by these MHD waves of a plasma gives a good description of the acceleration of ions by quasiparallel shock waves if we use the wave spectral density measured on satellites.[8] For the electrons, on the other hand, these waves constitute essentially adiabatic perturbations, and the electron acceleration efficiency is sharply lower because of the reduced scattering efficiency, Indeed, analysis of the measured fluxes of high-energy electrons and ions ahead of a shock wave in the supersonic solar wind near the earth has shown that the high-energy electrons come from those regions of the shock front where the magnetic lines of force run nearly parallel to the front, i.e., where the wave is quasiperpendicular, rather than quasiparallel.[9] In other words, the electrons are accelerated more effectively by quasiperpendicular shock waves. The acceleration mechanism in this case[10,11] turns out to be completely different from that in quasiparallel shock waves: At large Mach numbers the Lorentz force and the electric field cause up to 25% of the ions of the incoming plasma stream of the solar wind to be reflected from the front of a quasiperpendicular shock wave.[12,13] Since the energy of the ions increases severalfold upon reflection, the beams of reflected ions carry off a significant fraction of the energy of the incoming stream, and their relaxation in the plasma dominates the overall energy dissipation at the shock front.[13] It is during this relaxation of the beam of reflected ions that the electrons are accelerated. The entities which transfer the energy from the ion beam to the electrons are oblique plasma waves, which convert into whistlers in a high-β plasma (β is the ratio of the plasma pressure to the magnetic pressure). Whistlers have highly anisotropic phase velocities: The phase velocity is low in the direction across the magnetic field, and the waves reach a resonance with the beam ions, while along the magnetic field the phase velocity can vary over broad range, reaching and even exceeding the velocity of light. The waves excited by an ion beam can thus easily reach a Čerenko resonance with the motion of electrons along the magnetic field, accelerating these electrons to ultrarelativistic velocities.

Our purpose in the present paper is to solve the problem of electron acceleration in the limit of subrelativistic shock waves (by which we mean plasma velocities $\gtrsim 0.1c$, where c is the velocity of light), where electrons can be accelerated effectively to ultrarelativistic energies. In contrast with the nonrelativistic case discussed in Ref. 11, we find it possible in this case to derive analytic expressions for the angular distribution and energy spectrum of the accelerated electrons, which we can then use to calculate the synchrotron radiation of these electrons.

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2. BASIC EQUATIONS

A self-consistent calculation of electron acceleration by quasiperpendicular shock waves should include the following steps.

1. Determination of the angular distribution and energy spectrum of the ions reflected from the shock front as functions of the properties of the medium and the characteristics of the wave.

2. Study of the relaxation of the distribution of the reflected ions under the influence of the oblique plasma waves which they excite.

3. Calculation of the angular distribution and energy spectrum of the electrons in the region ahead of the shock wave, where these electrons are resonantly accelerated by waves excited by the reflected ions.

4. Determination of the angular distribution of the accelerated electrons behind the shock front, where plasma instabilities substantially change the originally highly anisotropic angular distribution.[14–16]

The first of these steps is a problem in its own right and lies outside the scope of the present paper. We will accordingly make use of the extensive theoretical and experimental research on ion reflection from shock waves in the interplanetary medium and in the space environment of the earth (see Ref. 12, for example) and assume that about a tenth of the ions of the incoming plasma stream, with an energy three or four times the kinetic energy of the plasma ions, are reflected from the front of a strong shock wave. This appears to be a reasonable assumption if we wish to apply the theory derived here to the synchrotron radiation of the plasma ejections from the cores of active galaxies, since the Mach number for the motion of a plasma formation in the interstellar medium is just as high as for shock waves near the earth[17] ($M \approx 5$–10).

We will concentrate our effort in this paper on solving the last three of these problems. The energetics of the acceleration can be determined easily by making use of the fact that the energy dissipation of the beam of reflected ions ahead of a strong shock dominates the total energy dissipation at the shock.[13]

The problem of the relaxation of the distribution of reflected ions and the problem of the ion acceleration should be solved jointly on the basis of the system of well-known quasilinear equations for the reflected ions and the accelerated electrons, along with a kinetic equation for the excited waves:

$$v_\parallel \frac{\partial f_b}{\partial x} = \frac{\partial}{v_\perp \partial p_\perp} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{e^2 |E_{\perp,\mathbf{k}}|^2 \omega_\mathbf{k}^2}{k_\perp^2 (k_\perp^2 v_\perp^2 - \omega_\mathbf{k}^2)^{1/2}} \frac{\partial f_b}{v_\perp \partial p_\perp}, \tag{1}$$

$$v_\parallel \frac{\partial f_e}{\partial x} = e^2 \sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{k}}{(2\pi)^3} \left(n m_e \omega_{ce} \frac{\partial}{p_\perp \partial p_\perp} + k_\parallel \frac{\partial}{\partial p_\parallel} \right) \frac{|E_{\perp,\mathbf{k}}|^2}{k_\perp^2} \zeta_n^2 \pi \delta \left(\omega_\mathbf{k} - k_\parallel v_\parallel - \frac{n\omega_{ce}}{\Gamma} \right) \left(n m_e \omega_{ce} \frac{\partial}{p_\perp \partial p_\perp} + k_\parallel \frac{\partial}{\partial p_\parallel} \right) f_e, \tag{2}$$

$$\frac{\partial \omega_\mathbf{k}}{\partial k_\parallel} \frac{\partial}{\partial x} \frac{\omega_{pe}^2}{\omega_{ce}^2} \left(1 + \frac{\omega_{pe}^2}{k_\perp^2 c^2} \right) |E_{\perp,\mathbf{k}}|^2$$

$$= \left\{ \frac{4\pi e^2}{k^2} \int \frac{\omega_\mathbf{k}}{(k_\perp^2 v_\perp^2 - \omega_\mathbf{k}^2)^{1/2}} \frac{\partial f_b}{v_\perp \partial p_\perp} d\mathbf{p} + \frac{4\pi e^2}{k^2} \sum_{n=-\infty}^{\infty} \int \left(n m_e \omega_{ce} \frac{\partial f_e}{p_\perp \partial p_\perp} + k_\parallel \frac{\partial f_e}{\partial p_\parallel} \right) \zeta_n^2 \pi \delta \left(\omega_\mathbf{k} - k_\parallel v_\parallel - \frac{n\omega_{ce}}{\Gamma} \right) d\mathbf{p} \right.$$

$$\left. + \frac{\pi \omega_{pi}^2}{\omega_{LH}^2} \frac{c^2}{H_0^2} \frac{1}{(\partial \omega_\mathbf{k} / \partial k_\perp)} \left[\frac{\partial}{\partial k_\perp'} \int \frac{k_\perp' |E_{\perp,\mathbf{k}'}|^2}{(\partial \omega_{\mathbf{k}'} / \partial k_\perp')} \left(k_\perp k_\perp' + \frac{\omega_{pe}^2}{c^2} \right) \frac{dk_\parallel'}{(2\pi)^2} \right]_{k_\perp' = k_\perp} \right\} \omega_\mathbf{k} |E_{\perp,\mathbf{k}}|^2, \tag{3}$$

where

$$\zeta_n = J_n \left(\frac{k_\perp p_\perp}{m_e \omega_{ce}} \right) \left(1 + \frac{\omega_{pe}^2}{k_\perp^2 c^2} \right)^{-1} \left(1 + \frac{n\omega_{ce}}{\omega_\mathbf{k} \Gamma} \frac{\omega_{pe}^2}{k_\perp^2 c^2} \right) + \frac{\omega_{pe}^2}{k^2 c^2} \frac{k_\perp p_\perp}{m_e \omega_{ce}} J_n' \left(\frac{k_\perp p_\perp}{m_e \omega_{ce}} \right).$$

The x axis is directed along the unperturbed magnetic field $\mathbf{H}_0$; $v_\parallel$, $p_\parallel$ and $v_\perp$, $p_\perp$ are the velocities and momenta of the particles respectively along and across the magnetic field; $f_b(x, p_\parallel, p_\perp)$ and $f_e(x, p_\parallel, p_\perp)$ are the distribution functions of the reflected ions and the accelerated electrons, respectively; $|E_\mathbf{k}|^2/8\pi$ is the spectral energy density of the electric field of the excited waves, with frequency $\omega_\mathbf{k}$ and wave vector $\mathbf{k}$; $k_\parallel$ and $k_\perp$ are the components of the wave vector respectively along and across the magnetic field; $\Gamma = (1 + p^2/m_e^2 c^2)^{1/2}$ is the Lorentz factor of the accelerated electrons; J_n is the Bessel function of index n; $\omega_{pj} = (4\pi e_j^2 n_j / m_j)^{1/2}$ is the plasma frequency of plasma component j, with density n_j, particle charge e_j, and particle mass m_j; $\omega_{cj} = |e_j| H_0 / m_j c$ is the cyclotron frequency of the nonrelativistic particles of species j; and $\omega_{LH} = (\omega_{ci} \omega_{ce})^{1/2}$ is the frequency of the lower hybrid resonance in the two-component plasma. The frequency of the waves which are excited is determined from the well-known dispersion relation for oblique plasma waves with the electromagnetic corrections (see Ref. 18, for example):

$$\omega_\mathbf{k}^2 = \frac{\omega_{LH}^2}{1 + \omega_{pe}^2 / k_\perp^2 c^2} \left[1 + \frac{m_i k_\parallel^2}{m_e k_\perp^2 (1 + \omega_{pe}^2 / k_\perp^2 c^2)} \right]. \tag{4}$$

In the limit $\omega_{pe}^2 \ll k^2 c^2$ this relation describes oblique electrostatic plasma waves, while in the opposite limit $\omega_{pe}^2 \gg k^2 c^2$ it describes electromagnetic whistlers. We restrict the present discussion to the case $k_{\parallel}^2 \ll k_{\perp}^2$, which corresponds to large phase velocities for the wave propagation along the magnetic field. This simplification is justified by the circumstance that the energy density of the particles in plasmas in space is usually comparable in magnitude to the energy density of the magnetic field, so that waves with $k_{\parallel} \gtrsim k_{\perp}(m_e/m_i)^{1/2}$ are subject to intense Landau damping by thermal electrons. In this limit the phase velocity for the wave propagation across the magnetic field does not exceed the Alfvén velocity, which is $(m_i/m_e)^{1/2}$ times smaller than the velocity of light in a comparatively low-density plasma ($\omega_{pe}^2 \sim \omega_{ce}^2$); the phase velocity is thus an order of magnitude below the velocity of the reflected ions. The instability growth rate in this case is significantly lower than the ion cyclotron frequency, so that formally, the individual cyclotron resonances would have to be taken into account. However, even a slight spread in beam velocities will allow us to switch to a summation over the individual cyclotron resonances in the dispersion relation for the waves and in the quasilinear equation for the ions. This approach corresponds to the well-known approximation of a weak magnetic field in a description of ion motion; this approximation was used in Ref. 19 to derive the equations which we need here, Eqs. (1) and (3).

The term with $n = 0$ on the right side of the quasilinear equation for the electrons describes a resonant acceleration of electrons at Čerenkov resonance with the natural waves of the plasma which have been excited. We should also take into account the pitch-angle scattering of the accelerated electrons which reach cyclotron resonance with the waves because of the normal ($n > 0$) and anomalous ($n < 0$) Doppler effects. Equation (2) can be extended to the relativistic case, with allowance for the nonelectrostatic nature of the waves, through a simple generalization of the equations of Refs. 14 and 15.

Finally, the wave growth caused by the beam of reflected ions [the first term on the right in Eq. (3) for the spectral energy density of the excited waves] is offset by the damping of these waves due to Čerenkov ($n = 0$) and cyclotron ($n \neq 0$) resonances with electrons and the induced scattering of waves by thermal ions of the plasma (the second and third terms, respectively, on the right) and also by the propagation of the waves out of the excitation region [the right side of Eq. (3)]. The induced scattering of waves by plasma ions was found from Ref. 19 in the so-called differential approximation, which holds in the limit

$$\omega_{\mathbf{k}} - \omega_{\mathbf{k}'} \approx \Delta\omega_{\mathbf{k}} = \Delta k_{\perp} v_A \gg \Delta k_{\perp} v_{Ti},$$

where $v_A = H_0(4\pi n_i m_i)^{-1/2}$ is the Alfvén velocity, and $v_{Ti} = (2T_i/m_i)^{1/2}$ is the thermal velocity of the ions. This condition usually holds in a plasma in space because of the slight difference between the electron and ion temperatures ($T_e \gtrsim T_i$) and because of the condition $\beta \approx 8\pi n_e T_e/H_0^2 \sim 1$. We turn now to an analysis of system (1)–(3).

3. RELAXATION OF THE BEAM OF REFLECTED IONS

Since the electrons are accelerated by energy taken form the beam of reflected ions, the acceleration efficiency is determined by the extent to which the reflected-ion distribution relaxes in a bounded volume ahead of the shock front. Here we can use the approximation that the plasma is infinite and homogeneous if the scale dimension of the region ahead of the shock wave (which is evidently smaller than or on the order of the radius of curvature of the shock front) is much smaller than the relaxation length for the beam of reflected ions. To determine the relaxation length from quasilinear equation (1) for the ions we need to know the spectral energy density of the waves which are excited. In general, this energy density depends on the momentum distribution of the electrons which reach resonance with the waves. We will show below that the Čerenkov interaction of waves with electrons in the relativistic limit is stronger than the cyclotron interaction. We would thus naturally expect that the wave level will be severely suppressed by Landau damping over the entire phase-velocity range $\omega_{\mathbf{k}}/k_{\parallel} < v_{\parallel,\max}$ ($v_{\parallel,\max}$ is the maximum velocity of the accelerated electrons) in which these waves can interact with electrons. At higher phase velocities, the mechanism which primarily limits the growth of the energy of the natural waves of the plasma is the induced scattering of these waves by thermal ions in the plasma, not cyclotron damping due to the normal Doppler effect. The propagation of waves out of the excitation zone is also of minor importance. Equating the first and last terms in Eq. (3), we find the following wave energy distribution in the transverse component of the wave vector:

$$\frac{\omega_{pe}^2}{2\omega_{ce}^2}\left(1+\frac{\omega_{pe}^2}{k_{\perp}^2c^2}\right)\int\limits_{-\omega/c}^{\omega/c}\frac{dk_{\parallel}}{(2\pi)^3}|E_{\perp,\mathbf{k}}|^2$$

$$\approx \frac{m_i m_e v_A^5}{k_{\perp}^2}\int v_{\perp}^{-2}\frac{\partial f_b}{\partial p_{\perp}}d\mathbf{p}\,\frac{(\omega_{pe}/k_{\perp}c)^3}{1+\omega_{pe}^2/k_{\perp}^2c^2}\int\limits_0^{(\omega_{pe}/k_{\perp}c)^2}\frac{x^2\,dx}{(1+x)^{5/2}}. \quad (5)$$

We see that the spectral energy density of the waves increases at long wavelengths. Correspondingly, the relaxation of the reflected-ion distribution is dominated by the longest waves which can grow in a plasma with a finite β. An upper limit is imposed on the wavelength of the unstable waves by the condition for an overlap of the individual cyclotron resonances in the expression of the growth rate of the ion-beam instability:

$$k_{\parallel}\Delta V_{b,\parallel} \approx \omega_{\mathbf{k}}\Delta V_{b,\parallel}/c \gtrsim \omega_{ci}. \quad (6)$$

We will accordingly use the approximation $k_{\min} \approx \omega_{pi}/\Delta V_b$, where V_b is the velocity of the reflected ions.

The scale length for momentum relaxation of the beam of reflected ions is found from quasilinear equation (1) with the help of expression (5) for the spectral density of the wave energy:

$$L_R = \frac{k_{min}\pi c^2 n_i}{\omega_{pe}^2 n_b}\left(\frac{V_b^2\Delta p_b}{m_e v_A^3}\right)^3\frac{m_e^2}{m_i^2}, \quad (7)$$

where n_b and Δp_b are the density of the reflected ions and their momentum spread due to scattering by the excited waves. Actually, the region ahead of the shock wave is not always large enough for a complete dissipation of the beam energy in this region, even in astrophysical objects. Expression (7) can then be used to determine the extent of the energy relaxation of a beam in a region of scale dimension L:

$$\frac{V_b^2\Delta p_b}{m_e c^3} \simeq \left(\frac{n_b L\omega_{pe}^2}{\pi n_i k_{min} c^2}\right)^{1/3} \left(\frac{m_e}{m_i}\right)^{5/6} \left(\frac{\omega_{ce}}{\omega_{pe}}\right)^3. \qquad (8)$$

As we will show below, it is this dimensionless parameter which characterizes the efficiency of the acceleration to ultrarelativistic energies. We note that this parameter decreases with increasing density of the plasma ahead of the shock wave.

4. ACCELERATION OF ELECTRONS

The resonant acceleration of electrons is described by the term with $n = 0$ on the right side of quasilinear equation (2) for the electron distribution function. This process is essentially a one-dimensional quasilinear diffusion of electrons in longitudinal velocity, which results in the formation of superthermal (nonMaxwellian) tails on their energy distribution. Here we would expect that the Landau damping by the electrons of the superthermal tail would lower the intensity of the resonant waves, and that as the tail was formed a balance would be struck between the rate at which waves are excited by the beam of reflected ions and the rate of the Landau damping by the superthermal electrons. The low energy of the resonant waves would then justify our ignoring the nonlinear terms in Eq. (3) for the waves. Equation (3) then takes the simple form

$$\left[\int \frac{\omega_{\mathbf{k}}}{(k_\perp^2 v_\perp^2-\omega_{\mathbf{k}}^2)^{1/2}} \frac{\partial f_b}{v_\perp \partial p_\perp} d\mathbf{p} + \int k_\parallel \frac{\partial f_e}{\partial p_\parallel} \zeta_0^2 \pi\delta(\omega_{\mathbf{k}} - k_\parallel v_\parallel)\, d\mathbf{p}\right] \times |E_{\mathbf{k}}|^2 \simeq 0. \qquad (9)$$

A balance between the excitation and absorption of waves can be reached over the entire range of longitudinal phase velocities $\omega_{\mathbf{k}}/k_\parallel \leqslant v_{\parallel,\max}$ only under the obvious condition

$$\int_0^\infty \frac{\partial f_e(p_\parallel, p_\perp)}{\partial p_\parallel} \left|\frac{\partial p_\parallel}{\partial v_\parallel}\right| \zeta_0^2 \left(\frac{k_\perp p_\perp}{m_e \omega_{ce}}\right) p_\perp\, dp_\perp \approx \text{const}, \qquad (10)$$

i.e., under the condition that the quantity on the left is independent of $p_\parallel$ and thus of $k_\parallel$. If we ignore processes tending to reduce the anisotropy of the distribution, i.e., if we assume $\langle p_\perp \rangle = 0$, then we can easily derive a one-dimensional distribution of superthermal electrons in longitudinal momentum:

$$f_e(p_\parallel) = \frac{n_{Te}}{2m_e c[1-m_e c/(p^2_{max}+m_e^2c^2)^{1/2}]} \times \left[\frac{p_{max}}{(p^2_{max}+m_e^2c^2)^{1/2}} - \frac{|p_\parallel|}{(p_\parallel^2+m_e^2c^2)^{1/2}}\right], \qquad (11)$$

where n_{Te} is the density of the accelerated electrons, and $p_{\max}$ is their maximum momentum.

This acceleration of electrons by shock waves thus leads to a universal power-law energy spectrum for the electrons with an exponent $\gamma_e = 2$. Substituting (11) into (9), we find that the region where it is most difficult to reach a balance between the excitation and damping of waves is at the greatest wavelengths, since the excitation rate increases with increasing wavelength there, while the damping rate decreases. We would thus expect the spectrum of excited waves to be of the nature of a "jet":

$$|E_{\perp,\mathbf{k}}|^2 \approx 2E^2(k_\parallel)\delta(k_\perp^2 - k_{min}), \qquad (12)$$

where $k_{\min}$ is determined by condition (6). Making use of this circumstance, we find from (9) the following expression for the density of accelerated electrons ($p_{\max} \gg m_e c$):

$$n_{Te} \approx \frac{n_b m_e c^3}{V_b^2 \Delta p_b}\left(\frac{v_A}{c}\right)\left(1+\frac{\omega_{pe}^2}{k^2_{min}c^2}\right)^2. \qquad (13)$$

We find the wave distribution in the longitudinal component of the wave vector from quasilinear equation (2) for the electrons, in which we again ignore the slow processes tending to make the distribution less anisotropic:

$$0 = -\frac{|v_\parallel|}{L_R} f_e + \frac{\partial}{\partial p_\parallel} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{e^2 k_\parallel^2 |E_{\perp,\mathbf{k}}|^2}{k_\perp^2} \times \zeta_0^2\left(\frac{k_\perp p_\perp}{m_e \omega_{ce}}\right) \pi\delta(\omega_{\mathbf{k}} - k_\parallel v_\parallel) \frac{\partial f_e}{\partial p_\parallel}. \qquad (14)$$

The first term on the right gives an approximate description of the loss of electrons which results from their escape at the velocity $v_\parallel$ from the acceleration region, whose dimension is L_R and in which the beam of reflected ions excites resonant waves. Using (11) for the distribution function of the accelerated electrons, we find a solution of this equation (ignoring processes tending to render the distribution isotropic):

$$E^2(k_\parallel) = \frac{4\pi m_e^2 c^4 k_\perp^2}{e^2 L_R k_\parallel^2}\left(1+\frac{\omega_{pe}^2}{k_\perp^2 c^2}\right)^2 \left(1-\frac{\omega_{\mathbf{k}}^2}{k_\parallel^2 c^2}\right)^{-3/2} \times \left\{\left[\left|\frac{\omega_{\mathbf{k}}}{k_\parallel c}\right| - \frac{p_{max}}{(p^2_{max}+m_e^2c^2)^{1/2}}\right]\left(1-\frac{\omega_{\mathbf{k}}^2}{k_\parallel^2 c^2}\right)^{-1/2} + \operatorname{arctg}(k_\parallel^2 c^2/\omega_{\mathbf{k}}^2 - 1)^{-1/2} - \operatorname{arctg}\left(\frac{p_{max}}{m_e c}\right) + C_1\left|\frac{\omega_{\mathbf{k}}}{k_\parallel c}\right|\right\}, \qquad (15)$$

where the integration constant C_1 can be found by joining solution (15) for the region of resonant phase velocities with solution (5) outside this region.

There is an important point to be noted regarding the slowness of the tendency of the electrons toward an isotropic distribution during their one-dimensional acceleration. The process is slow because the rate of the one-dimensional acceleration is determined by the spectral energy density of the waves at the point $k_\parallel = \omega_{\mathbf{k}}/v_\parallel$, which corresponds to the resonant phase velocity, while the processes tending to make the distribution isotropic depend on the integrated wave energy density [see Eq. (19) below]. Therefore, because of the sharp increase in the spectral energy density of the waves at high phase velocities, $|E_{\mathbf{k}}|^2 \sim |k_\parallel c - \omega_{\mathbf{k}}|^{-1}$ [see Eq. (15)], the acceleration rate will fall off with increasing electron energy, in contrast to the rate at which the distribution tends to become isotropic. This circumstance represents a fundamental distinction between the model used here and the models of acceleration by an isotropic plasma turbulence.[20]

The maximum energy to which the electrons are accelerated can be found by requiring that the energy flux carried into the acceleration region by the reflected ions, $\sim n_b V_b^2 \Delta p_b$, must exceed the energy flux carried out of this region by the accelerated electrons, $\sim n_{Te} m_e c^3 \ln(p_{\max}/m_e c)$. Using (13) for the density of accelerated electrons, we can write this condition as

$$\left(\frac{V_b^2\Delta p_b}{m_e c^3}\right)^2 \frac{c}{v_A} \geqslant \left(1+\frac{\omega_{pe}^2}{k_{min}^2 c^2}\right)^2 \ln\frac{p_{max}}{m_e c}. \quad (16)$$

The integrated energy density of the resonant waves is lower than that of the nonresonant waves. This circumstance justifies our use of the quasilinear approximation in calculating the spectral energy density of the resonant waves. This point can be verified easily by comparing (15) with (16).

5. EFFECT OF THE TENDENCY TOWARD AN ISOTROPIC DISTRIBUTION OF ACCELERATED ELECTRONS ON THE ENERGY SPECTRUM OF THESE ELECTRONS

Up to this point we have ignored the pitch-angle scattering of electrons due to the cyclotron interaction of the electrons with nonresonant waves ($\omega_{\mathbf{k}}/k_\parallel > v_{\parallel,\max}$) in the normal Doppler effect ($n>0$) and also with resonant waves in the anomalous Doppler effect. Because of the high intensity of the nonresonant waves, we will at this point consider the scattering of electrons by these waves alone. Since the pitch-angle scattering of electrons slow in comparison with their acceleration, we need consider this scattering only at modest electron energies, where even a slight increment in the transverse momenta of the electrons leads to a significantly less anistropic electron distribution. We immediately note that the cyclotron interaction itself occurs only for relativistic electrons with a Lorentz factor

$$\Gamma \approx p_\parallel/m_e c > \omega_{ce}/\omega_{\mathbf{k}}. \quad (17)$$

Since we expect to find a significant decrease in the anisotropy of the electron distribution, we consider the limiting case

$$k_\perp p_\perp/m_e\omega_{ce} \gtrsim \Gamma(\omega_{\mathbf{k}}-k_\parallel c)/\omega_{ce} \approx n_{res}, \quad (18)$$

where $n_{\rm res}$ is the number of the resonant cyclotron harmonic. In this limit we can carry out calculations on the pitch-angle scattering of electrons, described by the second term on the right side of Eq. (2), by working from

$$v_\parallel \frac{\partial f_e}{\partial x} = \frac{\partial}{p_\perp \partial p_\perp} e^2 \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{\omega_{pe}^4 |E_{\perp,\mathbf{k}}|^2}{k_\perp^6 c^4} \frac{n_{ce}^2 m_e^3 \omega_{ce}^2 \Gamma}{2\pi k_\perp p_\perp} \frac{\partial f_e}{p_\perp \partial p_\perp}. \quad (19)$$

Comparing this equation with Eq. (1) for the ion relaxation, in which we again find this intensity of nonresonant waves, we find an estimate of the transverse-momentum spread:

$$\frac{\langle p_\perp\rangle}{m_e c} \approx \left(\frac{|p_\parallel|}{m_e c}\right)^{3/5} \left(\frac{V_b^2\Delta p_b}{m_e c^3}\right)^{2/5} \left(\frac{\omega_{pe}}{k_{min} c}\right)^{4/5}. \quad (20)$$

Under condition (16), the pitch-angle scattering by nonresonant waves at comparatively modest energies, (18), causes a significant reduction of the anisotropy ($\langle p_\perp\rangle \gg m_e c$), which should be taken into account both in determining the electron spectrum from Eq. (10) and in finding the spectral energy density of the waves from Eq. (14). It is easy to see that incorporating in (10) the pitch-angle spread, which depends on the electron energy, changes the energy spectrum of the electrons:

$$f_e(p_\parallel) \approx \frac{n_{Te}}{|p_\parallel|^{1/5}(m_e c)^{4/5}},$$
$$n_{Te} \approx \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{V_b^2\Delta p_b}{m_e c^3}\right)^{1/5} \left(\frac{\omega_{pe}}{k_{min} c}\right)^{32/5} n_b. \quad (21)$$

This change in the spectrum, however, requires a large expenditure of energy, so that it can be implemented in a comparatively narrow energy interval. The size of this interval is found from the energy considerations which we have already formulated [see inequality (16)]:

$$n_b V_b^2 \Delta p_b > n_{Te} m_e c^3 \left(\frac{p_{max}}{m_e c}\right)^{9/5}, \quad (22)$$

i.e.,

$$\frac{p_{max}}{m_e c} < \left(\frac{V_b^2\Delta p_b}{m_e c^3}\right)^{4/9} \left(\frac{m_i}{m_e}\right)^{5/9} \left(\frac{k_{min} c}{\omega_{pe}}\right)^{32/9}. \quad (23)$$

Since this interval is extremely narrow in comparison with the maximum electron energies ($\Gamma \gtrsim 10^7$), result (21) is of no major interest. Furthermore, we will show below that for realistic properties of a plasma in space condition (22) contradicts inequality (18), so that solution (21) does not hold at all. In specific applications we are therefore justified in ignoring the quasilinear diffusion of electrons in transverse momentum in a first approximation and taking the electron distribution to be one-dimensional.

6. SPECTRUM OF THE SYNCHROTRON RADIATION BY ELECTRONS BEHIND A SHOCK FRONT

One of the most obvious manifestations of acceleration in a magnetized plasma in space is the synchrotron radiation of the accelerated electrons in various frequency ranges. In particular, the synchrotron radiation of magnetized plasma formations ejected from the cores of active galaxies[21] apparently reflects the acceleration of electrons by shock waves which arise in the supersonic (and super-Alfvén) motion of plasmas through the interplanetary medium.[22,23] This interpretation is supported by the fact that the exponent of the power-law frequency spectrum of the synchrotron radio emission is approximately equal to the value ($\alpha = 0.5$) which corresponds to the exponent found earlier for the power-law electron energy distribution, $\gamma_e = 2$, under the assumption that the pitch-angle distribution of the electrons subsequently becomes isotropic.

Before we go into the possibility that the accelerated electrons become isotropic, however, it is useful to make some quantitative estimates regarding the characteristic relaxation lengths of the beam of reflected ions, the maximum energy of the accelerated electrons, and the extent to which the pitch-angle distribution of these electrons becomes isotropic. Let us apply our theory to the plasma jet from the core of the nearby active galaxy M87. The density of the interstallar plasma and the temperature of the plasma electrons near the jet are known from x-ray measurements[24]: $n_0 \approx 0.01\ \mathrm{cm}^{-3}$ and $T_e \sim 3$ keV. The interstellar magnetic field can be estimated by assuming that energy is distributed equally between the thermal energy of the plasma, $n_0 T_e$, and the energy of the magnetic field, $H_0^2/8\pi$. Adopting this assumption, we find $H_0 \approx 3\cdot 10^{-5}$ G. The velocity of the ejected plasmas has not yet been measured. If we assume that the presence of a jet in only one direction means that the brightness of this jet is increased because of motion toward the observer along the line of sight, we can put the velocity of the plasmas at $V \approx 0.6c$ (Ref. 21). Substituting these values of the properties of the plasma and the magnetic field into expression (7) for the relaxation length of the reflected-ion beam, we easily find that this relaxation length is almost ten orders of magnitude greater than the size of the plasma formation[25]: $L \approx 3 \cdot 10^{19}$ cm. This result means that the shock wave which arises during the intrusion of the first plasma jet into the unperturbed interstellar medium cannot effectively accelerate electrons. Subsequent jets, however, will move through a medium of lower density, since the wake left by the first jet will not yet have been filled with interstellar plasma. The velocity at which plasma flows into the wake, which is on the order of the Alfvén velocity, falls off rapidly as the wake becomes filled with plasma. The limiting plasma density in the wave can be estimated from the obvious relation

$$L/v_A \approx d/V_b, \tag{24}$$

where d is the distance between two successive plasma formations in an ejection, which is about $d \approx 20L$. We then immediately find $v_A \approx c/30$, so that the plasma density in the wake is $n_0 \approx 10^{-4}$ cm^{-3}. As a result, the relaxation length of the beam of ions reflected from the front of the shock wave is reduced by eight orders of magnitude, and the extent of the beam relaxation, (8), over the characteristic dimension of the plasma formation, L, becomes quite large:

$$V_b^2 \Delta p_b / m_e c^3 \approx 5-10. \tag{25}$$

It follows from (16) that energy considerations impose no serious limitation on the highest energy to which the electrons can be accelerated. To find a more accurate estimate of this maximum energy we need to examine in detail the singular behavior of the spectral energy density of the resonant waves in the plasma, (15), taking into account the exchange of energy between resonant and nonresonant waves. This singularity is responsible for the acceleration of electrons over a distance on the order of the relaxation length of the reflected-ion beam [see Eq. (15)]. The maximum electron energy is determined by that wave phase velocity at which this singularity is eliminated. As for our approximation of a one-dimensional acceleration, we note that it holds quite well for the extent of the relaxation of the ion beam which we have found, (25).

The processes tending to reduce the anisotropy of the distribution of accelerated electrons cannot be ignored behind the shock front, however, where only half of these electrons arrive with a velocity directed away from the front toward the plasma. We know that a highly anisotropic, single-sided electron tail of this sort would be unstable with respect to the excitation of plasma waves.[16] To determine the pitch-angle distribution of electrons behind the front we use the following equation for the number density (N_e) of electrons in the energy interval (ε, ε, $+ d\varepsilon$) and the pitch-angle (θ, $\theta + d\theta$):

$$v_{\parallel} \frac{\partial N_e}{\partial x} = \frac{1}{\varepsilon^2} \frac{\partial}{\theta \partial \theta} \theta D \frac{\partial N_e}{\partial \theta} + \frac{\partial}{\partial \varepsilon} (\beta \varepsilon^2 \sin^2 \theta N_e). \tag{26}$$

The pitch-angle diffusion coefficient can be estimated from $D \sim \omega_{pe} (n_{Te}/n_0) m_e^2 c^4$, since this diffusion is caused by waves of frequency ω_{pe} which grow under the influence of the small fraction of accelerated electrons. The second term on the right side of this equation describes the synchrotron loss. Here we have $\beta = 2e^4 H_0^2 / 3 m_e^4 c^7$ (Ref. 26). Equation (26) must be supplemented with a boundary condition on the trailing edge of the shock front:

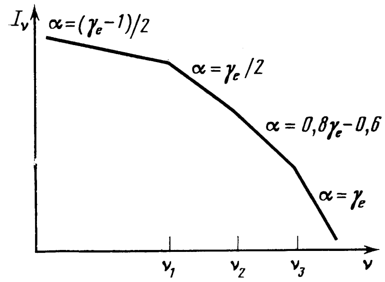

FIG. 1. Spectrum of the synchrotron radiation of electrons accelerated by a shock wave (the values of the parameters ν_i are given in the text proper). Here $I_\nu \propto \nu^{-\alpha}$.

$$N_e(x=0, \varepsilon, \theta) = \frac{n_{Te} (m_e c^2)^{\gamma_e - 1}}{\pi (\gamma_e - 1) \varepsilon^{\gamma_e}} \delta(\theta^2). \tag{27}$$

The theoretical exponent calculated above is $\gamma_e = 2$.

It follows from Eq. (26) that at modest energies the electron distribution easily becomes isotropic. Correspondingly, the low-frequency synchrotron radiation of the electrons accelerated by the shock wave is described by a power-law frequency spectrum with an exponent $\alpha = (\gamma_e - 1)/2$ (Fig. 1).

As the energy of the electrons increases, however, the decrease in the anisotropy of their pitch-angle distribution slows, and the synchrotron energy loss increases. Each of these effects tends to reduce the synchrotron radiation at high frequencies. The exponent of the frequency spectrum of this radiation depends strongly on the relative importance of these effects.

If the plasma formations are quite large,

$$L > c / \beta^{2/3} D^{1/3} \tag{28}$$

the synchrotron loss becomes important even at relatively low energies, $\varepsilon > \varepsilon_1 = c/\beta L$, where the decrease in the anisotropy has not yet come into play. We know that in this case there is a change in the slope of the emission spectrum at the frequency $\nu_1 = \nu_{ce} / \beta^2 m_e^2 c^2 L^2$, and the exponent of the spectrum becomes $\alpha_1 = \gamma_e/2$ (Ref. 27; see Fig. 1 of the present paper). At even higher energies, $\varepsilon > \varepsilon_2 + (D/\beta)^{1/3}$, the erasure of the anisotropy becomes incomplete because of the synchrotron energy loss of electrons with large pitch angles $[\langle \theta \rangle \sim (\varepsilon_2/\varepsilon)^{3/4}]$. The spectral density of the synchrotron radiation of the electrons in this case can be calculated approximately from

$$I_\nu = \int \beta \varepsilon^2 \sin^2 \theta N_e(\varepsilon, \theta) \, \delta\left(\nu - \frac{\nu_{ce} \varepsilon^2 \sin \theta}{2 m_e^2 c^4}\right) \sin \theta \, d\theta \, d\varepsilon. \tag{29}$$

Here we have made use of the circumstance that an electron with a given energy and a given pitch angle radiates in a narrow frequency range. Hence, using (26), we easily find that the radiation spectrum becomes steeper ($\alpha_2 = 0.8\gamma_e - 0.6$) at $\nu > \nu_2 = \nu_{ce} D^{2/3} / 2 \beta^{2/3} m_e^2 c^4$.

Finally, at very high energies, $\varepsilon > \varepsilon_3 = \beta D L^2 / c^2$, the extent to which the distribution becomes isotropic is determined by the finite time taken for the electrons to traverse the plasma ($\langle\theta\rangle\varepsilon \sim (DL/c)^{1/2}$), and the exponent of the spectrum changes for the last time ($\alpha_3 = \gamma_e$ at $\nu > \nu_3 = \nu_{ce}\beta DL^2/2m_e^2c^6$).

If inequality (28) does not hold, there is only a single slope change ($\alpha = \gamma_e$ at $\nu > \nu_{ce}DL/2m_e^2c^5$) in the synchrotron-radiation spectrum of the accelerated electrons. This slope change results from the slowing of the decrease in the anisotropy of the electron distribution the time required to traverse the plasma. It is this latter case which we are apparently seeing in the jet in M87, where the characteristic dimension of theplasma formations is[25] $L \approx 3\cdot 10^{19}$ cm, and the magnetic field in these formations is[28] $H_0 = 3\cdot 10^{-4}$ G. We then see why the exponent of the emission spectrum is unusually steep,[24] $\alpha = 1.8$, between the optical and x-ray frequency ranges. It is also interesting to note here that condition (27) holds in the case of another well-known jet, from the quasar 3C273, because of the strong magnetic field $H \sim 10^{-3}$ G and the far greater dimensions of the plasma formation. Correspondingly, the first slope change in the radiation spectrum of the jet is not as abrupt[29]: from $\alpha = 0.8$ to $\alpha_1 = 1.3$.

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