Magnetization of type-II superconductors near the upper critical field H_{c2}

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The magnetization of V_3Ge and of Nb + 5.7% Mo single crystals is investigated experimentally in the vicinity of the second critical field H_{c2}. It is shown that the resistive transitions occurs in a field weaker than H_{c2} and determined from the magnetization curve $M(H)$. Above the field H_{cr} corresponding to the resistive transition, the $M(H)$ dependence is described by theories that consider the fluctuational superconductivity. A singularity in the electrodynamic characteristics is observed below H_{cr}.

INTRODUCTION

It is known that a diamagnetism due to fluctuational superconductivity exists near the interface between the normal and superconducting states in a superconductor that is the normal state. This phenomenon was studied[1] in the temperature region near T_{c0} (the critical temperature at $H = 0$) and in weak magnetic fields. The region of temperature noticeably different from T_{c0}, at $T < T_{c0}$, was insufficiently investigated. Yet this region is of interest because the spectrum of the fluctuations near H_{c2} (at $H - H_{c2} \ll H_{c2}$) is effectively one-dimensional.[2] A mixed state (ordered vortex lattice) is known to exist below H_{c2}, but the location of the transition into the mixed state remains a moot question. The conclusion that follows from the self-consistent field approximation,[3] that the transition takes place at H_{c2}, is inaccurate, since this approximation does not hold in the immediate vicinity of H_{c2}.

The plot of magnetization $M(H)$ of type-II superconductors in the mixed state near H_{c2} is described, in the region where the self-consistent-field approximation holds, by a straight line drawn from H_{c2} (Ref. 3), as was experimentally confirmed in most studies. In the immediate vicinity of H_{c2}, where the self-consistent field approximation is not valid, the behavior of the magnetization was not investigated. Measurements of the heat capacity[4] and of the excess electric conductivity[5] in this magnetic-field region indicate that these quantities can be described in the vicinity of H_{c2} by a one-dimensional model.

The second critical field H_{c2} is revealed most frequently in experiment by the vanishing of the resistance as the magnetic field is decreased (resistive transition), or by the appearance of the critical current. However, the relation between the values of H_{c2} and the field corresponding to the resistive transition or to the vanishing of the critical current has not been sufficiently investigated.

We study in this paper the $M(H)$ dependences of the single crystals V_3Ge and Nb:5.7% Mo. The position of the resistive transition at a current perpendicular to the magnetic field is compared with H_{c2} determined from the $M(H)$ curve. The $M(H)$ dependence above the field H_{cr} corresponding to the resistive transition is compared with the theoretical relations obtained in the linear approximation[2] and in the Hartree-Fock approximation with a screened potential.[6] The singularities of the electrodynamic characteristics observed below H_{cr} are discussed.

EXPERIMENT

The measurements were performed on single-crystal V_3Ge and Nb:5.7% Mo samples obtained by zone melting, at temperatures 2–4.2 K, in magnetic fields produced by superconducting solenoids. After electric-spark machining, an additional ~0.5 mm of surface distorted by the machining was removed by electrochemical treatment (the Nb:5.7% Mo was rid of hydrogen by heating in vacuum (800 °C, 1 hr, 10^{-8}–10^{-9} Torr). The samples had various shapes (spheres, cylinders, plates) with characteristic dimensions 0.5–5 mm. The parameters of the investigated samples ($\varkappa$, T_{c0}, $H_{c2}(0)$) of the V_3Ge and Nb:5.7% Mo where respectively 13, 6.1 K, 70 kOe and 3, 7.5 K, and 7.5 kOe ($\varkappa$ is the parameter of he Ginzburg-Landau theory, $H_{c2}(0)$ is the second critical field at $T = 0$).

The electric resistance was measured by a four-contact method, and the field dependence of the magnetization $M(H)$ with a vibration magnetometer. A detailed description and the operating principle of the latter are given in Ref. 7. Here we indicate only the main design features needed for the understanding of the measurement results. The stray field due to the magnetic moment of the sample induces in two oppositely wound coils rigidly secured to the superconducting-solenoid coil form a signal proportional to $M(H)$ at the sample vibration frequency. This signal was amplified, passed through asynchronous detector, and fed to an automatic x-y plotter. The signal for the second axis of the plotter was proportional to the magnetic field of the solenoid.

Measurements with a vibration magnetometer are made complicated by the fact that in the presence of magnetic-field gradients there is induced in the observed sample a magnetic moment due to the eddy currents. This moment depends on the field gradient, on the frequency and amplitude of the vibrations, and on the conductivity of the sample. It is estimated that for our samples, at the employed frequencies (20–200 Hz) and amplitudes (~0.2 mm), in the normal state ($\rho = \rho_n$) and in the flux state ($\rho = \rho_f$) the moment due to the eddy currents is negligibly small compared with the static moment. However, the presence of highly nonlinear current-voltage characteristics in fields somewhat weaker

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than H_{c2}, with initial sections in which $\partial u/\partial I \ll \rho_n$, can lead to an anomalous increase of the moment. To eliminate the ac component of the magnetic field at the investigated samples, we obtained control plots of $M(H)$ using a container of highly purified aluminum having a wall thickness larger than the magnetic-field penetration depth at the vibration frequency. Simultaneously with the measurements of $M(H)$ we measured $j_c(H)$ (the critical current was defined as the one producing a voltage drop 0.5 μV on the length $\sim$2–4 mm) and plotted the resistive transition.

To study the response to an alternating magnetic field, we placed coaxially inside the superconducting solenoid the modulation coils and two receiving coils, with an immobile sample placed inside one of the latter. This arrangement permitted the passage of a current perpendicular to the field direction through the sample. The amplitude of the alternating current was varied from 0.1 to 10 Oe, and the frequency from 20 to 100 Hz.

MEASUREMENT RESULTS AND DISCUSSION

Figure 1 shows plots of the critical current $j_c(H)$, of the voltage on the sample, of the magnetization $M(H)$, and of the real part $\alpha'(H)$ of the magnetic susceptibility of the investigated samples in the vicinity of H_{c2} at 4.2 K.

Above the H_{cr} value corresponding to the vanishing of the bulk critical current (marked by the symbol ε_{cr} in the figures), the measured value of $M(H)$ is independent of the position of the sample in the solenoid, i.e., of the magnetic field gradient) and is fully reversible. Below this field, an anomaly appears on the $M(H)$ curve [see the $M(H)$ plots in Fig. 1a, lower curve, and Fig. 1b]. The magnitude and sign of the anomaly depend on the location of the sample relative to the center of the solenoid. For samples with low critical current comparable with the values given in Fig. 1, the $M(H)$ dependence is reversible within the error limit also at $H < H_{cr}$. If the sample is placed in the screening container, the anomaly of the $M(H)$ curve vanishes [see $M(H)$ in Fig. 1a, upper curve]. The slopes of the $M(H)$ curve change somewhat when the container is used, owing to the contribution made to the measured signal by the response from the dynamic moment of the container. Except for the immediate vicinity of H_{c2}, the $M(H)$ plot comprises two straight lines corresponding to the mixed ($H < H_{c2}$) and normal ($H > H_{c2}$) states. The point of intersection of the linear sections of $M(H)$ yields the value of H_{c2} (Ref. 3). The finite slope of $M(H)$ at $H > H_{c2}$ is connected with the paramagnetism of the normal state.

The value of H_{cr} is determined by extrapolating to zero the linear section of the right-hand slope of the peak of the $j_c(H)$ plot. The same value of H_{cr} is obtained also by measuring the voltage across the sample at low (0.1–1 A/cm^2) currents in the case of V_3Ge (Fig. 1b). In the Nb:5.7% Mo samples, surface superconductivity is observed and H_{cr} can be determined only from the $j_c(H)$ plot. As can be seen from Fig. 1, H_{cr} is smaller than the H_{c2} determined from the $M(H)$ plot. For V_3Ge the difference is $H_{c2}-H_{cr} \approx 500$ Oe, which agrees, within the experimental error, with the value obtained by us for H_{c2} by comparing the experimental and theoretical dependences of the excess electric conductivity in a magnetic field stronger than H_{c2} and perpendicular to the current.[8] For Nb:5.7% Mo we have $H_{c2}-H_{cr} = 20$ Oe.

In the immediate vicinity of H_{c2} at $H > H_{cr}$ the $M(H)$ plot becomes curved. For sufficiently homogeneous samples, the $M(H)$ dependence is universal for the various samples, so that the extra diamagnetism relative to the straight lines that characterize the mixed and normal states can be attributed to the fluctuational superconductivity.

Figure 2 shows the $M(H)$ plot near H_{c2} for the Nb:5.7% Mo samples. It can be seen that above H_{c2} the experimental $M(H)$ plot agrees with the theoretical one obtained in the linear approximation [see Eq. (11) of Ref. 2]. It must be noted that in the considered range of $\varepsilon_H = H/H_{c2} - 1$, as can be

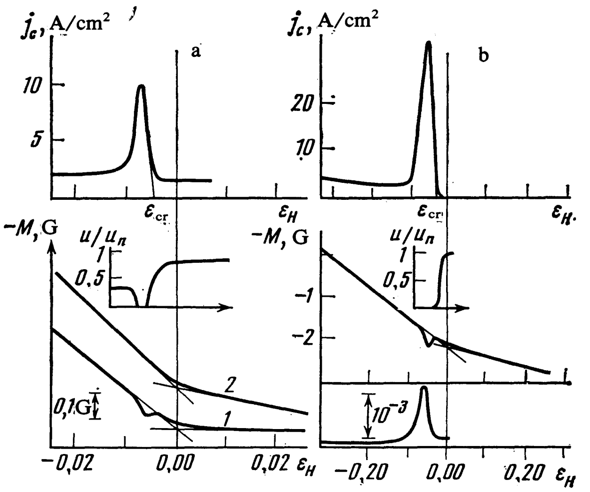

FIG. 1. Dependences of the critical-current density j_c, of the voltages u on the samples (at 7 A/cm^2 for Nb:57% Mo and 0.1–1 A/cm^2 for V_3Ge), and of the magnetization M on the relative magnetic field $\varepsilon_H = (H - H_{c2})/H_{c2}$ at $T = 4.2$ K. a—for Nb—5.7 Mo, b—for V_3Ge. $H_{c2} = 4.03$ and 26.4 kOe respectively. Curve 1—$M(H)$ measured without a screening container, 2—with a container. α'—real part of the dynamic susceptibility shown on Fig. b below [sic!].

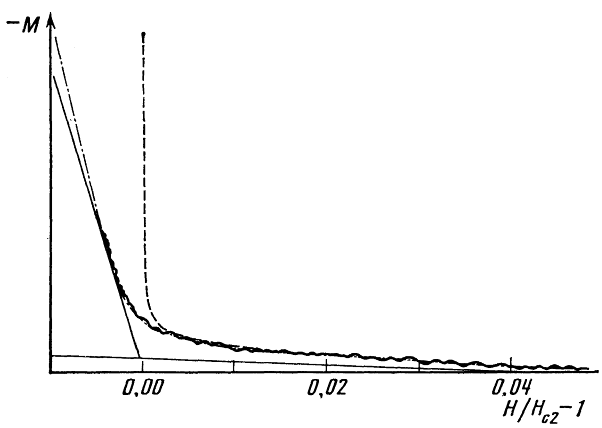

FIG. 2. Dependence of the magnetization on the magnetic field (Nb:5.7% Mo, $T = 4.2$ K): dashed curve—linear approximation, dash-dot—Hartree-Fock approximation with screened potential, solid—self-consistent-field approximation, wavy—experimental dependence.

seen from Ref. 2, the Ginzburg-Landau approximation is valid with sufficient accuracy and the $M(H)$ dependence can be expressed by the equation

$$M=-D\varepsilon_H^{-1/3}, \tag{1}$$

where

$$D=(8\pi^2 k_B T/\Phi_0^2)\varkappa^2\xi(T),$$

k_B is the Boltzmann constant, Φ_0 is the flux quantum, and ξ is the correlation radius. The magnetization M is normalized such that in the mixed state $M=\varepsilon_H/\beta_A$, where β_A is the Abrikosov constant.

The linear approximation does not hold near H_{c2}, and this explains the disparity between the experimental and theoretical relations near H_{c2}. Allowance for the interaction of the fluctuations in the Hartree-Fock approximation with screened potential leads to the following $M(\varepsilon_H)$ dependence:

$$\varepsilon_H=M+\frac{D^2}{M^2}\left[1-2\ln\left(1+\frac{|M|^3}{2D^2}\right)\right], \tag{2}$$

which can be obtained from Eq. (3.9) of Ref. 6 by recognizing that near H_{c2} the magnetization is proportional to the density of the superconducting electrons.[2] The use of the Hartree-Fock approximation with screened potential extends to $H_{\rm cr}$ the region where the theory agrees with experiment (Fig. 2). This fact agrees with the results of investigations of the heat capacity[4] and of the excess electric conductivity in a magnetic field perpendicular to the current,[5] so that the relation obtained in the Hartree-Fock approximation with screened potential[6] is close to that obtained from the exact solution of the one-dimensional model.[4] The foregoing confirms the assumption made by us in Ref. 5 that in the region of H_{c2} above $H_{\rm cr}$ the superconductor is in a state that can be conditionally called one-dimensional.

It can be seen from Eqs. (1) and (2) that the parameter D determines the width of the fluctuation region, which is proportional to $D^{2/3}$. For V_3Ge we have $D=2.4\times10^{-4}$, and for Nb:5.7% Mo we have $D=2.9\times10^{-5}$; we get hence that the width of the fluctuation region is four times larger in V_3Ge than in Nb:5.7% Mo, in agreement with experiment.

Below $\varepsilon_{\rm cr}$, a narrow and high peak is observed on the plot of the critical current $j_c(H)$ (revealed by the appearance of a voltage 0.5 μV across the sample) versus the magnetic field. The usual explanation of peaks of this kind[9] is based on allowance for the loss of long-range order in the vortex structure because of the decrease of its elastic moduli as H_{c2} is approached, and on the ensuing ability of the fluxoids to become matched to the pinning centers. We note in this connection that, as follows from an examination of the influence of the fluctuations on the properties of the mixed state,[10] the fluctuations corrections to the order parameters exceed their values above H_{c2} by a factor $\ln(L/\xi)$, where L is the characteristic dimension of the sample in the direction perpendicular to the magnetic field, or the distance between the pinning centers. [In our case the only possible volume pinning centers are dislocations with density $N\approx5\times10^5$ cm^{-2}, and $\ln(L/\xi)\approx7$.]

The presence of this factor extends the region of fields lower than H_{c2} in which the fluctuations are substantial, compared with the region above H_{c2}, and the peak of $j_c(H)$ turns out to be in those fields where the fluctuations must be taken into account. The results of the measurement of the field dependence of the reluctance to the flux[8] confirm this assumption. In this case the elastic-moduli calculations that do not take the fluctuations into account may turn out to be inapplicable, since it is necessary to allow also for the loss of the long-range order (the decrease of the correlation regions of the vortex lattice) as H_{c2} is approached on account of the fluctuations.

The current-voltage characteristics (CVC) of the samples in fields in which the peak effect is observed are nonlinear in a wide range of currents (they are convex downward). The anomaly in the dependence of the response of the sample to a low-amplitude alternating field appears in the same region (see Fig. 1). When current perpendicular to the magnetic field is made to flow through the sample, the singularities of the response to alternating current and of the $M(H)$ dependence measured without a screening container vanish only at current values corresponding to the linear section of the CVC. At currents exceeding the critical, but corresponding to the nonlinearity region of the CVC, the amplitude of the singularities decreases gradually. The increase of the dynamic susceptibility α' in the region of the singularity means that the alternating field does not penetrate into the sample, i.e., in the employed frequency range the resistance of the vortex lattice is substantially lower than the reluctance to the flux, whereas the differential resistivity $\rho_d=\partial u/\partial I$ is comparable with the normal one.

The reversibility of the magnetization curves in the region of fields corresponding to the peak of $j_c(H)$ at 4.2 K is evidence that the "true" critical current (taken to mean the current at which the voltage vanishes identically) is small. Within the experimental error limit, its upper-bound estimate from the magnetization curves yields for V_3Ge $j_c\leqslant15$ A/cm^2, much lower than the values obtained from the appearance of the 0.5 μV voltage. The presence of a voltage on the CVC at low values of the static current and the effective screening of the alternating magnetic fields signify apparently that a low-velocity flux is produced in the current by the static Lorentz force, and no fast motions are realized. This behavior agrees with the known[11] flux creep which is of activation origin. Measurements at lower temperature (2.37 K) have shown a substantial increase of the critical current and the onset of nonreversibility of the magnetization curves, thus confirming qualitatively this analogy.

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