Theory of double stimulated Mandel'shtam-Brillouin scattering in a plasma with a reflecting boundary

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A theory is developed of stimulated Mandel'shtam-Brillouin scattering (SMBS) in a plasma in the presence of a reflection surface for the pump wave and the scattered waves. In the model of a homogeneous layer of plasma with a reflecting rear wall, a five-wave process of joint scattering of two pairs of electromagnetic waves by a common sound wave (double SMBS) is shown to be an absolute parametric instability whose threshold lies below the familiar threshold for ordinary convective SMBS instability. In the case of oblique incidence of the pump wave, the double SMBS processes leads to collimated backward as well as specular reflection of the radiation. An exact nonlinear solution for the dependence of the nonliner reflection coefficient on the intensity of the incident radiation is found for the process with the lowest double SMBS backward threshold. It is shown that in the limit of high pump wave intensities, the reflection coefficient saturates at a level determined only by the specular reflection coefficient from a reflecting surface in the plasma. Inhomogeneity of the plasma density in a direction perpendicular to the reflection surface does not affect the double SMBS backward, but does lead to an increase in the double SMBS threshold in the specular direction.

1. Great interest in the study of stimulated Mandel'shtam-Brillouin scattering (SMBS) in a plasma is due to the fact that this process can lead to a limitation on the effectiveness of electromagnetic-energy input to the plasma. A characteristic feature of the interaction of intense radiation with a plasma is the presence in the plasma of a region of critical density, from which both the pump wave and the scattered electromagnetic waves are reflected. Thus, the SMBS process takes place in the field of two coherent pump waves—the incident and the reflected. At the same time, the coherent effect of the reflected component of the pump wave on the SMBS process in the plasma has not yet been investigated.

We shall show below that in the field of two pump waves, a process of double SMBS is possible, corresponding to the joint scattering of the incident and reflected components of the pump field from a common sound wave. Such a process represents an absolute parametric instability whose threshold is below the familiar threshold for ordinary convective SMBS instability. We emphase that although the threshold of the predicted phenomenon of double SMBS depends significantly on the reflection coefficient of the electromagnetic waves from the dense plasma layers and on the angle of incidence of the pump wave, it is always below the effective threshold of convective amplification in the case of ordinary SMBS.

In the present work, we have found the stationary regime of reflection of radiation in the case of double SMBS and have shown that this process consists in the fact that the ordinary specular reflection of the pump is replaced, with increase in pump intensity, by back reflection at the frequency of the Stokes satellite. Five waves take part in the interaction here: the two pump waves (incident and reflected), two Stokes waves (incident and reflected) and a single sound wave. Absolute instability arises because of the distributed feedback due to interaction of the opposing scattered waves through the common sound wave, and also because of lumped feedback, which consists of the reflection of electromagnetic waves from the dense layers of the plasma. Such a combination of feedbacks allows us to consider a certain analogy between double SMBS and the process of stimulated Raman scattering in the field of two pump waves in a nonlinear medium, in which five waves are also coupled. This process was considered by Akhmanov and Lyakhov.[1] However, the coupling of the waves that arises in Ref. 1 does not hold for the theory of SMBS in a plasma and does not allow us to obtain the nonlinear regime revealed by us below, of double SMBS with the lowest threshold of excitation. Processes of five-wave interaction were also considered in Refs. 2 and 3 for backward SMBS in the field of two antiparallel pump waves. However, the necessity of excitation of the anti-Stokes satellite distinguishes qualitatively the phenomena considered in Refs. 2 and 3 from the phenomena predicted by us, the conversion of the specular reflection of the pump into back reflection of the Stokes satellite.

At the same time, under conditions of reflection of electromagnetic waves from one of the boundaries of the region of interaction, as is shown in Ref. 3, an instability is realized that can also be called double SMBS and in which the maximum number of waves is coupled—eight, to wit, the incident and reflected components of the pump wave, the Stokes and the anti-Stokes satellites, and also two sound waves. Such an eight-wave interaction corresponds to two five-wave interactions, coupled by common pump waves and boundary conditions of specular reflection. As is shown below, under conditions of oblique incidence of the pump waves, a process of double SMBS that is similar to that considered in Ref. 3, with excitation of the anti-Stokes components, leads to scattering in the specular direction, but has a higher threshold than the five-wave double SMBS.

In this paper, we have also investigated the effect of the

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inhomogeneity of the density, which is characteristic for plasma, in a direction perpendicular to the reflecting surface, on the double SMBS process. It is shown that the density inhomogeneity does not affect backward double SMBS, but does lead to an increase in the threshold of specular double SMBS because of the narrowing of the region of spatial synchronism of the interacting waves.

2. As a simplest model, we consider a uniform plasma layer of thickness l $(0<x<l)$, one of the boundaries of which $(x=0)$ is transparent for electromagnetic waves, while the other $(x=l)$ partially reflects them in the specular direction. An s-polarized pump wave of frequency ω_0 is incident on such a layer from the left. In the approximation that is linear in the amplitude of the sound field, the SMBS process is described by the following pair of equations for the field

$$\mathscr{E}_z(\mathbf{r},t)=\operatorname{Re}E(\mathbf{r},t)e^{-i\omega_0 t}$$

and for the low-frequency perturbation of the density δn_e:

$$\left(2i\omega_0\frac{\partial}{\partial t}+c^2\nabla^2+\omega_0^2-\omega_{Le}^2\right)E=\omega_{Le}^2\frac{\delta n_e}{n_e}E,$$
$$\left(2\gamma_s\frac{\partial}{\partial t}+\frac{\partial^2}{\partial t^2}-v_s^2\nabla^2\right)\frac{\delta n_e}{n_e}=\frac{v_s^2}{16\pi n_c\varkappa_B T}\nabla^2|E|^2, \tag{2.1}$$

where ω_{Le} is the electron Langmuir frequency, n_e is the plasma density, n_c is the critical density, c is the speed of light, v_s is the speed of sound, γ_s is the damping decrement of the sound wave, $\varkappa_B$ is Boltzmann's constant and T is the temperature.

The double SMBS process corresponds to the scattering of two pairs of electromagnetic waves by a single sound wave. For the case of oblique incidence of the pump wave, this is possible if the sound propagates along the surface of the plasma layer (along the y axis). Such sound is excited in the scattering of the pump wave with wave vector $\mathbf{k}_0=(k_0\cos\theta,\ k_0\sin\theta)$ $(k_0=(\omega_0/c)\ (1-n_e/n_c)^{1/2}$ is the wave number of the pump into a Stokes scattered wave with wave vector $\mathbf{k}_{-1}=(k_0\cos\theta,\ -k_0\sin\theta)$ and also in the scattering of a specularly reflected pump wave $\mathbf{k}_0'=(-k_0\cos\theta,\ k_0\sin\theta)$ into a reflected Stokes wave $\mathbf{k}'_{-1}=(-k_0\cos\theta,\ -k_0\sin\theta)$. It is obvious that this process corresponds to stimulated backward reflection, since the scattered Stokes waves have a y component of the wave vector that is opposite in sign to the pump wave and, consequently, they are propagated counter to it. In accord with what was said above, we represent the high-frequency field R and the low-frequency perturbation of the density δn_e in the form

$$E(x,y,t)=\sum_{\sigma=\pm1}[E_{0\sigma}(x,t)\exp(i\sigma k_0x\cos\theta+ik_0y\sin\theta)$$
$$+E_{-1\sigma}(x,t)\exp(i\sigma k_0x\cos\theta-ik_0y\sin\theta+i\omega t)],$$
$$\delta n_e(x,y,t)=-in_e v_1(x,t)\exp(2ik_0y\sin\theta-i\omega t)+\text{c.c.} \tag{2.2}$$

The index $\sigma=+1$ corresponds to waves traveling in the direction of the pump wave, $\sigma=-1$ to opposite waves, $\omega\simeq 2k_0v_s\sin\theta$ is the sound frequency. The boundary conditions for the electric field

$$E_{01}(0,t)=E_0,\quad E_{-11}(0,t)=0;$$
$$E_{\mu-1}(l,t)=re^{i\varphi}E_{\mu1}(l,t)\quad(\mu=0,-1) \tag{2.3}$$

take partial reflection $(0<r\leqslant 1)$ on the rear boundary of the layer into account. In addition, we consider the limit of a sufficiently strong attenuation of the sound, when the derivatives in the acoustic equation (2.1) with respect to x can be neglected (the corresponding condition is obtained in Sec. 3). We then obtain from (2.1) the set of simplified equations:

$$\sigma\frac{\partial}{\partial x}E_{0\sigma}=-\frac{\alpha k_0}{2\cos\theta}E_{-1\sigma}v_1,\quad \sigma\frac{\partial}{\partial x}E_{-1\sigma}=\frac{\alpha k_0}{2\cos\theta}E_{0\sigma}v_1^*, \tag{2.4}$$
$$\left(\frac{\partial}{\partial t}+\gamma_s-i\Delta\omega\right)v_1=\frac{\omega}{32\pi n_c\varkappa_B T}(E_{01}E_{-11}^*+E_{0-1}E_{-1-1}^*), \tag{2.5}$$

where $\alpha=n_e/(n_c-n_e)$, $\Delta\omega=\omega-2k_0v_s\sin\theta$, and the time derivatives are neglected in (2.4) since $v_s/c\ll 1$.

There is significant interest in the stationary solution of the set of nonlinear equations (2.4) and (2.5). They allow us to obtain the values of the coefficient of back reflection: $R_{-1}=E_{-1-1}(0)/E_0$, and also for reflection in the specular direction: $R_0=E_{0-1}(0)/E_0$.

The set of equations (2.4) has three first integrals, which can be written in the following form with the help of the boundary conditions (2.3):

$$|E_{01}|^2+|E_{-11}|^2=|E_0|^2,$$
$$|E_{0-1}|^2+|E_{-1-1}|^2=|E_0|^2(|R_0|^2+|R_{-1}|^2)=|E_0|^2r^2,$$
$$E_{01}E_{-1-1}+E_{0-1}E_{-11}=E_0^2R_{-1}. \tag{2.6}$$

We note that even this indicates that the reflection coefficients R_0 and R_{-1} cannot exceed the limiting value r that is determined by the reflection coefficient of the rear wall. The equations (2.6) allow us to transform from the system (2.4) to an equation for the function

$$u(x)=(E_{01}E_{-11}^*+E_{0-1}E_{-1-1}^*)/|E_0|^2,$$

that enters in the right side of Eq. (1.5), which has the form

$$\frac{du}{dx}=\frac{\alpha k_0}{\cos\theta}v_1\left[\frac{(1-r^2)^2}{4}+|R_{-1}|^2-|u|^2\right]^{1/2},$$
$$u(0)=R_0R_{-1}^*,\qquad |u(l)|^2=\frac{|R_{-1}|^2}{4r^2}(1+r^2)^2. \tag{2.7}$$

The solution of the boundary-value problem (2.7), supplemented by stationary solution of Eq. (12.5) at $\Delta\omega=0$,

$$v_1=\omega u(x)|E_0|^2/32\pi n_c\varkappa_B T\gamma_s,$$

leads to the equation

$$\exp(2\varkappa q)=\frac{2(r^2-|R_{-1}|^2)^{1/2}[2qr-(1-r^2)(r^2-|R_{-1}|^2)^{1/2}]}{(1+r^2)(2q-1+r^2-2|R_{-1}|^2)}, \tag{2.8}$$

that determines the reflection coefficient $|R_{-1}|^2$. Here

$$q=[{}^1/_4(1-r^2)^2+|R_{-1}|^2]^{1/2},$$

while $\varkappa$ is the usual coefficient of convective SMBS amplification:

$$\varkappa=\frac{\alpha\omega Ik_0l}{8\gamma_s\cos\theta},\qquad I=\frac{|E_0|^2}{8\pi n_c\varkappa_B T}. \tag{2.9}$$

The simplest form of Eq. (2.8) occurs in the case $r=1$, when

$$|R_{-1}|=\text{th}(2\varkappa|R_{-1}|),\qquad |R_0|=1/\text{ch}(2\varkappa|R_{-1}|).$$

Figure 1 shows a graph of the dependence of $|R_{-1}|^2/r^2$ on $\varkappa$, i.e., on the thickness of the layer and on the intensity of the

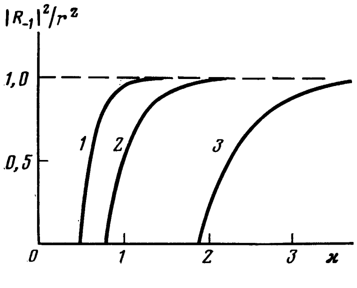

Fig. 1. Dependence of the coefficient of back reflection $|R_{-1}|^2/r_2$ on the parameter $\varkappa = \alpha k_0 I\omega l/8\gamma_s \cos\theta$ for $r = 1(1); 0.5(2); 0.1(3)$. Allowance for pump exhaustion.

radiation. The presence of the threshold of stimulated back reflection

$$\varkappa_{th} = \frac{1}{1-r^2}\ln\frac{1+r^2}{2r^2},$$

follows from (2.8). It achieves its minimum value $\varkappa'_{\min} = 1/2$ at $r=1$. In the near-threshold region, we have $|R_{-1}|^2 \sim (\varkappa - \varkappa_{\rm th})$. The limitation of the back reflection upon significant excess over threshold is manifest in that $|R_{-1}|^2 \to r^2$ according to the law

$$|R_{-1}|^2 = r^2 - \left(1+\frac{1}{r^2}\right)\exp[-2\varkappa(1+r^2)].$$

Thus, Eq. (2.8) describes the switching over from specular reflection at the fundamental frequency to back reflection at the shifted frequency. The effectiveness of such switching increases with increase in the angle of incidence.

Even for a small reflection coefficient $r \ll 1$, the double SMBS threshold $\varkappa_{\rm th} \approx \ln(1/2r^2)$ is small in comparison with the logarithm of the ratio of the intensity of the pump to the intensity of the thermal fluctuations in the plasma. This allows us to suppose that it is precisely the double SMBS that is the reason for the back reflection in the plasma at moderate fluxes of laser radiation. Thus, for example, if the absorption is 95%, then $\varkappa_{\rm th} = \ln 10 \approx 2.3$, and at $\varkappa > \varkappa_{\rm th}$ the back-reflection coefficient should saturate at a level of $\sim 5\%$ (cf., for example, Ref. 4).

The solution of Eqs. (2.7) and (2.5) (at $\partial/\partial t = 0, \Delta\omega = 0$) allows us to find the distribution of the sound field in the layer:

$$u(x) = 2qs\exp(-2q\varkappa x/l)\,[1+s^2\exp(-4q\varkappa x/l)]^{-1},$$
$$s = [q + {}^1/_2(1-r^2) + |R_{-1}|^2]/|R_0 R_{-1}|^2.$$

The amplitude of the sound field increases monotonically with the coordinate, reaching its maximum value

$$|v_1(l)| = \frac{I}{8}\frac{\omega}{\gamma_s}\frac{|R_{-1}}{r}(1+r^2) \tag{2.10}$$

at the rear boundary of the layer. Here the intensity of the pump wave falls off monotonically from $|E_0|^2$ at $x=0$ to

$$|E_{01}(l)|^2 = \frac{1}{2}|E_0|^2[1+(1-|R_{-1}|^2/r^2)^{1/2}].$$

It is important to note that even under the conditions of maximum reflection ($R_{-1} = r$), the intensity of the pump at the rear boundary of the layer is less than half its intensity at the input. This means that the double SMBS does not hinder effective absorption of the pump wave in the plasma.

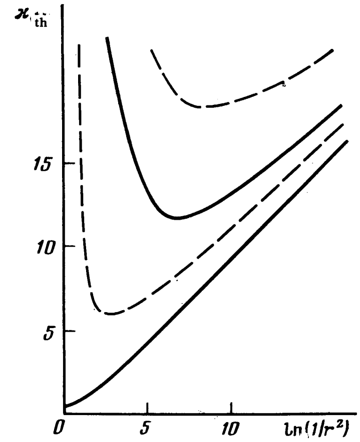

Fig. 2. Dependence of the thresholds of the absolute instability on the reflection coefficient r^2 at the rear boundary of the layer. The solid curves correspond to double SMBS backward, for the lower number of the excited mode, $n=0$, for the upper, $n = \pm 1$. The dashed curves correspond to double SMBS in the specular direction, for the lower curve, $n = 0$; 1, for the upper, $n = 2, -1$.

3. The phenomenon of stimulated reflection under conditions of strong sound damping that is discussed here corresponds to an absolute parametric instability if we assume the pump waves E_{01} and E_{0-1} to be specified.[1)] Actually, a test of the system (2.4) and (2.5) in the linear approximation [$E_{01} = E_0 = $ const, $E_{0-1} = r\exp(i\varphi)E_0$] for stability [$v_1 \propto \exp(\lambda\gamma_s t)$] leads to the following dispersion equation:

$$\varkappa\frac{1-r^2}{1+\lambda-i\Delta\omega/\gamma_s} = 2i\pi n + \ln\frac{1+r^2}{2r^2},$$

where n is an integer. The threshold of the absolute instability ($\lambda = 0$) which represents the threshold of generation of the Stokes component in the plasma layer, is given by the equation

$$\varkappa_{\rm th} = \frac{1}{1-r^2}\left[\ln\frac{1+r^2}{2r^2} + (2\pi n)^2\ln^{-1}\frac{1+r^2}{2r^2}\right],$$
$$\Delta\omega_{\rm th} = 2\pi n\gamma_s\ln^{-1}\frac{1+r^2}{2r^2}. \tag{3.1}$$

The nonlinear solution obtained above corresponds to the excitation of a mode with $n=0$ when $\Delta\omega = 0$. The presence of higher modes ($n \neq 0$, $\Delta\omega \neq 0$) indicates the possibility of nonstationary nonlinear states corresponding to scattering at the several frequencies $\omega_n = 2k_0 v_s \sin\theta + \Delta\omega_n$. It follows from (3.1) that for the mode with number $n \neq 0$ the maximum threshold is achieved at $r^2 \sim \exp(-2\pi|n|)$ and is equal to $\varkappa_{\min} \sim 4\pi|n|$. Graphs of the dependence of the threshold of the absolute instability, determined by Eq. (3.1), as functions of the reflection coefficient r for the modes with $n=0$ and $n=1$ are shown in Fig. 2.

The phenomenon of double SMBS is realized only for waves that are scattered almost along the direction of the pump ($\Delta\theta \sim \varkappa/k_0 l \ll 1$), when the beats between both pairs of electromagnetic waves excite a single sound wave, i.e., it corresponds to the collimated stimulated back reflection.

The condition for neglect of the spatial derivatives ($\partial^2/\partial x^2$) in the equations of ν_1 (2.5) has the form

$$8\gamma_s/\omega > (\alpha I/\sin\theta\cos\theta)^{2/3},$$

which indicates a violation of the strongly dissipative approximation at small incidence angles. We note that upon decrease in θ, the DSMBS threshold increases: $I_{\rm th} \propto \theta^{-2}$.

The considered effect is also preserved in a spatially inhomogeneous plasma, if its density depends only on the coordinate x. Since the x components of the wave vectors of the interacting electromagnetic waves are equal, the inhomogeneity of the density does not lead to a disruption of the spatial synchronism. All the formulas given above are preserved here with the replacement of $\varkappa$ (2.9) by the following expression:

$$\varkappa = \frac{1}{8} I_0 \frac{\omega}{\gamma_s} \frac{\omega_0 L_0}{c} \cos\theta_0 \ln\left(\frac{\omega_0 L_0}{c}\cos\theta_0\right)^{2/3},$$

$$\omega = 2\omega_0 \frac{v_s}{c} \sin\theta_0, \tag{3.2}$$

where $I_0 = E^2_{\rm vac}/8\pi n_c \varkappa_B T$ is the intensity of the pump field in vacuum, θ_0 is the angle of incidence of the pump wave, L_0 is the scale of the inhomogeneity of the plasma density at the turning point of the electromagnetic waves.

An inhomogeneous flow of plasma along the x axis has no effect on the considered phenomenon since the sound is propagated in the perpendicular direction and therefore does not experience a Doppler frequency shift.

The considered effect is preserved also when account is taken of the motion of the reflecting surface along the x axis. Such motion leads to equal Doppler shifts of frequencies of the reflected pump wave and Stokes satellite, keeping their difference unchanged. Since the ponderomotive force in (2.5) is created by the beats between the waves traveling into the interior the plasma and the reflected waves, its form does not depend on the velocity of the reflecting surface.

4. The nonlinear state found in Sec. 2 in the case of double SMBS corresponds to allowance for only a single nonlinear effect—exhaustion of the pump wave. In this section, we discuss the effect of the nonlinearity of the sound wave on the stimulated backward reflection. In a sufficiently dense plasma, in which the sound wavelength is much greater than the Debye electron radius $r_{D_e}(k_0 r_{D_e} \ll 1)$, the equation of δn_e in the set (2.1) must be replaced by the equation of nonlinear acoustics (see Refs. 7 and 8):

$$\left(2\gamma_s \frac{\partial}{\partial t} + \frac{\partial^2}{\partial t^2} - v_s^2 \nabla^2\right) \frac{\delta n_e}{n_e} = \frac{v_s^2}{16\pi n_c \varkappa_B T} \nabla^2 |E|^2 + v_s^2 \nabla^2 \left(\frac{\delta n_e}{n_e}\right)^2. \tag{4.1}$$

Account of the last term in this equation, which describes the generation of the higher harmonics of the sound, becomes significant if $|\delta n_e/n_e| > 2\gamma_s/\omega$. A comparison of this inequality with the estimate (2.1) of the maximum sound amplitude gives the condition for the necessity of consideration of the acoustic nonlinearity.

In particular, at $|R_{-1}| \sim r$, we have

$$I > 8(\gamma_s/\omega)^2. \tag{4.2}$$

Consequently, the exhaustion of the pump determines the stimulated reflection only in the case of relatively low intensities of the pump and relatively strong damping. In the opposite limit, the sound wave becomes strongly nonlinear almost immediately after exceeding the threshold of absolute instability. Here the nonlinear term on the right side of (4.1) turns out to be significantly larger than the linear terms on the left side. Therefore, following Ref. 8, we solve Eq. (4.1) with neglect of the left side when, for $\delta n_e(x, y - v_s t)$, we have

$$I \frac{\partial}{\partial y} \mathrm{Re}\{u(x)\exp(2ik_0 y\sin\theta - i\omega t)\} + \frac{\partial}{\partial y}\left(\frac{\delta n_e}{n_e}\right)^2 = 0. \tag{4.3}$$

As an additional condition for Eq. (4.3) we have the requirement of the constancy of the plasma density over the period of the sound field:

$$\int_0^{\pi/k_0\cos\theta} dy\, \delta n_e(x, y - v_s t) = 0. \tag{4.4}$$

The solution of (4.3) with the condition (4.4) gives a sawtooth sound wave (cf. Ref. 8):

$$\delta n_e = n_e (2I|u(x)|)^{1/2} \sin[k_0 y \sin\theta - {}^1/_2\omega t + {}^1/_2\psi(x) - n\pi],$$
$$\psi(x) = \arg u(x), \quad -\pi/2 < k_0 y\sin\theta - {}^1/_2\omega t + {}^1/_2\psi - n\pi < \pi/2. \tag{4.5}$$

According to Refs. 7 and 8, Eq. (4.5) must be made more precise close to the surface of discontinuity

$$k_0 y \sin\theta - {}^1/_2\omega t + {}^1/_2\psi = \pi(n + {}^1/_2).$$

However, this fact turns out to be insignificant for the problem of the value of the reflection coefficient of interest to us, since only ν_1 enters into the equation for the high-frequency field (2.7); here ν_1 is the amplitude of the fundamental of the nonlinear field (4.5):

$$\nu_1 = \frac{i}{\pi} k_0 \sin\theta \int_{a_0}^{a_0+\pi/k_0\sin\theta} da \frac{\delta n_e(x,a)}{n_e} \exp(-2ik_0 a\sin\theta) = \frac{4}{3\pi}(2I|u|)^{1/2} e^{i\psi}. \tag{4.6}$$

Substituting Eq. (4.6) in (2.7), we obtain a closed equation for $|u(x)|$:

$$\frac{d|u|}{dx} = \frac{w}{l}\left(\frac{2}{q}\right)^{1/2} [|u|(q^2 - |u|^2)]^{1/2}, \quad w = \frac{4\alpha k_0 l (qI)^{1/2}}{3\pi\cos\theta}. \tag{4.7}$$

The solution of this equation has the form

$$|u(x)| = q\,\mathrm{cn}^2\left(\eta - w\frac{x}{l}, \frac{1}{\sqrt{2}}\right), \tag{4.8}$$

where cn is the elliptical cosine. From (4.8), with account of the boundary conditions (2.7), we obtain a set of two equations for the determination of the constant η and the reflection coefficient $|R_{-1}|$ as functions of the parameter w:

$$q\,\mathrm{cn}^2\left(\eta,\frac{1}{\sqrt{2}}\right)=|R_{-1}|\,(r^2-|R_{-1}|^2)^{1/2},$$
$$q\,\mathrm{cn}^2\left(\eta-w,\frac{1}{\sqrt{2}}\right)=\frac{|R_{-1}|}{2r}(1+r^2), \tag{4.9}$$

where $0<\eta$, $\eta-\omega<K(1/\sqrt{2})$, K is a complete elliptic integral of the first kind. In the simplest case $r=1$, we get from (4.9)

$$|R_{-1}|^2=1-\mathrm{cn}^4(w,\,1/\sqrt{2}).$$

It must be emphasized that Eqs. (4.9) describe the dependence of the back reflection coefficient on the thickness of the layer only in the case in which the sound field obeys the equation of nonlinear acoustics (4.3), which corresponds to satisfaction of the inequality $u(x)>8(\gamma_s/\omega)^2/I$. This condition is violated at small reflection coefficients $|R_{-1}|<8(\gamma_s/\omega)^2/rI$, when we must use Eq. (2.8) instead of Eq. (4.9). It is important to emphasize that such a violation also occurs as $|R_{-1}|\to r$, i.e., upon a significant excess over threshold. Namely, at

$$(r^2-|R_{-1}|^2)^{1/2}<8(\gamma_s/\omega)^2/rI \tag{4.10}$$

the set of equations (4.9) determining $|R_{-1}|$ should be modified, since the sound amplitude is so small near the left boundary of the layer that the solution (4.5) is inapplicable, and we must use the expression $v_1=\omega Iu/4\gamma_s$ in Eq. (2.7), an expression that follows from (2.5). The solution of (2.7) in this case yields

$$|u(x)|=r(r^2-|R_{-1}|^2)^{1/2}\exp\left[(1+r^2)\varkappa x/l\right]. \tag{4.11}$$

The region described by such an approximation is bounded by the coordinate of the point x_0 at which the amplitude of the sound wave reaches the value $u(x_0)\approx 8(\gamma_s/\omega)^2/I$ and therefore the sound wave becomes strongly nonlinear. The coordinate x_0, in accord with (4.11), is determined by the relation

$$x_0\approx\frac{l}{\varkappa(1+r^2)}\ln\left[8\left(\frac{\gamma_s}{\omega}\right)^2(rI)^{-1}(r^2-|R_{-1}|^2)^{-1/2}\right]. \tag{4.12}$$

At $x>x_0$ the sound wave is described by the relation (4.8) with a certain value of the constant $\eta=\eta_1$ that differs from the value following from the set (4.9). The condition of matching the solutions (4.8) and (4.11) at the point x_0, and the boundary condition for the amplitude of the sound wave at the right end of the layer yield a pair of equations for the determination of the constant η_1 and of the reflection coefficient $|R_{-1}|$:

$$q\,\mathrm{cn}^2\left(\eta_1-w\frac{x_0}{l},\frac{1}{\sqrt{2}}\right)=\frac{8}{I}\left(\frac{\gamma_s}{\omega}\right)^2,$$
$$q\,\mathrm{cn}^2\left(\eta_1-w,\frac{1}{\sqrt{2}}\right)=\frac{|R_{-1}|}{2r}(1+r^2). \tag{4.13}$$

By virtue of the relation (4.2), the first of the equations (4.13) yields $\eta_1-wx_0/l\approx K(1/\sqrt{2})$, while it follows from the second [by virtue of (4.10)] that $\eta_1\propto w$. We then find the length of the region $l_*=l-x_0$, B in which the sound wave is strongly nonlinear:

$$l_*\approx 3\pi K(1/\sqrt{2})\cos\theta/2\alpha k_0[(1+r^2)I]^{1/2}. \tag{4.14}$$

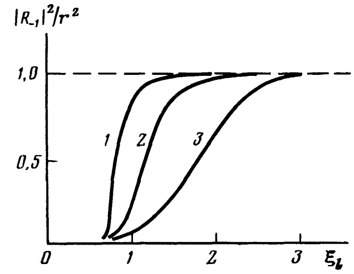

Fig. 3. Dependence of $|R_{-1}|^2/r^2$ on the parameter $\xi_1=4\alpha k_0 l\sqrt{I}/3\pi\cos\theta_0$ at $r^2=1(1)$; 0.5 (2); 0.1 (3). Allowance for generation of higher sound harmonics.

Upon satisfaction of (4.10) we have $x_0\gg l$, as is seen from (4.12) and (4.14). That is, on the greater part of the region the sound wave is linear, and only in a comparatively small region, adjoining the right boundary of the layer, do higher harmonics of the sound appear. The expression for the reflection coefficient $|R_{-1}|$ in the vicinity of its maximum value r, under the conditions of (4.10), has the form

$$|R_{-1}|^2=r^2-\left(\frac{8}{rI}\right)^2\left(\frac{\gamma_s}{\omega}\right)^4\exp[-2(1+r^2)\varkappa]. \tag{4.15}$$

Plots of the back reflection coefficient $|R_{-1}|^2$ as functions of the layer thickness l under conditions of a developed acoustic nonlinearity are shown in Fig. 3.

For estimates of the acoustic nonlinearity, we remark that the characteristic length of the layer l_B over which, because of the pump exhaustion, saturation of the reflections takes place, is found from the condition $\varkappa\sim 1$:

$$k_0 l_B\sim 8\gamma_s\cos\theta/\alpha\omega I.$$

The ratio $l_*/l_B\sim\sqrt{I}\,\omega/\gamma_s\gg 1$. Therefore, the back reflection coefficient tends to the saturation level $|R_{-1}|=r$ at significantly higher intensities of the pump in comparison with the case considered in the second section, in which only the pump source was taken into account.

5. Along with the double SMBS process considered above, which is connected with the excitation of sound along the plane of the layer, another process of double SMBS is also possible, associated with the excitation of a sound wave in the direction perpendicular to the plane of the layer. It corresponds to the scattering of electromagnetic waves with a change in the x component of the wave vector and, consequently, to radiation of scattered waves in the specular direction. In the case of normal incidence of the pump wave, such a process has already been considered in Ref. 3; it is characterized by the presence in the scattered radiation of not only Stokes, but also anti-Stokes satellites.

We present the high-frequency electric field in the plasma layer in the following form:

$$E_z(x,y,t)=\sum_{\sigma=\pm1}(E_{0\sigma}+E_{-1\sigma}e^{i\omega t}+E_{1,\sigma}e^{-i\omega t})$$
$$\times\exp(i\sigma k_0 x\cos\theta+ik_0 y\sin\theta). \tag{5.1}$$

The beats of the Stokes (E_{-10}) and anti-Stokes (E_{10}) waves with the pump wave excite the sound waves

$$\delta n_e = -in_e \sum_{\sigma=\pm 1} \nu_\sigma \exp(2i\sigma k_0 x \cos\theta - i\omega t) + \text{c.c.} \tag{5.2}$$

Under the assumption of sufficiently high sound damping, $\gamma_s/\omega > \sqrt{\alpha I}/4\cos\theta$, the following set of simplified equations follows from the set of equations (2.1) for the amplitude of the scattered fields (5.1) and the density perturbation (5.2):

$$\sigma\frac{\partial}{\partial x}E_{1\sigma} = -\frac{\alpha k_0}{2\cos\theta}E_{0-\sigma}\nu_\sigma, \qquad \sigma\frac{\partial}{\partial x}E_{-1\sigma} = \frac{\alpha k_0}{2\cos\theta}E_{0-\sigma}\nu_{-\sigma}^*; \tag{5.3}$$

$$\left(\frac{\partial}{\partial t} + \gamma_s - i\Delta\omega\right)\nu_\sigma = \frac{\omega}{32\pi n_c \varkappa_B T}(E_{1\sigma}E_{0-\sigma}^* + E_{0\sigma}E_{-1-\sigma}^*), \tag{5.4}$$

where $\Delta\omega = \omega - 2k_\theta v_s \cos\theta$. The boundary conditions for this set of equations are similar to (2.3):

$$E_{\pm 11}(0) = 0, \quad E_{\pm 1-1}(l) = re^{i\varphi}E_{\pm 11}(l), \quad E_{0-1}(l) = r_0 e^{i\varphi_0}E_{01}(l). \tag{5.5}$$

For purposes of calculation of the nonlinear properties of the reflection, it is assumed that the reflection coefficient of the intense pump wave $r_0 e^{i\varphi_0}$ can differ from the reflection coefficients of the weak scattered waves.

The set of equations (5.3) has two first integrals:

$$E_{1\sigma}E_{0\sigma}^* - E_{-1-\sigma}^* E_{0-\sigma} = \text{const.} \tag{5.6}$$

This allows us [by setting $\nu_0 \propto \exp(\lambda\gamma_s t)$, and by assuming a given field of the pump wave] to obtain the following dispersion equation from the set (5.3), (5.4) with the boundary conditions (5.5):

$$\operatorname{ch}\left(\frac{1+r_0^2}{1+\lambda-i\Delta\omega/\gamma_s}\varkappa\right) = -f = -\frac{1-2r^2r_0^2+r_0^4}{2r_0^2(1+r^2)}, \tag{5.7}$$

this determines the growth rate $\lambda\gamma_s$ and the frequency $\Delta\omega$ of the excited waves:

$$\begin{aligned} \lambda &= -1 + \varkappa(1+r_0^2)\ln U[\ln^2 U + (2n-1)^2\pi^2]^{-1}, \\ \Delta\omega &= (2n-1)\pi\varkappa\gamma_s(1+r_0^2)[\ln^2 U + (2n-1)^2\pi^2]^{-1}, \end{aligned} \tag{5.8}$$

where $U = f + (f^2-1)^{1/2}$ and n is an integer.

The instability threshold for a mode with number n is, according to (5.8), equal to

$$\varkappa_{th} = [\ln^2 U(r, r_0) + (2n-1)^2\pi^2]/(1+r_0^2)\ln U(r, r_0). \tag{5.9}$$

This expression corresponds for the case of normal incidence of the pump wave $\theta = 0$ to the result obtained in Ref. 3. The case of normal incidence ($\theta = 0$) and of the absence of reflection for the scattered waves ($r = 0$) was considered in Ref. 2.

It follows from Eq. (5.9) that the instability does not occur at all values of the reflection coefficients r_0 and r. The criterion of the possibility of the development of an instability is the inequality $f > 1$, which leads to the condition $r < (1 - r_0^2)/2r_0$. In particular, for equal reflection coefficients $r = r_0$, an instability is possible at $r_0^2 < 1/3$; under conditions of the absence of reflection of scattered waves ($r = 0$), an instability is possible at $0 < r_0 < 1$. Plots of the dependence of the threshold amplification coefficient $\varkappa_{th}$ (5.9) of $r_0^2 = r^2$ for the first few modes with the lowest thresholds ($n = 0$; 1 and $n = 2$; 1) are shown in Fig. 2. It is seen that these curves are located significantly above the threshold curves for double SMBS that corresponds to back reflection. The minimum threshold of "specular" double SMBS

$$\varkappa_{min} \approx 2\pi, \quad I_{min} \approx 16\pi\gamma_s\cos\theta/\alpha k_0 l\omega \tag{5.10}$$

is achieved under the condition $U = e^\pi$ or $r_{0*}^2 e^{-\pi}$. By virtue of the smallness of $r_{0*}^2 \ll 1$, the value of the minimun threshold (5.10) is practically independent of the quantity r and therefore, increasing with decrease in θ, it is identical at $\theta = 0$ with the minimum threshold obtained by Zel'dovich and Shkunov.[2]

6. The inhomogeneity of the plasma density $n_e(x)$ depends essentially on the threshold of specular double SMBS, because the dependence of the wave vectors of the pump ($\mathbf{k}_0$) and the scattered ($\mathbf{k}_{\pm 1}$) waves on the coordinate x does not permit us to satisfy the conditions of spatial synchronism of the beats of the electromagnetic waves with the sound simultaneously over the entire region of the plasma. We now consider the conditions of excitation of specular double SMBS in the case in which the density $n_e(x)$ increases monotonically from $n_e(0) = 0$, such that there are turning points both for the pump wave $[k_{0x}(x_t) = 0]$ and for the scattered waves. Here we neglect the difference between the x components of the wave vectors of the electromagnetic waves $k_{0x} \approx k_{1x} \approx k_{-1x} = k_x$. This corresponds to satisfaction of the inequality $(v_s/c)k_0 x_t \ll 1$ which is usually satisfied under the condition of the effect of radiation on the plasma.

Similar to (5.1) and (5.2), we represent the high-frequency electric field $E_z(x, y, t)$ and the perturbation of the density $\delta n_e(x, y, t)$ in the following form:

$$E_z = \left(\frac{k_x(0)}{k_x(x)}\right)^{1/2} \sum_{\sigma=\pm 1}(E_{0\sigma} + E_{-1\sigma}e^{i\omega t} + E_{1\sigma}e^{-i\omega t}) \times \exp\left(i\sigma\int^x k_x\,dx' + ik_y y\right),$$

$$\delta n_e = -in_e(x)\sum_{\sigma=\pm 1}\nu_\sigma(x,t)\exp\left(2i\sigma\int^x k_x\,dx' - i\omega t\right) + \text{c.c.} \tag{6.1}$$

$$k_y = \frac{\omega_0}{c}\sin\theta_0, \quad k_x = \frac{\omega_0}{c}\left(\cos^2\theta_0 - \frac{n_e}{n_c}\right)^{1/2}.$$

Here θ_0 is the angle of incidence of the pump wave on the plasma. The amplitudes of the pump field are assumed to be given: $E_{01} = E_0$, $E_{0-1} = r_0 e^{i\varphi_0}E_0$. For the amplitudes of the scattered waves, we impose boundary conditions similar to (5.5):

$$E_{\pm 11}(0) = 0, \quad E_{\pm 1-1}(x_t) = re^{i\varphi}E_{\pm 11}(x_t), \quad k_x(x_t) = 0. \tag{6.2}$$

Using (6.1), we obtain simplified equations from Eq. (2.1) for the amplitudes of the interacting waves:

$$\sigma\frac{\partial}{\partial x}E_{-1\sigma} = \frac{\omega_{Le}^2}{2k_x c^2}E_{0-\sigma}\nu_{-\sigma}^*, \quad \sigma\frac{\partial}{\partial x}E_{1\sigma} = -\frac{\omega_{Le}^2}{2k_x c^2}E_{0-\sigma}\nu_\sigma, \tag{6.3}$$

$$\left(\frac{\partial}{\partial t} + \gamma_s + 2\sigma v_s^2\frac{k_x}{\omega}\frac{\partial}{\partial x} - i\frac{\omega^2 - 4k_x^2 v_s^2}{2\omega}\right)\nu_\sigma = \frac{k_x k_x(0)v_s^2}{8\pi n_c \varkappa_B T\omega}(E_{0\sigma}E_{-1-\sigma}^* + E_{0-\sigma}^* E_{1\sigma}). \tag{6.4}$$

The equations for the electromagnetic waves (6.3) differ from the Eqs. (5.3) only in the fact that ω_{Le} and k_x depend on the coordinate x. Consequently, the first integrals (5.6) occur also for the system (6.3). It follows from Eq. (6.4) that scattering with specified frequency ω takes place in the vicinity of the resonance point $x_r(\omega)$ at which $\omega^2 = 4k_x^2(x_r)v_s^2$. We consider the case of sufficiently strong damping of the sound:

$$\gamma_s/\omega > \omega_{Le}^2/2k_x^{3/2}(x_r)cL^{1/2}, \tag{6.5}$$

where $L(\omega)$ is the scale of the inhomogeneity of the density at the resonance point x_r. Here the spatial derivative in Eq. (6.4) can be neglected, and the system (6.3) and (6.4) at the instability threshold ($\partial/\partial t = 0$) reduces to two equations for the amplitudes of the Stokes and anti-Stokes waves:

$$\begin{aligned} \frac{dE^*_{-1-1}(x)}{dx} &= -iQ[E^*_{-1-1}(x)(1+r_0^2) - r_0^2E^*_{-1-1}(0)], \\ \frac{dE_{1-1}(x)}{dx} &= iQ[E_{1-1}(x)(1+r_0^2) - r_0^2E_{1-1}(0)], \end{aligned} \tag{6.6}$$

where the function $Q(x)$ describes the interaction of the waves:

$$Q(x) = \frac{I_0\omega_{Le}^2v_s^2k_x(0)}{c^2[\omega^2-4k_x^2v_s^2+2i\omega\gamma_s]}, \quad I_0 = \frac{|E_0|^2}{8\pi n_c\varkappa_BT}. \tag{6.7}$$

Solution of the system (6.6) with the boundary conditions (6.2) leads to the dispersion equation [compare with (5.7)]

$$\mathrm{ch}\left[i(1+r_0^2)\int_0^{x_t} Q\,dx\right] = -\frac{1-2r^2r_0^2+r_0^4}{2r_0^2(1+r^2)} = -f(r, r_0). \tag{6.8}$$

We emphasize that the condition (6.5) is not a necessary one for obtaining (6.8). It was used by us only for simplicity of exposition. Under conditions of weak damping of the sound, the system (6.3) and (6.4) reduces to two equations of second order for the amplitudes E^*_{-1-1} and E_{1-1}. Far from the resonance point x_r, these equations are solved in the approximation of geometric optics. In the vicinity of the resonance point, where the density can be approximated by a linear function of the coordinate, the solutions of these equations are expressed in terms of parabolic cylinder functions. The matching of the obtained solutions (this was described by us in detail earlier[9] in application to another problem) and the use of the boundary conditions (6.2) again lead to the dispersion equation (6.8).

Similar to (5.7), Eq. (6.8) can be represented in the form of two equations that determine the threshold intensity of the pump wave and the shift of the frequency of the scattered waves:

$$(\pi/4)(1+r_0^2)I_{0\,th}(\omega_0L/c)\cos\theta_0 = \ln U(r, r_0), \tag{6.9}$$

$$\Psi = (1+r_0^2)I_{0th}\frac{\omega_0v_s^2}{c^3}\cos\theta_0\,\mathrm{Re}\int_0^{x_t}\frac{\omega_{Le}^2(x)\,dx}{\omega^2-4k_x^2v_s^2+2i\omega\gamma_s} = \pi(2n-1). \tag{6.10}$$

Comparing (6.9) with the threshold of specular double SMBS in a homogeneous layer (5.9), we note that the inhomogeneity of the plasma turns out to have a two fold effect. On the one hand, the absence of the large factor ω/γ_s on the right side of (6.9) leads to an increase in the threshold due to a narrowing of the resonance region but, on the other hand, the term $(2n-1)^2/\ln U$ is absent on the right side of (6.9), which lowers the threshold. The latter circumstance is connected with the fact that, as is seen from (6.10), in an inhomogeneous plasma the region of change of phase of the interacting waves is significantly wider than the resonance region.

According to (6.9) and (6.10), specular double SMBS is possible only under the condition $r<(1-r_0^2)/2r_0$, when $f>1$, in particular, in the case $r=r_0$ at $r_2<1/3$. However, in contrast to the model of a homogeneous layer, the minimum threshold is achieved not at $r_{0*}^2 = e^{-\pi}$, but at the maximum possible coefficient r_0, when $U \sim 1$. A more accurate condition of generation for specular double SMBS depends on the form of the plasma density distribution over the entire region occupied by the electromagnetic field.

For example, in a plasma with an exponential density profile,

$$n_e = n_c\cos^2\theta_0\exp(x/L_0),$$

assuming $r=r_0$, we have for the minimum threshold of specular double SMBS

$$I_{0\,min} \approx \frac{3\pi c}{\omega_0L_0\cos\theta_0}\ln^{-1}\frac{\omega}{2\gamma_s}. \tag{6.11}$$

This is realized for waves with a frequency shift $\omega \approx 2\frac{\omega_0}{c}v_s\cos\theta_0$ and for

$$\ln U = \frac{\pi^2}{2}\ln^{-1}\frac{\omega}{2\gamma_s}.$$

A comparison of the threshold of specular double SMBS (6.11) with the threshold of backward double SMBS [Eq. (3.1) at $n=0$ with account of (3.2)] shows that just this latter process should determine the reflection in experiments on SMBS under conditions of oblique incidence of the pump wave in moderate energy fluxes, since its threshold is approximately an order of magnitude smaller.

7. Much lower values of the thresholds for backward double SMBS follow from Eqs. (3.1) and (3.2). For example, for a plasma with an inhomogeneity $\sim 100\,\mu$, created by the radiation of a neodymium laser, the threshold of backward double SMBS amounts to not more than 10^{12} W/cm^2. Practically all the published experiments on the interaction of laser radiation with a plasma have been carried out at large energy fluxes. Starting out from the theory developed above, we assume that a radical revision of the interpretation of already published experiments is necessary. The effects of collimated back reflection and saturation of the coefficient of SMBS reflection find simple explanations. There is no need for introducing an assumption that the plasma contains intense sources of above-thermal noise[10] for the explanation of the observed level of reflection. Since the double SMBS threshold is significantly lower than the effective threshold of convective SMBS, it can be thought that either the estimates made earlier of the scale of the inhomogeneity of the plasma density are much too high, or the values of the temperature of the plasma are too low. It follows from Eq. (3.2) that the shift in the frequency of the back scattered radiation gives a direct possibility of the determination of the plasma temperature in the vicinity of the point of reflection at least in the case of moderate energy fluxes.

Finally, an extremely important consequence of the our theory is the fact that processes of double SMBS actually do not prevent effective absorption of the heating radiation, because more than half of the energy of the pump wave passes into the dense plasma layer even under conditions of developed double SMBS. Here the back reflection coefficient (which is equal in order of magnitude to the reflection coefficient of the radiation from the dense layers of the plasma) provides a direct estimate of the absorption coefficient.

On the basis of what has been said above, we can draw the conclusion that the pessimistic estimate of the role of stimulated scattering processes for the estimate of the absorption of intense electromagnetic radiation [see, for example, Ref. 11], based on the results of the convective theory, is invalid.

For an experimental confirmation of the important role of double SMBS processes, it is desirable to make special measurements of the dependence of the back reflection coefficient from the energy flux of the pump and of the dependence of the back scattered spectrum of radiation on the angle of incidence of the pump wave for moderate energy fluxes (for the neodymium laser, $\lesssim 10^{14}$ W/cm^2). Limitation of the reflection coefficient should be observed here, while the frequency shift of the Stokes satellite $\omega(3.2)$ should increase with increase in the angle of incidence θ_0.

It should be noted that the recent experiments in Garching on the iodine laser[12] correspond to the phenomenon of double SMBS with conversion of the specular reflection into collimated backward reflection. Here the SMBS threshold actually turns out to be significantly lower than follows from the convective theory and agrees satisfactorily with the threshold (3.1) of our absolute SMBS instability.

[1)]We emphasize here the fact that under conditions of strong sound damping, the ordinary SMBS instability is convective. Only under conditions of negligibly small sound damping did the theory of SMBS for a layer with transparent boundaries lead earlier to absolute instability.[5,6]

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