Electron-phonon interaction and sound propagation in one-dimensional conductors

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We give a derivation from first principles of the interaction Hamiltonian of electrons and long-wavelength phonons in 1D-metals. Due to the almost complete compensation of the usual deformation and Coulomb mechanisms the interaction between electrons and sound turns out to be appreciably weaker than in a 3D-metal. The main role is played by the so-called cross-deformation interaction and the inertial mechanism connected with the Stewart-Tolman effect. The cross-deformation interaction arises as the result of the modulation of the random scattered field of the sound wave and turns out to be off-diagonal in the electron momentum on the Fermi surface. We find the equations of elasticity theory. We propose a new technique for calculating dynamic correlators which enables us automatically to take into account multiple scattering and the electron localization effect. We analyze the frequency dependence of the sound speed and damping under conditions both of weak and of strong spatial dispersion. We predict a geometric-resonance-like effect of oscillations of the absorption, which is caused by the jumplike nature of the electron motion in the inhomogeneous field of the sound wave with a fixed length of the jump. Due to the absence of Landau damping the frequency dependence of the absorption turns out to be quadratic (rather than linear as in 3D metals) in the range of strong spatial dispersion and is determined by the quantum nature of the scattering by individual impurities oscillating with the lattice.

1. INTRODUCTION

The one-dimensional nature of the electron motion leads to a number of anomalies in the physical properties of 1D-conductors. In them the electrons are strongly coupled to the phonons with a momentum equal to $2p_0$ (p_0 is the Fermi momentum) which is the cause of the Peierls instability.[1] The temperature dependence of the conductivity and the electron-phonon interaction near such a structural transition were studied in Refs. 2, 3. Another peculiarity of 1D-metals is the localization of all electron states for arbitrarily weak scattering by impurities.[4] The change in the conductivity with temperature and the effect of phonons on the localization were studied in Refs. 5,6. As to the electron-phonon interaction and the sound propagation in 1D-conductors in the long-wavelength limit, this problem has up to now not been studied either theoretically or experimentally.

Meanwhile, in a one-dimensional system the interaction mechanism between electrons and long-wavelength phonons turns out to be basically different from that in the three-dimensional case. Firstly, the deformation potential at the Fermi surface reduces to a constant and when there is no band overlap it is almost completely screened. For that reason the usual deformation mechanism which plays the main role in a three-dimensional metal does not operate here and the electron-phonon interaction turns out to be appreciably weaker. Secondly, in a 1D-conductor there is no Landau damping as the electrons move along a single axis. Damping of sound therefore occurs only when the electrons collide. Thirdly and finally, a one-dimensional system of electrons is appreciably disordered upon arbitrarily weak scattering by impurities. It is therefore necessary to elucidate the electron-sound interaction mechanism in a disordered system and take into account the electron state localization effect which appreciably affects the spectrum and damping of low-frequency phonons.

The present paper is the first to obtain the electron-phonon interaction Hamiltonian in 1D-conductors, to write down the elasticity theory equations, and to find the frequency dependence of the absorption and speed of long-wavelength sound for weak and for strong spatial dispersion (at $T = 0$).

2. ELECTRON-ION HAMILTONIAN

As a model of a metal with one-dimensional conductivity we take a three-dimensional crystal with a simple lattice in which the ions are positioned in the lattice points $\{\mathbf{R}\}$ and have a density n. The volume of the crystal is V_0, the total number of ions N, the charge of an ion $Z|e|$, and their mass M. We shall assume that the electrons have a one-dimensional dispersion law $\varepsilon(p)$ ($p = p_x$) and can move only along straight-line chains of ions in the x-direction. Let these chains form a quadratic lattice with lattice constant a; we neglect transitions of electrons from one chain to another. The electron density is $n_e = Zn$. Such a model is very close to the structure of compounds like KCP in which the anisotropy of the conductivity reaches 10^5. We neglect the possibility of a Peierls transition.

We write the Hamiltonian of the electrons and ions in the form

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$$H=\sum_{\alpha}\frac{\mathbf{p}_\alpha{}^2}{2m_0}+\sum_{i,\alpha}\Phi(\mathbf{r}_\alpha-\mathbf{R}_i)+\frac{1}{2}\sum_{\alpha\neq\beta}\frac{e^2}{|\mathbf{r}_\alpha-\mathbf{r}_\beta|}$$

$$+\sum_i\frac{\mathbf{P}_i{}^2}{2M}+\frac{1}{2}\sum_{i,j}A(\mathbf{R}_i-\mathbf{R}_j)+\sum_{\alpha,l}V_{imp}(\mathbf{r}_\alpha-\mathbf{R}_l).\quad(2.1)$$

Here $\mathbf{r}_\alpha$ and $\mathbf{p}_\alpha$ are the radius-vector and momentum of electron number α, m_0 is the free electron mass; $\mathbf{R}_i$ and $\mathbf{P}_i$ are the coordinates and momentum of ion number i; $\Phi(\mathbf{r}_\alpha-\mathbf{R}_i)$ is the electron-ion interaction energy (pseudo-potential): $A(\mathbf{R})=g(\mathbf{R})+Z^2e^2/R$ is the ion-ion interaction energy, where $g(\mathbf{R})$ is its short-range part. The last term in (2.1) is the potential for the interaction between electrons and impurities which are randomly positioned in the sites $\mathbf{R}_l$.

The coordinates $\mathbf{R}_i$ give the positions of the ions (and impurities) in the deformed lattice:

$$\mathbf{R}_i=\mathbf{R}+\mathbf{u}(\mathbf{R},\ t),\quad(2.2)$$

where $\mathbf{u}(\mathbf{R},t)$ is the ion displacement field.

We transform in the Hamiltonian (2.1) to new electron coordinates $\mathbf{r}'$:

$$\mathbf{r}=\mathbf{r}'+\mathbf{u}(\mathbf{r}',\ t).\quad(2.3)$$

This transformation corresponds to a change to a coordinate system K' which is connected with the deformed lattice. The need for changing to the system K' is dictated by the fact that just in that system the electron density n_e remains unchanged under the deformation and, hence, the Fermi momentum p_0 is unchanged.

We describe the scheme for obtaining from (2.1) the electron-ion Hamiltonian in the system K'. We perform a canonical transformation of H corresponding to (2.3); we expand all terms in (2.1) in powers of the tensor $\partial u_i/\partial x_k$ up to quadratic terms inclusively; after that we change to the second-quantization representation in the Bloch functions $\Psi_{p'}(\mathbf{r}')$ of the single-electron Hamiltonian

$$\left\{\frac{\mathbf{p}'^2}{2m_0}+\sum_{\mathbf{R}}\Phi(\mathbf{r}'-\mathbf{R})\right\}\Psi_{p'}(\mathbf{r}')=\varepsilon_{p'}\Psi_{p'}(\mathbf{r}').\quad(2.4)$$

The electron-ion Hamiltonian in K' can then be written in the form (we drop the primes everywhere in what follows):

$$H=\sum_p\varepsilon_p a_p{}^+a_p+\frac{Mn}{2}\int d^3x\dot{\mathbf{u}}^2+\frac{1}{2}\int d^3x\gamma_{iklm}\frac{\partial u_i}{\partial x_l}\frac{\partial u_k}{\partial x_m}$$

$$+\sum_{p,\mathbf{q}}\lambda_{ik}(p)\overset{+}{a}_{p+q}a_p u_{ik}(\mathbf{q})+\frac{V_0}{2}\sum_{\mathbf{q}\neq0}\frac{4\pi e^2}{q^2}\rho_{\mathbf{q}}\rho_{-\mathbf{q}}$$

$$-\sum_{p,\mathbf{q}}m_0\mathbf{v}_p\dot{\mathbf{u}}(\mathbf{q})\overset{+}{a}_{p+q/2}a_{p-q/2}+\sum_{\alpha,\mathbf{R}_l}V_{imp}\left(\mathbf{r}_\alpha-\mathbf{R}_l+(x_k-R_{kl})\frac{\partial\mathbf{u}}{\partial x_k}\right).$$

(2.5)

Here a_p^+ and a_p are the creation and annihilation operators for a Bloch electron with quasi-momentum p (in the system K'); in the sum over momenta we include summation over all chains; $\lambda_{ik}(p)$ is the deformation potenial tensor:

$$\lambda_{ik}(p)=\langle p|-\frac{p_ip_k}{m_0}+\sum_{\mathbf{R}}(x_k-R_k)\frac{\partial\Phi(\mathbf{r}-\mathbf{R})}{\partial x_i}|p\rangle,\quad(2.6)$$

which is the same as the corresponding expression in Ref. 7.

The Fourier component of the electron density is

$$\rho_{\mathbf{q}}=\frac{1}{V_0}\sum_\alpha\exp(-i\mathbf{q}\mathbf{r}_\alpha).\quad(2.7)$$

The interaction with the impurities in (2.5) is obtained from (2.1) if we take into account the coordinate transformations (2.2), (2.3), and the short-range nature of the impurity potential. We do not give the explicit expression for the bare elastic moduli γ_{iklm}, as we shall write down the final formula for the adiabatic moduli.

The electron part of the Hamiltonian (2.5), neglecting impurities, corresponds to the usual form of the single-electron dispersion law in the system K':

$$\varepsilon(p,\ \mathbf{r},\ t)=\varepsilon_p+\lambda_{ik}u_{ik}(\mathbf{r},\ t)+e\varphi(\mathbf{r},\ t)-m_0\mathbf{v}\dot{\mathbf{u}}(\mathbf{r},\ t).\quad(2.8)$$

In a 1D-conductor where the Fermi surface consists just of the two points $\pm p_0$ the second and third terms in (2.8) cancel each other up to terms of order $(qa)^2\ll1$, where $\mathbf{q}$ is the phonon wave vector. This cancellation is a consequence of the screening of the term with λ_{ik} in (2.5) due to the Coulomb interaction between the electrons. There is thus no usual deformation interaction in K'. The condition for neglecting small corrections of order $(qa)^2$ from the incomplete cancellation of the deformation interaction is given below.

Hence, the electron-phonon interaction in a 1D-metal is described by the last two terms in (2.5). The first of them is caused by the Stewart-Tolman effect, and the second arises as a result of the scattering of electrons by impurities which are displaced during the lattice vibrations. The energy of these interactions is small compared to the Fermi energy and they are in the three-dimensional case neglected as a rule against the strong deformation interaction. In the one-dimensional case these two mechanisms become the basic ones and thus the interaction between the electrons and the long-wavelength phonons turns out to be appreciably weaker than in the three-dimensional case.

We shall assume that the scattering by impurities is weak so that

$$\varepsilon_F\tau\gg1,\quad(2.9)$$

where τ is the electron free flight time relative to backwards scattering; we neglect forward scattering (the impurity potential is short range). It is then sufficient to take in the Hamiltonian (2.5) into account only electron states with momenta in the vicinity of the points $\pm p_0$.

We introduce the operators

$$\psi_\alpha(x)=\frac{1}{L^{1/2}}\sum_k e^{ikx}a_{\alpha p_0+k},\quad\psi_\alpha{}^+(x)=\frac{1}{L^{1/2}}\sum_k e^{-ikx}\overset{+}{a}_{\alpha p_0+k},\quad(2.10)$$

where L is the length of the chain, the index $\alpha=\pm$, and $|k|\ll p_0$. We write the first term and the term arising from the Stewart-Tolman effect in (2.5) in the form

$$\varepsilon_{p_0}ZN+\int dx\left(\psi^+(x)\sigma_3\left(-iv\frac{\partial}{\partial x}-m_0v\dot{u}_x\right)\psi(x)\right),\quad(2.11)$$

where σ_3 is a Pauli matrix and the round brackets containing the operators ψ^+ and ψ indicate summation over the chains and contraction over the "spinor" indexes α.

We now turn to the impurity potential in (2.5). Scattering by impurities leads not only to damping but also to an appreciable rearrangement of the electron states (localization effect). We write the last term in (2.5) such that we leave in the Hamiltonian only backward scattering and take into account the change in the matrix element of such scattering through the lattice deformation. To do this we first of all find the correlation function of the impurity potential

$$D(\mathbf{r},\ \mathbf{r}_1)=\langle H_{imp}(\mathbf{r})H_{imp}(\mathbf{r}_1)\rangle, \tag{2.12}$$

where $H_{\rm imp}(\mathbf{r})$ is the last term in (2.5) without the sum over α and angular brackets indicate averaging over the impurity positions. In the approximation which is linear in u_{ik}

$$D(\mathbf{r},\mathbf{r}_1)=\frac{n_{imp}}{V_0}\sum_{\varkappa}[V_\varkappa(\mathbf{r})-V_\varkappa{}^{ik}(\mathbf{r})u_{ik}(\mathbf{r},t)]$$

$$\times[V_\varkappa(\mathbf{r}_1)-V_\varkappa{}^{lm}(\mathbf{r}_1)u_{lm}(\mathbf{r}_1,t_1)]^*, \tag{2.13}$$

where the summation over the momentum transferred $\varkappa$ extends over the whole of the first Brillouin zone; $n_{\rm imp}$ is the volume density of the impurities,

$$V_\varkappa(\mathbf{r})=\sum_{\mathbf{B}}V_{imp}(\varkappa+\mathbf{B})e^{i(\varkappa+\mathbf{B})\mathbf{r}},\quad V_{imp}(\varkappa)=\int d^3r\,V_{imp}(\mathbf{r})e^{-i\varkappa\mathbf{r}},$$

$$V_\varkappa{}^{lm}(\mathbf{r})=\sum_{\mathbf{B}}\frac{\partial}{\partial\varkappa_m}[(\varkappa+\mathbf{B})_lV_{imp}(\varkappa+\mathbf{B})]e^{i(\varkappa+\mathbf{B})\mathbf{r}}, \tag{2.14}$$

where $\mathbf{B}$ is the reciprocal lattice vector. In the diagram technique the function $D(\mathbf{r},\mathbf{r}_1)$ corresponds to an impurity line describing the scattering of two electrons due to the effective pair interaction through impurities. In order to take only backward scattering into account it is sufficient to retain in $D(\mathbf{r},\mathbf{r}_1)$ terms with $\varkappa_x=\pm2p_0+k_1$, $|k_1|\ll p_0$.

We now introduce in the Hamiltonian, instead of the field of the randomly positioned impurities, an equivalent random complex field $\zeta(x)$ that leads to the same spatial correlations as the original impurity potential (cf. Ref. 6). In terms of $\zeta(x)$ the last term in (2.5) will correspond to two terms:

$$\sum_{\alpha,\mathbf{R}_l}V_{imp}\left(\mathbf{r}_\alpha-\mathbf{R}_l+(x_k-R_{lk})\frac{\partial\mathbf{u}}{\partial x_k}\right)\to\frac{1}{\tau}\int dx[\zeta^+(x)(1-\Delta_{ik}{}^*u_{ik})$$

$$\times(\psi^+(x)\hat{a}\psi(x))+\zeta(x)(1-\Delta_{ik}u_{ik})(\psi^+(x)\hat{a}^+\psi(x))]. \tag{2.15}$$

The 2×2 matrices here are

$$\hat{a}=\begin{pmatrix}0&0\\1&0\end{pmatrix},\quad\hat{a}^+=\begin{pmatrix}0&1\\0&0\end{pmatrix},$$

and the probability for backward scattering per unit time is

$$\frac{1}{\tau}=\frac{n_{imp}}{va^2}|U_{-p_0}(2p_0)|^2,$$

$$U_{-p_0}(2p_0)=\sum_{\mathbf{B}}V_{imp}(\varkappa+\mathbf{B})M_{-p_0}(\varkappa_x+B_x)|_{\varkappa_x=2p_0r} \tag{2.16}$$

$$M_p(\varkappa)=\int dx\,\Psi^*_{p+\varkappa}(x)e^{i\varkappa x}\Psi_p(x).$$

The dimensionless complex tensor Δ_{ik} is analogous to the deformation potential tensor and arises due to the interaction of an electron with an impurity in the deformed crystal in exactly the same way as the tensor λ_{ik} arises when there is interaction with ions. It is equal to

$$\Delta_{ik}=\frac{V_{-p_0}^{ik}(2p_0)}{U_{-p_0}(2p_0)}=\delta_{ik}+U_{-p_0}^{-1}(2p_0)$$

$$\times\sum_{\mathbf{B}}B_i\frac{\partial V_{imp}(\varkappa+\mathbf{B})}{\partial B_k}M_{-p_0}(\varkappa_x+B_x)|_{\varkappa_x=2p_0}. \tag{2.17}$$

We shall call the tensor Δ_{ik} the "cross-deformation" potential emphasizing thereby that (2.15) is off-diagonal in the "spinor" indexes α. The second term in (2.17) has the scale of the product of m_0v and the range of the impurity potential.

Later $\zeta(x)$ is Gaussian and δ-correlated:

$$\langle\zeta(x)\zeta^+(x_1)\rangle=l\delta(x-x_1),$$

$$\langle\zeta(x)\zeta(x_1)\rangle=\langle\zeta^+(x)\zeta^+(x_1)\rangle=0,\ l=v\tau. \tag{2.18}$$

Using the methods of functional integration, expanded in Ref. 13, one can derive Eqs. (2.15) to (2.18) with mathematical rigor.

The Hamiltonian of the electron-ion system taking into acount that part of the impurity potential which describes backwards scattering, is therefore in the long-wavelength limit of the form $H=H_0+H_{e-ph}$,

$$H_0=\int dx\left(\psi^+(x)\left[-iv\sigma_3\frac{\partial}{\partial x}+\frac{1}{\tau}(\zeta^+(x)\hat{a}+\zeta(x)\hat{a}^+)\right]\psi(x)\right)$$

$$+\frac{1}{2}\int d^3r(Mn\dot{\mathbf{u}}^2+K_{iklm}u_{ik}u_{lm}). \tag{2.19}$$

The electron-phonon interaction operator in a 1D-metal is

$$H_{e-ph}=-\int dx\left(\psi^+(x)\left[\frac{1}{\tau}(\zeta^+(x)\hat{a}\Delta_{ik}{}^*\right.\right.$$

$$\left.\left.+\zeta(x)\hat{a}^+\Delta_{ik})u_{ik}+m_0v\sigma_3\dot{u}_x\right]\psi(x)\right). \tag{2.20}$$

The first term in the square brackets in (2.20), arising as a result of the modulation of the random scattering field of a sound wave, will be called the "cross-deformation" interaction, and the second term the inertial or Stewart-Tolman one.

The adiabatic elastic moduli when there is no scattering K_{iklm} in (2.19) can be obtained from the general formulae of Ref. 7 by taking the limit to a one-dimensional electron dispersion law:

$$K_{iklm}=\lambda_{iklm}-m_0v^2\nu(\varepsilon_F)[\delta_{ix}\delta_{kx}\delta_{lx}\delta_{mx}m_0v^2+\delta_{ix}\delta_{kx}\lambda_{lm}+\delta_{lx}\delta_{mx}\lambda_{ik}],$$

$$\nu(\varepsilon_F)=(\pi va^2)^{-1}, \tag{2.21}$$

where λ_{iklm} includes contributions from the lattice and the interband electron transition (for details see Ref. 7). The anisotropy of the electron contribution to K_{iklm} is clear from (2.21).

Formula (2.20) proves the foregoing statement that the interaction between the electrons and long-wavelength phonons turns out to be weak in a 1D-conductor and is caused by the cross-deformation mechanism and the Stewart-Tolman effect. As compared to the usual deformation interaction which is characterized by the quantity λ_{ik}

(of the order of the Fermi energy ε_F), the first mechanism is weaker by a factor $\varepsilon_F\tau$ and the second one has a relative smallness of order s/v, where s is the sound speed. As both interactions are characterized by different constants they can compete with one another. Moreover, these interactions have a different symmetry under the change $v \to -v$ and a different matrix structure.

We give a criterion for dropping, as was done above, terms of the incomplete cancellation of the usual deformation interaction [of order $\varepsilon_F(qa)^2$] as being small compared to (2.20). It is shown in section 5 that the condition $(qa)^2 \ll 1$ is sufficient for the energy of the cross-deformation interaction to dominate over the "unscreened" deformation potential. The latter can be neglected also in comparison with the inertial mechanism, if $(qa)^4 \ll m/M$. On the other hand, the relative role of the cross-deformation interaction and the Stewart-Tolman effect is determined by the ratio of m/M to $(p_0 l)^{-1}$.

One can assume that the electron relaxation time for scattering by long-wavelength phonons which determines the low-temperature conductivity will be determined by the Hamiltonian (2.20) with a small effective electron-phonon coupling constant.

3. ELASTICITY THEORY EQUATIONS

The equations of motion for the lattice are obtained from the Hamiltonian (2.19), (2.20) using the usual formulae of mechanics. To do this we must, strictly speaking, replace in $h\,\dot{\mathbf{u}}$ by $\mathbf{P}/M$ and formulate next the Hamiltonian equations of motion. As there is in (2.20) a term which is linear in the lattice momentum (from the Stewart-Tolman effect), the connection between the velocity and the momentum is given by the formula

$$M\dot{\mathbf{u}}=\mathbf{P}-m_0\mathbf{v}\,\mathrm{Sp}(\psi^+\sigma_3\psi)=\mathbf{P}-(m_0/e)\,\mathbf{j}, \tag{3.1}$$

where Sp includes averaging over the random field $\zeta(x)$, the trace over the spinor index, and thermodynamic averaging, $\mathbf{j}=\mathbf{j}(\mathbf{r},t)$ is the electric current density ($\mathbf{j}\,||\,x$). The force $\mathbf{F}$ in the elasticity equations $\dot{\mathbf{P}}=\mathbf{F}$ is found from the formula $\mathbf{F}=-\delta H/\delta\mathbf{u}$ and as a result:

$$Mn\ddot{u}_i=K_{iklm}\frac{\partial^2 u_m}{\partial x_k\,\partial x_l}-\frac{m_0}{e}\frac{\partial j_i}{\partial t}-\frac{1}{a^2}\frac{\partial T_{ik}}{\partial x_k}, \tag{3.2}$$

$$\begin{gathered} T_{ik}(r,t)=\mathrm{Sp}(\psi^+(x)\,t_{ik}(x)\,\psi(x)), \\ t_{ik}(x)=\frac{1}{\tau}\Big[\zeta(x)\,\hat{a}^+\Delta_{ik}+\zeta^+(x)\,\hat{a}\Delta_{ik}^{*}\Big]\,. \end{gathered} \tag{3.3}$$

The current density $\mathbf{j}$ and the strain tensor T_{ik} can be found using linear response theory:[8]

$$j_x(q,\omega)=-\frac{m_0}{e}\sigma(q,\omega)\,\ddot{u}_x-\langle jT_{lm}\rangle_{q\omega}u_{lm}, \tag{3.4}$$

$$T_{ik}(q,\omega)=-\frac{m_0}{e}\langle T_{ik}j\rangle_{q\omega}\dot{u}_x-\langle T_{ik}T_{lm}\rangle_{q\omega}u_{lm}. \tag{3.5}$$

Here $\sigma(q,\omega)$ is the conductivity, the angle brackets $\langle AB\rangle_{q\omega}$ indicate the Fourier components of retarded correlators of the form

$$\langle AB\rangle_{q\omega}=-i\int_0^\infty dt\,e^{i\omega t}\int_{-L/2}^{L/2}dx\,e^{-iqx}\,\mathrm{Tr}\,\rho_0[\hat{A}(x,t),\hat{B}], \tag{3.6}$$

where ρ_0 is the equilibrium density matrix with Hamiltonian H_0 of (2.19), $\hat{A}(x,t)$ is an operator in the Heisenberg representation, Tr indicates the trace over the electron quantum states for a fixed realization of the field ζ and subsequent averaging over realizations.

Substituting (3.4) and (3.5) into Eq. (3.2) we get the elasticity equation for a 1D-metal:

$$\begin{aligned} &Mn\omega^2u_i \\ &=\Big[K_{iklm}+\frac{1}{a^2}\langle T_{ik}T_{lm}\rangle_{q\omega}\Big]q_kq_lu_m-i\omega^3\left(\frac{m_0}{e}\right)^2\sigma(q,\omega)\,u_x\delta_{ix}\cdot \\ &\qquad-\frac{m_0\omega}{a^2e}[\delta_{ix}\langle jT_{lm}\rangle_{q\omega}q_lu_m+\langle T_{ik}j\rangle_{q\omega}q_ku_x]. \end{aligned} \tag{3.7}$$

The problem has thus been reduced to evaluating correlators of the type (3.6) with the operators $\hat{T}_{ik}$ and $\hat{\mathbf{j}}$ taking into account the rearrangement of the electron states in the scattering field $\zeta(x)$.

4. TECHNIQUE FOR EVALUATING THE CORRELATORS

In the present section we give a new technique for evaluating the correlators occurring in Eq. (3.7). Well known methods[9,6,10] of averaging over impurity configurations in 1D-conductors which up to now have been applied to evaluate only current-current and density-density correlators are inconvenient for the calculation of the quantities we are interested in because of the tensor nature, the complexity, the different matrix structure, and the dependence of the vertex operators $\hat{T}_{ik}$ on the random field $\zeta(x)$. The method proposed below differs, on the one hand, by the high degree of automatism and, on the other hand, enables us to evaluate correlators of any order or kind, and not only binary ones.

One shows easily that for the binary retarded correlator (3.6) the expression

$$\begin{aligned} \langle AB\rangle_{q\omega}=&\int\frac{d\varepsilon}{2\pi i}\frac{\partial f_0}{\partial\varepsilon}\,\mathrm{Sp}\,\mathscr{A}\mathscr{B}[G^a(\varepsilon)-G^r(\varepsilon)] \\ &+\frac{i\omega}{2\pi}\int d\varepsilon\,\frac{\partial f_0}{\partial\varepsilon}\,\mathrm{Sp}\,\mathscr{A}e^{-iq\hat{x}}G^r(\varepsilon+\omega)\,e^{iq\hat{x}}\mathscr{B}[G^a(\varepsilon)-G^r(\varepsilon)], \end{aligned} \tag{4.1}$$

is valid, where $G^a(\varepsilon)$ and $G^r(\varepsilon)$ are the advanced and retarded Green functions, $\mathscr{A}$ and $\mathscr{B}$ are single-electron operators corresponding to the quantities A and B, and $f_0(\varepsilon)$ is the Fermi function. The operator corresponding, for instance, to $G^r(z)$ is, according to (2.19) equal to

$$\begin{gathered} G^r(z)=\Big[z+iv\sigma_3\frac{\partial}{\partial x}-\frac{1}{\tau}(\zeta^+(x)\,\hat{a}+\zeta(x)\,\hat{a}^+)\Big]^{-1}, \quad \mathrm{Im}\,z>0, \\ \times\exp(-iq\hat{x})\,G^r(z)\,\exp(iq\hat{x})=G^r(z-qv\sigma_3). \end{gathered} \tag{4.2}$$

Using the technique for expanding in an operator base for two-level systems[11] we can obtain the following representation for the retarded and advanced single-electron Green functions:

$$\langle x|G^r(z+\omega-qv\sigma_3)|y\rangle$$

$$=-\frac{1}{v}[1+\Gamma_+^\omega(y)\Gamma_-^\omega(y)]^{-1}\begin{pmatrix} G_2^{xy} & G_4^{xy} \\ G_3^{xy} & G_1^{xy}\end{pmatrix}, \qquad (4.3)$$

$$G_1^{xy}=ie^{-ip_+(x-y)}\left[-\theta_+\frac{\gamma_+^\omega(x)}{\pi_+^\omega(y)}\Gamma_-^\omega(y)e^{ip_\omega(x-y)}+\theta_-\frac{\pi_-^\omega(x)}{\pi_-^\omega(y)}\right],$$

$$G_2^{xy}=ie^{ip_-(x-y)}\left[\theta_+\frac{\pi_+^\omega(x)}{\pi_+^\omega(y)}-\theta_-\frac{\gamma_-^\omega(x)}{\pi_-^\omega(y)}\Gamma_+^\omega(y)e^{ip_\omega(y-x)}\right],$$

$$G_3^{xy}=e^{-i(p_+x+p_-y)}\left[\theta_+\frac{\gamma_+^\omega(x)}{\pi_+^\omega(y)}e^{ip_\omega x}+\theta_-\frac{\pi_-^\omega(x)}{\pi_-^\omega(y)}\Gamma_+^\omega(y)e^{ip_\omega y}\right],$$

$$(4.4)$$

$$G_4^{xy}=e^{i(p_-x+p_+y)}\left[\theta_+\frac{\pi_+^\omega(x)}{\pi_+^\omega(y)}\Gamma_-^\omega(y)e^{-ip_\omega y}+\theta_-\frac{\gamma_-^\omega(x)}{\pi_-^\omega(y)}e^{-ip_\omega x}\right],$$

$$\langle y|G^a(z)|x\rangle=[\langle x|G^r(z)|y\rangle]^+, \qquad \Gamma_\pm^\omega(x)=\frac{\gamma_\pm^\omega(x)}{\pi_\pm^\omega(x)}.$$

We have here introduced the notation:

$$p_\omega=\frac{2\omega}{v}, \qquad p_\pm=\frac{z+\omega}{v}\pm q, \qquad \theta_\pm=\theta[\pm(x-y)],$$

$\theta(x)$ is the unit step function.

The functionals of the random field $\zeta(x)$ occurring in (4.3) and (4.4) satisfy the following equations:

$$\frac{d\Gamma_+^\omega(x)}{dx}=-2i\frac{\omega}{v}\Gamma_+^\omega(x)-\frac{1}{l}\zeta^+(x)+\frac{1}{l}\zeta(x)[\Gamma_+^\omega(x)]^2,$$

$$\frac{d\Gamma_-^\omega(x)}{dx}=2i\frac{\omega}{v}\Gamma_-^\omega(x)+\frac{1}{l}\zeta(x)-\frac{1}{l}\zeta^+(x)[\Gamma_-^\omega(x)]^2,$$

$$\frac{d}{dx}\frac{1}{\pi_+^\omega(x)}=\frac{1}{l}\frac{\zeta(x)}{\pi_+^\omega(x)}\Gamma_+^\omega(x),$$

$$\frac{d}{dx}\frac{1}{\pi_-^\omega(x)}=-\frac{1}{l}\frac{\zeta^+(x)}{\pi_-^\omega(x)}\Gamma_-^\omega(x); \qquad (4.5)$$

$$\gamma_+^\omega(x)=\frac{1}{l}\int_x^\infty dy e^{ip_\omega(y-x)}\zeta^+(y)\pi_+^\omega(y),$$

$$\gamma_-^\omega(x)=\frac{1}{l}\int_{-\infty} dy e^{ip_\omega(x-y)}\zeta(y)\pi_-^\omega(y).$$

The boundary conditions for these equations are the following:

$$\Gamma_+^\omega(x)\to 0,\ \pi_+^\omega(x)\to 1 \text{ as } x\to+\infty;$$

$$\Gamma_-^\omega(x)\to 0,\ \pi_-^\omega(x)\to 1 \text{ as } x\to-\infty.$$

All these equations have a structure of the form

$$\frac{dw_i}{dx}=\varphi_i(w)+b_{ik}(w)f_k(x), \qquad f_k(x)=(\zeta(x),\zeta^+(x)), \qquad (4.6)$$

where summation over repeated indexes is understood. The functionals with an index + depend only on values of the random field $\zeta(x')$ for $x'>x$ and functionals with index − are determined by values of $\zeta(x')$ for $x'<x$. The representation (4.4) and Eqs. (4.5) are valid for any statistical property of the field $\zeta(x)$.

If we now substitute Eqs. (4.3), (4.4) for the Green functions into (4.1) and then expand denominators of the kind $(1+\Gamma_+^\omega\Gamma_-^\omega)^{-1}$ in a power series in $\Gamma_+^\omega\Gamma_-^\omega$, we can express the correlator $\langle AB\rangle_{q\omega}$ in terms of a sum of average values of pair products of some functionals R_+ with index + and functionals R_- with index −. These products under the sign for averaging over the realizations of the field $\zeta(x)$ are constructed from blocks of $\pi_\pm^\omega(x)$, $\Gamma_\pm^\omega(x)$, and $\gamma_\pm^\omega(x)$, and the same blocks with $\omega=0$. Each of such products splits on averaging into a product of average values of factors, as the functionals R_+ and R_- are statistically independent due to the fact that the field $\zeta(x)$ is δ-correlated. On the other hand, averages of functionals of one kind (R_+ or R_-) can easily be calculated using recurrence relations which in the general form can be written in the form:

$$\left\langle \varphi_i\frac{\partial R_\pm}{\partial w_i}\right\rangle\mp\frac{1}{2}D_{kl}\left\langle b_{jl}\frac{\partial}{\partial w_j}\left(b_{ik}\frac{\partial R_\pm}{\partial w_i}\right)\right\rangle=0, \qquad (4.7)$$

where the corresponding block w_i satisfies one of Eqs. (4.5) while the quantities D_{kl} are taken from the relations

$$\langle f_k(x)f_l(y)\rangle=D_{kl}\delta(x-y).$$

The technique described which will be applied to the calculation of the conductivity of a 1D-metal at once leads to Berezinskiĭ's equations.[9] For the correlators in which we are interested we obtain the following results.

The cross correlators are connected through symmetry relations:

$$\langle jT_{ik}\rangle_{q\omega}=-\langle T_{ik}j\rangle_{-q\omega}. \qquad (4.8)$$

Because of this connection the two last terms in Eq. (3.7) cancel one another, if the vector **q** is directed along or at right angles to the x-axis. This means that the two basic mechanisms for the electron-phonon interaction—the cross-deformation and the inertial mechanisms—do not interfere in the elasticity equations, i.e., they act independently. This conclusion is the consequence of the different symmetry of the corresponding terms in the Hamiltonian (2.20) under the substitution $v\to-v$.

The correlator $\langle T_{ik}T_{lm}\rangle_{q\omega}$ is equal to:

$$\langle T_{ik}T_{lm}\rangle_{q\omega}=-\frac{\Delta_{ik}\Delta_{lm}^*+\Delta_{ik}^*\Delta_{lm}}{4\pi a\tau}\left(1+i\frac{\omega a}{v}\right)$$

$$+\frac{\omega}{8\pi il}(\Delta_{ik}-\Delta_{ik}^*)(\Delta_{lm}-\Delta_{lm}^*)[S(\omega,q)+S(\omega,-q)]. \qquad (4.9)$$

The function $S(\omega,q)$ is expressed through the integral:

$$S(\omega,q)=\frac{1}{\beta}e^\beta\int_\beta^\infty d\xi e^{-\xi}\xi(\xi-\beta)y(\xi), \qquad \beta=-2i\omega\tau, \qquad (4.10)$$

where $y(\xi)$ is the solution of the equation

$$[\hat{\nu}-i(\omega-qv)\tau]y(\xi)=\beta f(\xi)+2(\xi-\beta)\frac{d}{d\xi}(\xi f(\xi)), \qquad (4.11)$$

$$f(\xi)=-e^\xi\,\mathrm{Ei}(-\xi),$$

$$\hat{\nu}=-\frac{d}{d\xi}\left(\xi^2\frac{d}{d\xi}\right)+\xi\frac{d}{d\xi}\xi+\beta\frac{d}{d\xi}\xi\left(\frac{d}{d\xi}-1\right) \qquad (4.12)$$

and satisfies the boundary conditions of finiteness at $\xi=\beta$ and $y(\zeta)\to 0$ as $\xi\to+\infty$. The differential operator φ can be treated as the operator of the dimensionless collision frequency as the left-hand side of (4.11) recalls the linearized kinetic equation.[12]

The first term in the correlator (4.9) is independent of the localization effect and can be obtained in the Born approximation in the cross-deformation interaction. However, the second term in (4.9), containing $S(\omega,q)$ and proportional to $\mathrm{Im}\Delta_{ik}\,\mathrm{Im}\Delta_{lm}$, is caused by the effect of localization as the effect of multiple scattering by impurities leads to the characteristic Eq. (4.11).

We calculate $S(\omega,q)$. Firstly we consider the high-frequency region, $\omega\tau \gg 1$. According to (4.10) in that case we must find $y(\xi)$ for large ξ. It follows from (4.11) that here $y(\xi) = A\xi^{-1}$ and the integral in (4.10) equals

$$S(\omega,\ q) = y(\beta) = [1 - i(\omega - qv)\tau]^{-1}. \tag{4.13}$$

In the limit the operator $\hat{v}$ reduces to multiplication by unity.

We now turn to the low-frequency limiting case, $\omega\tau \ll 1$. We can look for the solution of (4.11) in the form of a power expansion in β. In the main approximation we put $\beta = 0$ in Eqs. (4.11) and (4.12) and using the Green function of the equation obtained

$$G(\xi,\xi') = \frac{1}{2\pi^2}\int_0^\infty d\mu\,\frac{\mu\,\mathrm{sh}(\pi\mu/2)}{\nu(\mu)+iql}$$
$$\times \exp\left(\frac{\xi-\xi'}{2}\right)(\xi\xi')^{-1/2}K_{i\mu/2}\left(\frac{\xi}{2}\right)K_{i\mu/2}\left(\frac{\xi'}{2}\right) \tag{4.14}$$

we easily find the solution in the form:

$$y_0(\xi) = \frac{1}{2}\left(\frac{\pi}{\xi}\right)^{1/2} e^{\xi/2}\int_0^\infty d\mu\,\frac{\mu\,\mathrm{sh}(\pi\mu/2)}{\mathrm{ch}^2(\pi\mu/2)}\,\frac{\nu^2(\mu)}{\nu(\mu)+iql}\,K_{i\mu/2}\left(\frac{\xi}{2}\right). \tag{4.15}$$

Here $\nu(\mu) = (1+\mu^2)/4$ is the dimensionless collision frequency which depends on the quantum number μ over whose fluctuations we must average in (4.15) (details in Ref. 12).

Substituting (4.15) into (4.10) we get the main term in the function $S(\omega,q)$:

$$S_0(\omega,q) = \frac{\pi^2}{2\beta}\int_0^\infty d\mu\,\frac{\mu\,\mathrm{sh}(\pi\mu/2)}{\mathrm{ch}^3(\pi\mu/2)}\,\frac{\nu^3(\mu)\,(1+\nu(\mu)/2)}{\nu(\mu)+iql}. \tag{4.16}$$

The first order correction $y_1(\xi)$ to (4.15) is given by the equation

$$-\frac{d}{d\xi}\left(\xi^2\frac{dy_1}{d\xi}\right) + \xi\frac{d}{d\xi}(\xi y_1) + iqly_1 = \beta g(\xi), \tag{4.17}$$

$$g(\xi) = -\frac{1}{2}y_0(\xi) - \frac{d}{d\xi}\left(\xi\frac{dy_0}{d\xi} - \xi y_0\right) + f(\xi) - 2\frac{d}{d\xi}(\xi f(\xi)) \tag{4.18}$$

with boundary conditions of decreasing at infinity and $y_1 \approx 0$ at $\xi = \beta$. Using (4.14) we get the solution of (4.17) in the form

$$y_1(\xi) = B(x)(\pi\xi)^{-1/2}\exp\left(\frac{\xi}{2}\right)K_{\eta/2}\left(\frac{\xi}{2}\right)$$
$$+\beta\int_\beta^\infty d\xi'\,G(\xi,\xi')\,g(\xi'), \tag{4.19}$$
$$\eta = (1+4iql)^{1/2},\ \mathrm{Re}\,\eta > 0.$$

Analysis shows that we can neglect the first term in (4.19) and as a result we easily get for the correction term to $S_0(\omega,q)$:

$$S_1(\omega,q) = \frac{11}{24},\qquad ql \ll \frac{1}{2|\ln\omega\tau|}, \tag{4.20}$$

$$S_1(\omega,q)$$
$$= \frac{\pi^2}{4}\int_0^\infty d\mu\,\frac{\mu\,\mathrm{sh}(\pi\mu/2)}{\mathrm{ch}^3(\pi\mu/2)}\left\{\frac{\nu(\mu)\,(\nu^3(\mu)+2\nu^2(\mu)+3\nu(\mu)-2)}{\nu(\mu)+iql}\right.$$
$$\left. - \frac{\nu^3(\mu)\,(\nu(\mu)+2)}{2(\nu(\mu)+iql)^2}\right\}. \tag{4.21}$$

Formula (4.21) is valid for $2ql\,|\ln\omega\tau| \gg 1$. In contrast to the conductivity $\sigma(\omega,q)$ the expansion of the correlator $S(\omega,q)$ for $\omega\tau \ll 1$ does not contain powers of $\ln\omega\tau$ even when $q = 0$.

5. DAMPING AND DISPERSION OF SOUND

The characteristic feature of 1D-conductors is that in them there is no collisionless Landau damping mechanism which is normally connected with the spread in electron velocities in magnitude and direction. All absorption mechanisms therefore are collisional in nature.

The general Eqs. (3.7) and (4.9) enable us to study the propagation of long-wavelength sound of arbitrary polarization and in any direction in a 1D-metal. We restrict ourselves here to longitudinal sound with a wave vector $\mathbf{q}$ along the direction of high conductivity. We introduce the adiabatic longitudinal sound speed s: $K_{xxxx} = Mns^2$. As the dispersion $s(\omega)$ of the speed is small as is the relative damping Γ of the sound we can easily write down from (3.7) the following formulae for them $[\Delta s = s(\omega) - s]$:

$$\frac{\Delta s}{s} = -Z\left(\frac{mv^2}{2Ms^2}\right)\left\{\left(\frac{m_0 s}{mv}\right)^2(\omega\tau)^2\alpha(\omega,\mathrm{q})\right.$$
$$\left. + \frac{1}{2(p_0 l)}\left[\frac{|\Delta|^2}{p_0 a} + \frac{\omega}{2\varepsilon_F}\Delta_2^2\,\mathrm{Im}\,S_+(\omega,q)\right]\right\}, \tag{5.1}$$

$$\Gamma = (\omega\tau)Z\left(\frac{mv^2}{2Ms^2}\right)$$
$$\times\left\{\left(\frac{m_0 s}{mv}\right)^2\mathrm{Re}\,\frac{\sigma(\omega,q)}{\sigma_0} + \frac{1}{2(p_0 l)^2}[|\Delta|^2 - \Delta_2^2\,\mathrm{Re}\,S_+(\omega,q)]\right\}. \tag{5.2}$$

Here $\alpha(\omega,q) = -\mathrm{Im}\sigma(\omega,q)/\omega\tau\sigma_0$ is a quantity proportional to the polarizability of a 1D-conductor; $\sigma_0 = n_e e^2\tau/m$; $\Delta_{xx} = \Delta = \Delta_1 + i\Delta_2$, $S_+(\omega,q) = S(\omega,q) + S(\omega,-q)$, $m = p_0/v$ is the effective electron mass. On the right-hand side of (5.1) and (5.2) we must replace the wave number q by ω/s. Taking spatial dispersion into account, the conductivity $\sigma(\omega,q)$ was found in Ref. 12. We analyze the frequency dependence of the sound speed and damping in different regions. We shall then neglect the first term in the square brackets in (5.1) which is independent of the frequency and is a relatively small collisional correction to the adiabatic elasticity moduli.

Low frequencies, $\omega\tau \ll 1$. There are here several regions depending on the magnitude of the spatial dispersion parameter $\varkappa = ql = \omega l/s$.

1. When $\omega\tau|\ln\omega\tau| < s/v$ one can put $q = 0$ in Eqs. (5.1) and (5.2) and we get for the speed and the damping

$$\frac{\Delta s}{s}=-Z\left(\frac{mv^2}{2Ms^2}\right)\left\{\left(\frac{m_0 s}{mv}\right)^2 4\zeta(3)(\omega\tau)^2+\frac{11}{48}\left(\frac{\Delta_2}{p_0 l}\right)^2\right\}, \tag{5.3}$$

$$\Gamma=(\omega\tau)Z\left(\frac{mv^2}{2Ms^2}\right)\times\left\{\left(2\frac{m_0 s}{mv}\omega\tau\ln\omega\tau\right)^2+\frac{1}{2(p_0 l)^2}\left(|\Delta|^2-\frac{11}{12}\Delta_2^2\right)\right\}. \tag{5.4}$$

2. In this region where $\omega\tau|\ln\omega\tau|>s/v$, but $\omega\tau<s/v$ the change in the velocity Δs is described by the same Eq. (5.3), but the damping Γ is given by the expression

$$\Gamma=(\omega\tau)Z\left(\frac{mv^2}{2Ms^2}\right)\times\left\{2\left(\frac{m_0 s^2}{mv^2}\right)^2\left[1-\left(\frac{\omega\tau}{2}\right)^{2\varkappa^2}\cos\left(2\frac{v}{s}\omega\tau\ln\frac{\omega\tau}{2}\right)\right]+\frac{1}{2(p_0 l)^2}\left(|\Delta|^2+\frac{1}{4}\Delta_2^2\right)\right\}. \tag{5.5}$$

In this region the damping due to the Stewart-Tolman effect turns out to be an oscillating function of the frequency with a slowly varying period $\pi s/v\tau|\ln(\omega\tau/2)|$. The oscillations are caused by geometric resonance in $\mathrm{Re}\sigma(\omega,q)$ in the field of the inhomogeneous sound wave which arises due to the jumplike nature of the conductivity with a fixed jump length $2l\,|\ln(\omega\tau/2)|$.[12] Similar oscillations exist also in the sound speed dispersion but they have a small amplitude ($\sim s/v$) relative to the first term in (5.3).

3. In the region of strong spatial dispersion, where $\omega\tau>s/v$, but $\omega\tau\ll 1$ we have

$$\frac{\Delta s}{s}=-Z\frac{m}{2M}\left[\left(\frac{m_0 s}{mv}\right)^2+\frac{17}{120}\left(\frac{\Delta_2}{p_0 l\omega\tau}\right)^2\right], \tag{5.6}$$

$$\Gamma=(\omega\tau)Z\left(\frac{mv^2}{Ms^2}\right)\left[\left(\frac{m_0 s^3}{mv^3\omega\tau}\right)^2+\left(\frac{|\Delta|}{2p_0 l}\right)^2\right]. \tag{5.7}$$

High frequencies, $\omega\tau\gg 1$. Here the sound absorption is given by the same Eq. (5.7), but the velocity dispersion is given by the formula

$$\frac{\Delta s}{s}=-Z\frac{m}{2M}\left[\left(\frac{m_0 s}{mv}\right)^2-\left(\frac{\Delta_2}{p_0 l}\right)^2\right]. \tag{5.8}$$

In the figure we show the frequency dependence of the relative damping Γ in the case of most interest, when the parameter

$$\alpha=(2m_0/|\Delta|m)(s/v)^2 p_0 l>1. \tag{5.9}$$

The quantity α characterizes the relative role of the Stewart-Tolman effect in comparison to the cross-deformation interaction: if $\alpha>1$ these two mechanisms compete with one another.

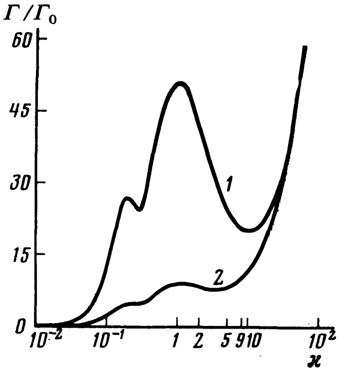

FIG. 1. Calculated frequency dependence of the relative damping of longitudinal sound in a 1D-conductor. The quantity Γ in units $\Gamma_0=Z(mv/Ms)(|\Delta|/2p_0 l)^2$; $\Delta_2=0$; along the abscissa axis we plot $\varkappa=\omega l/s$; for curve 1—$\alpha=10$, $v/s=10^3$; for curve 2—$\alpha=4$, $v/s=0.5\times10^3$.

The region 1 of very low frequencies where the quantity Γ depends linearly on ω and where the role of the inertial mechanism is negligibly small cannot be seen in Fig. 1. This is connected with the fact that for the chosen values of the parameter α the region 1 corresponds to $\omega l/s\ll 10^{-2}$. For higher frequencies up to $\omega l/s\sim\alpha$ the main contribution to the sound damping comes from the inertial mechanism. Here Γ initially increases proportional to $\omega^3\ln^2\omega\tau$ and after that one oscillation of Γ in the region $0.15<\omega l/s<0.25$ is clearly visible. The fact that no further periods manifest themselves is explained by the fact that for $s/v\sim 10^{-3}$ already in the second period $2|\ln(\omega\tau/2)|\approx 20$ and the parameter $\omega l/s$ approaches unity and the amplitude of the oscillating term in (5.5) becomes very small, $\sim 10^{-4}$. After that in the region $\omega l/s>1$ the contribution of the inertial mechanism decreases as ω^{-1} and the absorption reaches a minimum

$$\Gamma_{min}=Z\frac{m_0 s}{Mv}\frac{|\Delta|}{p_0 l}\quad\text{when}\quad\omega=\alpha\frac{s}{l}. \tag{5.10}$$

This minimum is caused by the above mentioned competition of the two damping mechanisms and can exist only when $\alpha\gtrsim 1$. The absorption beyond the minimum reaches the linear dependence which is caused by the cross-deformation mechanism.

The role of the Stewart-Tolman effect leading to the oscillations of $\Gamma(\omega)$ weakens as the quantity α^2 decreases. As to the dispersion of the sound speed, for $|\Delta_2|\approx 1$ and at low frequencies it is caused by the cross-deformation interaction and at high frequencies $\Delta s/s$ is, generally speaking, determined by both mechanisms for the electron-phonon interaction.

We explain the physical nature of that part of the impurity absorption which is described by the term with $|\Delta|^2$ in (5.2) and is not affected by spatial dispersion. This damping has a quantal character and occurs when the electron is scattered by a single impurity oscillating with the lattice absorbing at that moment a quantum $\hbar\omega$. The local character of such scattering makes it independent of the macroscopic scales of the electron motion in the crystal and, in particular, of the localization of the electron states in the 1D-conductor. Just therefore there is in it no effect of the spatial dispersion. Such a treatment is also confirmed by a formal calculation which shows that the contribution to the corresponding correlator (4.9) is determined by the electron Green function for equal spatial arguments (because of the short-range nature of the impurity potential). At the same time the absorption proportional to Δ_2^2 in (5.2) is caused by multiple scattering by different impurities and thus depends on the localization and spatial dispersion effects.

We show that the condition $(qa)^2 \ll 1$ is sufficient to neglect the incomplete screening of the direct deformation interaction. Indeed, the second term in (5.7) can be obtained from the following simple considerations. The relative absorption Γ_λ due to the direct deformation interaction is as to order of magnitude equal to $(\lambda \approx \varepsilon_F q^2 a^2)$

$$\Gamma_\lambda \approx \frac{|\lambda \operatorname{div} \dot{u}|^2 \tau \nu(\varepsilon_F)}{Mn\omega^3 |u|^2 (1+q^2 l^2)} \approx \frac{\varepsilon_F (qa)^4}{Ms^2} \frac{\omega\tau}{1+q^2l^2}. \tag{5.11}$$

An estimate of the absorption from the cross-deformation interaction is obtained from (5.11) if in that formula we replace the quantity λ by $|\Delta|/\tau$ and neglect $q^2 l^2$ in the denominator (for the reason given above). Comparison of these two quantities gives the estimate

$$\frac{(qa)^4 (p_0 l)^2}{1+q^2l^2} = (qa)^2 (p_0 a)^2 \frac{(ql)^2}{1+(ql)^2} \leqslant (qa)^2 \ll 1. \tag{5.12}$$

For long-wavelength sound the unscreened remainder of the direct deformation interaction thus turns out to be negligibly small.

6. CONCLUDING REMARKS

We emphasize that the elasticity Eqs. (3.7) are suitable for any direction of propagation and arbitrary polarization of the sound in a three-dimensional crystal with one-dimensional conductivity. It is clear from those equations that the absorption and velocity dispersion of the sound depend in an essential way on the orientation of its wave vector and the polarization vector relative to the conducting filaments. The statement about the absence of a strong deformation interaction between the electrons and the sound in a 1D-metal is valid independent of the direction of propagation, and also when $\mathbf{q} \perp x$. Indeed, that fact is a consequence of the dispersion law being one-dimensional and the electron density n_e being constant in the comoving reference frame K'. When longitudinal sound propagates along the conducting filaments or at angles of the order unity n_e being constant $i\delta$ maintained by the Coulomb forces. If, however, longitudinal sound is propagating at right angles to the filaments, even though the electron plasma frequency turns to zero, the unchanging electron density is guaranteed by those forces which contain the electrons to the conducting filaments.

We give a criterion for the validity of the neglect of three-dimensional effects when describing the interaction between the electrons and long-wavelength sound (the role of three-dimensional effects in the conductivity has been discussed, e.g., in Ref. 6). Collisional absorption of sound by the electrons with a (due to three-dimensionality) weakly curved Fermi surface can be estimated using Eq. (5.11) in which we must replace $(qa)^4$ by θ where the parameter $\theta \sim (p_0/P) \sim (m_\parallel / m_\perp)^{1/2}$ characterizes the deviation of the shape of the Fermi surface from a plane (P is a characteristic radius of curvature, $m_\parallel$ and $m_\perp$ are the longitudinal and transverse (relative to the x-axis) electron effective masses). In the case of collisionless damping the quantity $(1+q^2l^2)$ in (5.11) must be replaced by ql. As a result we get the condition

$$\theta^2 \sim \frac{m_\parallel}{m_\perp} \ll \frac{1}{p_0 l} + qa, \tag{6.1}$$

under which the effects of three-dimensionality are unimportant in the electron-phonon interaction.

We note finally that the possibility for an experimental observation, at sufficiently low temperatures, of the frequency dependence of the sound absorption and speed obtained in the present paper is completely realistic notwithstanding the presence of a Peierls transition in many well known 1D-conductors. We draw in this connection attention to two facts. Firstly, among the quasi-one-dimensional organic conductors there are substances in which such a structural transition has not been observed experimentally down to the lowest temperatures. As examples we mention $Qn(TCNQ)_2$ and $Adz(TCNQ)_2$, crystals with a rather high degree of internal disorder which, apparently, suppresses a Peierls transition. Secondly, even if there occurs a Peierls transition in a 1D-conductor, in the presence of impurities and at temperatures below the transition point (in the Peierls dielectric phase) the electron conductivity may have the same nature as in a 1D-metal with localized electron states. For instance, Larkin and Lee[14] have shown that for a sufficiently weak potential for the scattering of electrons by impurities the soliton conductivity mechanism, characteristic for a Peierls dielectric, is replaced by a jump conductivity with electrons jumping between localized states; this is described by the Berezinskiĭ equations.

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