On the quantum theory of transmission and radiation of low-energy relativistic electrons in crystal channels

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A study is made of the lifetime of electrons in a crystal channel. The kinetic equations describing the evolution of the populations of the channeled-electron bands are solved, and the linewidths of the Kumakhov radiation are calculated. It is shown that it should be possible to obtain stimulated emission of the channeled electrons in the optical region and to control the population of the bound states with the aid of laser radiation. The theory is found to be in good agreement with experiment. The possibility of detecting the anomalous Doppler effect is discussed.

1. INTRODUCTION

In recent years the channeling of light relativistic charged particles —emissions and positrons—has aroused considerable interest. This was occasioned by Kumakhov's discovery[1] of intense x-ray and γ emission accompanying the motion of a channeled particle in a single crystal. This type of emission is now the subject of active theoretical and experimental research (reviewed in Refs. 2–5). The spectral-angular characteristics of the emission are affected substantially by the scattering of the channeled particle by the nuclei and electrons of the crystal. At high energies (~ 1 GeV) the dechanneling of light particles (and also that of heavy particles such as α particles, protons, and ions) is described by classical kinetic equations of the Fokker-Planck type, and the change in the transverse energy of the particle is of a diffusional nature.[6–8] At low energies of the channeled emissions (1–50 MeV) the number of transverse-energy levels is small, and the classical diffusion approximation therefore becomes inapplicable. To study the transmission of electrons in a channel in this energy range one must solve the quantum kinetic equations for the density matrix of the particle in the crystal[8]; here the probabilities for the transitions between the energy levels of the channeled electron will determine the width of the level and, consequently, the width of the emission line observed at a definite angle. Such a study has never been done.

In Ref. 9 the linewidths for emission during channeling were estimated by introducing an additional imaginary term to the continuous potential of the atomic chain, by analogy with the semiempirical theory for scattering during electron diffraction.[10] However, the size of the imaginary part of the potential cannot be rigorously calculated theoretically. In Ref. 11, perturbation theory was used to obtain an expression for the probability for the transition of an axially channeled electron from the $1s$ state to the continuum as a result of scattering by thermal vibrations of the atoms of the chain. The formula obtained in that paper does not work for other electron energy levels and, furthermore, the main contribution to the level width is due to transitions between neighboring states. The effects of scattering processes involving the excitation of the electron shells of the atom were also ignored in Ref. 11. Ryabov[12] has considered the problem of scattering by an isolated atom, i.e., without allowance for the ordered arrangement of atoms in the channeling plane; this greatly exaggerates the contribution of nuclear scattering to the dechanneling and understates the lifetimes of the channeled electron, even if one neglects the scattering by electrons.

A consistent quantum theory of elastic and inelastic scattering of electrons in crystal channels has been developed by Bazylev and Goloviznin.[13,14] We shall outline the main ideas of these papers below.

In another recent paper,[15] Andersen and co-workers obtained results which do not agree with those of Bazylev and Goloviznin.[13,14] We shall also analyze that paper.

In the present paper we inviestigate in detail the elastic and inelastic scattering of low-energy electrons moving in a planar single-crystal channel. We solve the kinetic equations for the diagonal elements of the density matrix. These equations, which describe the evolution of the populations of the electron levels in the channel, take into account not only the departure of electrons from bound states as a result of scattering but also the replenishment of the bound states at the expense of the bands above the barrier. We elucidate the influence of various factors (scattering, band broadening, the thickness of the crystal) on the linewidth of the Kumakhov emission and show that in certain cases the band broadening gives the predominant contribution to the linewidth. We calculate the spectral-angular distributions of the channeling emission and find good agreement with the experimental data. We discuss the possibility of controlling the population of the levels with the aid of intense laser radiation.

2. SCATTERING BY THERMAL VIBRATIONS OF THE CRYSTAL ATOMS AND ELECTRONS

The motion of an electron in a planar channel is described by the Schrödinger equation with a relativistic particle mass (see Refs. 2–5). Allowance for the periodicity of the interplanar potential gives rise to a band structure in the energy spectrum of the channeled electron, and the wave function of the transverse motion can be represented as an expansion in plane waves[16]:

$$\psi_i(x) = \sum_m C_m^i \exp\{i(k_\perp + mg)x\}, \tag{1}$$

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where $k_\perp$ is the electron quasimomentum and $g = 2\pi/d_p$ (d_p is the distance between planes).

The probability of an electronic transition from initial state i to final state f due to scattering by thermal vibrations of the atoms of the plane was obtained in Ref. 13:

$$W_{if} = \frac{n}{\pi\hbar^2 c}\int_0^{p_i} dq\,[\langle|(V_q)_{if}|^2\rangle_{th} - \exp(-q^2u_t^2)\,|(\langle V_q\rangle_{th})_{if}|^2], \tag{2}$$

where q is the change in the momentum of the electron in the longitudinal direction,

$$q^2 = (\Delta E_\perp/\hbar c)^2 + k_i k_f \theta^2, \quad k_{i,f} = p_{i,f}/\hbar,$$

$p_{i,f}$ is the momentum of the electron in the initial (final) state, $V_q(x - x_a)$ is the Fourier transform of the atomic potential, the parentheses with subscripts if denote the operation

$$(V_q)_{if} = \int dx\,\psi_i(x)\,\psi_f^*(x)\,V_q(x - x_a), \tag{3}$$

n is the density of atoms in the plane, u_t is the amplitude of the thermal vibrations of the atoms, and θ is the angle between k_i and k_f.

Expression (2) corresponds to incoherent scattering; the coherent (phononless) contribution to the scattering probability for motion in a plane far from the directions of the principal axes is strongly suppressed.[13]

Let us express the Fourier transform of the potential $V_q(x - x_a)$ in terms of the atomic scattering factors calculated[17] in the Hartree-Fock approximation:

$$V_q(x - x_a) = \frac{4\pi^{3/2}\hbar^2}{m_0}\sum_j a_j b_j^{-3/2}\exp\left(\frac{-b_j q^2}{16\pi^2} - \frac{4\pi^2(x - x_a)^2}{b_j}\right), \tag{4}$$

where a_j and b_j are tabulated in Ref. 17. Using the wave functions of the transverse motion of the electron in the form (1) and making the necessary evaluations, we obtain the transition probability W_{if} as

$$W_{if} = \frac{8\pi^{3/2}\hbar^2 n}{m_0^2 c d_p^2}\sum_{j,m,n}\sum_{j',m',n'} a_j a_{j'} C_m^{\,i} C_{m'}^{\,i} C_n^{\,f} C_{n'}^{\,f}$$
$$\times\exp\left[-(16\pi^2u_t^2 + b_j + b_{j'})\left(\frac{\Delta E_\perp}{4\pi\hbar c}\right)^2\right]$$
$$\times\exp\left[-\frac{1}{16\pi^2}(\Delta K_\perp^2 + \Delta K_\perp'^2)\,(b_j + b_{j'})\right]$$
$$\times\left\{\frac{\exp[-(\Delta K_\perp - \Delta K_\perp')^2 u_t^2/2]}{(b_j + b_{j'})^{1/2}} - \frac{\exp[-(\Delta K_\perp^2 + \Delta K_\perp'^2)\,u_t^2]}{(16\pi^2u_t^2 + b_j + b_{j'})^{1/2}}\right\}, \tag{5}$$
$$\Delta K_\perp = k_\perp^{\,i} - k_\perp^{\,f} + (m - n)\,g, \quad \Delta K_\perp' = k_\perp^{\,i} - k_\perp^{\,f} + (m' - n')\,g.$$

The first term in the curly brackets in formula (5) corresponds to scattering by atoms arranged in a disordered fashion in the channeling plane (an amorphous target), while the second term takes into account the order in the arrangement of atoms in the plane and leads to suppression of the incoherent scattering. The level width Γ_i is obtained from (5) by summing over all final state $f \neq i$.

In addition to a change in the state of the transverse motion of the electron, scattering can involve processes in which the particle does not change its level of transverse energy but changes the magnitude of its momentum in the plane of the channel, i.e., scatters by some angle in the channeling plane. This process does not lead to dechanneling and does not affect the population of the bound states, but it does contribute to the observed linewidths. The beam averaged probability for scattering in the plane of the channel is given in Ref. 13:

$$\frac{dW}{dq} = \frac{n}{\pi\hbar^2 c}\int dx\,Q(x)\,(\langle V_q^2\rangle_{th} - \exp(-q^2u_t^2)\,|\langle V_q\rangle_{th}|^2), \tag{6}$$
$$Q(x) = \sum_i P_i|\psi_i(x)|^2,$$

where $q(x)$ is the particle number density, and P_i is the population of the ith level.[16] From this expression, using (1) and (4), we obtain an expression for the mean-square scattering angle of an electron in the channeling plane:

$$\overline{\theta^2} = \frac{8\pi^2}{p^2}N\sum_{i,j,j',m,n} P_i C_m^{\,i} C_n^{\,i} a_j a_{j'}$$
$$\times\left\{\frac{\exp[-\Delta K_\perp^2(8\pi^2u_t^2(b_j + b_{j'}) + b_j b_{j'})/16\pi^2(b_j + b_{j'})]}{(b_j + b_{j'})^2} - \frac{\exp[-\Delta K_\perp^2(8\pi^2u_t^2 + b_j)\,(8\pi^2u_t^2 + b_{j'})/16\pi^2(16\pi^2u_t^2 + b_j + b_{j'})]}{(16\pi^2u_t^2 + b_j + b_{j'})^2}\right\}, \tag{7}$$

where $n = 32\pi^3\hbar^2 n l P_{ch}/m_0^2 c d_p$, l is the thickness of the crystal, and P_{ch} is the average number of particles populating the bound states (determined from the solution of the kinetic equations).

Expressions (2) and (5) do not agree with the corresponding formulas in Ref. 15. The problem is that in Ref. 15 an error was made in writing out the coefficients of the initial wave function of the electron and crystal in the expansion of the electron wave function after the switching on of the perturbation. Consequently, two additional terms appear in the formula for the coherence length [cf. Eq. (4.18) in Ref. 15] and in subsequent expressions. When the operation is correctly carried out these terms cancel.

The probability for inelastic scattering of a channeled electron by the electron shells of the crystal atoms was obtained in Ref. 14:

$$W_{if} = \frac{2e^4 n}{\pi\hbar^2 c}\int\frac{d\lambda\,d\mu\,d\varkappa}{(\lambda^2 + \varkappa^2)\,(\lambda^2 + \mu^2)}\exp\left(-\frac{1}{2}u_t^2(\varkappa - \mu)^2\right)$$
$$\times b_{if}(\varkappa)\,b_{fi}(-\mu)\,\{[B(\varkappa,\lambda)B^*(\mu,\lambda)]^{00} - B^{00}(\varkappa,\lambda)B^{*00}(\mu,\lambda)\}, \tag{8}$$

where $\lambda = p_i\theta$, $\varkappa = p_{fx} - p_{ix}$, μ is an analogous parameter,

$$b_{if} = \int dx \psi_f^*(x) \psi_i(x) \exp(-i\varkappa x), \tag{9}$$

$$B^{00}(\varkappa, \lambda) = \sum_{l=1}^{z} \int dv |\varphi_0(\{\mathbf{r}_a\})|^2 \exp(i\varkappa x_l - i\lambda y_l), \tag{10}$$

$$dv = \prod_{i=1}^{z} d^3 r_i,$$

$\varphi_0(\{\mathbf{r}_\alpha\})$ is the ground-state wave function of the atom, $\{\mathbf{r}_a\}$ is the set of coordinates of all the electrons of the atom, and z is the number of electrons in the atom. Since the inelastic scattering processes for different atoms of the plane are incoherent, it is single-atom scattering that is considered in (8).

To evaluate the transition probability it is convenient to separate (8) into two terms: The first term corresponds to the excitation of inner-shell electrons and the second to the excitation of valence electrons. For inner electrons we have the condition $u_t^2 \lesssim \langle x^2 \rangle_l \ll \langle x^2 \rangle_i$ ($\langle x^2 \rangle_i$ is the average of the square of the coordinate of the channeled electron), which makes it possible to series expand the exponentials in the formula for $B^{00}(\varkappa, \lambda)$. Since the field inside the atom is radially symmetric, one can assume that [14]

$$\langle x^2 \rangle_l = \langle y^2 \rangle_t = \tfrac{1}{3} \langle r^2 \rangle_l \quad \langle \mathbf{r}_l \rangle = 0.$$

Expansion then gives

$$[B(\varkappa, \lambda) B^*(\mu, \lambda)]^{00} - B^{00}(\varkappa, \lambda) B^{*00}(\mu, \lambda) = \frac{\mu\varkappa + \lambda^2}{3} \langle d^2 \rangle,$$

$$\langle d^2 \rangle = \int dv \sum_{l=i}^{z-z_v} r_l^2 |\varphi_0(\{\mathbf{r}_a\})|^2,$$

where z_0 is the number of valence electrons. Substituting this expression into (8) with wave functions of the form (1) and performing the integration, we obtain a formula for the transition probability:

$$W_{if} = \frac{16\pi^2 e^4 n \langle d^2 \rangle}{3 d_p^2 \hbar c} \times \sum_{m,n,m',n'} C_m^i C_{m'}^i C_n^f C_{n'}^f \frac{\exp(-u_t^2 (m-m'+n-n')^2 g^2)}{2\Delta k_\perp + (n+n'-m-m') g}. \tag{11}$$

The values of $\langle d^2 \rangle$ have been calculated [18] to high accuracy using Hartree-Fock wave functions.

For the higher shells the wave functions of the atomic electrons can in a first approximation be described by plane waves, and the expression for the transition probability of a channeled electron becomes

$$W_{if} = \frac{4\pi^2 e^4 n z_v}{\hbar^2 c d_p^2} \sum_{m,n,m',n'} \frac{C_m^i C_n^f C_{m'}^i C_{n'}^f}{|\Delta k_\perp + (m-n) g|^3} \delta_{m-n, m'-n'}. \tag{12}$$

Andersen *et al.*[15] considered only scattering in a free-electron gas, i.e., without allowance for the excitation of the inner shells of the atom. Calculations show, however, that the contribution to the scattering probability from the excitation of inner shells is not small. As to the formula for the coherence length in a free-electron gas [see Eq. (4.54) in Ref. 15], it cannot be obtained from Eq. (4.53) of Ref. 15 by summation of the squares of the matrix elements.

3. EMISSION LINEWIDTHS AND THE KINETICS OF THE BAND POPULATIONS

In experimental studies of low-energy channeled electrons[19,20] the emission spectrum is characterized by the presence of discrete lines corresponding to transitions between transverse-energy levels in the potential well. The width of the observed lines is governed by the following factors: the band-broadening width of the levels, the finite lifetime of the level because of scattering by thermal vibrations of the nuclei and atomic electrons, the divergence of the electron beam along the crystallographic plane because of scattering by atom planes, and the finite thickness of the crystal. The band broadening of the levels was investigated in detail by the author in a previous paper.[16] The level width due to scattering processes involving transitions of the channeled electron to other transverse-motion levels is obtained by summing (5), (11), and (12) over all final states of the electron. Scattering which involves a change in the direction of motion of the electron in the channeling plane does not change the state of transverse motion but does carry an effective contribution $\Delta\hbar\omega = \hbar\omega\gamma^2 \overline{\theta^2}$ to the width of the forward-observed line, where $\overline{\theta^2}$ is given by (7).

To elucidate the influence of the aforementioned factors on the linewidth of the channeling emission, computer calculations were carried out for the experiments of Refs. 19 and 20. Gouanere *et al.*[19] studied the emission from 54-MeV electrons channeled by various planes in a diamond crystal 20 μm thick. This study, I believe, is the best of all the experiments on the radiation of low-energy channeled electrons: The crystal was rather thin, the divergence and nonmonochromaticity of the beam were small, the spectrometer had a rather good energy resolution, etc. The experiment of Ref. 20 studied the emission from 4-MeV electrons in a silicon single crystal; the resolution of the spectrometer was inadequate ($\sim$300 eV) for precise measurement of the linewidths.

Table I gives the theoretical and experimental values of the energy of the emitted photons, the contributions to the linewidths due to scattering by nuclei, crystal electrons, band broadening, and scattering in the channeling plane, the total theoretical linewidths, and the observed linewidths. It is seen from the table that under the experimental conditions of Ref. 20 the linewidth is determined first and foremost by the thickness of the crystal (0.3–1 μm): $\Gamma_l = 2\gamma^2 2\pi\hbar c/l$. In the experiment of Ref. 19 $l = 20\,\mu$m, and this factor has only a small influence on the linewidth. The main contribution in this case come from scattering in the channeling plane, scattering by crystal electrons, and also the band-broadening level widths if the initial state of the particle is sufficiently close to the top of the potential barrier. Furthermore, in some cases [e.g., C(100), transition 2–1] the band-broadening width exceeds all the remaining contributions. The result found for $\Gamma_{\overline{\theta^2}}$ means that a mean-square scattering angle $\overline{\theta^2} \sim 0.03(1/\gamma^2)$ is accumulated over a length of 20 μm for particles found in the channel. For an amorphous target this angle comes out larger. For a thinner crystal the contribution to the linewidth from $\Gamma_{\overline{\theta^2}}$ will be smaller, but the finiteness of the transit time through the crystal will come into play.

TABLE I. Energies and widths of the emission lines during planar channeling.

E, MeV, crystal	Transition	$\hbar\omega$ exp, keV	$\hbar\omega$ theo, keV	$2\gamma^2\Gamma$ nuc. keV	$2\gamma^2\Gamma$ el, keV	$2\gamma^2\Gamma$ band keV	$\Gamma_{\vartheta\vartheta}$, keV	Γ^{tot} theo keV	Γ exp, keV
54 C(110)	1–0	161.8	158.3	0.73	3.54	–	5,07	9.34	12,1
	2–1	104.4	101.1	0.79	3.03	0.02	3,27	7.11	8.7
	3–2	78.4	76.6	0.64	2.73	0,44	2.46	6,27	7.6
	4–3	58.0	58.5	0,5	2,49	4,56	1,82	9,37	8.8
54 C(100)	1–0	120,9	124.8	0,52	2.53	0,94	4,06	8,04	8,0
	2–1	65,4	68:4	0.51	1,73	11,47	3.69	18,4	24.0
4 Si(110)	1–0	1,6	1.6	0.02	0.05	0,14	0.01	0.22	0,6–1,0

Thus it is not possible to obtain information about the probability of departure of the electron to other transverse-emission states directly from the emission linewidths observed experimentally during planar channeling, since the level lifetime associated with these processes turns out to be longer than the lifetime due to the finiteness of the crystal (for thin slabs) or longer than the time for departure to states with a different value of p_y.

In a diamond single crystal the inelastic scattering by electrons exceeds the scattering by thermal vibrations of the nuclei (see Table I) because the nuclear scattering is suppressed by the periodicity of the arrangement of the atoms in the crystallographic plane.[13] With increasing nuclear charge of the crystal atom the contribution due to scattering by electrons decreases in accordance with the fact that $W_{if}^{\text{nuc}} \sim z^2$, while $W_{if}^{\text{el}} \sim z$. As the number of the level increases, both the scattering by nuclei and the scattering by electrons fall off.

The dependence of the level populations on the depth of penetration of the electron into the crystal is determined from the solution of the kinetic equations

$$\frac{dP_i}{dz} = \sum_f W_{if}(P_f - P_i), \tag{13}$$

where the probability W_{if} is the sum of expressions (5), (11), and (12). As initial conditions here we must use the population P_i^0 (Ref. 16). Equations (13) are nothing more than the law of conservation of particle number.

In the dipole approximation the probability for the emission of a photon by a channeled electron into an element of solid angle $d\Omega$ and in the spectral interval $d\omega$ is given by the formula (see, e.g., Ref. 3)

$$\frac{d^2w_{if}}{d\Omega\, d\omega} = \frac{e^2\omega^3 p_{if}^2}{2\pi\hbar m_0^2 c^3 \gamma^2 \Omega_{if}^2}\left[\frac{\Omega_{if}^2}{\omega^2}\sin^2\xi + \cos^2\xi \times\left(\beta_\parallel \sin^2\vartheta - \frac{\Omega_{if}}{\omega}\cos\vartheta\right)^2\right] \frac{\Gamma/2\pi}{[\omega(1-\beta_\parallel\cos\vartheta)-\Omega_{if}]^2+\Gamma^2/4}, \tag{14}$$

where $\Gamma = (\Gamma_i + \Gamma_f)$, p_{if} is the dipole matrix element of the transition, $\Omega_{if} = (E_{\perp f} - E_{\perp i})\hbar$, $\Omega_{if} = (E_{\perp f} - E_{\perp i})\hbar$, ϑ is the angle between $\omega \cdot \mathbf{n}$ and the z axis, $\mathbf{n}$ is the direction of propagation of the photon, ξ is the azimuthal angle, and $\beta_\parallel = v_\parallel / c$. Then the number of photons emitted by the electron will be equal to the product of probability (14) and the solution of kinetic equations (13) for the ith level.

Let us consider the kinetics of the band populations and the emission spectra under the experimental conditions of Ref. 19. Calculations were carried out in accordance with Ref. 16, and a Moliere potential with allowance for the thermal vibrations was used. In both cases a small number of bands form in the potential well, and transitions between them give rise to discrete lines in the emission spectrum. To determine the z dependence of the population of the bands, the probabilities (5), (11), and (13) for scattering with a change in the energy level were calculated and the kinetic equations (13) were solved numerically. Figure 1 shows the dependence of the band populations on the depth of penetration of the electrons into the crystal. Allowance was made in the calculations for the divergence of the incident beam (0.1 mrad). The solution of system (13) was found for a finite number of equations, equal to the number of bands inside the potential well plus the number of bands above the barrier in an arbitrarily chosen quasichanneling region. It was assumed that the quasichanneling region corresponds to a transverse energy $0 \leqslant E_\perp \leqslant 2U_0$, where U_0 is the depth of the potential well, while the electrons with transverse energies $E_\perp > 2U_0$ do not return to the channeling regime. This corresponds to the addition of a term $-P_i\Gamma_i'$, in (13), where $\Gamma_i' = \Gamma_i - \Sigma_f W_{if}$ and the summation on f is over the region within the potential well and the quasichanneling region. Expansion of the quasichanneling region does substantially alter the results.

As is seen in Fig. 1, the populations of the bands equalize rather rapidly, and the more bands in the potential well, the faster the populations level off (cf. Figs. 1a and 1b). It follows that if at the initial time the angle of incidence is chosen so as the bring about an inversion of the populations of two states, then the inversion decreases as one goes further into the crystal. Thus, at least for the planar case, scattering cannot be used to obtain a population inversion. The initial capture under the experimental conditions of Ref. 19 turns out to be rather large: ~80% of the electrons populate levels in the potential well. This is due to the small divergence of the beam in comparison with the critical angle for channeling. At a depth of 10 μm a rather large number of particles (~25%) still remain in the potential well. At a depth of 20 μm around 10% of the electrons remain in the channel. It follows that only the first 10 μm contribute effectively to the emission.

Figure 1c shows the kinetics of the band populations for electrons channeled by (110) planes of a silicon single crystal

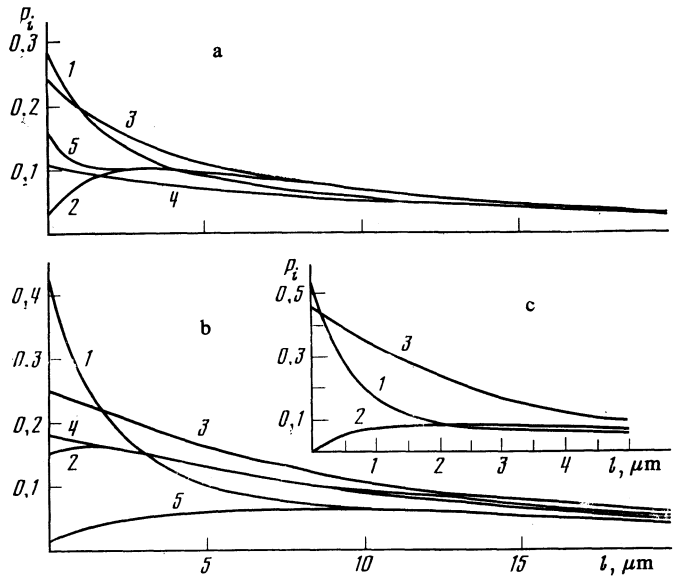

FIG. 1. Kinetics of band populations in terms of the depth of penetration of the particle into the crystal: a) C(110), with six bands in the potential well; b) C(100), with three bands, $E = 54$ MeV; the curves are labeled with the numbers of the bands; c) Si(110), $E = 4$ MeV; curves 1 and 2 are levels inside the potential well, curve 3 is an above-barrier level.

for a zero angle of incidence ($\varphi = 0^0$) of the particles with respect to the planes. This case is interesting in that the second band is initially unpopulated (odd wave function), but as one goes into the crystal its population begins to grow, reaches a maximum, and then falls off. This means that in order to obtain the maximum emission intensity under these conditions one must choose a suitable thickness of the single-crystal slab.

The solution of kinetic equations (13) permits a detailed comparison of the experimentally observed emission spectrum with the theoretical curves. Figure 2 shows the results of the experiment of Ref. 19 and the present calculation. In the calculations the transitions between ten bands were taken into account using formula (14), while for the electrons occupying the higher above-barrier bands the formula for bremsstrahlung was used:

$$d_2N_\tau/dzd\hbar\omega = 4/3\hbar\omega L_{rad},$$

where L_{rad} is the radiation length. It is seen in Fig. 2 that the theoretical and experimental curves are in good agreement, a certain amount of disagreement concerning the heights of the peaks being attributable to additional dechanneling by lattice defects, mosaic structure in the crystal, etc.

Let us now consider stimulated emission during channeling, the possible existence of which was first pointed out by Kumakhov.[21] Let us assume for the sake of definiteness that the resonance condition $\omega = \Omega_{if}/(1 - \beta_\parallel \cos\vartheta)$ is satisfied. Then the gain G will be

$$G = \sigma_{if}\Delta N_e = \frac{12\pi c^2 A_{if}}{\omega_{if}^2\Gamma}\Delta N_{if}, \qquad (15)$$

where σ_{if} is the cross section for stimulated emission, A_{if} is the Einstein coefficient, ΔN_{if} is the population inversion. We distinguish two characteristic cases.

1. The angle of emission of the photon is $\vartheta \sim \pi/2$ and the wavelength lies in the optical region. Then for observation of stimulated emission is is necessary that the crystal used be optically transparent and that two levels be formed in the potential well in such a way that the transition energy $\hbar\Omega_{if}$ lies in the visible region. These conditions are satisfied, for example, in the channeling of 20-MeV electrons by the (110) planes of diamond: in this case three bands form in the potential well: $\hbar\Omega_{if} = 4.8$ eV, $2\gamma^2\Gamma = 2.5$ keV. Determining the inversion from the solution of the kinetic equations and finding the dipole matrix element of the transition according to Ref. 16, we find that in order to satisfy the condition $G > 10^{-2}$ cm^{-1} (in the optical region a Fabry-Perot type res-

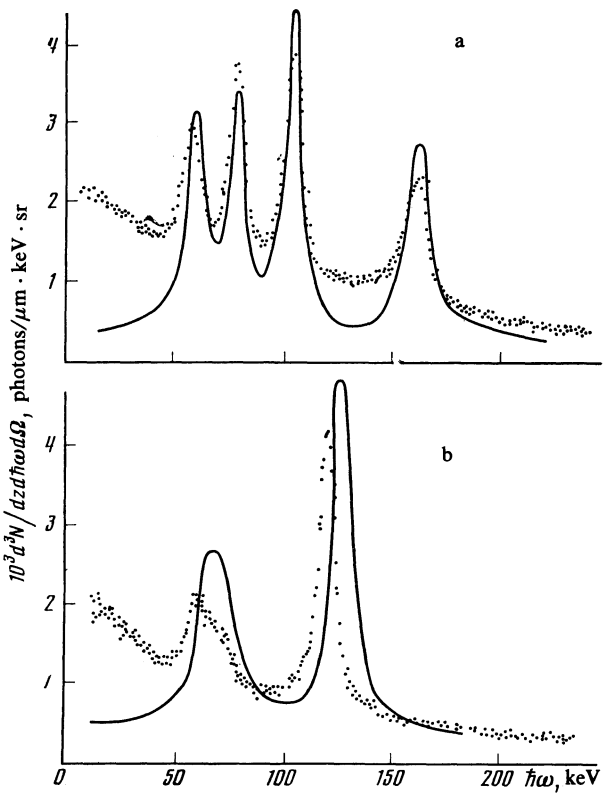

FIG. 2. Emission spectrum of 54-MeV electrons in diamond: a) the (110) plane; b) the (100) plane; the points are the experimental results of Ref. 19, the curves are theoretical.

onator can be used) it is necessary that the current density of the electron beam be $j \sim 10^5$ A/cm^2 (it was assumed in the calculations that the incident electron beam had a divergence of $\varphi_{cr}/3$ and that the crystal was 5 μm thick). Such a current density cannot be achieved for a 20-MeV accelerator. A current density of ~ 100 kA/cm^2 can in principle be achieved, however, in linear induction accelerators at lower energies. In order to obtain optical transitions at $E \sim 5$ MeV it is necessary to use transparent single crystals of heavier elements (e.g., quartz) in the case of planar channeling or else to consider axial channeling in the same crystal (diamond). The second approach is preferable, since the system of energy levels will be substantially simpler than in the first case. And since the emission in axial channeling is somewhat more intense than in planar channeling, approximately the same current density will be needed for axial channeling at 5 MeV as was obtained above for planar channeling at 20 MeV.

2. The photon is emitted at $\vartheta = 0°$ with an energy in the x-ray region. Let us consider, for example, the channeling of electrons by the (110) planes in silicon at $E = 4$ MeV. Here $\hbar\omega = 1.6$ keV, $2\gamma^2\Gamma = 200$ eV. In view of the fact that the absorption coefficient in the x-ray region is rather large, $\sim 1.4 \cdot 10^3$ cm^{-1} (Ref. 22), in order for amplification to prevail over absorption it is necessary to have an electron current density $j > 3 \cdot 10^{11}$ A/cm^2.

4. CONTROL OF THE BAND POPULATIONS BY MEANS OF LASER RADIATION

The scattering of laser photons by a beam of relativistic channeled electrons and positrons has been studied in Refs. 23–25. Owing to the resonant character of the interaction, the scattering cross section increases by several orders of magnitude in comparison with the case of free-particle beams. High-power laser radiation will alter the population kinetics for the channeled particles.

The cross section for absorption of a laser photon $\hbar\omega$ by a channeled electron is related to the probability of spontaneous emission by the expression

$$\sigma_{abs}(\omega, \mathbf{n}) = \frac{8\pi^3 c^2}{\omega^2} w(\omega, \mathbf{n}), \tag{16}$$

where $\mathbf{n}$ is the direction of propagation of the photon and $w(\omega,\mathbf{n})$ is given by expression (14). Suppose that the laser beam is monochromatic, has an exact direction in space, and is linearly polarized; we can then choose the angle of incidence ϑ of the laser beam in such a way that the photon $\hbar\omega$ is exactly at resonance with the transition $i \to f$ of the channeled electron.

Let us study the possibility of controlling the kinetics of the band populations for 54-MeV electrons channeled by the (110) planes of a diamond single crystal. Under these conditions there are six bands in the potential well. Let us choose the angle of incidence to be such that the population of the fourth band is maximum: $\varphi = 1.4 \cdot 10^{-4}$ rad at a beam divergence of 0.1 mrad. Under these conditions electrons can be pumped into this band through absorption of neodymium-glass laser radiation at a wavelength of 1.06 μm in the electronic transition 3→4. Furthermore, one can make use

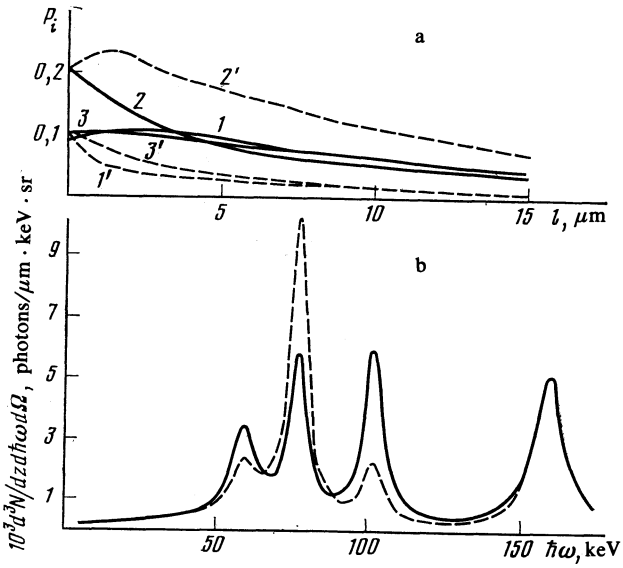

FIG. 3. a) The influence of laser radiation on the kinetics of the band populations; curves 1, 2, and 3 correspond to bands 3, 4, and 5; b) the change in the spectrum. The solid and dashed curves are for the cases with and without laser radiation, respectively. C(110), $E = 54$ MeV.

of the anomalous Doppler effect,[26] in which the absorption of a photon leads to a transition to a lower-lying level. In practice one can use a ruby-laser pulse to bring about the transition 5→4($\lambda = 694.3$ nm). Naturally, the energy flux density of the laser radiation must be rather large. Calculations show that there is no appreciable change in the band populations at $P_{las} = 1$ MW/cm^2 even when two lasers are used. However, the power of the two lasers can be substantially higher, ~ 1 GW/cm^2, and the radiation can be focused to a spot of area $\sim 10^{-3}$ cm^2; this leads to a substantial change in the populations (Fig. 3a). Bands 3 and 5 are emptied much faster, and the population of state 4 grows by about a factor of two, as a result of the laser pumping. This leads to a transformation of the emission spectrum (Fig. 3b); line 4–3 becomes roughly twice as intense as before, while the transitions 5–4 and 3–2 become substantially weaker. Thus, instead of a series of lines of approximately equal intensity, we obtain one sharp peak in the emission spectrum. The observation of a change in the emission spectrum when only a ruby laser is used can serve as experimental confirmation of the existence of the anomalous Doppler effect.

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