Interaction between electrons in a light wave

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A potential is derived for the interaction between two electrons in a circularly polarized light wave of arbitrary intensity. The potential has local minima at large distances. Bound states can form at these minima in a sufficiently strong field. In the near zone, the potential in a strong field is non-Coulomb. That component of the radiation pressure exerted on the two electrons which stems from coherence effects is calculated.

Let us examine the interaction between two electrons in an electromagnetic wave of arbitrary intensity. The quantum-mechanical problem of the scattering of an electron by an electron in a strong field was studied in Refs. 1 and 2. The external field was taken into account exactly, while the interaction between electrons was treated by perturbation theory. The general expression for a Møller-scattering matrix element is extremely complicated, but it simplifies in the nonrelativistic limit, which was studied in Refs. 3–5.

If, however, we do not concern ourselves with the subtleties of the quantum-mechanical structure, the interaction problem can be handled by a classical description of both the motion of the electrons and the scattered radiation field over a broad range of the intensity of the external field. Below we derive an effective classical interaction potential averaged over the period of the light wave. We show that this potential is quite different from a Coulomb potential in a sufficiently strong field.

1. EFFECTIVE POTENTIAL FOR THE INTERACTION BETWEEN ELECTRONS

The motion of an electron in a monochromatic electromagnetic wave is a superposition of two motions: a uniform translational motion of the center of the orbit at some average velocity and an oscillatory motion whose characteristics are determined by the frequency and amplitude of the external field. The relative velocity of the motion of the centers of mass of the electrons is not changed substantially by the imposition of the electromagnetic field, so that the case of a nonrelativistic average relative motion is quite general. This situation can arise, for example, when a laser beam is applied to a plasma or to an electron beam. We will accordingly restrict the discussion below to the simplest and most interesting case: that in which the electrons are at rest on the average and have an arbitrary oscillation velocity. In this case the interaction between electrons is described by an adiabatic potential.

The motion of an electron is described by the classical equation ($c = 1$)

$$m\frac{d}{dt}[\mathbf{v}(1-\mathbf{v}^2)^{-1/2}]=e(\mathbf{E}+[\mathbf{v}\times\mathbf{H}])+\mathbf{g}_r, \tag{1}$$

which incorporates the radiation reaction[6] $\mathbf{g}_r$. In strong fields the reaction $\mathbf{g}_r$ is generally comparable to the force acting between charges. In lowest-order perturbation theory, however, the two forces enter the average equation of motion in an additive manner, so that $\mathbf{g}_r$ can be ignored if our problem is to find the force acting between the charges.

The field acting on an electron consists of the external field $\mathbf{E}_0(\xi)$ (where $\xi = t - z$) of a plane wave which is propagating along the z axis and the field $\mathbf{E}_1$ of the other electron:

$$\mathbf{E}(\mathbf{r},\ t)=\mathbf{E}_0(\xi)+\mathbf{E}_1(\mathbf{r},\ t). \tag{2}$$

The external field $E_0(\xi)$ and the static part of the field $\mathbf{E}_1(\mathbf{r}, t)$ are treated rigorously, and the alternating part of $\mathbf{E}_1$ (the scattered field, $\mathbf{E}_{1s}$) by perturbation theory. The change caused in the motion of an electron by the scattered field gives rise to an average force exerted on the electron by the external field. At distances large in comparison with the classical radius of the electron the scattered field is much weaker than the external field,

$$E_{1s}\ll E_0. \tag{3}$$

We represent the motion of a particle as the superposition

$$\mathbf{R}(t)=\mathbf{r}(t)+\boldsymbol{\rho}_0(t)+\boldsymbol{\rho}_1(t), \tag{4}$$

where $\boldsymbol{\rho}_0(t)$ is the trajectory of the electron in the external field $\mathbf{E}_0(\xi)$, and the term $\boldsymbol{\rho}_1(t)$ describes fast oscillations due to the scattered field $\mathbf{E}_{1s}$ ($\rho_1\ll\rho_0$). The slow motion of the center of mass is described by the term $\mathbf{r}(t)$. We thus have

$$\langle\mathbf{R}(t)\rangle=\mathbf{r}(t),\quad\langle\boldsymbol{\rho}_0(t)\rangle=\langle\boldsymbol{\rho}_1(t)\rangle=0,$$

where the angle brackets denote an average over the oscillation period of the external field. Correspondingly, we write the electron velocity as

$$\mathbf{v}=d\mathbf{r}/dt+\mathbf{v}_0+\mathbf{v}_1,\quad\mathbf{v}_{0,1}=d\boldsymbol{\rho}_{0,1}/dt, \tag{5}$$

so that we have $\langle\mathbf{v}_{0,1}\rangle = 0$. By virtue of condition (3), ρ_1 and v_1 are small in comparison with ρ_0 and v_0 and can be sought by perturbation theory. We now assume that the external field is circularly polarized:

$$\mathbf{E}_0(\xi)=E_0(\cos\psi,\ \sin\psi,\ 0),\quad\psi=\omega\xi. \tag{6}$$

This assumption puts the expressions for the unperturbed motion in their simplest form:

$$\begin{gathered}\boldsymbol{\rho}_0=-\rho_0(\cos\psi,\ \sin\psi,\ 0),\quad\rho_0=eE_0/m\gamma\omega^2,\\ \mathbf{v}_0=v_0(\sin\psi,\ -\cos\psi,\ 0),\quad v_0=\rho_0\omega,\\ \gamma^2=(1-v_0{}^2)^{-1}=1+(eE_0/m\omega)^2.\end{gathered} \tag{7}$$

Linearizing Eq. (1) in terms of the scattered field, we find

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$$\frac{d}{dt}[\gamma\mathbf{u}+\gamma^3(\mathbf{v}_0\mathbf{u})\mathbf{v}_0]=(\boldsymbol{\rho}_1\nabla)\mathbf{F}_0+[\mathbf{u}\mathbf{H}_0]+\mathbf{F}_1, \quad \mathbf{u}=\frac{d\mathbf{r}}{dt}+\mathbf{v}_1, \tag{8}$$

where the Lorentz forces $\mathbf{F}_{0,1}$ are caused by the fields $\mathbf{E}_0$ and $\mathbf{E}_1$, taken at the point $\mathbf{r}+\boldsymbol{\rho}_0(t)$.

Using (6) and (7), we can rewrite the first two terms on the right side of (8) as

$$-\frac{d}{dt}(\mathbf{E}_0 z_1)+\mathbf{n}_z\left[(\mathbf{u}\mathbf{E}_0)-z_1\left(\mathbf{v}_0\frac{d\mathbf{E}_0}{d\xi}\right)\right], \tag{9}$$

where $\mathbf{n}_z$ is a unit vector along the z axis, and z_1 reflects the change in the phase ξ due to the scattered field. We then have the following equation for the transverse velocity component (in the xy plane):

$$\gamma\mathbf{u}_\perp+\mathbf{E}_0 z_1=\mathbf{G}_\perp-\mathbf{v}_0(\mathbf{v}_0\mathbf{G}), \quad \mathbf{G}=\int_{-\infty}^{t} dt'\,\mathbf{F}_1(t'). \tag{10}$$

From (8)–(10) we find

$$\frac{d}{dt}\gamma u_z=F_{1z}+\frac{1}{\gamma}(\mathbf{E}_0\mathbf{G}). \tag{11}$$

From this point on the problem is one of averaging Eqs. (10) and (11) over the period of the field. For this purpose we calculate the average value $\langle \mathbf{E}_0(\xi)z_1\rangle$, using (11):

$$\langle\mathbf{E}_0 z_1\rangle=-\frac{1}{\gamma\omega^2}\left\langle\mathbf{E}_0\left(F_{1z}+\frac{1}{\gamma}(\mathbf{E}_0\mathbf{G})\right)\right\rangle. \tag{12}$$

After a sufficiently long time the second term becomes dominant, since G is proportional to t. This proportionality arises upon the integration of the zeroth harmonic in the expansion of $F_1(t)$ in a Fourier series in the period of the external field.

Omitting the term with F_{1z} from (12), we find the average equation

$$\gamma(d^2\mathbf{r}/dt^2)=\langle\mathbf{F}_1\rangle-\mathbf{n}_z\langle(\mathbf{v}_0\mathbf{F}_1)\rangle. \tag{13}$$

We see that there is a force in addition to the average perturbing force $\langle F_{1z}\rangle$ acting along the longitudinal direction on the particle. This additional force is caused by the change of the motion of the particle in the strong field $\mathbf{E}_0$ by the scattered field $\mathbf{E}_{1s}$. This force is determined by the average Lorentz force exerted on the particle by the external field, $\langle\mathbf{v}_1\times\mathbf{H}_0\rangle$. We might note that a similar increment in the force was analyzed in Ref. 7, where the average radiation-pressure force exerted on one particle as a result of radiation friction was calculated. If we adopt the radiation-friction force as the perturbing force here, we find from (13) the same average radiation-pressure force as was found in Ref. 7.

We turn now to the calculations of the average interaction forces between two electrons. We denote by a the center of the orbit of the first electron, and we place it at the origin of coordinates. This electron is revolving uniformly along a circle in the x, y plane, and its instantaneous position is described by the vector $\mathbf{R}_a(t)=\boldsymbol{\rho}_{0a}(t)$. The position of the second electron, b, is described by the vector $\mathbf{R}_b(t)=\mathbf{r}+\boldsymbol{\rho}_{0b}(t)$, where $\mathbf{r}$ is the distance between the centers of the orbits of the two electrons (Fig. 1).

The force exerted on charge b by charge a is

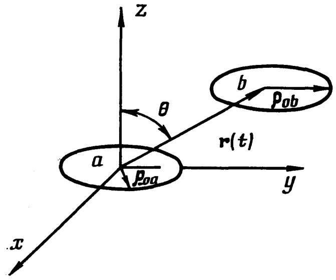

FIG. 1. Motion of electrons in a circularly polarized plane electromagnetic wave which is propagating along the z axis.

$$\mathbf{F}_b(t)=-e\int d\mathbf{k}\exp[i\mathbf{k}\mathbf{R}_b(t)]$$
$$\times\{i\mathbf{k}(\varphi_\mathbf{k}(t)-(\mathbf{v}_b(t)\mathbf{A}_\mathbf{k}(t)))+(\partial\mathbf{A}_\mathbf{k}/\partial t+i\mathbf{A}_\mathbf{k}(\mathbf{v}_b\mathbf{k}))\}, \tag{14}$$

where $\varphi_\mathbf{k}$ and $\mathbf{A}_\mathbf{k}$ are Fourier components of the scalar and vector potentials of the field produced by particle a.

The term in the second set of parentheses has the form of a total time derivative. Consequently, after an average is taken over the time this term contributes only to the increment of the longitudinal force:

$$\langle(\mathbf{v}_b\mathbf{F}_b)\rangle=-e\int d\mathbf{k}\exp(i\mathbf{k}\mathbf{R}_b)\left\{i(\mathbf{k}\mathbf{v}_b)\varphi_\mathbf{k}+\left(\frac{\partial\mathbf{A}_\mathbf{k}}{\partial t}\mathbf{v}_b\right)\right\}. \tag{15}$$

It can be seen from (14) that the transverse component of the force (directed perpendicular to the z axis) is of the form of a gradient. It follows from the relation

$$\frac{\partial}{\partial t}[\exp(i\mathbf{k}\mathbf{R}_b)]=i\exp[i\mathbf{k}(\mathbf{r}+\boldsymbol{\rho}_{0b})][k_z-(\mathbf{v}_b\mathbf{k})]$$

that the longitudinal component of the force [with (15) taken into account] is also of the form of a gradient. The resultant force (13) is thus a potential force:

$$\mathbf{f}_b=\langle\mathbf{F}_b\rangle-\mathbf{n}_z\langle\mathbf{v}_b\mathbf{F}_b\rangle=-\nabla V(\mathbf{r}).$$

Using the expressions for the Liénard-Wiechert potentials, we can write $V(\mathbf{r})$ as

$$V(\mathbf{r})=e^2\int_0^\infty d\tau\left\langle\frac{\delta(R(t,\tau)-\tau)}{R(t,\tau)}[1-(\mathbf{v}_a(t-\tau)\mathbf{v}_b(t))]\right\rangle,$$
$$\mathbf{R}^2(t,\tau)=[\mathbf{r}+\boldsymbol{\rho}_{0b}(t)-\boldsymbol{\rho}_{0a}(t-\tau)]^2. \tag{16}$$

From the equation $\mathbf{R}^2(t,\tau)=\tau^2$ we can determine both the dependence of the retardation τ on the time t and the inverse function $t(\tau)$. For the motion of a charge in a circularly polarized wave, the function $t(\tau)$ takes a simple form, allowing us to take an average over the time. Using expression (7) for the trajectory of the unperturbed motion and making use of the δ-function in integral (16), we find the following expression for the average potential:

$$V=\frac{2e^2}{\pi}\int d\varepsilon\,(1-v_0^2\cos\omega\varepsilon)(A_+A_-)^{-1/2}, \quad \varepsilon=\tau-z;$$
$$A_\pm=(\varepsilon+z)^2-r^2-\left(\frac{2v_0}{\omega}\right)^2\sin\frac{\omega\varepsilon}{2}\left(\sin\frac{\omega\varepsilon}{2}\pm1\right). \tag{17}$$

Here the integration is carried out over all $\varepsilon > 0$ for which the expression in the radical is positive.

Expression (17) is a solution of our problem. It determines the average potential of the interaction between two charges in a circularly polarized electromagnetic wave. This potential is asymmetric with respect to the replacement of $\mathbf{r}$ by $-\mathbf{r}$, and we have $\mathbf{f}_b \neq -\mathbf{f}_a$; in other words, Newton's third law does not hold ($\mathbf{f}_a$ is the force exerted on charge a by charge b).

We introduce the interaction force $\mathbf{f}$ and the force $\mathbf{g}$ that acts on the center of mass of charges a and b:

$$\mathbf{f}={}^1/_2(\mathbf{F}_b-\mathbf{F}_a)=-\nabla V_s, \quad \mathbf{g}=\mathbf{F}_a+\mathbf{F}_b=-2\nabla V_a+2g_r\mathbf{n}_z, \tag{18}$$

$$V=V_a+V_s, \quad V_{s,a}={}^1/_2[V(\mathbf{r})\pm V(-\mathbf{r})].$$

Here g_r is the radiation reaction of one electron (the radiation pressure). The symmetric part of the potential thus determines the interaction force between the particles, while the antisymmtric part determines the increment in the radiation pressure due to the coherence effects which arise in the scattering of light by the two charges. The radiation-pressure force exerted by the external field on an individual electron is[6]

$$g_r={}^2/_3(e\omega v)^2\gamma^4. \tag{19}$$

We turn now to the characteristic limiting cases of weak and strong fields and the near and far zones.

2. WEAK FIELD

We first consider the case of a weak field and distances which are not too small:

$$E_0 \ll E_{rel}=m\omega/e, \quad v_0 \ll 1.$$

In this case the retardation effects can be treated by perturbation theory. It is more convenient here to work directly from Eq. (16). We find

$$V=\frac{e^2}{r}\left\{1-v_0^2\left[\cos\varphi\left(1-\frac{1}{2}\sin^2\theta\right)\right.\right.$$
$$\left.\left.+\left(\frac{\sin\varphi}{\omega r}-\frac{1-\cos\varphi}{(\omega r)^2}\right)\left(\frac{3}{2}\sin^2\theta-1\right)\right]\right\}, \tag{20}$$

where $\varphi=\omega(r-z)$ and θ is the angle between $\mathbf{r}$ and $\mathbf{n}_z$.

Near zone

In the near zone $\omega r \ll 1$ expression (20) simplifies, and the interaction potential becomes

$$V=\frac{e^2}{r}\left[1-v_0^2\left(\cos^2\theta+\frac{3}{4}\sin^4\theta\right)\right]. \tag{21}$$

This result agrees with the Darwin potential[6] for the interaction between electrons with equal velocities, $v_1=v_2=v_0$:

$$V(R)=\left\langle\frac{e^2}{R_{ab}}\right\rangle\left[1-\frac{v_0^2}{2}\left(1+\frac{1}{2}\sin^2\theta\right)\right],$$

if we expand $R_{ab}=|\mathbf{r}+\boldsymbol{\rho}_b-\boldsymbol{\rho}_a|$ in this expression in powers of ρ.

The radiation-pressure force is

$$\mathbf{g}=4g_0\mathbf{n}_z, \quad g_0={}^2/_3(e\omega v_0)^2,$$

as it must be in the near zone, where it is proportional to the square of the number of scattering charges.[6]

Far zone ($\omega r \gg 1$)

From (20) we find the interaction potential to be

$$V=\frac{e^2}{r}\left[1-v_0^2\cos\varphi\left(1-\frac{1}{2}\sin^2\theta\right)\right]. \tag{22}$$

We see that the increment in the Coulomb potential is oscillatory. The difference between the oscillation phases of two charges in a plane wave is ωz. The phase φ which appears in the potential results from an interference between the spherical scattered wave $e^{i\omega r}/r$ and the plane wave of the external field. The force corresponding to potential (22) is then

$$\mathbf{f}=e^2\left[\frac{\mathbf{n}}{r^2}-\frac{\omega v_0^2}{2r}\left(1-\frac{1}{2}\sin^2\theta\right)\right.$$
$$\left.\times(\sin\varphi_+(\mathbf{n}+\mathbf{n}_z)+\sin\varphi_-(\mathbf{n}-\mathbf{n}_z))\right],$$
$$\varphi_\pm=\omega(r\pm z). \tag{23}$$

Here we have omitted a correction to the Coulomb force $(\sim v_0^2/r^2)$.

We thus see that at large distances the retarded interaction varies as $r^{-1}\sin\varphi$ and falls off more slowly than the Coulomb force. At sufficiently short distances,

$$r>r_0, \quad r_0=(\omega v_0)^{-1}$$

the interaction force becomes oscillatory. Oscillations in the retarded interaction of particles in an external field are common. Zhukova *et al.*[8] have shown that the interaction of neutral atoms in an external light field is similar in form.

Bound states of charges in a light wave

At large distances, $r>r_0$, local minima appear in potential (22), and bound states can form at these minima. Ignoring the Coulomb part of the interaction, we find that the potential has extrema at the points where $\sin\varphi_+=\sin\varphi_-=0$. To determine the nature of the extremum we write an expression for the second variation of the potential,

$$\delta V_s=\frac{1}{4}\frac{e^2v_0^2}{r}\left(1-\frac{1}{2}\sin^2\theta\right)$$
$$\times\{\cos\varphi_+[\delta\rho\sin\theta+(1+\cos\theta)\delta z]^2$$
$$+\cos\varphi_-[\delta\rho\sin\theta+(1-\cos\theta)\delta z]^2\}$$

for small displacements $(\delta\rho, \delta z)$ from the extremum. Here $(\delta\rho, \delta z)$ are the components of the displacement vector in cylindrical coordinates. Under the condition $\cos\varphi_+=\cos\varphi_-=1$ the second variation δV_s becomes positive definite, corresponding to a potential minimum. This minimum occurs at intersections of the paraboloids of revolution $\varphi_\pm=2\pi n_\pm$ or $z=\pm(\lambda n_\pm-\rho^2/\lambda n_\pm)$, where $n_\pm$ are positive integers, and $\lambda=2\pi/\omega$ is the wavelength of the external field.

The depth of the potential well is proportional to the intensity of the electromagnetic wave. The minimum intensity at which bound states arise, $I_0=E_0^2/4\pi$, can be found from the condition

$e^2v_0^2/r>(\hbar\omega)^2/m.$

At $r\sim r_0$ we then find where

$$I>(\omega a)^{1/2}I_{rel},\quad I_{rel}=(m\omega/e)^2/4\pi, \tag{24}$$

a is the Bohr radius. In the visible part of the spectrum we would have $\omega a\sim 10^{-3}$–10^{-4}, and condition (24) would become the weak-field condition $v_0^2=I_0^2(1+I_0^2)^{-1}\ll 1$.

We thus see that in a weak field and at a short distance, $r<r_0$, the interaction potential differs only slightly from a Coulomb potential. We do not see any qualitative changes in the potential until we move out to large distances.

3. STRONG FIELD

In a strong field, $I>I_{\rm rel}$, the oscillatory motion of the electrons becomes relativistic ($\gamma>1$), and the retardation effects cannot be treated by perturbation theory. In this case it is more convenient to use expression (17) to find the interaction potential. For this purpose we must solve the transcendental equations

$$A_\pm(\varepsilon)=0,$$

which determine the interval of retardation times which contributes to the interaction between the charges. These equations can be solved explicitly in certain limiting cases.

Near zone

At $\omega r\ll 1$ the expression for $A_\pm$ simplifies, since $\sin(\omega\varepsilon/2)$ can be expanded in a series in ε, and the expressions for $A_\pm$ take the following form in a cylindrical coordinate system:

$$A_\pm(\varepsilon)=2\varepsilon a_\pm\pm\rho^2\pm\gamma^{-2}\varepsilon^2-{}^1/_{12}v_0^2\omega^2\rho\varepsilon^3\pm{}^1/_{12}\omega^2\varepsilon^4, \tag{25}$$
$$a_\pm=v_0\rho\mp z.$$

Expansion (25) has been carried out to ε^4, so that the term $(\varepsilon/\gamma)^2$ is generally small in the ultrarelativistic case.

A charge revolving in the field of an electromagnetic wave radiates at a characteristic frequency $\sim\omega\gamma^3$ in the ultrarelativistic case.[6] Consequently, the near zone can be broken up into two subzones

$r\ll\lambda\gamma^{-3}$ and $\lambda\gamma^{-3}\ll r\ll\lambda$.

The case $r\ll\lambda\gamma^{-3}$

In this case, it is sufficient to retain terms up to ε^2 in expression (25) for $A_\pm$, and in first order in the parameter $\omega r\gamma^{-3}\ll 1$ the integral in (17) becomes

$$V=\frac{2e^2}{\pi}\int_{\varepsilon_1^-}^{\varepsilon_1^+}d\varepsilon[(\varepsilon_1^+-\varepsilon)(\varepsilon-\varepsilon_1^-)(\varepsilon-\varepsilon_2^+)(\varepsilon-\varepsilon_2^-)]^{-1/2},$$
$$\varepsilon_1^\pm=r\gamma^2[\pm v_0\sin\theta-\cos\theta+(1\mp v_0\sin 2\theta)^{1/2}], \tag{26}$$
$$\varepsilon_2^\pm=r\gamma^2[\pm v_0\sin\theta-\cos\theta-(1\mp v_0\sin 2\theta)^{1/2}].$$

Here $\varepsilon_{1,2}^\pm$ are the roots of the equations $A_\pm(\varepsilon)=0$; $\varepsilon_1^\pm>0$ and $\varepsilon_2^\pm<0$. The integration over positive ε between ε_1^+ and ε_1^- in (26) corresponds to a particular choice of the retarded

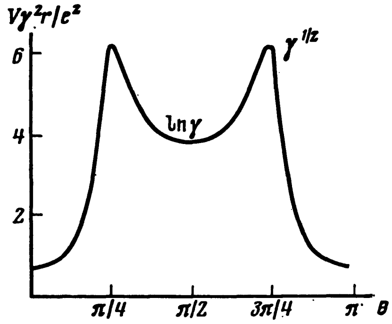

FIG. 2. Angular dependence of the effective interaction potential in the case $r\gamma^3<\lambda$.

potential for the scattered field. Expression (26) reduces to the complete elliptic integral of the first kind, $K(q)$:

$$V=\frac{2e^2}{\pi r\gamma^2}(1-v_0^2\sin^2 2\theta)^{-1/4}K(q);$$
$$q^2=\frac{1}{2}\left[1-\frac{1-2v_0^2\sin^2\theta}{(1-v_0^2\sin^2 2\theta)^{1/2}}\right]. \tag{27}$$

This expression is valid in both the nonrelativistic and ultrarelativistic cases. Under the condition $v_0\ll 1$ it becomes the same as (21). The potential (27) is symmetric with respect to the replacement $\theta\to\pi-\theta$, so that there is no radiation pressure in this approximation.

Under the condition $\gamma\gg 1$, expression (27) takes the simpler form

$$V=\frac{e^2}{r\gamma^2|\cos 2\theta|^{1/2}}\begin{cases}1, & 0\leqslant\theta<\pi/4\\ \dfrac{2}{\pi}\ln\left|\dfrac{4\gamma\cos 2\theta}{\sin^2\theta}\right|, & \dfrac{\pi}{4}<\theta\leqslant\dfrac{\pi}{2}\end{cases}. \tag{28}$$

The angular dependence of potential (28) (at $r=$ const) is plotted in Fig. 2. The potential rises sharply near the angles $\theta_0=\pi/4$ and $\theta_1=3\pi/4$; here $|\cos 2\theta|$ in (28) should be replaced by γ^{-1}. The reason for the increase in the potential by a factor of $\gamma^{1/2}$ near these angles is that the coefficient $a_\pm(\theta)$ becomes small, and the four roots $\varepsilon_{1,2}^\pm$ are all approximately the same.

We see thus that at short distances and in the relativistic case the interaction potential is different from a Coulomb potential: There is a strong angular dependence, and the effective charge is reduced by a factor of γ. The reason for this decrease is that an attraction arises between the electrons because of the magnetic interaction of the two identically directed electron currents.

The case $\lambda\gamma^{-3}\ll r\ll\lambda$

This case obviously can arise only in an ultrarelativistic field. The potential (17) can be written in this case as

$$V=\frac{e^2}{\pi}\int\frac{(2\gamma^{-2}+\omega^2\varepsilon^2)d\varepsilon}{(A_+A_-)^{1/2}},$$
$$A_+(\varepsilon)=\begin{cases}2\varepsilon a_++\rho^2, & 0\leqslant\theta<\pi/4\\ 2\varepsilon a_+-{}^1/_{12}\,\omega^2\varepsilon^4, & \pi/4<\theta<\pi\end{cases}, \tag{29}$$
$$A_-(\varepsilon)=\begin{cases}2\varepsilon a_--\rho^2, & 0\leqslant\theta<3\pi/4\\ 2\varepsilon a_-+{}^1/_{12}\,\omega^2\varepsilon^4, & 3\pi/4<\theta\leqslant\pi.\end{cases}$$

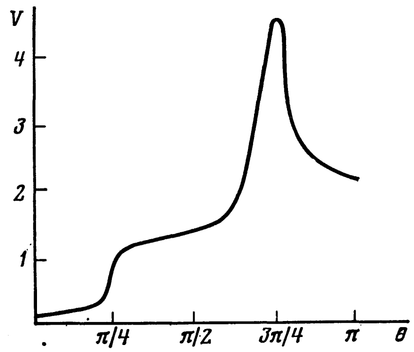

FIG. 3. Angular dependence of the interaction potential in the case $\lambda\gamma^{-3} \ll r \ll \lambda$. Here the potential is highly anisotropic, and the radiation-pressure force is comparable in magnitude to the interaction force.

At angles $0 \leqslant \theta < \pi/4$ the potential differs from (28) by a term which is linear in r:

$$V=\frac{e^2}{\gamma^2 r(\cos 2\theta)^{1/2}}\left[1+\frac{\omega^2}{8}\gamma^2 r^2\left(1-\frac{1}{2}\sin^2\theta\right)\frac{\sin^4\theta}{\cos^2 2\theta}\right]. \quad (30)$$

Under the condition $\omega\gamma r > 1$, and for angles which are not too small, the second term becomes dominant, and the effective potential becomes attractive.

At angles in the interval $\pi/4 < \theta < 3\pi/4$ the potential is

$$V=\frac{e^2\omega^{2/3}C}{r^{1/3}}\sin^{1/6}\alpha\cos^{-1/2}\alpha\,{}_2F_1(1,2,1;-\mathrm{tg}\,\alpha),\quad \alpha=\theta-\frac{\pi}{4}, \quad (31)$$

where ${}_2F_1(\alpha,\beta,\gamma;\ x)$ is a hypergeometric function, and $C \approx 2.8$.

For the rest of the angle interval, $3\pi/4 < \theta \leqslant \pi$, we find

$$V=\frac{e^2(2\omega)^{2/3}}{(3r)^{1/3}}\left|\cos\left(\theta-\frac{\pi}{4}\right)\right|^{-1/3} \times {}_2F_1\left(\frac{1}{3},\frac{1}{2},1;\frac{2\sin\theta}{\sin\theta-\cos\theta}\right). \quad (32)$$

The potential is plotted in Fig. 3; in this case it is highly anisotropic. The radiation-pressure force g is thus comparable in magnitude to the interaction force f but much weaker than g_r; specifically, $g \sim g_r(r\omega\gamma^3)^{-4/3}$.

Figures 4 and 5 are plots of the forces f and g found from

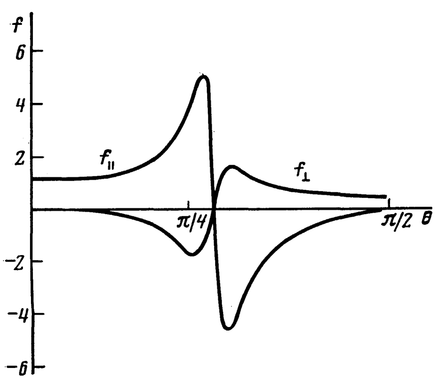

FIG. 4. Longitudinal and transverse components of the interaction force in the case $\lambda\gamma^{-3} \ll r \ll \lambda$.

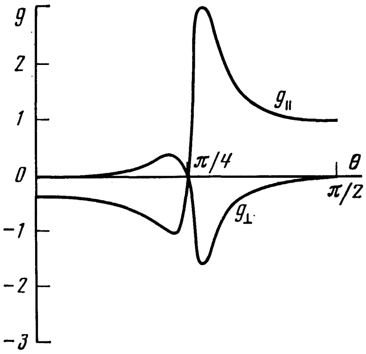

FIG. 5. Radiation-pressure force exerted on the two electrons in an intense light wave.

potential (30)–(32). Depending on the orientation of the charges with respect to the ray propagation direction, the interaction force tends to align the charges either along the ray, under the condition $\theta < \pi/4$, or across the ray, if $\pi/4 < \theta < \pi/2$. The interaction force is weaker than the Coulomb force e^2/r^2 by a factor on the order of the parameter $(\omega r)^{2/3} \ll 1$.

Interestingly, the longitudinal radiation-pressure force $g_\parallel$ is an alternating-sign force, although we are treating the case of the near zone. The reason is that the oscillation of the electrons is in phase only at distances $r < \gamma\lambda^{-3}$. Outside this range the oscillations go out of phase, and the longitudinal radiation-pressure force exerted on the two electrons may be either greater or less than $2g_r$.

The interaction of the particles in the external field gives rise to a new component of the radiation-pressure force: $\mathbf{g}_\perp$. This component drives the center of mass of the charges into motion along $\mathbf{r}_\perp$; the sign of this force component is determined by the angle θ.

Far zone

At $\omega r \gg 1$ we can simplify (17) by noting that the radius of the resolution orbit, ρ_0, is always less than r. As a result we find

$$V=\frac{e^2}{\pi r}\int\frac{d\varepsilon[1-\cos(2\varepsilon\omega)]}{[(\rho_0\sin\theta\sin\omega\varepsilon)^2-(\varepsilon-r+z)^2]^{1/2}}. \quad (33)$$

The integration is again carried out over those values $\varepsilon > 0$ for which the expression in the radical would be positive.

The integral in (33) can be evaluated easily in the case $\sin\theta \ll 1$:

$$V=(e^2/r)(1-\cos\varphi).$$

This potential varies slowly at $\theta \ll 1$ and oscillates rapidly (over a wavelength) at values of θ near π. This coordinate dependence gives rise to potential wells with a characteristic depth $\sim e^2/r$ and a width $\sim\lambda$.

4. CONCLUSION

In this analysis of the interaction between electrons in an external electromagnetic field of arbitrary intensity, we have leaned heavily on the condition that the relative motion of the centers of the revolution orbits of the electrons in the light wave is slow. In this case the interaction can be described by a time-average potential. It follows from (17) that in a sufficiently strong field this potential becomes different from a Coulomb potential, and the difference may be so pronounced that the two electrons form bound states in the external field. A potential of this sort might be used in the problem of the scattering of slow electrons in a non-Born approximation and also to study bound states in classical and quantum theories. We should point out that collective effects might cause substantial changes in the effective potential of the interaction between electrons in a light wave.

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