Photoinduced nonequilibrium states of a two-temperature electron-hole plasma in a semiconductor

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Possible steady and time-varying nonequilibrium states of an electron-hole plasma heated uniformly by interband optical absorption are analyzed in the two-temperature approximation. A possible dependence of the gap width on the electron and hole temperatures and on the charge-carrier density is taken into account. Under certain conditions the steady state of the plasma is unstable, and self-oscillations arise in the electron and hole temperatures and in the carrier density. The conditions corresponding to this instability are found. When the relaxation times of the carrier energy are related in a certain way to the characteristic time for electron scattering by holes, two stable steady states and a single unstable state arise in this system. The system may thus operate as a flip-flop.

§ 1. INTRODUCTION

Vladimirov and Gorshkov[1] have analyzed the onset of spontaneous oscillations in an electron-hole plasma, using a dc power supply. In the present paper we analyze the nonequilibrium states of a plasma which arise when a plasma is heated by interband absorption of light. We assume that the light intensity is high enough to make the electron and hole densities n and p considerably higher than their equilibrium values (and therefore to satisfy the condition $n \approx p$). We furthermore assume that we can use the concepts of electron and hole temperatures T_n and T_p (the corresponding criteria are well known[2,3]), but that the electron and hole gases are nevertheless nondegenerate. The positive-feedback mechanism which is responsible in particular for the possible self-oscillations of T_n and T_p, consists of a dependence of the gap width E_g and of the effective masses m_n and m_p on the density and temperatures of the charge carriers. Another important factor is the density dependence of the characteristic time for energy exchange between electrons and holes.

To simplify the problem we assume a uniform excitation. This is of course a rather restrictive assumption for the conditions in which we are interested here; generally speaking it would be valid only for sufficiently thin films. Nevertheless, it does appear to bring out the essential features of the problem. We might also note that if the period of the self-oscillations is short enough our problem may not apply to the entire sample (which may be thick) but only to a surface layer in which all the significant light absorption occurs [specifically, the oscillation period must be short in comparison with the characteristic times for the diffusion of particles and energy $t_1 = \gamma^{-2} D^{-1}$ and $t_2 = \gamma^{-2} \varkappa^{-1}$, where γ, D, and $\varkappa$ are the absorption coefficient for the light, the ambipolar diffusion coefficient, and the electron (or hole) thermal diffusivity, respectively].

§ 2. BASIC EQUATIONS

In the problem as formulated here we should use the recombination-kinetics equation and energy-balance equations for the electrons and holes.

We denote by R_r and g the rates of interband recombination and the optical generation of electrons and holes. The recombination kinetic equation for the case of interest here can then be written

$$\dot{n} = g - R_r, \tag{1}$$

where

$$g = \nu\gamma J, \quad R_r = \alpha n^2. \tag{2}$$

Here ν is the quantum yield, J is the photon flux density, and α is the interband recombination coefficient (the recombination may be radiative or radiationless). To avoid unnecessary complications we assume that α and ν depend on only the lattice temperature T_0, but not on T_n, T_p, or n (this assumption means, in particular, that we are ignoring the possible collisional recombination). For the optical absorption coefficient, on the other hand, we

$$\gamma = \gamma_0 \left(\frac{\hbar\omega - E_g^*}{\hbar\omega - E_g}\right)^{1/2+q} \left(\frac{m^*}{m}\right)^{3/2} \left(\frac{m_n^* + m_p^*}{m_n + m_p}\right)^{q}. \tag{3}$$

Here $q = 0$ and $q = 3/2$ correspond to direct and indirect optical transitions, respectively; $\gamma_0 \propto (\hbar\omega - E_g)^{1/2+q}$ is the absorption coefficient calculated neglecting the dependence of the parameters of the energy spectrum on T_n, T_p, and n; and m and E_g are the corresponding values of the reduced effective mass of the electron and hole and the gap width (minus the energy of the phonon emitted during the absorption of the light). E_g^* and m^* denote the renormalized values of the corresponding properties. Their dependence on T_n, T_p, and n may result from, for example, an interaction of charge carriers with each other and with phonons. This dependence was studied in Refs. 4 and 5, in particular, for the case of a nondegenerate gas under equilibrium conditions ($T_n = T_p = T_0$). Those papers used perturbation theory, however, which would hardly be justified under the conditions of interest here. It thus seems more convenient to use the phenomenological equation

$$E_g^* = E_g[1 - \bar{\psi}(T_n, T_p, n)], \tag{4}$$

where

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$$\bar{\psi}=a_n(T_n/T_0)^{q'}+a_p(T_p/T_0)^{q''}+bn^p, \quad (5)$$

and a_n, a_p, b, p, q', and q'' are numbers. These numbers are characteristics of the energy spectrum which are just as fundamental as the gap width or the effective masses. In principle, these numbers could be derived theoretically, but unless we use perturbation theory the problem is no simpler than, for example, that of calculating E_g "from first principles." For the time being it is more convenient to treat these quantities as parameters to be determined experimentally. The latter approach might prove less hopeless than it appears at first glance, since the density and temperature of the charge carriers can be measured independently (by optical methods, for example). For our purposes it is sufficient to note that in the absence of small parameters the numbers a_n and a_p, which are dimensionless, should be on the order of unity. If the ratios $(T_n - T_0)/T_0$ and $(T_p - T_0)/T_0$ are small, an equation of the type in (4) could easily be guessed, as we will now show. Indeed, under equilibrium conditions (at low carrier density) we have the known empirical formula

$$E_g(T_0)=E_{g,0}(1-aT_0),$$

where $E_{g,0}$ corresponds to the limit $T_0\to 0$. If there is a small deviation from the electron equilibrium (under conditions such that we can use the two-temperature approximation), it would be natural to use the formula

$$E_g^*=E_g(T_0)\{1-\bar{a}_n(T_n-T_0)/T_0-\bar{a}_p(T_p-T_0)/T_0-\bar{b}n^p\}$$
$$=E_g(T_0)(1+\bar{a}_n+\bar{a}_p)\{1-(a_nT_n/T_0+a_pT_p/T_0+bn^p)\},$$
$$a_{n,p}=\bar{a}_{n,p}(\bar{a}_n+\bar{a}_p+1)^{-1},\ b=\bar{b}(\bar{a}_n+\bar{a}_p+1)^{-1}.$$

In the case of large deviations from equilibrium, these arguments of course lose force and become merely suggestions.

A few comments are in order regarding Eqs. (4) and (5). First, both a_n and a_p, on the one hand, and b, on the other, may depend on T_0. The numbers q' and q'' may be either positive or negative. The first case arises under the conditions assumed in Ref. 5 (where $q'=q''=1/2$), while the second arises under conditions to which the results of Ref. 4 apply. Furthermore, according to Ref. 4 and 5, the number p seems to lie between 1/4 and 1 (see the Appendix).

Second, Eq. (5) simplifies slightly: Repeating the calculations of Refs. 4 and 5 for the case $T_n\neq T_p$, we easily find that these quantities, just as n, generally do not enter in $\bar{\psi}$ as separate terms. For the discussion below, however, this point is unimportant, since we are interested primarily in the case $T_n\gg T_p$. Furthermore, the dependence of E_g^* on n or on T_n and T_p, is important only in the regions $T_n\approx T_0$ and $n\approx n_0=(g_0/\alpha)^{1/2}$, respectively (the generation rate g_0 corresponds to the absorption coefficient γ_0).

Third, expression (4) can be meaningful only under the condition $\bar{\psi}<1$ [see the discussion following Eq. (13) below for more details].

Fourth, under the conditions $q'=q''=1$, $T_n=T_p$, and $bn^p\ll 1$, the right side of (4) converts into the expression proposed by Van Vechten and Wautelet.[6] This circumstance should not assigned any great importance, however, since Van Vechten and Wautelet were dealing with the thermal rather than optical gap width. The optical gap width, however, should also depend on T_n, T_p, and n, as can be seen from perturbation-theory calculations, for example.

Fifth and finally, we should not be confused by the circumstance that the first two terms on the right side of (5) do not vanish in the limit $n\to 0$: The two-temperature approximation itself can be justified only if the carrier density is sufficiently high. We note in this connection that our ultimate results remain qualitatively the same when we choose a different form of the function $\bar{\psi}$:

$$\bar{\psi}=[a_n(T_n/T_0)^{q'}+a_p(T_p/T_0)^{q''}](n/n_0)^p.$$

By virtue of (2)–(4) we can write

$$g=g_0[1+E_g(\hbar\omega-E_g)^{-1}\bar{\psi}]^{1/2+q}\left(\frac{m^*}{m}\right)^{3/2}\left(\frac{m_n^*+m_p^*}{m_n+m_p}\right)^q \quad (4')$$

Here we are assuming that $\hbar\omega>E_g$.

The energy-balance equations are (we are expressing the temperatures in energy units)

$$\frac{3}{2}\frac{d}{dt}(nT_n)=-nR_{np}-nR_n+G_n, \quad (6a)$$

$$\frac{3}{2}\frac{d}{dt}(nT_p)=nR_{np}-nR_p+G_p. \quad (6b)$$

Here R_{np}, R_n, and R_p are the rates at which energy is transferred from electrons to holes, from electrons to the lattice, and from holes to the lattice; G_n (G_p) is the energy acquired by electrons (or holes) from the light wave per unit time per unit volume. According to Ref. 7, we have

$$nR_{np}=4(2\pi)^{1/2}\frac{ne^4L}{m_pm_n\varepsilon^2}\left(\frac{T_n}{m_n}+\frac{T_p}{m_p}\right)^{-3/2}(T_n-T_p), \quad (7)$$

where ε is the dielectric constant of the lattice, and L is the Coulomb logarithm. Strictly speaking, this logarithm depends on T_n, T_p, and n but since Eq. (7) has an exact meaning only at $L\gg 1$ we may ignore this dependence, treating L as a constant.

Furthermore, for strain scattering of energy by acoustic phonons we have[8]

$$nR_{n,p}=8^{1/2}m_{n,p}^{5/2}E_{n,p}^2(\pi^2\hbar^4\rho)^{-1}nT_{n,p}^{1/2}(T_{n,p}-T_0), \quad (8)$$

where E_n (E_p) is the strain potential for the electrons (holes), and ρ is the crystal density. This energy relaxation mechanism is dominant both below and well above the threshold for the production of optical phonons. We will consider this mechanism exclusively below, ignoring the comparatively narrow energy interval, slightly above the threshold for the production of optical phonons, in which the emission of these phonons is substantial (in principle, this interval might not be reached at all).

Finally, we have

$$G_{n,p}=g(\hbar\omega-E_g^*)\lambda_{n,p}, \quad (9)$$

where λ_n and λ_p are the fractions of the energy which are transferred to electrons and phonons, respectively ($\lambda_n+\lambda_p=1$). We will restrict the discussion below to the case in which the electron and hole have different masses; we will assume, for example, $m_n\ll m_p$. It is easy to show that for direct transitions we would have $\lambda_n/\lambda_p=m_p/m_n$, while for indirect transitions (and under the condition $m_n\ll m_p$) we

would have $\lambda_n/\lambda_p = 2m_p T_n/p_{ph}^2$, where p_{ph} is the quasimomentum transferred to phonons. In the former case we would have $\lambda_n \gg \lambda_p$, i.e., $\lambda_n \approx 1$. The situation is the same in the case of indirect transitions if $2m_p T_n \gg p_{ph}^2$.

At this point it is convenient to introduce dimensional variables:

$$T_n = T_0 x, \quad T_p = T_0 y, \quad n = (g_0/\alpha)^{1/2} z. \tag{10}$$

From (4') and (9) we then have

$$G_{n,p} = g_0(\hbar\omega - E_g) f(x, y, z) \lambda_{n,p}, \tag{11}$$

where

$$f = [1+\psi(x,y,z)]^{3/2+q} \left(\frac{m^*}{m}\right)^{3/2} \left(\frac{m_n^* + m_p^*}{m_n + m_p}\right)^q, \tag{12}$$

$$\psi = \alpha_n x^{q'} + \alpha_p y^{q''} + \beta z^p; \tag{13}$$

the coefficients α_n and α_p are found by multiplying a_n and a_p by $E_g(\hbar\omega - E_g)^{-1}$, while β is found by multiplying b by $E_g(\hbar\omega - E_g)^{-1}(g_0/\alpha)^{p/2}$.

So far, the only method which has been developed for finding the explicit dependences of the effective mass on the electron and hole temperatures and on the carrier density is to use perturbation theory. This approach is not very reliable in the situation of interest here. We will accordingly take a phenomenological approach, simply discarding from (12) the factors with the effective masses and treating a_n, a_p, and b as simply parameters which are not necessarily the same as those in Eqs. (4) and (5)[1]; we will correspondingly ignore the temperature dependence of the effective masses in Eqs. (7) and (8). As we mentioned earlier, the quantities a_n and a_p are not necessarily small, but they could hardly exceed unity. Furthermore, expression (13) itself, like (5), could be meaningful under the conditions $q' > 0$, $q'' > 0$, and $p > 0$ only in limited intervals of the temperature and the densities, since we are working from the condition $E_g > T_0$. If $E_g \lesssim T_0$ (and this situation itself might be of some interest), the electron and hole gases cannot be regarded as nondegenerate. The behavior of the function $\psi(x, y, z)$ under the condition $E_g \lesssim T_0$ (or in the limit $E_g \to 0$ deserves a special study. We will restrict the present paper to a very simple approximation, assuming that in the limit $x \to \infty$ and/or $y \to \infty$, and $z^p \to \infty$ the function ψ, which is positive, has an upper bound ($\psi \leqslant \psi_\infty$), and its derivatives ψ'_x, ψ'_y and ψ'_z are nonnegative.

According to Eq. (5) and the relationship betwen ψ and $\overline{\psi}$ specified above, the parameter ψ_∞ must satisfy the condition

$$\psi_\infty \leqslant (E_g - T_0)(\hbar\omega - E_g)^{-1}. \tag{13'}$$

Equations (1), (6a), and (6b) take the following form in terms of the new variables:

$$z\dot{x} + x\dot{z} = -\frac{z^2(x-y)}{\tau_{np}(x\mu_n + y\mu_p)^{3/2}} - \frac{zx^{1/2}(x-1)}{\tau_n} + \overline{G}_n, \tag{14}$$

$$z\dot{y} + y\dot{z} = \frac{z^2(x-y)}{\tau_{np}(x\mu_n + y\mu_p)^{3/2}} - \frac{zy^{1/2}(y-1)}{\tau_p} + \overline{G}_p, \tag{15}$$

$$\dot{z} = (z_0^2 - z^2)\tau_r^{-1}, \tag{16}$$

where

$$\tau_{np} = 3T_0^{3/2}\varepsilon^2 (m_n + m_p)^{3/2} \times (\alpha/g_0)^{1/2} [8Le^4 (2\pi m_n m_p)^{3/2}]^{-1}, \tag{17}$$

$$\mu_{n,p} = m_{p,n}(m_n + m_p)^{-1},$$

$$\tau_{n,p} = 3\pi^4\hbar^4\rho[32E_{n,p}^2 (2m_{n,p}^5 T_0)^{1/2}]^{-1} f, \tag{18}$$

$$\tau_r = (g_0\alpha)^{-1/2}, \quad z_0^2 = (1+\psi)^{1/2+q}, \tag{19}$$

$$\overline{G}_{n,p} = 2(\hbar\omega - E_g)\lambda_{np}(3\tau_r T_0)^{-1}. \tag{20}$$

We see from (18) that a situation corresponding to the condition $\tau_p < \tau_n$ is quite probable, and it is this situation which we will be discussing below.

A frequently encountered situation is

$$\tau_r |z_0^2 - z^2|^{-1} \gg z^{-2}\tau_{np}, \quad z^{-1}\tau_n, \quad z^{-1}\tau_p. \tag{21}$$

If the condition $(\hbar\omega - E_g)/T_0 \gtrsim 1$ also holds, we can use the adiabatic approximation, treating z as a slowly varying quantity. We can therefore begin with a "truncated" version of system (14), (15), treating z there as some arbitrary given number.

§ 3. SINGULARITIES OF THE TRUNCATED SYSTEM OF EQUATIONS

We begin by showing that infinity is absolutely unstable in this case. The truncated system of equations is

$$\begin{aligned} \dot{x} &= -z\tau_{np}^{-1}(x-y)(x\mu_n + y\mu_p)^{-3/2} - \tau_n^{-1}x^{1/2}(x-1) + z^{-1}\overline{G}_n \equiv P(x, y), \\ \dot{y} &= z\tau_{np}^{-1}(x-y)(x\mu_n + y\mu_p)^{-3/2} - \tau_p^{-1}y^{1/2}(y-1) + z^{-1}\overline{G}_p \equiv Q(x, y). \end{aligned} \tag{22}$$

Multiplying the first and second equations in (22) by x and y, respectively, and summing the results, we find the following in the limit $x \to \infty$ and/or $y \to \infty$ (we are using the arguments of §2):

$$\frac{1}{2}\frac{d}{dt}(x^2 + y^2) \to -(\tau_n^{-1}x^{5/2} + \tau_p^{-1}y^{5/2}) < 0. \tag{23}$$

This proves our assertion of instability. It is easy to see that this assertion also applies to the complete system (14)–(16).

The singularities in the finite part of the x, y phase plane are determined by the equations

$$P(x, y) = 0, \quad Q(x, y) = 0. \tag{24}$$

The solutions of these equations, x_s and y_s, determine the steady-state electron and hole temperatures. It follows from (23) that all the singularities in the finite part of the plane can be circled by a limit cycle without any points of tangency; all the integral curves which intersect the cycle are directed inward. It follows that the number of singularities must be an odd number; if there is a single singularity then it would have to be a node or a focus, as expected, while if there are three singularities then one will be a saddle point and the two others will be nodes or foci.[9]

Equations (24) can be solved easily in the case of a slight heating, in which case we can set

$$x_s = 1 + \xi, \quad y_s = 1 + \eta \tag{25}$$

under the conditions $\xi \ll 1$ and $\eta \ll 1$, and we can ignore the

dependence of $\overline{G}_n$ and $\overline{G}_p$ on x and y. We find

$$\xi=(\overline{G}\tau_{np}^{-1}\tau_n\tau_p+z^{-1}\tau_n\overline{G}_n)\Delta^{-1},$$
$$\eta=(\overline{G}\tau_{np}^{-1}\tau_n\tau_p+z^{-1}\tau_p\overline{G}_p)\Delta^{-1},$$
$$\overline{G}=\overline{G}_n+\overline{G}_p, \quad \Delta=1+z(\tau_n+\tau_p)/\tau_{np}. \tag{26}$$

Under the conditions $\tau_p \ll \tau_n, \tau_{np}$, the condition for a slight heating reduces to the inequality

$$z^{-1}(\tau_n/\tau_r)(\hbar\omega-E_g)/T_0 \ll 1. \tag{27}$$

The appearance of the factor $(\hbar\omega - E_g)/T_0$ should not be surprising: It is the relationship between T_0 and the difference between the photon energy and the gap width which determines the heating of the electron gas.

We turn now to the case of a more pronounced heating of the electrons, with

$$z^{-1}\tau_n\overline{G}_n \geqslant 1, \tag{28}$$

and the function ψ on the right side of (12) cannot be ignored. We begin by showing that Eqs. (24) impose certain restrictions on the electron and hole temperatures. Using (20), we easily see that under the conditions $x_s > y_s$ we have the inequalities

$$y_s^{1/2}(y_s-1) > \frac{2\tau_p}{3z\tau_r}\frac{\hbar\omega-E_g}{T_0} f\lambda_p,$$
$$x_s^{1/2}(x_s-1) < \frac{2\tau_n}{3z\tau_r}\frac{\hbar\omega-E_g}{T_0} f\lambda_n, \tag{29}$$

and the function $f \geqslant 1$ has an upper bound.

The meaning of inequalities (29) is obvious: The lower temperature (the hole temperature) must still exceed T_0, strictly speaking, while the higher temperature (the electron temperature) remains bounded. If $\tau_p \ll \tau_r$, the first of these restrictions is inconsequential, however, and under the conditions $\tau_p \ll \tau_{np}$ and $m_n \ll m_p$ we may assume $y_s - 1 \ll 1$, provided that the ratio $(\hbar\omega - E_g)/T_0$ is not too large. Incidentally, in this situaton we can assume $x_s > y_s$ as long as inequality (27) does not hold.

If $y_s - 1 \ll 1$, the first of Eqs. (24) becomes

$$\varphi(x_s) \equiv \lambda(x_s-1)x_s^{-1/2} + x_s^{1/2}(1-x_s^{-1}) = C(x), \tag{30}$$

where

$$\lambda = z\tau_n\tau_{np}^{-1}, \quad C(x) = \tau_n\overline{G}_0 f(x) z^{-1} \equiv \frac{2\tau_n(\hbar\omega-E_g)}{3z\tau_r T_0} f, \tag{31}$$

and we have introduced the notation $f(x) = f(x, 1; z)$ for brevity. The quantity $\overline{G}_0$ is determined by the second relation in (31). Here we do not need the second equation, which determines the small difference $y_s - 1$. In accordance with the discussion above, the function $C(x)$ is monotonic in the region of interest here (if $q' > 0$).

At $x_s < 1$, $x_s = 1$, and $x_s \to \infty$ we have $\varphi < 0$, $\varphi = 0$, and $\varphi \to \infty$, respectively. Since the function $C(x)$ is bounded, $C_{\min} \leqslant C(x) \leqslant C_{\max}$, Eq. (30) has an odd number of roots in the region of interest here. The actual number of roots depends on the parameter λ: If $\lambda < \lambda_c = 169$, there is only one singularity, which is a node or focus. At $\lambda = \lambda_c$, two real roots of the derivative φ' arise: x_1' and x_2', both greater than unity. Correspondingly, the function $\varphi(x)$ becomes nonmonotonic at $x > 1$. If the inequalities

$$\varphi(x_2') \leqslant C_{min} = z^{-1}\tau_n\overline{G}_0(1+\alpha_n+\alpha_p+\beta)^{1/2+q},$$
$$\varphi(x_1') \geqslant C_{max} = z^{-1}\tau_n\overline{G}_0(1+\psi_\infty)^{1/2+q} \tag{32}$$

also hold, Eq. (30) has three real roots in this interval. Under these conditions, the value $\lambda = \lambda_c$ determines a bifurcation point. The node or focus which exists at $\lambda < \lambda_c$ converts into a saddle point ($x_s = x_2$), and two more singularities appear. These new singularities are nodes or foci; one of them, $x_s = x_1$, lies to the left of the point x_1', while the other, $x_s = x_3$, lies to the right of x_2'.

Under the conditions $\lambda \gg \lambda_c$ and $\lambda - \lambda_c \ll \lambda_c$ we find, respectively,

$$x_2' = (\lambda/3)^{1/2} + 9 + O(\lambda^{-1/2}), \quad x_1' = 3,$$
$$\varphi(x_2') = 1{,}92\lambda^{3/4} + 4{,}43\lambda^{1/4}, \quad \varphi(x_1') = 0{,}385\lambda \tag{33a}$$

and

$$x_2' = 4{,}22 + 7{,}5\eta, \quad x_1' = 4{,}22 - 7{,}5\eta, \tag{33b}$$
$$\varphi(x_2') = 69{,}1 - 0{,}175\eta^{3/2},$$
$$\varphi(x_1') = 69{,}1 + 0{,}175\eta^{3/2}, \quad \eta = (\lambda-\lambda_c)\lambda_c^{-1}.$$

As expected, inequalities (32) define a rather narrow interval in the case (33b); the case (33a), on the other hand, requires either an intense photoinjection of charge carriers or the condition $\tau_n \gg \tau_{np}$.

Since the second of inequalities (32) is based on a very simple model ($\psi \leqslant \psi_\infty$), it may not be altogether convenient. However, this inequality is not necessary if the point x_3 also lies in the region in which Eq. (3) is applicable (in the case the quantity $\alpha_n x_3^{q'}$ may also be greater than unity). If $q' = q'' = 1$ and $\lambda \gg \lambda_c$, the situation is the same for direct optical transitions, provided that the following inequalities hold:

$$\frac{\hbar\omega-E_g}{E_g}(\tau_n\overline{G}_0)^{-3/2} < \alpha_n < \min\left\{2\lambda^{-1/2}, \frac{\hbar\omega-E_g}{E_g}\left(\frac{4}{\tau_n\overline{G}_0}\right)^{3/2}\right\}. \tag{32a}$$

In this case we have

$$x_3 = [\lambda(\overline{G}_0\tau_n\alpha_n^{3/2} - 1)^{-1}]^{1/2}. \tag{34a}$$

If

$$2\lambda^{1/2} < [(\hbar\omega-E_g)/E_g](4/\tau_n\overline{G}_0)^{2/3},$$

then inequalites (32a) can be satisfied only under the auxiliary condition

$$[(\hbar\omega-E_g)/E_g](4/\tau_n\overline{G}_0)^{2/3} < 2^{7/3}\lambda^{-1/2}.$$

On the other hand, it is not difficult to show that in the case of indirect optical transitions (and under the conditions $q' = q'' = 1$, as before), the point x_3 must lie outside the range of applicability of Eq. (13). This situation arises under the inequalities

$$(\tau_n\overline{G}_0)^{3/2} > \left(\frac{\lambda}{3}\right)^{1/2}\left(\frac{\hbar\omega-E_g}{\hbar\omega-T_0}\right)^2,$$
$$(\tau_n\overline{G}_0)^{3/2} \gg \max\left\{\left(\frac{\hbar\omega-E_g}{\hbar\omega-T_0}\right)^2, \frac{(\hbar\omega-E_g)^2}{E_g(E_g-T_0)}a_n^{-1}\right\} \tag{32b}$$

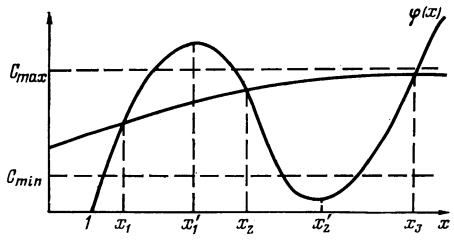

FIG. 1. Sketch of the function $\varphi(x)$ under conditions (32).

Here we have

$$x_3=(\tau_n\bar{G}_0)^{3/5}(\hbar\omega-T_0)^2/(\hbar\omega-E_g)^2. \tag{34b}$$

Under conditions (32), (32a), and (32b), with $\lambda>169$, this system thus has three stationary states, with different electron temperatures x_1, x_2, and x_3 (Fig. 1). In this connection we note three circumstances.

First, we have been working on the basis of only some rather general properties of the function $\psi(x, y, z)$, not the specific form of this function. The assumption that the heating of the holes is slight is also not crucial: A generalization to higher hole temperatures requires only some more complicated calculations, but the ultimate results are not changed significantly. Furthermore, inequalities (32) may hold even if we ignore the dependence of the gap width and of the effective masses on the electron and hole temperatures [in which case we would have $f=1$ and $C(x)=\text{const}$]. The role played by the dependence of E_g on the electron temperature and on the carrier density becomes clear, however, when we examine the stability of the singularities and also—a particularly important point—when we analyze Eq. (16) [§5].

Second, according to our arguments one of the three singularities ($x=x_2$) is a satellite point, so that there is no corresponding stationary state. The two other singularities correspond to nodes or foci.

Third, the number and positions of the stationary states of the system are controlled by the frequency and intensity of the light.

§ 4. STABILITY OF THE SINGULARITIES

The question of the stability of a singularity arises if the singularity is a node or focus. Carrying out a standard linearization of Eqs. (22) around a singular point (x_s, y_s) of this type, we conclude that the singularity is stable (unstable) if $\Gamma<0$ ($\Gamma>0$), where

$$2\Gamma=P_x'+Q_y', \tag{35}$$

and x and y should be replaced by x_s and y_s.

Since infinity is absolutely unstable, the condition $\Gamma>0$ may imply the appearance of a stable limit cycle, i.e., self-oscillations of the electron and hole temperatures (in the absence of other—stable—singularities, this would necessarily be the case).

Using Eqs. (22) and (24), we find

$$2\Gamma=Az/\tau_{np}+B/\tau_n+C/\tau_p, \tag{35'}$$

where (we are omitting the subscripts "s" from x and y for brevity)

$$A=(x\mu_n+y\mu_p)^{-1/2}[(1{,}5+q)(x-y)(\psi_x'-\psi_y')(1+\psi)^{-1}$$
$$-2+1{,}5(x-y)(\mu_n-\mu_p)(x\mu_n+y\mu_p)^{-1}],$$
$$B=(1{,}5+q)x^{1/2}[(x-1)\psi_x'(1+\psi)^{-1}-(1-1/3x)(1+{}^2/_3q)^{-1}],$$
$$C=(1{,}5+q)y^{1/2}[(y-1)\psi_y'(1+\psi)^{-1}-(1-1/3y)(1+{}^2/_3q)^{-1}].$$

It is easy to see that in the case of a slight heating we would have $\Gamma<0$, as expected. The situation is the same in the limit $x\to\infty$, $y\to\infty$. By virtue of the assumed boundedness of the function ψ, the derivatives, ψ_x' and ψ_y' must tend to zero (the inverse relationship disappears). It is meaningful, however, to consider the case in which, on the one hand, the heating is intense ($x\gg y\gg 1$) while, on the other, Eqs. (12) and (13) still hold. We assume that the parameter $\lambda=z\tau_n/\tau_{np}$ is not too large (thereby eliminating the case of three singularities). The only singularity in whose stability we are interested is therefore a node or focus.

Under these assumptions the term Az/τ_{np} on the right side of (35') can be ignored, and if $q'=q''=1$ the condition for the instability of the singularity becomes

$$a<{}^2/_3q,\quad 1+\sigma a>(1+a)({}^2/_3q-a)^{-1}[\rho+\sigma(1-a)], \tag{36}$$

where

$$\sigma=\frac{\alpha_p y}{\alpha_n x},\quad a=\frac{\tau_n}{\tau_p}\left(\frac{y}{x}\right)^{1/2},\quad \rho=\frac{1+\beta z^p}{\alpha_n x}. \tag{37}$$

The quantities σ and a can be calculated from Eqs. (24) (the terms with z/τ_{np} are ignored). Using (18), we find

$$\sigma=\frac{\alpha_p m_n^{3/5}E_n^{4/5}}{\alpha_n m_p^{3/5}E_p^{4/5}},\quad a=\left(\frac{m_pE_p}{m_nE_n}\right)^{4/5}. \tag{37'}$$

We see from (36) that no instability occurs if $q=0$. If $q=1.5$, condition (36) reduces to the system of two inequalities

$$a<(1-\sigma-\rho)(1-\sigma+\rho),\quad \rho+\sigma<1. \tag{38}$$

We will now show, however, that these inequalities cannot be satisfied. According to (24), with $(z/\tau_{np})x^{-1/2}\ll 1$, and $q=1.5$ we have

$$(\alpha_n x)^{1/2}=c(1+\beta z^p)+c(1+\sigma)\alpha_n x,\quad c=\alpha_n^{1/2}(z^{-1}\tau_n\bar{G}_0)^{1/2},$$

and in the limit $c\to 0$ we have $\alpha_n x\to 0$. Hence

$$(\alpha_n x)^{1/2}=\{1-[1-4c^2(1+\beta z^p)(1+\sigma)]^{1/2}\}$$
$$\times[2c(1+\sigma)]^{-1}\leqslant[2c(1+\sigma)]^{-1},$$

where the equality holds under the condition

$$c^2(1+\sigma)(1+\beta z^p)={}^1/_4.$$

We thus find

$$\rho>(1+\beta z^p)4c^2(1+\sigma)=1,$$

in contradiction of (38).

Under the assumptions adopted above, the unique singularity of the truncated system of equations must necessarily be stable. This result means that for any fixed value of z there will be no spontaneous oscillations of the electron and hole temperatures. A more interesting situation arises when we take into account the time dependence of the electron

density under conditions such that there are three singularities in the truncated system of equations. This case is discussed in the following section.

§ 5. TIME DEPENDENCE OF THE ELECTRON DENSITY

We turn now to Eq. (16), considering case (32), in which there are three singularities in the truncated system of equations, and we have $q = 1.5$ (indirect transitions). In the adiabatic approximation, adopted above, the quantity z_0^2 is given by the second of Eqs. (19), in which x and y should be understood as the coordinates x_s and y_s of one of the stable singularities. We then find two equations, which we assign indices $i = 1, 3$, denoting the values of x and y in them as $x_i(z)$ and $y_i(z)$. We then find, in place of (16), the equations

$$\tau_r z=[1+\psi_i(z)]^2-z^2\equiv R_i(z), \tag{39}$$

where the functions $R_i(z)$ are determined by this equation. According to §3, ψ_1 can be found from (13) by replacing x, y by $x_1(z), y_1(z)$ and by setting $\psi_3 = \psi_\infty$.

The singularities of the complete system of equations are found from equations

$$R_1=0, \quad R_3=0.$$

Generally speaking, however, these equations may not be satisfied. In particular, we might have the case

$$R_1<0, \quad R_3>0. \tag{40}$$

Sufficient conditions for this case with $\lambda\geqslant\lambda_c$ can be found easily by noting that we have $x_1\leqslant 3$ and $x_3\geqslant(\lambda/3)^{1/2}$ according to (33a), and we can use Eq. (30) to find upper and lower estimates of z. We find

$$\tau_{np}(\hbar\omega-E_g)/\tau_r T_0>\beta^{(1-3p)/2(1-p)}, \tag{41a}$$

where

$$\beta^{2(1-2p)/(1-p)}\gg 1 \tag{41b}$$

and

$$\beta>[(E_g-T_0)/(\hbar\omega-E_g)]^{1-p}. \tag{41c}$$

Condition (41b) was adopted to simplify the final results; inequalities (41a) and (41c) ensure the satisfaction of the first and second inequalities in (40), respectively. In this case we find spontaneous oscillations of the electron and hole temperatures, with a period apparently on the order of τ_r. Let us assume that at some initial time the state of the system corresponds to the point $x = x_1$; i.e., z decreases as time elapses. According to Eq. (30), as z decreases the value of x_1 will increase, ultimately reaching x_1'. There will be an abrupt transition (Fig. 1) to the point x_3, and by virtue of (40) the value of z will begin to increase, while x_3 will decrease. When x_3 reaches x_2', there will be an abrupt transition to the state $x = x_1$, etc.

Let us examine inequalities (41a)–(41c) in somewhat more detail for the case $p = 1/4$, which corresponds to Ref. 5 (see the Appendix). In this case we have $(1-3p)/2(1-p) = 1/6$, and for all reasonable values of $\beta(\gg 1)$ the right side of (41a) can be replaced by unity without introducing any great error. This inequality nevertheless remains the most restrictive one, since we would most frequently have $\tau_{np}\ll\tau_r=(g_0\alpha)^{-1/2}$. At the highest energy flux densities and lowest temperatures which are actually attainable, inequality (41a) could apparently be satisfied only in a narrow-gap semiconductor (with a gap width no more than a few hundreths of an electron volt).

Inequalities (41b) and (41c) are considerably less stringent. We first note that under the conditions assumed here we would have $E_g - T_0\approx E_g\gg\hbar\omega - E_g$. We thus need to examine only the second of these conditions. At $p = 1/4$, it can easily be rewritten

$$bn_0^{1/4}>[(\hbar\omega-E_g)/E_g]^{1/4}, \tag{42}$$

where, as before, $n_0=(g_0/\alpha)^{1/2}$.

For a rough estimate of the parameter b we will use (for lack of a better approach) Eq. (A3), found from the perturbation theory of Ref. 5. We thus write

$$bn_0^{1/4}=0.016T_0^{1/4}E_g^{-1}[n_0\cdot 10^{-18}(12\varepsilon^{-1})^3]^{1/4}, \tag{43}$$

where E_g and T_0 are expressed in electron volts, and n_0 in particles per cubic centimeter. Noting that the condition $T_0\ll E_g$ implies $\hbar\omega - E_g\geqslant T_0$, we can rewrite inequality (42) as

$$T_0(\hbar\omega-E_g)^{-1}>(E_gE_{g,\mathrm{cr}}^{-1})^3, \tag{44}$$

where

$$E_{g,\mathrm{cr}}=0.00403[n_0\cdot 10^{-18}(12\varepsilon^{-1})^3]^{4/3}\ \text{eV}\quad E_g\leqslant E_{g,\mathrm{cr}}.$$

In particular, with $\hbar\omega - E_g = 2T_0$, $n_0 = 10^{18}$ cm^{-3}, and $\varepsilon = 12$, inequality (44) yields $E_g\leqslant 0.0032$ eV.

§ 6. PHYSICAL CONSEQUENCES

For obvious reasons, spontaneous oscillations of the electron and hole temperatures must imply oscillations of any other kinetic and optical properties which are related to these temperatures. We would expect, for example, to find oscillations in the current in the circuit connecting the sample to the dc power supply (or in a voltage in this circuit), and we would expect to find oscillations in the absorption and reflection coefficients for intraband absorption of light.

Furthermore, case (32) seems to be interesting even if condtions (40a) and (40b) do not hold, and the complete system of equations has two singularities which are stable (on a small scale). Here we would expect to find flip-flop effects. Formally, the initial conditions determine just which of these two states will occur experimentally (Fig. 2), but in practice the initial conditions can be adjusted, by varying the frequency and intensity of the light, for example.

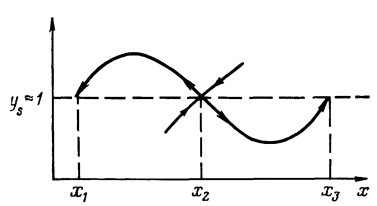

FIG. 2. Phase plane of a system with three singularities under conditions such that the singularities x_1 and x_3 are stable. The curves nominally show separatrices of a saddle point (for simplicity it is assumed that the condition $y_s - 1\ll 1$ holds for all three singularities).

APPENDIX

The effect of the electron-electron interaction on the energy distribution of the electrons (or holes) in a nondegenerate semiconductor was analyzed in Refs. 4 and 5. If the interband interaction is ignored, the equations derived in those papers can be used immediately to find the corresponding change in the gap width. We are interested here in the case with $T_n \approx T_p \approx T_0$ and $m_p \ll m_n$.

We denote by $r_0 = (\varepsilon T_0/8\pi ne^2)^{1/2}$ the plasma frequency. We introduce the parameter

$$\lambda = e^2 T_0/\varepsilon r_0 \hbar^2 \omega_0^2 \equiv a_B^{-1}(\varepsilon T_0/2\pi ne^2)^{1/2},$$

where $a_B = \varepsilon\hbar^2/m_n e^2$.

It is easy to see that the equations of Refs. 4 and 5 are valid under the conditions $\lambda \ll 1$ and $\lambda \gg 1$, respectively.

For these two cases we find, respectively,

$$\Delta E_g = -(\pi e^2\hbar^2/2\varepsilon T_0 m_n)\, n \equiv -|\Delta E_g|_1 \qquad \text{(A1)}$$

and

$$\Delta E_g = -2(e^2 T_0/\varepsilon r_0)^{1/2} \equiv -|\Delta E_g|_2. \qquad \text{(A2)}$$

Assuming $\Delta E_g = -E_g b n^p$, we find from (A2)

$$p = 1/4, \quad b = [2^7 \pi T_0 (e^2/\varepsilon)^3]^{1/4} E_g^{-1}. \qquad \text{(A3)}$$

The inequalities $\lambda \ll 1$ and $\lambda \gg 1$ eliminate an extremely interesting region of parameters (as expected!). For lack of a better approach, we might use the simple interpolation formula

$$|\Delta E_g| = \frac{\lambda}{\lambda+1}|\Delta E_g|_2 + \frac{1}{\lambda+1}|\Delta E_g|_1. \qquad \text{(A4)}$$

However, we know of no reliable experimental data for comparison with results from this formula.

[1)] If Eq. (13) holds at all experimentally, then it will be the coefficients a_n, a_p, and b introduced here which will be determined experimentally.

[1]V. V. Vladimirov and V. N. Gorshkov, Fiz. Tekh. Poluprovodn. **14**, 417 (1980) [Sov. Phys. Semicond. **14**, 247 (1980)].

Translated by Dave Parsons