Low-temperature properties and phase transition in a three-dimensional magnet with random anisotropy

Vik. S. Dotsenko and M. V. Feĭgelman

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

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We show that the low-temperature phase of a three-dimensional magnet with an easy plane type anisotropy is a spin-glass kind of disordered phase when there are weak random fields or random anisotropies present. This phase is characterized by a finite correlation length (larger in weak random fields) and a finite spin rigidity which is the same as the rigidity of the ferromagnetic phase of a pure magnet. We study the critical behavior of the system near the phase transition point. We find the critical exponents of the static susceptibility of the system in a zero external field for various orders of anisotropy. We study the behavior of the susceptibility in zero external field in the low-temperature phase. We show that there exists a range of fields H for which the susceptibility varies as $H^{-3/2}$.

1. INTRODUCTION

In problems connected with the spin glass problem one usually considers systems with random spin-spin exchange (e.g., the RKKY interaction). However, in real magnets there may be present a totally different kind of disorder in the form of frozen-in random fields and random anisotropies.[1] Systems in which there is an additional random off-diagonal exchange between spins can also be considered to be magnets with a random anisotropy axis.[2] It is now already well known that in real spin systems of dimensionality $d = 2$ and 3 for arbitrarily small magnitude of the random fields this kind of disorder destroys the given order.[3,4] It would therefore be very interesting to understand the nature of the low-temperature state of such systems and also how random fields change the critical behavior near the phase transition temperature T_c.

The simplest system for studying these problems is the XY-model (rather than an Ising magnet for which up to now no reliable lower critical dimensionality has been established). Moreover, in a well-defined sense the situation in a three-dimensional XY-model turns out to be simpler than in the two-dimensional system with this kind of disorder. This is connected with the fact that the low-temperature phase in a three-dimensional XY-model without random fields is ferromagnetic and there exists a scale R_c (the size of the core of the vortex line) beyond which the system is described by a single free scalar field.

$$H_0=\frac{1}{2}\rho_s\int d^3x(\nabla\varphi)^2. \tag{1.1}$$

The perturbation-theory expansion in corrections connected with random fields relative to $H_0\{\varphi\}$ looks comparatively simple in contrast to the two-dimensional case where in such an expansion there is a difficulty connected with the divergence of the quantity $\int d^2k\,\langle\varphi_k\varphi_{-k}\rangle$.

In an Ising magnet there is also a ferromagnetic state but here the perturbation of that ground state is described by perturbations—the domain walls—which are non-linear in an essential way. The properties of these domain walls in the presence of random fields remains so far unexplained.

In previous papers by us[5,6] we described the properties of a two-dimensional or quasi-two-dimensional (layered) planar magnet with random anisotropies. We noted that a layered magnet displays when the temperature is lowered essentially three-dimensional properties and a "tendency" to form a spin glass when there is an arbitrarily weak coupling between the layers.[6]

In the present paper we consider a real three-dimensional magnet with random anisotropies of any order:

$$\mathscr{H}=\mathscr{H}_0+\beta D\sum_{\mathbf{x}}s^{i_1\ldots i_n}(\mathbf{x})\,h_{i_1}(\mathbf{x})\ldots h_{i_n}(\mathbf{x}), \tag{1.2}$$

where $\mathscr{H}_0$ is the Hamiltonian of the pure magnet. We shall study a Heisenberg magnet in which there is an easy plane type anisotropy.

$$\mathscr{H}_0=\beta\sum_{\mathbf{x}\hat{\alpha}}[J(\mathbf{s}(\mathbf{x}),\mathbf{s}(\mathbf{x}+\hat{\alpha}))+\Lambda(s^z(\mathbf{x}))^2]. \tag{1.3}$$

Here $\beta = 1/T$ and J is the constant of the interaction between nearest neighbors. Spins of unit length are given in the sites of the three-dimensional lattice and $\hat{\alpha}$ are the unit vectors of the three-dimensional lattice.

The second term in the Hamiltonian (1.2) describes a random n-th order anisotropy. Here $s^{i_1\ldots i_n}$ is an nth order irreducible tensor composed out of the vectors $\mathbf{s}^i$ and $\mathbf{h}(\mathbf{x})$ is a unit vector which is directed randomly in different points:

$$\overline{h_i(\mathbf{x})h_j(\mathbf{x}')}=\delta_{ij}\delta(\mathbf{x}-\mathbf{x}'). \tag{1.4}$$

In a magnet with random fields the second term in the Hamiltonian (1.2) has the form

$$\beta D\sum_{\mathbf{x}}(\mathbf{s}(\mathbf{x}),\mathbf{h}(x)); \tag{1.5}$$

in a magnet with a random axis of a second order anisotropy—

$$\beta D\sum_{\mathbf{x}}(s^i(\mathbf{x})s^j(\mathbf{x})-{}^1/_3\delta^{ij})\,h_i(\mathbf{x})\,h_j(\mathbf{x}); \tag{1.6}$$

in a magnet with a random axis of a third order anisotropy—

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$$\beta D \sum_{\mathbf{x}} (s^i(\mathbf{x}) s^j(\mathbf{x}) s^k(\mathbf{x}) - {}^1/_5 \delta^{ij} s^k(\mathbf{x}) - {}^1/_5 \delta^{ik} s^j(\mathbf{x}) - {}^1/_5 \delta^{jk} s^i(\mathbf{x})) h_i(\mathbf{x}) h_j(\mathbf{x}) h_k(\mathbf{x}) \quad (1.7)$$

and so on. The random fields are assumed to be weak: $D \ll J$.

The fact that even for an arbitrarily small magnitude of the random forces there is no long-range order in the system considered is clear already from the following qualitative considerations. Let the characteristic length over which the long-range order is destroyed be equal to L. In a volume of linear size L the exchange energy connected with deformations of the structure will then be of the order of $L^3/L^2 \sim L$ while the gain in energy in the same volume due to the interaction with the random field will be of the order of the mean-square quantity $(D^2L^3)^{1/2}$. Therefore for sufficiently large L the gain in energy from the adjustment to the random field is larger than the loss in exchange energy. The formation of a randomly-inhomogeneous structure with a characteristic correlation range $L \sim D^{-2}$ is therefore advantageous.

We see that the low-temperature phase of such a system is a spin glass kind of phase (section 2). This phase is characterized by a finite correlation length and a finite spin rigidity which is the same as the rigidity of the ferromagnetic phase of the pure magnet.

We study in section 3 the critical region near the phase transition point. At temperatures not too close to T_c the phase transition in the spin glass is described by the exponents of the ferromagnetic phase transition in the pure magnet. However, in the cases $n = 1$ and 2 (random field and random second-order anisotropy axis) there is near the transition temperature T_c a narrow region $\tau_n^*(D)$ in which the critical behavior is changed and which is not described by the indexes of the pure magnet.

We find in section 4 for various orders of anisotropy (outside the temperature interval $\tau_n^*(D)$) the critical exponents of the static susceptibility in zero external field. We also study the behavior of the susceptibility in a non-zero external field. We show that when the external field is strengthened before reaching the law $\chi \propto H^{-1/2}$ characteristic for a pure magnet there is an intermediate region where the susceptibility behaves as $H^{-3/2}$.

2. LOW-TEMPERATURE PHASE

Because of the presence of an easy-plane type anisotropy at dimensions $L_\Lambda \sim \Lambda^{-1/2}$ the spins "lie down" in the plane and therefore, if the length at which the destruction of the long-range order occurs $L \sim D^{-2}$ is much longer than L_Λ the system can effectively be described by the XY-model:

$$\mathscr{H} = \mathscr{H}_0 + \beta D \int d^3x \cos(n\varphi(x) + \theta(x)), \quad (2.1)$$

where $\mathscr{H}_0$ is the Hamiltonian of the pure $3D$ XY-model:

$$\mathscr{H}_0 = \beta J \int d^3x (\nabla s_\gamma(\mathbf{x}))^2. \quad (2.2)$$

Here s_γ is the planar spin with a direction described by the phase $\varphi(\mathbf{x})$ $(0 \leqslant \varphi \leqslant 2\pi)$. The frozen-in fields $\theta(\mathbf{x})$ which with equal probability are distributed over the interval $0 \leqslant \theta \leqslant 2\pi$ describe the directions of the random anisotropy fields. The parameter $n = 1, 2, 3, \ldots$ is the order of the anisotropy, $n = 1$ corresponds to a random magnetic field, $n = 2$ to a random axis type anisotropy.

The free energy of the system has the form

$$\mathscr{F} = -\frac{1}{\beta} \int D[\theta] \ln(Z\{\theta\}) \equiv -\frac{1}{\beta} \overline{(\ln Z)}, \quad (2.3)$$

where the partition function

$$Z\{\theta\} = \int_0^{2\pi} D[\varphi] \exp(-\mathscr{H}\{\varphi, \theta\}). \quad (2.4)$$

We consider the low-temperature region, i.e., that region where the correlation radius R_c of a pure $3D$ XY-model, determined by the size of vortex excitations is small compared to the length of the spin-spin correlations $L \sim D^{-2}$ arising due to the presence of random fields.[3,4]

When we expand the partition function (2.4) in a perturbation theory series in $\beta D \Sigma \cos(n\varphi + \theta)$ in Nth order there arise averages of the form

$$\left\langle \exp\left\{ in \sum_{i=1}^{N} q_i \varphi(\mathbf{x}_i) \right\} \right\rangle \quad (2.5)$$

($q_i = \pm 1$) where the averaging is performed using the Hamiltonian (2.2) of the pure $3D$ XY-model. One easily understands that if all distances $|\mathbf{x}_i - \mathbf{x}_j| \gg R_c$ in the correlator (2.5) we can rewrite it in the form

$$(K_n(T))^N \left\langle \exp\left\{ in \sum_{i=1}^{N} q_i \varphi(\mathbf{x}_i) \right\} \right\rangle , \quad (2.6)$$

where the averaging is performed using the Hamiltonian of the free field

$$\mathscr{H}_f = {}^1/_2 \beta \rho_s(T) \int d^3x (\nabla \varphi(\mathbf{x}))^2 \quad (2.7)$$

and where we have introduced the notation

$$K_n(T) = \langle \cos(n\varphi) \rangle_0. \quad (2.8)$$

At low temperatures far from the phase transition point $R_c(T) \approx 1$, $\rho_s(T) \approx J$, $K_n(T) \approx 1$. On the other hand, in the scaling region near the transition point $(\tau \equiv (T_c - T)/T_c \ll 1)$:

$$R_c(T) \sim \left(\frac{T_c - T}{T_c} \right)^{-\nu}, \quad (2.9)$$

$$K_n(T) \sim \left(\frac{T_c - T}{T_c} \right)^{\varkappa_n} \quad (2.10)$$

and neglecting the anomalous critical dimensionality $\eta \approx 0.02$ for the 3D XY-model[7] we get

$$\rho_s(T) = K_1^2(T) \sim \left(\frac{T_c - T}{T_c} \right)^{2\beta}. \quad (2.11)$$

Thanks to having written the correlator (2.5) in the form (2.16) we can instead of the Hamiltonian (2.1) use the Hamiltonian

$$\mathscr{H}_c = \frac{1}{2} \beta \rho_s \int d^3x (\nabla \varphi(\mathbf{x}))^2 + \beta D K_n(T) \int d^3x \cos(n\varphi(\mathbf{x}) + \theta), \quad (2.12)$$

in which all length scales are assumed to be much larger than R_c.

In terms of the Hamiltonian (2.12) one can prove the following exact results:

$$\overline{\langle\varphi_k\rangle\langle\varphi_{-k}\rangle}=n^2D^2\left(\frac{K_n}{\rho_s}\right)^2\frac{1}{k^4}, \quad (2.13)$$

$$\overline{\langle\langle\varphi_k\varphi_{-k}\rangle\rangle}\equiv\overline{\langle\varphi_k\varphi_{-k}\rangle}-\overline{\langle\varphi_k\rangle\langle\varphi_{-k}\rangle}=\frac{1}{\beta\rho_s k^2}. \quad (2.14)$$

One can obtain Eqs. (2.13) and (2.14) by a direct summation of the perturbation theory series using the method proposed by Efetov and Larkin.[8] One can then show that only the first order correction gives a contribution to the correlator (2.13) and the correlator (2.14) therefore remains the same as in a pure system. Near T_c the results (2.13) and (2.14) can also be proved using the supersymmetric Lagrangian description of systems with random fields, proposed by Parisi and Sourlas.[9]

Using the results (2.13) and (2.14) we easily obtain asymptotic expressions for the spin-spin correlation functions

$$\overline{\langle(\mathbf{s}(\mathbf{x}),\mathbf{s}(\mathbf{x}'))\rangle}=\mathrm{Re}\overline{\langle\exp\{i(\varphi(\mathbf{x})-\varphi(\mathbf{x}'))\}\rangle}\approx\rho_s\exp\left\{-\frac{|\mathbf{x}-\mathbf{x}'|}{L_n}\right\} \quad (2.15)$$

($|\mathbf{x}-\mathbf{x}'|\gg R_c$) where the correlation length

$$L_n(T)=\frac{n^2}{8\pi}D^{-2}\left(\frac{K_n(T)}{\rho_s(T)}\right)^{-2} \quad (2.16)$$

On the other hand, the result (2.14) enables us to find the spin rigidity of the system with respect to spatially inhomogeneous perturbations:

$$\left.\frac{\delta^2\mathcal{F}}{\delta a_\mu(\mathbf{k})\delta a_\nu(-\mathbf{k})}\right|_{\mathbf{a}=0}. \quad (2.17)$$

The free energy $\mathcal{F}\{a\}$ is given by Eq. (2.3) and the Hamiltonian (2.12) in which we have made the substitution $\nabla\varphi(\mathbf{x})\rightarrow\nabla\varphi(\mathbf{x})+\mathbf{a}(\mathbf{x})$. One easily verifies that

$$\delta^2\mathcal{F}/\delta a_\mu(\mathbf{k})\,\delta a_\nu(-\mathbf{k})\,|_{\mathbf{a}=0}=\rho_s\delta_{\mu\nu}-\beta\rho_s^2k_\mu k_\nu\langle\overline{\langle\varphi_k\varphi_{-k}\rangle}\rangle=\rho_s(\delta_{\mu\nu}-k_\mu k_\nu/k^2) \quad (2.18)$$

($|\mathbf{k}|\ll R_c^{-1}$). The spin rigidity $\rho_s(T)$ of our spatially disordered system is thus the same as the rigidity of the ferromagnetic phase of a pure 3D XY-model.

3. PHASE TRANSITION

The description of our magnet in terms of the continuous Hamiltonian (2.12) and the results (2.15), (2.16), and (2.18) are true only under the condition

$$L_n(T)\gg R_c(T). \quad (3.1)$$

This condition may be violated near the phase transition point where $R_c(T)$ increases and the scaling relations (2.9) to (2.11) are realized. The magnitude of the "forbidden" temperature interval near T_c where condition (3.1) is violated depends on n and is determined by the actual magnitude of the critical indexes.

If we assume that for the 3D XY-model the specific heat index $\alpha=0$ and the anomalous critical dimensionality $\eta=0$,[7] we have

$$\nu=2/3,\quad \beta=\varkappa_1=1/3. \quad (3.2)$$

The critical exponents κ_n ($n\neq1$) are determined by the critical behavior of the irreducible nth order spin tensors:[10]

$$\varkappa_2\approx0.8,\quad \varkappa_3\approx1.4,\ldots. \quad (3.3)$$

It follows from (2.16) that $[\tau=(T_c-T)/T_c]$

$$L_n(\tau)\sim D^{-2}\tau^{-2(\varkappa_n-2\beta)}. \quad (3.4)$$

Therefore

$$L_1(\tau)\sim D^{-2}\tau^{2/3}, \quad (3.5)$$

$$L_2(\tau)\sim D^{-2}\tau^{-0,4}, \quad (3.6)$$

$$L_3(\tau)\sim D^{-2}\tau^{-1,5}. \quad (3.7)$$

Comparing these relations with the behavior $R_c(\tau)\sim\tau^{-2/3}$ one sees easily that for $n=1$ and 2 condition (3.1) is satisfied only for temperatures $\tau\gg\tau_n^*$, where

$$\tau_n^*\sim D^{\sigma_n}, \quad (3.8)$$

$$\sigma_n=\frac{2}{\nu+4\beta-\varkappa_n} \quad (3.9)$$

or

$$\sigma_1=3/2, \quad (3.10)$$

$$\sigma_2\approx6.4. \quad (3.11)$$

On the other hand, for $n=3$ (and for all $n>3$) condition (3.1) is always satisfied and here the phase transition can thus be described using the exponents of the ferromagnetic phase transition of the pure 3D XY-model.

4. SUSCEPTIBILITY

The static susceptibility of a system in zero external field is given by the integral

$$\int d^3x\langle\langle\overline{(\mathbf{s}(\mathbf{x}),\mathbf{s}(0))}\rangle\rangle, \quad (4.1)$$

where in the low-temperature phase the irreducible correlator

$$\langle\langle\overline{(\mathbf{s}(\mathbf{x}),\mathbf{s}(0))}\rangle\rangle\sim\frac{1}{|\mathbf{x}|}\exp\left\{-\frac{|\mathbf{x}|}{L_n}\right\} \quad (4.2)$$

for $|\mathbf{x}|\gg R_c$. Therefore $\chi_n(T)\sim L_n^2(T)$ and in the low-temperature phase far from the transition point [see (2.16)] we get

$$\chi_n\sim D^{-4}. \quad (4.3)$$

In the critical region close to the transition point ($T<T_c$) we have, for temperatures $\tau_n^*\ll\tau\ll1$, $n=1$ and 2,

$$\chi_n(\tau)\sim D^{-4}\tau^{-4(\varkappa_n-2\beta)} \quad (4.4)$$

or

$$\chi_1(\tau)\sim D^{-4}\tau^{4/3}, \quad (4.5)$$

$$\chi_2(\tau)\sim D^{-4}\tau^{-0,7}, \quad (4.6)$$

$$\chi_3(\tau)\sim D^{-4}\tau^{-3}. \quad (4.7)$$

In the paramagnetic phase $(T>T_c)\chi\sim R_c^2(\tau)\sim\tau^{4/3}$.

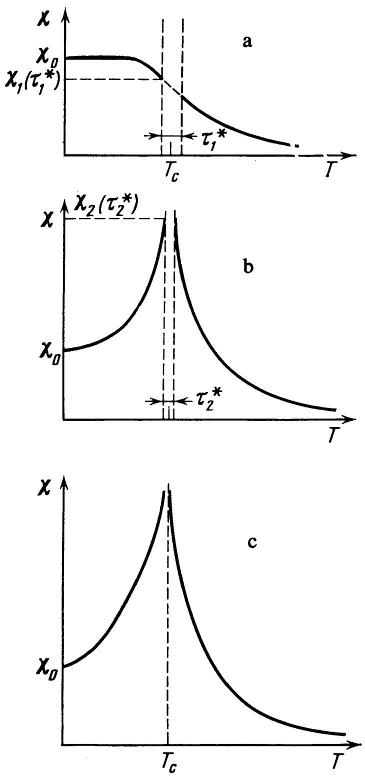

FIG. 1. Qualitative behavior of the susceptibility of a magnet (a) with random fields ($n = 1$) $\chi_0 \sim D^{-4}$, $\chi(\tau_1^*) \sim D^{-2}$, $\tau_1^* \sim D^{3/2}$; (b) with a random second order anisotropy axis: $\chi_0 \sim D^{-4}$, $\chi(\tau_2^*) \sim D^{-8}$, $\tau_2^* \sim D^{6.4}$; (c) with a random third order anisotropy axis.

The qualitative behavior of the static susceptibility is shown in Fig. 1.

For the spin rigidity (2.18) in the critical region we have

$$\rho_s(\tau) \sim \tau^{2/3}. \qquad (4.8)$$

We note that the quantities τ_n^* determining the limits of the temperature range where we can use the critical exponents of a pure magnet turn out to be equal to (3.8), (3.9) also in the high-temperature phase ($T > T_c$). One checks that easily using the effective dimensionality of the random anisotropy, following Ref. 11.

We now consider the susceptibility of the system in a non-zero (weak) external field H. If the spins are mainly directed along the field, i.e., if the declinations of the spins from that direction are small:

$$\langle \overline{\varphi^2} \rangle \ll 1, \qquad (4.9)$$

the magnet can in that case be described by the Hamiltonian

$$\mathscr{H} = \beta \int d^3x [{}^1/{}_2 \rho_s (\nabla\varphi)^2 + {}^1/{}_2 \rho_s m^2 \varphi^2 + DK_n \cos(n\varphi + \theta)], \qquad (4.10)$$

where

$$m^2 = \frac{H}{\rho_s^{1/2}} \ll 1. \qquad (4.11)$$

We estimate the condition (4.9):

$$\langle \overline{\varphi^2} \rangle \sim \frac{D^2 K_n^2}{\rho_s^2} \int d^3k \frac{1}{(k^2+m^2)^2} \sim \frac{D^2 K_n^2}{\rho_s^2} \frac{1}{m} \ll 1. \qquad (4.12)$$

Hence it follows that

$$m \gg L_n^{-1}(T), \qquad (4.13)$$

or

$$H \gg D^4 \left(\frac{K_n(T)}{\rho_s(T)} \right)^4 \rho_s^{1/2} \equiv H_c. \qquad (4.14)$$

In magnetic fields $H \lesssim H_c$ the spin correlations are destroyed over a length $L_n(T)$ and therefore in such fields the susceptibility of the system is the same as in a zero field (4.3), (4.4).

Let now the condition/$H \gg H_c$ be satisfied. The susceptibility is given by the integral

$$\chi = \int d^3x \langle\langle \overline{\cos\varphi(x)\cos\varphi(0)} \rangle\rangle.$$

When averaging over φ we now must use the Green functions

$$G(k) = 1/\rho_s(k^2+m^2). \qquad (4.15)$$

The result of the averaging has the form

$$\chi \approx \rho_s \int d^3x \{D(x)(e^{\Gamma_1(x)} - e^{\Gamma_2(x)}) + {}^1/{}_2 D^2(x)(e^{\Gamma_1(x)} - e^{\Gamma_2(x)})\}, \qquad (4.16)$$

where

$$D(\mathbf{x}) = [4\pi\rho_s|\mathbf{x}|]^{-1} e^{-m|\mathbf{x}|}, \qquad (4.17)$$

$$\Gamma_{1,2}(\mathbf{x}) = -\frac{\rho_s^2}{L_n} \int d^3k\, G(k)(1 \mp e^{i(\mathbf{k},\mathbf{x})}) \approx -\frac{1}{L_n m}(1 \mp e^{-m|\mathbf{x}|}). \qquad (4.18)$$

After integration we get

$$\chi \approx 1/8\pi\rho_s m + 2/L_n m^3. \qquad (4.19)$$

Therefore under the condition

$$1/L_n \ll m \ll (\rho_s/L_n)^{1/2}, \qquad (4.20)$$

which is equivalent to $D^4 \ll H \ll D^2$ far from the transition point ($T < T_c$)

$$D^4 \tau^{4\varkappa_n - 7\beta} \ll H \ll D^2 \tau^{2\varkappa_n - 3\beta} \text{ when } \tau_n^* \ll \tau \ll 1, \qquad (4.21)$$

the susceptibility behaves like

$$\chi \sim \begin{cases} D^2 H^{-3/2} & \text{far from the transition point } (T < T_c), \\ D^2 \tau^{2\varkappa_n - 5/2\beta} H^{-3/2} & \text{when } \tau_n^* \ll \tau \ll 1. \end{cases} \qquad (4.22)$$

For a magnet with a random second-order ($n = 2$) anisotropy axis we have [see (3.2), (3.3)]

$$\chi \sim D^2 \tau^{0.8} H^{-3/2} \qquad (4.23)$$

when $D^{6.4} \ll \tau \ll 1$, $D^4 \tau^{0.9} \ll H \ll D^2 \tau^{0.6}$.

Finally, in a "strong" magnetic field $m \gg (\rho_s/L_n)^{1/2}$ or $H \gg D^2$ far from the transition point ($T < T_c$)

$$H \gg D^2 \tau^{2\varkappa_n - 3\beta}, \text{ when } \tau_n^* \ll \tau \ll 1 \qquad (4.24)$$

the susceptibility behaves just as in a pure magnet:

$$\chi = (8\pi)^{-1} \rho_s^{-3/4} H^{-1/2}. \qquad (4.25)$$

The qualitative behavior of χ^{-1} as function of $H^{1/2}$ is shown in Fig. 2.

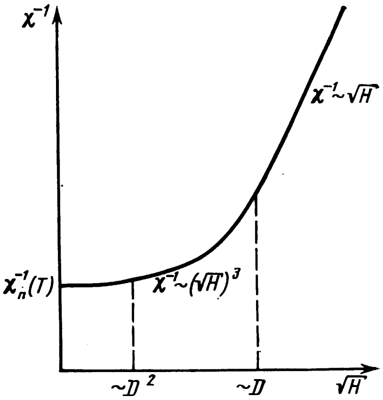

FIG. 2. The reciprocal susceptibility χ^{-1} as function of $H^{1/2}$. For $D^2 \ll H^{1/2} \ll D$ the relation $\chi^{-1} \sim D^{-2}(H^{1/2})^3$ holds; for $H^{1/2} \gg D$, $\chi^{-1} \sim H^{1/2}$.

5. CONCLUSION

In the present paper we have considered a three-dimensional magnet with an easy-plane type anisotropy in which there are weak random fields or random anisotropies. We showed that the low-temperature phase of such a magnet is a spin-glass kind of disordered phase with a large correlation range (when there is weak disorder).

We found for various orders of anisotropy the critical exponents of the susceptibility in zero external field using near the transition temperature the critical exponents of a pure magnet. We showed for the susceptibility in a non-zero external field that there exists a range of magnetic fields $D^4 \ll H \ll D^2$ in which the susceptibility behaves as $D^2 H^{-3/2}$.

At the same time a number of problems require further study. Although in the immediate vicinity of the transition point T_c when $\tau \ll \tau_n^*$ ($n = 1$ and 2) the critical behavior of the system may change, in the case of a magnet with a random field ($n = 1$) one must, apparently, not expect a large change in the magnitude of the susceptibility $\chi_1(\tau \ll \tau_1^*)$ as compared to its value $\chi(\tau \sim \tau_1) \sim D^{-2}$. On the other hand, in the case of a magnet with a random second order anisotropy axis ($n = 2$) the nature of the behavior of the susceptibility $\chi_2(\tau)$ in the region $\tau \ll \tau_2^*$ remains so far completely unexplained. This "negative" result is in agreement with the paper by Aharony[11] who demonstrated the absence of a fixed point in the renormalization group equations for a magnet with a random second-order anisotropy axis.

Recently Aharony and Pytte[12] have obtained equations of state for a degenerate magnet with a random field ($n = 1$) and with a random second order anisotropy axis ($n = 2$) in a weak external field. Their prediction that the susceptibility becomes infinite and the asymptotic power law behavior of the spin-spin correlation functions in the low-temperature phase ($T < T_c$) of a magnet with a random second order anisotropy axis are in contradiction with our results. From the exposition in the present paper one understands easily that an accuracy of order D^2 to which the calculations in Ref. 12 were in fact restricted is insufficient to obtain a correct result.

We note that in a real inhomogeneous magnet there also may occur effective spatial fluctuations of T_c (e.g., thanks to the inhomogeneity of the magnitude of the spin-spin exchange). There may then exist in the system an additional correlation length determined by the fluctuations in T_c and leading to the appearance of an additional temperature interval τ^* around T_c where the critical behavior is no longer described by the critical indexes of the pure 3D XY-model. In that case the results stated above require an additional condition $\tau \gg \tau^*$.

It is also necessary to note that the low-temperature behavior of an isotropic Heisenberg magnet with random anisotropies may turn out to be completely different due to the possible renormalization of the spin rigidity ρ_s of such a magnet.

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