Electromagnetic-wave emission in parametric excitation of magnons in antiferromagnets

A. S. Mikhaĭlov and A. V. Chubukov

Moscow State University
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The characteristics of a system of parametrically excited spin waves in an above-threshold stationary state are obtained for the case when the growth of their amplitude is nonlinearly restricted by a dissipative mechanism. The total intensity of the electromagnetic radiation emitted by the parametric waves is calculated, and the spectral composition of the radiation is determined. Comparison with the experimental data on parametric excitation of spin waves in the antiferromagnet $FeBO_3$ attests to the good agreement between the theoretical predictions and the experimental results.

Our aim here is to investigate theoretically the behavior of a system of parametrically excited spin waves (PESW) in antiferromagnets of the easy plane type (EPAFM), for which the Néel temperature is higher than the Debye temperature. Parametric excitation of spin waves (SW) in one such antiferromagnet, $FeBO_3$, was recently investigated experimentally.[1–4]

It is known that a stationary state of a PESW sets in above threshold through the action of nonlinear mechanisms that limit the amplitude of these waves (nonlinear dephasing of PSEW[5]), and through the effect of positive nonlinear damping.[6,7] The latter is caused by heating of the nonresonant SW and phonons, which increases under certain conditions the damping of the PESW as their number increases.

The strongest positive nonlinear damping results from three-particle PESW decay processes (see Refs. 6 and 7). Results of experiments with ferromagnetic YIG crystals offer evidence that when these processes are allowed their action suppresses the phase mechanism of the above-threshold limitation.[8] An evaluation reported in Ref. 9 shows that in antiferromagnetic crystals in which the spectra of low-frequency spin waves and phonons intersect[1)] (e.g., $MnCO_3$ and $CsNiF_3$) all three-particle processes capable of leading to positive nonlinear damping are forbidden, so that the principal role is assumed by the phase damping mechanism. This conclusion is confirmed by experimental data.[10,11] There is no spectrum intersection in $FeBO_3$, so that the decay of a PESW into an SW and a phonon is allowed by the conservation laws.[2)] One should thus expect the dissipative mechanism of above-threshold damping to be the most important in this case.

Parametric excitation of SW in $FeBO_3$ crystals was the subject of a number of recent experimental studies.[1–4] The results of these experiments indicate that the above-threshold limitation on the growth of the number of PESW is indeed imposed here by positive nonlinear damping. Measurements of the absorbed power at various pumping levels made it possible to determine the nonlinear-damping coefficient.[2] Within the limits of the measurement accuracy, this coefficient was found to be independent of the excess above threshold. Electromagnetic radiation emitted by PESW at half the pump frequency was first observed in Refs. 3 and 4, where its spectral characteristics were investigated.

In the present paper we present an explanation of the experimental results. In §1 we obtain a stationary state of the PESW system in an antiferromagnet for dissipative above-threshold damping. In §2 we solve the problem of electromagnetic radiation from a PESW system in an antiferromagnet. §3 is devoted to a comparison of the theoretical conclusions with the experimental data of Refs. 2–4. The nonlinear-damping coefficient is calculated in the Appendix.

§1. STATIONARY STATE OF A PESW SYSTEM

Low frequency SW in an EPAFM are characterized by the dispersion law[12]

$$\omega_{\mathbf{k}}^2=\Delta^2+s^2\mathbf{k}^2, \tag{1}$$

where $\Delta^2 = g^2H(H+H_D)$ is the gap in the spectrum, H_D is the Dzyaloshinskii field, H is the static magnetic field, and g is the gyromagnetic ratio.

The interaction of SW with an external pump is described by the Hamiltonian

$$\mathscr{H}_v=\frac{\hbar}{2}\sum_{\mathbf{k}}[hV\exp(-i\omega_p t)a_{\mathbf{k}}^*a_{-\mathbf{k}}^*+\text{H.c.}], \tag{2}$$

in which the coefficient of the coupling with the pump is given by[13]

$$V=(g^2/2\omega_p)(2H+H_D). \tag{3}$$

In Eq. (2) h is the amplitude of the magnetic field of the pump. We consider the case when the average occupation numbers of the magnon states are large compared with unity, so that a_k^* and a_k are classical complex amplitudes of the SW.

We write down now the equation of motion for the PESW amplitudes with allowance for their interaction with nonresonant SW and phonons. This interaction leads to two effects: on the one hand PESW damping sets in and can be described by introducing a term—$\gamma_k a_k$ in the equation of motion. However, if only damping were taken into account, the SW amplitude would decrease with time to zero, and we would be unable to describe the thermal fluctuations on the

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PESW. To take correctly into account the interaction of PESW with nonresonant SW and phonons it is necessary to include in the equations of motion, simultaneously with the damping, the random forces $f_k(t)$ that describe the noise action of the SW and phonons on the PESW.[3)] As a result, the equations of motion for the PESW amplitudes take the form

$$\dot{a}_{\mathbf{k}}=(-i\omega_{\mathbf{k}}+\gamma_{\mathbf{k}})a_{\mathbf{k}}-ihV\exp(-i\omega_p t)a_{-\mathbf{k}}^*+f_{\mathbf{k}}(t). \tag{4}$$

Here $f_k(t)$ is a Gaussian random force with correlators:

$$\langle f_{\mathbf{k}}(t)\rangle=0,\quad \langle f_{\mathbf{k}}(t)f_{\mathbf{k}'}(t')\rangle=0, \qquad \langle f_{\mathbf{k}}^*(t)f_{\mathbf{k}'}(t')\rangle=2D_{\mathbf{k}}\Delta_{\mathbf{k},\mathbf{k}'}\delta(t-t'), \tag{5}$$

where $\Delta_{k,k'}$ is the Kronecker delta and the coefficient D_k is connected with the damping γ_k (see below). With the aid of differential equations (4) we can obtain, by averaging over the realizations of the random force, the following equation for the average PESW density $n_k = \langle a_k^* a_k\rangle$:

$$^1/_2\dot{n}_{\mathbf{k}}=-\gamma_{\mathbf{k}}n_{\mathbf{k}}+D_{\mathbf{k}}+\mathrm{Im}\,[hV\exp(-i\omega_p t)\sigma_{\mathbf{k}}], \tag{6}$$

in which the anomalous correlator σ_k is defined as $\sigma_k = \langle a_k a_{-k}\rangle$. It follows from (6) that D_k has the form of the arrival term in the kinetic equation.

We consider below only the interaction of single PESW with resonant SW and phonons. In such processes the anomalous paired correlation in the PESW system cannot be related with the form of the collision integrals. Taking this into account, we can find the values of γ_k and D_k from the usual (i.e., without allowance for the anomalous correlators) kinetic equations.[4)] It is shown in the Appendix that in the case of interest to us the quantities γ_k and D_k are given by

$$\gamma_{\mathbf{k}}=\gamma_{\mathbf{k}}^{(0)}(1+\eta_{\mathbf{k}}\tilde{N}), \tag{7}$$

$$D_{\mathbf{k}}=\gamma_{\mathbf{k}}\Phi_{\mathbf{k}}. \tag{8}$$

Here $\tilde{N} = N/\mathcal{N}$ is the ratio of the total number of PESW to the number of sites in the crystal, while η_k and Φ_k depend little on $\tilde{N}$ and this dependence can be neglected.

Returning to Eq. (4), we note that the connection between the PESW amplitudes and the random forces is linear. Consequently the time variation of the complex amplitudes a_k is a Gaussian random process. With the aid of Eqs. (4) it is easy to obtain for the Fourier transform of the equal-time correlation function $S_k(\tau) = \langle a_k^*(t+\tau)a_k(t)\rangle$ the expression

$$S_{\mathbf{k}\omega}=2D_{\mathbf{k}}\frac{\gamma_{\mathbf{k}}^2+(\tilde{\omega}-\tilde{\omega}_{\mathbf{k}})^2+(hV)^2}{(\tilde{\omega}_{\mathbf{k}}^2-\tilde{\omega}^2+\gamma_{\mathbf{k}}^2-(hV)^2)^2+4\gamma_{\mathbf{k}}^2\tilde{\omega}^2}, \tag{9}$$

where $\tilde{\omega}_k = \omega_k - \omega_p/2$; $\tilde{\omega} = \omega - \omega_p/2$. We note that $S_{k\omega}$ determines the spectral intensity of the fluctuations of the complex PESW amplitude with specified wave vector **k**.

Integrating (9) with respect to the frequency ω we obtain the PESW density in **k**-space:

$$n_{\mathbf{k}}=\Phi_{\mathbf{k}}\frac{\gamma_{\mathbf{k}}^2+\tilde{\omega}_{\mathbf{k}}^2}{\tilde{\omega}_{\mathbf{k}}^2+\gamma_{\mathbf{k}}^2-(hV)^2}. \tag{10}$$

Equations (9) and (10) are general. We confine ourselves below to the isotropic situation and neglect the dependences of γ_k and D_k on the directions of the wave vector **k**.

Expressions (9) and (10) contain the still unknown total number of PESW [see (7)]. It can be determined by using the self-consistency condition:

$$\frac{1}{\mathcal{N}}\sum_{\mathbf{k}}n_{\mathbf{k}}=\tilde{N}, \tag{11}$$

where the summation is limited by the region of wave vectors close to the resonant surface $\omega_{\mathbf{k}0} = \omega_{p/2}$. Condition (11) leads to a nonlinear equation for the total number $\tilde{N}$ of the PESW:

$$\tilde{N}[1-\beta^2y^3/(\eta\tilde{N})^3]=(y-1)/\eta. \tag{12}$$

Here $y = h/h_c$, $h_c = \gamma_{k_0}^{(0)}/V$, h_c determines the threshold for parametric SW excitation, while the coefficient β is defined by the expression

$$\beta=\frac{1}{2\pi}\left(\frac{\gamma_0}{s_0k_0}\right)(ak_0)^3\eta\Phi, \tag{13}$$

in which

$$s_0=\left.\frac{\partial\omega_k}{\partial k}\right|_{k=k_0}=\frac{2s^2k_0}{\omega_p}$$

is the velocity of a spin wave with wave vector k_0, $\gamma_0\not\equiv\gamma_{k_0}^{(0)}$ is its damping, $a = v_0^{1/3}$ (v_0 is the volume of the unit cell), $\eta\equiv\eta_{k0}$ $\Phi\equiv\Phi_{k_0}$.

Estimates show (see §3) that $\beta\ll 1$. In this case the inequality $y-1\gg\beta^{2/3}$ is satisfied even at a small excess above threshold, and the solution of (12) is

$$\tilde{N}=\frac{y-1}{\eta}\left[1+\beta^2\left(\frac{y}{y-1}\right)^3\right]. \tag{14}$$

As can be seen from (10), the PESW are then concentrated in a narrow spherical layer of radius k_0 and thickness δk:

$$l\delta k=\beta y^2/(y-1). \tag{15}$$

Here $l = s_0/\gamma_0$ is the mean free path of a spin wave with wave vector k_0 in the absence of pumping. We see that at an appreciable excess above threshold ($y\gg 1$) the width δk of the PESW distribution in k-space increases in proportion to the amplitude of the pump magnetic field.

§2. ELECTROMAGNETIC RADIATION FROM PARAMETRIC SPIN WAVES

If the sample has a dipole magnetic moment $\mathbf{M}(t)$, it emits electromagnetic waves of intensity[19]

$$I(t)=\frac{2}{3c^3}(\ddot{\mathbf{M}}(t))^2. \tag{16}$$

But what is the dipole moment of a crystal in which a certain number of SW is excited? This question calls for an attentive analysis. On the one hand, this analysis can be based on the eigenmodes of the crystal vibrations, determined with account taken of its actual form and explicit boundary conditions on the surface. This is precisely how the theory of magnetostatic modes is formulated.[20] When SW is having a wavelength much shorter than the size of the sample, different assumptions are usually made, viz., the spin waves are produced with an arbitrary wave vector in the interior of the sample and are scattered or reflected on reaching the surface. It is precisely the scattering and reflection which indicate that the spin waves arse not eigenmodes of the crystal vibrations.

Let SW with complex amplitudes $a_k(t)$ be excited in the crystal at the instant of time t. The deviation $\delta \mathbf{M}(t)$ of the magnetic dipole moment of the sample from its static value $\mathbf{M}_0$ is then

$$\delta M^+(t)=\sum_{\mathbf{k}}\int d\mathbf{r} g\hbar(2S)^{1/2}[u_{\mathbf{k}}a_{\mathbf{k}}(t)+v_{-\mathbf{k}}a_{-\mathbf{k}}^*(t)]e^{i\mathbf{k}\mathbf{r}}, \qquad (17)$$

where u_k and v_k are the coefficients of the diagonalizing uv transformation. The spectral density of the magnetic dipole-moment fluctuations, which is defined as the Fourier transform of the correlation function $\langle \delta \mathbf{M}^*(t+\tau)\delta \mathbf{M}(t)\rangle$, is therefore given by

$$(\delta \mathbf{M}^2)_\omega=\left(\frac{gH_D}{\omega_k}\right)\left[\frac{4(g\hbar)^2S}{v_0}\right]\left(\frac{H_D}{H_E}\right)\Omega\sum_{\mathbf{k}}S_{\mathbf{k}\omega}G_{\mathbf{k}}^2. \qquad (18)$$

Here H_E is the exchange field, Ω is the sample volume, v_0 is the unit-cell volume, and $S_{k\omega}$ is the spectral intensity of the PESW [see (9)]. The function G_k is determined by an integral over the volume of the crystal:

$$G_{\mathbf{k}}=\frac{1}{\Omega}\int_\Omega e^{i\mathbf{k}\mathbf{r}}d\mathbf{r}. \qquad (19)$$

The value of the integral (19) is sensitive to the sample shape. For the sake of argument we present an expression obtained for a sample in the form of an ideal sphere of radius R:

$$G_{\mathbf{k}}=3(\sin kR)/(kR)^2. \qquad (20)$$

We assume in addition that the width of the PESW in space is small: $\delta kR \ll 1$. This condition is satisfied in the experiments of Refs. 3 and 4. Because of this condition we can use in the equations that follow the value of G_k at $\mathbf{k}=\mathbf{k}_0$.

According to (16), the spectral intensity $I(\omega)$ of the radiation is expressed in terms of the spectral intensity of the fluctuations of the magnetic dipole moment in the form

$$I(\omega)=(1/3\pi c^3)\omega^4(\delta\mathbf{M}^2)_\omega. \qquad (21)$$

With allowance for (18), we get

$$I(\omega)=(I/2\pi N)\sum_{\mathbf{k}}S_{\mathbf{k}\omega}. \qquad (22)$$

The quantity I in (22) defines the time-averaged intensity of the electromagnetic radiation. Its explicit form is

$$I=\int I(\omega)d\omega=\frac{N}{3c^3}\left(\frac{gH_D}{\omega_p}\right)\left[\frac{(g\hbar)^2S}{v_0}\right]\left(\frac{H_D}{H_E}\right)\omega_p^4G_{\mathbf{k}_0}^2. \qquad (23)$$

Here N is the total number of the PESW.

Integration of (9) with respect to the wave number $\mathbf{k}$ leads to the relation

$$I(\omega)\propto[A(A+B)^{1/2}]^{-1}, \qquad (24)$$

where

$$A=[(\gamma_0^2d^2-\tilde\omega^2)^2+4\gamma^2\tilde\omega^2]^{1/2},$$
$$B=\gamma_0^2d^2-\tilde\omega^2,$$
$$d=\beta y^2/(y-1), \quad \gamma=\gamma_0(1+\eta\tilde N). \qquad (25)$$

We have left out of (24) factors that do not depend on $\tilde\omega=\omega-\omega_p/2$. We note that allowance for the actual shape of the sample (i.e., for its deviation from spherical) can influence only the integral intensity of the radiation, but not its spectral composition.

It can be seen from (24) that the radiation line shape is quite complicated (see Fig. 1). Near the center, however, at $|\omega-\omega_p/2| \lesssim \gamma_0^2d^2/\gamma$, it can be approximated by the Lorentz expression

$$I(\omega)\propto[(\omega-\omega_p/2)^2+(\delta\omega)^2]^{-1} \qquad (26)$$

with half-width

$$\delta\omega=(2/5)^{1/2}\gamma_0\beta^2y^3/(y-1)^2, \quad y-1\gg\beta. \qquad (27)$$

Thus, at large excess above threshold ($y \gg 1$) the half-width of the radiation line increases in proportion to the amplitude of the pump magnetic field.

To conclude this section we must note that in certain cases, besides the foregoing mechanism of direct radiation from PESW, a significant role can be played by reradiation through a uniform-precession mode, which is connected with the PESW via the two-magnon process of scattering by impurities, defects, and the crystal surface:

$$\mathcal{H}_{imp}=\sum_{\mathbf{k}}(g_{0\mathbf{k}}a_0^*a_{\mathbf{k}}+\text{c.c.}). \qquad (28)$$

It was demonstrated in Ref. 21, for a ferromagnetic YIG, that this mechanism is decisive in the resonant case, i.e., when the frequency $\omega_p/2$ coincides with the uniform-precession frequency. It must be emphasized, however, that reradiation is possible also on the uniform-precession line wing via induced excitation of this oscillation at an unrelated frequency. The spectral intensity of radiation produced by such a process is of the form

$$I^{imp}(\omega)\propto\frac{|g_{0\mathbf{k}_0}|^2}{(\omega-\omega_0)^2}\sum_{\mathbf{k}}S_{\mathbf{k}\omega}. \qquad (29)$$

Comparison of (22) and (29) shows that when the condition $|\omega_p/2-\omega|\gg\delta\omega$, is satisfied, i.e., far from resonance with the uniform precession, the radiation line shape is not distorted and is determined, just as in the case considered above, by the sum contained in Eq. (29).

A quantitative estimate of the contribution made to the total radiation intensity by the processes indicated above is difficult, inasmuch as in the case of direct emission by para-

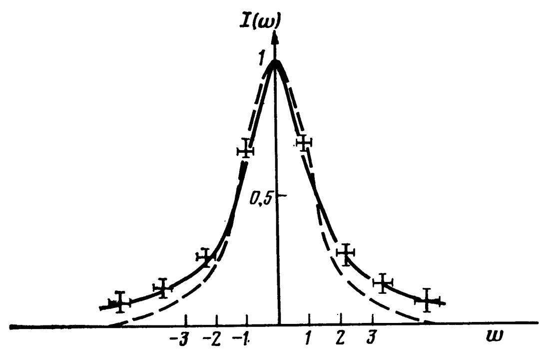

FIG. 1. Electromagnetic-radiation line shape calculated from Eq. (24). The abscissas are the parameter combinations $w=2(\omega-\frac{1}{2}\omega_p)/\gamma_0 y\beta^2$; the radiation intensity I is in arbitrary units. At $y \gg 1$ the $I(\omega)$ is universal. The line shape is Lorentzian near the radiation maximum but broadens into a "pedestal" far from the center. The dashed line shows for comparison a Lorentz line having the same width. The experimental points marked on the diagram were obtained by reduction of the electro-magnetic radiation spectrum.[4]

metric waves the total intensity depends substantially on the actual shape of the sample, as well as on its surface roughnesses of size $(\delta R)k \gtrsim 1$, whereas the intensity of the radiation due to elastic scattering with transformation into uniform precession contains as a factor the scattering amplitude g_{0k_0} which we do not know.

§3. COMPARISON WITH THE EXPERIMENTAL DATA

In this section we compare our theoretical results with the experimental data[2–4] on parametric excitation of SW in the antiferromagnet $FeBO_3$.

We shall use in the estimates the following parameter values[1]: the magnetoelastic constant $\Theta \approx 2 \cdot 10^{-15}$ erg, the exchange field $H_E \approx 3 \cdot 10^3$ kOe, the Dyaloshinskiĭ field H_D 10^2 kOe, the unit-cell volume $v_0 \approx 0.9 \cdot 10^{-22}$ cm^3, the magnon velocity $s \approx 1.4 \cdot 10^6$ cm/asec, the phonon velocity $c \approx 4.7 \cdot 10^5$ cm/asec, the site spin $S = 5/2$, $\omega_p = 35.7$ GHz, $Mc^2 = 10^5$ K, and the unit-cell mass M.

We verify first that the residual damping γ_0 of the PESW is indeed due to magnon-phonon decay. In the Appendix we derive Eq. (A9) for the damping on account of such a process. In the case $\alpha \gg 1$ this equation goes over into the expression obtained in Ref. 22 for the relaxation. Substituting in (A9) the numerical values of the $FeBO_3$ crystal parameters we find (at $H = 100$ Oe and $T = 1.2$ K) that $\gamma_0 \approx 0.1$ mHz, of the same order as the value obtained in Ref. 2. Just as in the experiment, the damping γ_0 obtained by us increases linearly with temperature and decreases with increasing magnetic field H.

In the same investigation of the depedence of the absorbed power on the excess above threshold, they measured also the nonlinear-damping coefficient η. The relation obtained at small H is

$$\eta^{\exp} \approx (10^7 - 10^8) T^{-1}, \tag{30}$$

where the temperature T is in degrees Kelvin. The theoretical expression for the coefficient η has a similar temperature dependence [see (A12)]. The numerical expression obtained by us at $T = 1.2$ K is of the same order as the experimental estimate (30). The electromagnetic radiation produced when parametric SW are excited in a crystal were investigated in Refs. 3 and 4. According to the data of Ref. 4 the profile of the spectral intensity of the radiation $I(\omega)$ consists of a low broad pedestal a narrow Lorentzian peak rising above it. Such a curve, as can be seen from the figure, is well described by Eq. (24). It follows next from (27) that the line width $\delta\omega$ outside the vicinity of the threshold increases in proportion to the pump-field amplitude. A similar dependence was observed in Ref. 3, and the linearity was preserved within the accuracy limits all the way to the kinetic-instability point. At the kinetic-instability threshold, the line width was seen to increase: this can be attributed to the abrupt increase of the PESW damping in the immediate vicinity of the value $\tilde{N} = N_{cr}$ [see (A11)].

According to our theoretical results [see (13) and (A11)], the coefficient β can be expressed in terms of the quantity y_{cr}, which determines the threshold of the kinetic instability ($\tilde{N} \approx N_{cr}$). In particular, at $H = 100$ Oe and $T = 1.2$ K we have

$$\beta \approx 4 \cdot 10^{-3} (y_{cr} - 1). \tag{a}$$

In the experiments,[3,4] the kinetic-instability threshold corresponded to $y_{cr} = 10$. In this case we obtain from (31) $\beta \approx 4 \times 10^{-3}$, which agrees in order of magnitude with the experimental result at the same values of the external magnetic field and temperature, $\beta^{\exp} \approx 2 \times 10^{-1}$. The value of y_{cr} can be calculated theoretically, using Eqs. (A3), (A5), and (A9). This yields $y_{cr} \approx 2$, in approximate accord with the experimental value. One should not expect better agreement in this case, since our calculation, in the Appendix, of the nonlinear damping coefficient is based on a number of simplifying assumptions.

We note in conclusion that it follows from (13) that the coefficient β increases linearly with temperature (since $\eta \propto T^{-1}$, $\Phi \propto T$, $\gamma_0 \propto T$), and therefore the line width $\delta\omega$ should increase like T^3 with rising temperature. The experiments[3,4] revealed an increase of $\delta\omega$ with increasing T, but no complete investigation of the temperature dependence of the line width was made.

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APPENDIX

The analysis presented above is based to a substantial degree on the assumption that the nonlinear-damping coefficient η and the value of Φ depend weakly [see (7) and (8)] on the number of the PESW. In this Appendix we justify the assumption and obtain quantitative estimates.

As already noted, positive nonlinear damping in antiferromagnets having a high Néel temperature is due to the possibility of decay of a parametric SW into an SW and a phonon. The effects observed here are close in principle to those considered by Gottlieb and Suhl,[5] who investigated PESW decay into two spin waves in ferromagnets. Our case differs mainly in the forms of the dispersion laws, in the different amplitudes of the analyzed processes, as well as in the possibility of a larger difference between the SW and phonon damping.

The experiments show[23] that at low temperatures the phonons attenuate in antiferromagnets more strongly than the spin waves, so that the deviation of the phonon distribution function from thermal equilibrium can be neglected. The rapid phonon damping is apparently due to the substantial contributions of processes within the phonon subsystem and of the heat transfer to the surrounding medium.

The main contribution to low-temperature magnon damping in the easy-plane antiferromagnets investigated in Refs. 1 and 2 is made by processes of relaxation on two-level impurities.[24] Above the resonance threshold, however, this main contribution to the damping drops out and the residual damping of the PESW is due to the decay of the PESW into a magnon and a phonon (Refs. 22 and 26).[3)]

The gist of the nonlinear damping in the decay process is that a definite group of nonresonant SW becomes heated, and this increases the PESW damping. Allowance for the conservation laws for the PESW decay process determines the k-space regions in which the secondary SW are concentrated. This region is characterized by the condition $k_1 < k < k_0$. The minimum wave vector of the secondary SW is

$$k_1=\frac{\omega_p}{2s(\alpha^2-1)}\left|\frac{2sk_0}{\omega_p}(\alpha^2+1)-2\alpha\right|, \tag{A1}$$

where $\alpha = s/c$ and c is the phonon velocity. PESW decay is forbidden if $k_0 < \omega_p/2s\alpha$. It is known (see Ref. 25) that the matrix element for the decay of an SW of frequency ω_k into a spin wave of frequency $\omega_{k'}$ and a phonon is proportional to $(\omega_k - \omega_{k'})$.[2] Therefore the most intense process is PESW decay into spin waves with small wave vectors, since the matrix element for these waves is the largest. Thus, the maximum heating is reached for secondary SW with wave vectors k close to k_1. Assuming that the width of the heated region is small enough, we disregard hereafter in the kinetic equation the interactions between the secondary SW, since the matrix elements for such processes are proportional to the square of the width of the heated region.

With allowance for the foregoing remarks, the solution of the kinetic equation for the secondary spin waves takes the form

$$n_k=W_1\frac{\tilde N}{N_{cr}}\left[1-\frac{\tilde N}{N_{cr}}+W_2\frac{\tilde N}{N_{cr}}\frac{k-k_1}{k_1}\right]^{-1}, \quad k>k_1. \tag{A2}$$

Here

$$W_1=2T/(\omega_p-2\omega_1); \tag{A3}$$

$$W_2=2\left(\frac{sk_1}{\omega_1}\right)^2\left[\frac{\omega_p}{\omega_p-2\omega_1}+\frac{2\alpha}{\alpha^2-1}\frac{\Delta^2}{\omega_1 sk_1}\frac{\alpha sk_1/\omega_1-1}{(\alpha^2+1-2\alpha sk_1/\omega_1)}\right] \tag{A4}$$

In addition

$$N_{cr}=\frac{T\omega_p}{2(sk_0)^2}\left(\frac{2\omega_1}{\omega_p-2\omega_1}\right)^2 Q(k_1)\frac{(ak_0)^3}{2\pi^2}; \tag{A5}$$

$$Q(k)=\ln\frac{\alpha^2-1}{\alpha^2+1-2\alpha sk/\omega_k}-2\frac{\alpha sk/\omega_k-1}{\alpha^2-1}. \tag{A6}$$

In (A3)–(A6) T is the sample temperature (i.e., the temperature of the phonon thermostat), and $\omega_1 \equiv \omega_{k_1}$.

The total number of secondary SW per crystal site is

$$\tilde N'=\frac{(ak_1)^3}{2\pi^2}\frac{W_1}{W_2}\left|\ln\left(1-\frac{\tilde N}{N_{cr}}\right)\right|, \quad \tilde N<N_{cr}. \tag{A7}$$

The PESW damping is given by

$$\gamma=\gamma_0\left[1+\frac{2\pi^2}{(ak_1)^3}W_3\tilde N'\right], \tag{A8}$$

where γ_0 is the equilibrium damping of the PESW via decay processes

$$\gamma_0=\frac{\alpha^2}{3\pi}\left(\frac{\Theta}{Mc^2}\right)\left(\frac{\Theta}{\hbar}\right)\left(\frac{gH_E}{S\omega_p}\right)^2\left(\frac{\omega_p a}{s}\right)^3\left(\frac{T}{sk_0}\right)Q(k_0), \tag{A9}$$

and the coefficient W_3 is given by the expression

$$W_3=\left(\frac{\omega_p-2\omega_1}{\omega_p}\right)^2\frac{sk_1}{T\omega_1}\frac{1}{Q(k_0)}. \tag{A10}$$

Substituting (A8) in (A7) we obtain

$$\gamma=\gamma_0\left(1+\frac{W_1W_3}{W_2}\ln\frac{N_{cr}}{N_{cr}-\tilde N}\right). \tag{A11}$$

It can be seen from (A7) and (A11) that at $\tilde N = N_{cr}$ the total number $\tilde N'$ of secondary spin waves and the PESW damping γ become infinite. The value $\tilde N = N_{cf}$ can be interpreted as the threshold of the kinetic instability[21,26] for the secondary spin waves at a given number of PESW. Below the kinetic-instability threshold, expanding (A11) in terms of the parameter $\tilde N/N_{cf}$, we arrive at expression (7) for the nonlinear damping. The nonlinear-damping coefficient is equal to

$$\eta=\frac{W_1W_3}{W_2N_{cr}}=\frac{\pi^2}{(ak_0)^3}\left(\frac{\omega_p-2\omega_1}{\omega_p}\right)^3\frac{(sk_0)^2}{\omega_1 T}\frac{1}{Q(k_0)}\frac{1}{Q(k_1)}\times\left[\frac{\omega_p}{\omega_p-2\omega_1}+\frac{2\alpha}{\alpha^2-1}\frac{\Delta^2}{\omega_1 sk_1}\frac{\alpha sk_1/\omega_1-1}{\alpha^2+1-2\alpha sk_1/\omega_1}\right]^{-1} \tag{A12}$$

In the same approximation, we obtain for the coefficient Φ [see (8)]

$$\Phi=\frac{2T}{(\omega_p-2\omega_1)}\left(1-2\frac{\omega_1}{\omega_p}\frac{\gamma_0}{\gamma}\right). \tag{A13}$$

[1]As a rule, the Néel temperature T_N in such crystals is lower than the Debye temperature T_D.

[2]We note that three-magnon processes with participation of SW from only the lower branch of the spectrum never occur in EPAFM, since the coefficient of the corresponding term in the Hamiltonian is identically zero. As for three-magnon processes with participation of SW of the upper branch of the spectrum, they are allowed only for SW with very large wave vectors.[9]

[3]A general theoretical foundation for the procedure used by us can be found in Refs. 14 and 15; as applied to the study of parametric excitation of SW the questions were discussed also in Refs. 16 and 17.

[4]This question was discussed also recently in Ref. 18.

[5]PESW–phonon coalescence, which contributes to the below-threshold damping of SW,[22] is eliminated simultaneously with the impurity relaxation.

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