Features of two-phonon resonant Raman scattering of light in polar semiconductors in a strong magnetic field

V. I. Belitskiĭ, A. V. Gol'tsev, I. G. Lang, and S. T. Pavlov

A. F. Ioffe Physicotechnical Institute, USSR Academy of Sciences
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We calculate the contribution made to the cross section for second-order Raman scattering of light (RS) by processes in which free (disregarding the Coulomb interaction) electron-hole pairs (EHP) participate in a strong magnetic field. It is shown that this contribution is of first order in the Fröhlich coupling constant α of the electrons with the LO phonons, whereas in a zero magnetic field it is of second order in α. The lowering of the order in α is due to the quasi-one-dimensional character of the electron motion in the strong magnetic field. For polar semiconductors with coupling constant $\alpha \ll 1$ (such as InSb) one should expect an abrupt increase of the secondary-radiation intensity when a strong magnetic field is applied. The dependence of the scattering cross section on the magnetic field **H** and (or) on the frequency ω_l of the exciting light is subject to oscillations caused by the fact that the density of states becomes infinite at the bottom of the Landau band. It is shown that in the vicinity of a number of magnetic-field values $H = 2H_0/L$, where $H_0 = m_e \omega_{LO} c/|e|$, m_e (e) is the effective mass (charge) of the electron, and L is an integer, the scattering cross section acquires additional contributions of interference origin. These lead to a resonant decrease of the scattered-light intensity.

1. INTRODUCTION

The secondary emission spectra of a number of polar semiconductors irradiated by light in the intrinsic absorption region contain line series with frequencies $\omega_s = \omega_l - N\omega_{LO}$, where ω_l is the frequency of the exciting light, ω_{LO} is the frequency of the longitudinal optical lattice vibrations, and N is an integer that reaches values 7–9 (Ref. 1). This phenomenon is called multiphonon resonant Raman scattering of light (MRRSL). Contributing to the MRRSL are processes of two types: scattering via excitonic states and scattering via states of free electron-hole pairs (EHP).

Processes in which hot Wannier-Mott excitons participate were investigated in a number of studies (see, e.g., Ref. 4 and the literature cited therein). This type of MRRSL can be qualitatively described as follows: when a primary photon $\hbar\omega_l$ is absorbed, a hot exciton is created indirectly (with simultaneous emission of an LO phonon); the exciton next goes through a cascade of $(N-2)$ transitions through real intermediate states with emission of $(N-2)$ LO phonons after which, finally, it annihilates indirectly and emits the secondary photon $\hbar\omega_s$ and the last LO phonon.[1)] Since the strongest interaction in polar semiconductors is that of electrons (holes) with LO phonons, the relative (dimensionless) probability of the exciton emiting an LO phonon is of zeroth order in the Fröhlich coupling constant α, while the contribution to the cross section of the LRRSL processes with participation of hot Wannier-Mott excitons is of first order for any scattering order N.

Processes with participation of free electron-hole pairs in a zero magnetic field were theoretically investigated in Refs. 5–11. At $N \geqslant 4$ these processes can be described as follows: on absorption of an incident photon $\hbar\omega_l$, an EHP is created (e.g., directly) and then goes through a cascade of $(N-1)$ transitions through real intermediate states with emission of $(N-1)$ LO phonons, and finally annihilates in indirect manner. The process with indirect creation and direct annihilation of an EHP can be similarly described.[2)] (At $N=2$ and 3 the main contribution to the cross section is made by processes in which all the intermediate states in the scattering process are virtual.) The difference from processes with participation of hot Wannier-Mott excitons is that in the course of scattering via the EHP states $(N \geqslant 4)$ one of the transitions (creation or annihilation) can be direct, whereas in the MRRSL both creation and annihilation of the exciton can be only indirect. It is therefore to be expected that the contribution to the cross section from scattering via the EHP state $(N \geqslant 4)$ is of zeroth order in α. It was shown in Ref. 6, however, there is no contribution of zeroth order in α to the cross section. The reason for the absence of this contribution is that the electron and hole are separated in space after emitting a finite number of LO phonons, and the probability of the EHP annihilation is inversely proportional to a certain characteristic volume V_{EHP} occupied by the EHP after the emission of N LO phonons. The size of this volume for a free electron and a free hole, in polar semiconductors at $N \geqslant 4$, is determined by the interaction with the LO phonons (the exciton annihilation probability is inversely proportional to the exciton volume, a quantity determined by the Coulomb interaction of the electron and hole and independent of the interaction with the LO phonons). It was shown in Refs. 5–11 that for a Fröhlich interaction of electrons (holes) with LO phonons the contribution made by processes with EHP participation to the cross section σ_N for N-th orders MRRLS, in the case of a zero magnetic field is $\sigma_2 \propto \alpha^2$ for $N=2$, $\sigma_3 \propto \alpha^3 \ln\alpha^{-1}$ for $N=3$, and $\sigma_N \propto \alpha^3$ for $N \geqslant 4$.

We investigate here the contribution made to the cross section for second-order RRSL by processes with participation of free electron-hole pairs in a strong magnetic field (see also Ref. 12). In Sec. 2 we obtain an expression for the cross

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section for the two-phonon scattering in an arbitrary strong magnetic field. We show that for N-th order MRRSL ($N \geqslant 2$) the scattering cross section is of first order in the Fröhlich coupling constant α and explain qualitatively the lowering of the power of α compared with the case of a zero magnetic field. In Sec. 3 are considered interference contributions to the scattering cross section. These become substantial in a small vicinity of a discrete set of values of the strong magnetic field.

2. TWO-PHONON RRSL IN AN ARBITRARY STRONG MAGNETIC FIELD

We consider in this section the contribution to the cross section for second-order MRRSL ($N = 2$) from processes with participation of free EHP in an arbitrary[3)] strong magnetic field H:

$$\omega_{eH}\tau \gg 1, \tag{2.1}$$

where $\omega_{eH} = |e|H/m_e c$ is the cyclotron frequency, m_e and e are the effective mass and the charge of the electron, c is the speed of light, and τ is the electron relaxation time. The frequency of the primary emission is $\omega_l > \omega_g + 3\omega_{LO} + \omega_{eH}/2$, where $\hbar\omega_g$ is the band gap. For simplicity, the effective hole mass m_h is assumed infinite, its kinetic energy is infinitely small, and it is clear that the hole can emit an LO phonon ony via a virtual transition. This assumption is justified for semiconductors with large ratio m_h/m_e under the condition that $\omega_l - \omega_g \sim N\omega_{LO}$. In the calculation of the scattering cross section we use the magnetic-field potential in the Landau gauge $\mathbf{A} = \mathbf{A}(0,xH,0)$.

As already noted in the Introduction, there are two types of MRRSL in which free EHP participate: 1) scattering with direct creation and indirect annihilation of EHP (Fig. 1a) and 2) scattering with direct creation and direct annihilation of EHP (Fig. 1b).

The Fröhlich Hamiltonian of the interaction of electrons with LO phonons is[13]

$$H_{int} = \sum_{\mathbf{q}} (C_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}} b_{\mathbf{q}} + C_{\mathbf{q}}^* e^{-i\mathbf{q}\mathbf{r}} b_{\mathbf{q}}^+), \tag{2.2}$$

$$C_{\mathbf{q}} = -i\hbar\omega_{LO}\left(\frac{4\pi\alpha l^3}{V_0}\right)^{1/2}\frac{1}{lq}, \tag{2.3}$$

$$\alpha = \frac{e^2}{2\hbar\omega_{LO} l}(\varkappa_\infty^{-1} - \varkappa_0^{-1}), \quad l = \left(\frac{\hbar}{2m_e\omega_{LO}}\right)^{1/2},$$

where $b_{\mathbf{q}}$ ($b_{\mathbf{q}}^+$) are the annihilation (creation) operators for phonons with wave vector $\mathbf{q}$,$\mathbf{r}$ is the radius vector of the electron, $\varkappa_0$ and $\varkappa_\infty$ are the static and high-frequency dielectric constants, V_0 is the normalization volume. The corresponding expression for the holes differs from (2.2) in that the electron effective mass m_e is replaced by the hole effective mass m_h and that the sign of the right-hand side of (2.3) is reversed (it is easily seen that the interaction C_q does not depend on the electron (hole) effective mass, even though this mass is contained in the definition of α). It is assumed that the temperature is much lower than the Debye temperature and that processes with LO-phonon absorption can be neglected.

According to Ref. 14, the scattering cross section can be

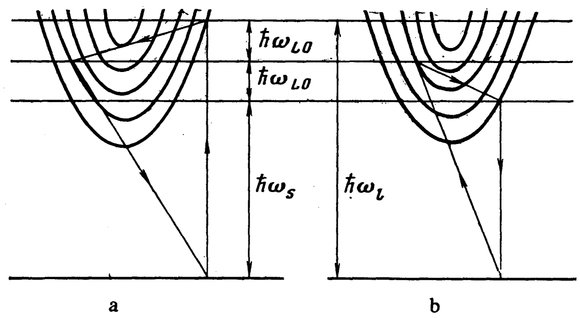

FIG. 1. Schematic representation of the second-order RRSL in a strong magnetic field with direct creation and indirect annihilation of EHP (a) and with indirect creation and direct annihilation of EHP (b).

expressed in the form

$$\frac{d^2\sigma}{d\Omega\, d\omega_s} = \frac{V_0\omega_s^3\omega_l}{c^4}\frac{n(\omega_s)}{n(\omega_l)} e_{s\alpha}^* e_{s\beta} e_{l\gamma} e_{l\lambda}^* S_{\alpha\gamma\beta\lambda}, \tag{2.4}$$

where Ω is the solid angle, $\mathbf{e}_l$ (e_s) is the polarization vector of the primary (secondary) light, $n(\omega)$ is the refractive index, and $S_{\alpha\gamma\beta\lambda}$ is the fourth-rank light-scattering tensor introduced in Ref. 14. We shall neglect hereafter the dispersion and the damping of the LO phonons. The scattering vector $S_{\alpha\gamma\beta\lambda}$, whose general form is obtained in Ref. 14, can then be written for the considered two-phonon process in the form

$$S_{\alpha\gamma\beta\lambda} = \frac{1}{V_0\hbar^2\omega_s^2\omega_l^2}\sum_{\mathbf{q}}\frac{1}{2}A_{\alpha\gamma}(\mathbf{q})A_{\beta\lambda}^*(\mathbf{q})\,\delta(\omega_l - \omega_s - 2\omega_{LO}), \tag{2.5}$$

where $A_{\alpha\gamma}(\mathbf{q})$ is the scattering amplitude (if the wave vectors of the primary and secondary light are neglected, the phonon wave vectors $\mathbf{q}_1$ and $\mathbf{q}_2$ in the two-phonon RRLS process are, by virtue of quasimomentum conservation, equal and opposite, $\mathbf{q}_1 = -\mathbf{q}_2 = \mathbf{q}$). The quantity $A_{\alpha\gamma}(\mathbf{q})$ was determined by a diagram procedure that is a particular case of the diagram technique for the light-scattering tensor $S_{\alpha\gamma\beta\lambda}$ (Ref. 14) with dispersion and damping of the LO phonons neglected.

All the graphs for the amplitude of the second-order MRRLS process are shown in Fig. 2. The solid lines above (below) the dash-dot line correspond to an electron (hole) and correspond to the expressions

$$iG_e(n_e, k_{ez}; \omega), \; iG_h(n_h, k_{hz}; \omega),$$

where

$$G_e(n_e, k_{ez}; \omega) = [\omega - (n_e + 1/2)\omega_{eH} - \hbar k_{ez}^2/2m_e + i\gamma_e(n_e, k_{ez})/2]^{-1}, \tag{2.6}$$

$$G_h(n_h, k_{hz}; \omega) = [\omega + i\gamma_h(n_h, k_{hz})/2]^{-1}$$

are respectively the Green's functions of the electron and hole, n is the number of the Landau band, k_z is the z-component of the wave vector, and $\gamma(n,k_z)$ is the reciprocal lifetime in the given quantum state. Since $m_h \gg m_e$, we can put $\gamma_h(n_h,k_{hz}) \ll \gamma_e(n_e,k_{ez})$. The wavy (dashed) lines correspond to phonons (photons) and are not associated with anything. The light circles denote the vertices of the interaction between the electron subsystem of the crystal and the light. The vertex with an incoming (outgoing) light line corre-

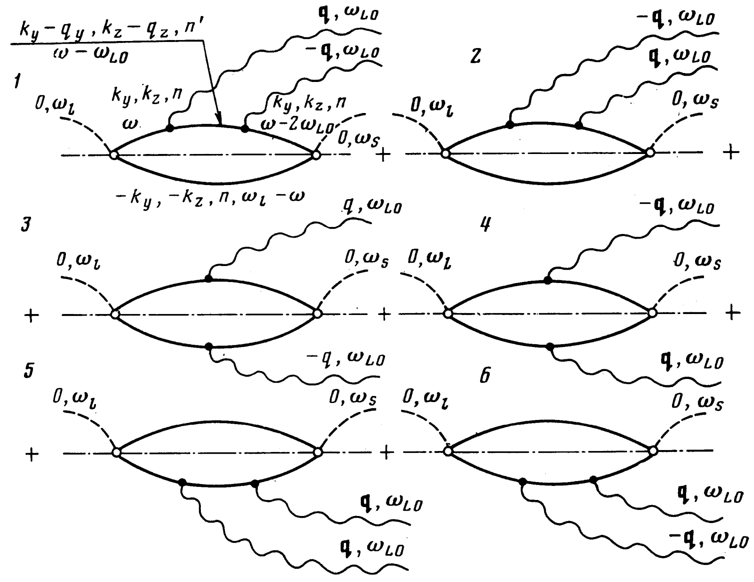

FIG. 2. Graphs that determine the amplitude of second-order RRSL.

sponds to the quantity $\mathscr{I}_\alpha$ ($\mathscr{I}_\alpha^+$) where

$$\mathscr{I}_\alpha=(e/m_0)\,p_{cv,\alpha}, \tag{2.7}$$

Here m_0 is the electron rest mass and $\mathbf{p}_{cv}$ is the interband matrix element of the momentum operator. The dark circles denote the vertices of the electron (outgoing hole) line (n,k_y,k_z) and outgoing electron (incoming hole) line (n',k_y',k_z'), the corresponding expression is

$$\pm iC_{\mathbf{q}}^*\exp\{il_H^2(\pm 2q_xk_y\mp q_xq_y+i(q_x^2+q_y^2)/2)\}$$
$$\times\begin{cases}\sqrt{\dfrac{n'!}{n!}}[l_H(\pm q_y-iq_x)]^{n-n'}L_{n'}^{n-n'}\,[l_H^2(q_x^2+q_y^2)], & n>n'\\ \sqrt{\dfrac{n!}{n'!}}[l_H(\mp q_y-iq_x)]^{n'-n}L_{n}^{n'-n}\,[l_H^2(q_x^2+q_y^2)], & n<n'\end{cases}, \tag{2.8}$$

where q_x and $q_y=k_y-k_y'$ are the components of the phonon wave vector, $l_H=(\hbar c/2eH)^{1/2}$ is the magnetic length, $L_m^p(x)$ is a Laguerre polynomial, and C_q is defined in (2.3). The upper and lower signs in (2.8) correspond to electrons and holes, respectively.

The conservation laws for frequency and for the y- and z-components of the wave vector are satisfied at all vertices. $n_e=n_h$ at the vertices of the interaction between the electron subsystem and the light. When calculating the expression that corresponds to a graph it is necessary to integrate with respect to ω with weight $(2\pi)^{-1}$ and sum over n, n', k_y, and k_z.

The total amplitude of the process is expressed by the sum

$$A_{\alpha\gamma}(\mathbf{q})=\sum_{i=1}^{6}A_{\alpha\gamma}^{(i)}(\mathbf{q}),$$

where each term of the sum corresponds to a graph in Fig. 2 and to an expression in accordance with the diagram-technique rules cited above.

The calculation results show that the following equalities hold:

$$A_{\alpha\gamma}^{(i)}(\mathbf{q})=A_{\alpha\gamma}^{(i)}(-\mathbf{q}),\quad A_{\alpha\gamma}^{(3)}(\mathbf{q})=-2A_{\alpha\gamma}^{(1)}(\mathbf{q}), \tag{2.9}$$

whence

$$A_{\alpha\gamma}(\mathbf{q})=2[A_{\alpha\gamma}^{(5)}(\mathbf{q})-A_{\alpha\gamma}^{(1)}(\mathbf{q})]. \tag{2.10}$$

We obtain ultimately for the total second order MRRSL amplitude in a strong magnetic field (as $m_h\to\infty$)

$$A_{\alpha\gamma}(\mathbf{q})=\sum_{nn'}2\left(\frac{e}{m_0}\right)^2 p_{cv,\alpha}\dot{p}_{cv,\gamma}\alpha\omega_{LO}ll_H^3\frac{m_e^2}{\hbar^2}$$
$$\times\frac{T(n,n',x)}{x+l_H^2q_z^2}\left\{-\frac{1}{\bar{k}_0(n)\bar{k}_1(n)[\bar{k}_0(n)+\bar{k}_1(n)]}\right.$$
$$+\frac{1}{\bar{k}_2(n)\bar{k}_1(n)[\bar{k}_2(n)+\bar{k}_1(n)]}$$
$$+\frac{\bar{k}_0(n)+\bar{k}_1(n')}{\bar{k}_0(n)\bar{k}_1(n')[(\bar{k}_0(n)+\bar{k}_1(n'))^2-q_z^2]}$$
$$\left.-\frac{\bar{k}_2(n)+\bar{k}_1(n')}{\bar{k}_2(n)\bar{k}_1(n')[(\bar{k}_2(n)+\bar{k}_1(n'))^2-q_z^2]}\right\}, \tag{2.11}$$

where

$$T(n,n',x)=\frac{\min(n!,n'!)}{\max(n!,n'!)}e^{-x}x^{|n-n'|}[L_{\min(n,n')}^{|n-n'|}(x)]^2,$$
$$x=l_H^2(q_x^2+q_y^2),\quad \bar{k}_s(n)/l_H$$
$$=\left[\frac{2m_e}{\hbar}(\omega_l-\omega_g-(n+{}^1/_2)\omega_{eH}-s\omega_{LO}+i\gamma_s(n)/2)\right]^{1/2},$$

and $\gamma_s(n)$ is the reciprocal lifetime of the electron produced upon absorption of a photon $\hbar\omega_l$ that emits s LO phonons and is situated in the Landau band numbered n. The first two terms in the curly brackets of (2.11) correspond to the contribution of processes in which both phonons are emitted (virtually) by a hole [plots 5 and 6 of Fig. 2; the first term in (2.10)], with the first and second terms corresponding to processes with direct production and direct annihilation of the EHP. The third and fourth terms in the curly brackets of

(2.11) correspond to the sum of the contributions of graphs 1–4 in Fig. 2 [the second term in (2.10)], with the third and fourth terms corresponding to the contributions of the processes with direct production and direct annihilation of the EHP.

We note that at $n = n'$ and as $q \to 0$ the contribution of each graph of Fig. 2, taken separately, diverges but the total amplitude $A_{\alpha\gamma}$ ($\mathbf{q} \to 0$) remains finite, as can be seen from (2.11), by virtue of the mutual cancellation of the contributions of the processes in which electrons and holes take part.

To calculate the scattering cross section, (2.11) must be substituted in (2.5) and (2.4). Analysis of the resultant rather lengthy expression permits separation of the terms that make the main contribution to the cross section:

$$\sigma_2=\sigma_2^{(0)}\sum_{nn'mm'}\int_0^\infty dx T(n,n',x)T(m,m',x)$$

$$\times 2\,\mathrm{Im}\left\{\frac{\chi_1^*}{\tilde{k}_0(n)\tilde{k}_1(n')\tilde{k}_0^*(m)\tilde{k}_1^*(m')(\beta_1^2+x)^2(\chi_1^{*2}-\beta_1^2)}\right.$$

$$+\frac{\chi_2^*}{\tilde{k}_2(n)\tilde{k}_1(n')\tilde{k}_2^*(m)\tilde{k}_1^*(m')(\beta_2^2+x)^2(\chi_2^{*2}-\beta_2^2)}$$

$$-\frac{\chi_1^*}{\tilde{k}_2(n)\tilde{k}_1(n')\tilde{k}_0^*(m)\tilde{k}_1^*(m')(\beta_2^2+x)^2(\chi_1^{*2}-\beta_2^2)}$$

$$\left.-\frac{\chi_2^*}{\tilde{k}_0(n)\tilde{k}_1(n')\tilde{k}_2^*(m)\tilde{k}_1^*(m')(\beta_1^2+x)^2(\chi_2^{*2}-\beta_1^2)}\right\} \quad (2.12)$$

$$\chi_1=\tilde{k}_0(m)+\tilde{k}_1(m'),\quad \chi_2=\tilde{k}_2(m)+\tilde{k}_1(m'),$$

$$\beta_1=\tilde{k}_0(n)+\tilde{k}_1(n'),\quad \beta_2=\tilde{k}_2(n)+\tilde{k}_1(n'), \quad (2.13)$$

$$\sigma_2^{(0)}=\frac{\omega_s}{\omega_l}\frac{n(\omega_s)}{n(\omega_l)}V_0|\mathbf{p}_{cv}\mathbf{e}_l|^2|\mathbf{p}_{cv}\mathbf{e}_s|^2\alpha^2\frac{|e|^{5/2}\omega_{LO}m_e^3}{\hbar^{7/2}c^{5/2}m_0^4\cdot 2^{5/2}H^{3/2}}.$$

The quantity $\sigma_2^{(0)}$ with dimension of cross section is of second order in the Fröhlich coupling constant α. Equation (2.12) does not hold at magnetic fields H and incident radiation frequencies ω_l at which the electron reciprocal relaxation time is determined, even in one of the intermediate states in the course of scattering, by the possibility of a transition to a small vicinity $\sim\alpha\omega_{LO}$ of the bottom of one of the Landau bands. To obtain the correct result account must be taken of the influence exerted on the excitation spectrum by the interaction of the electrons and holes with the LO phonons, so that the spectrum is renormalized as the result.[15] We shall assume hereafter that ω_l and H are such that (2.12) is valid. We assume in addition that ω_l and H exclude the possibility of an electron landing in the vicinity $\sim\alpha\omega_{LO}$ of the bottom of one of the Landau bands in the course of the EHP production. It can then be noted that under the condition $n = m$, $n' = m'$ we obtain $\chi_1 = \beta_1$, $\chi_2 = \beta_2$, and

$$\beta_1^2-\chi_1^{*2}\approx i\frac{k_0(n)+k_1(n')}{\omega_{eH}}\frac{k_0(n)\gamma_1(n')+k_1(n')\gamma_0(n)}{k_0(n)k_1(n')} \quad (2.14)$$

with an analogous relation for $\beta_2 - \chi_2^{*2}$

$$k_s(n)/l_H=\left[\frac{2m_e}{\hbar}(\omega_l-\omega_g-(n+{}^1/_2)\omega_{eH}-s\omega_{LO})\right]^{1/2}.$$

But since the reciprocal relaxation time of an electron in a polar semiconductor is $\gamma\propto\alpha$, it follows from (2.14) and (2.12) that the main contribution to the second order cross section σ_2 in an arbitrary strong magnetic field is made by the first two terms in the curly brackets of (2.12), and this contribution is of first order in the Fröhlich coupling constant α. In an arbitrary strong magnetic field the third and fourth terms in (2.12) make a contribution of second order in α to the cross section, and we shall neglect them in this section (we shall return to these contributions in the next section, where we shall show that close to a number of magnetic-field values the contributions of the third and fourth terms of (2.12) become of first order in α). Analysis shows that in the calculation of (2.12) the main contribution to the sum over q_z in (2.5) is made by a small vicinity of the discrete series of values $q_z = k_0(n) + k_1(n')$ and $q_z = k_2(n) + k_1(n')$. It is imporant to note that expression (2.12) for the cross section σ_2 corresponds to the third and fourth terms in the curly brackets of expression (2.11) for the total amplitude of the process. The first and second terms in the curly brackets in (2.11) correspond to processes in which both phonons are emitted by a hole and, as can be seen by direct calculation of the cross section and will be seen later from qualitative considerations, the contribution of such processes to the scattering cross section is of second order in the coupling constant α.

The final form of the MRRSL cross section of second order in an arbitrary strong magnetic field (excluding values of ω_l and H for which at least one transition into the vicinity $\sim\alpha\omega_{LO}$ of one of the Landau bands is possible) can be written as

$$\sigma_2^{(a)}\approx 2\sigma_2^{(0)}\omega_{eH}\int_0^\infty dx[T(n,n',x)]^2$$

$$\times\left\{\sum_{\substack{0\leqslant n\leqslant s_1\\0\leqslant n'\leqslant s_2}}\frac{f_0(n,n',x)}{k_0(n)k_1(n')[k_0(n)\gamma_1(n')+k_1(n')\gamma_0(n)]}\right.$$

$$\left.+\sum_{\substack{0\leqslant n\leqslant s_3\\0\leqslant n'\leqslant s_2}}\frac{f_2(n,n',x)}{k_2(n)k_1(n')[k_2(n)\gamma_1(n')+k_1(n')\gamma_2(n)]}\right\}; \quad (2.15)$$

$$s_1\leqslant\omega_{0e}/\omega_{eH},\ s_2\leqslant(\omega_{0e}-\omega_{LO})/\omega_{eH},$$

$$s_3\leqslant(\omega_{0e}-2\omega_{LO})/\omega_{eH},\quad f_i=[x+l_H^2(k_i(n)+k_1(n'))^2]^{-2},\ i=0,\ 2;$$

$$\omega_{0e}=\omega_l-\omega_g-\omega_{eH}/2.$$

The first term in the curly brackets in (2.15) corresponds to the contribution made to the cross section for the scattering of processes with direct EHP creation, the second to the contribution of processes with direct EHP annihilation. The dependence of the cross section on the magnetic field H at fixed frequency ω_l (or of the frequency ω_l of the primary light at fixed H) should exhibit characteristic oscillations (see Fig. 4 below) the locations of whose maxima is determined by the condition

$$H=H_{ns}=[(\omega_l-\omega_g)/\omega_{LO}-s]H_0/(n+{}^1/_2),\ H_0=m_e\omega_{LO}c/|e|,$$

n is the number of the Landau band on the bottom of which the electron lands after emitting $s = 0$, 1, *or* 2 phonons. These oscillations are connected with the increase (in proportion to k_2^{-1}) of the density of states near the bottom of the Landau band. As already indicated, Eq. (2.15) is not valid in the vicinity $\sim\alpha H_0$ of the values $H = H_{ns}$. We note that the

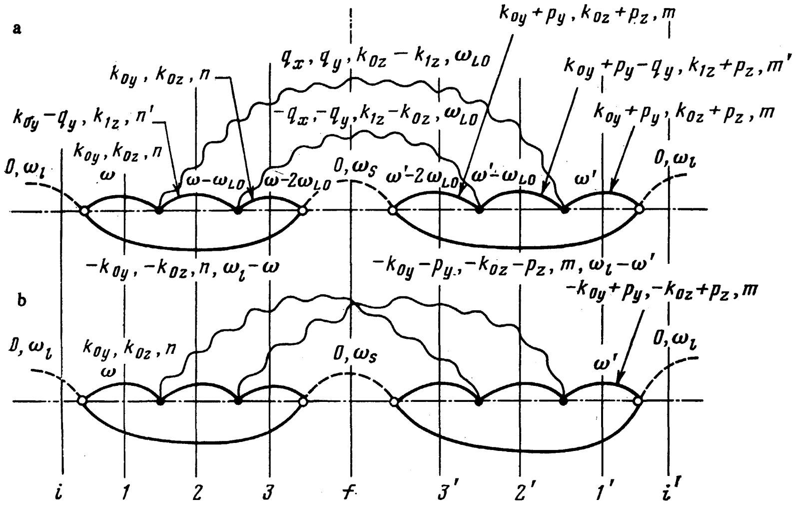

FIG. 3. Two of the eight graphs that make the main contribution to the expressions that correspond to the light-scattering tensor $S_{\alpha\gamma\beta\lambda}$ for second-order RRSL in a strong magnetic field.

renormalization of the spectrum should lead to a smoothing of the oscillations.

Thus, when a strong magnetic field is turned on the degree of the Fröhlich coupling constant is lowered in the expression for the second-order cross section for the MRRSL process ($\sigma_2 \propto \alpha^2$, $H=0$). We shall give below a qualitative explanation of the proportionality $\sigma_2 \propto \alpha$ in a strong magnetic field. We use for this purpose a diagram technique for direct calculation of the scattering tensor $S_{\alpha\gamma\beta\lambda}$ (Ref. 14). For a second-order scattering process the tensor $S_{\alpha\gamma\beta\lambda}$ has only eight graphs corresponding to cross-section contributions of first order in the coupling constant α. All these graphs correspond to processes in which the hole emits not more than ne phonon (these graphs can be picked out by calculating the cross section as a function of the scattering amplitude). Figures 3a and 3b show two of the eight graphs of this type. The rules of the diagram technique for these graphs, neglecting dispersion and damping of the *LO* phonons, differ from the corresponding rules for graphs corresponding to the scattering amplitude (see Fig. 2) only in that all the relations set in correspondence to all the Green's functions and vertices to the right of the section f that corresponds to the final state in the scattering process (see Fig. 3) are the complex-conjugates of (2.6)–(2.8). The graph in Fig. 3b corresponds to interference between processes with different sequences of *LO*-phonon emission.

We shall demonstrate the calculation of such graphs using 3a as the example. The integration with respect to the frequencies ω and ω' is elementary. Next, from the forms of the expressions that are set in correspondence with the lines and vertices of the graph it is clear that the summation over k_{0z}, k_{1z}, and p_z reduces to calculation of the expression

$$P=\sum_{p_z}\Big[\sum_{k_{0z}} G_e(n,k_{0z};\omega_l)G_e(n,k_{0z};\omega_l-2\omega_{LO})$$
$$\times G_e^*(m,k_{0z}+p_z;\omega_l)G_e^*(m,k_{0z}+p_z;\omega_l-2\omega_{LO})\Big]$$
$$\times\Big[\sum_{k_{1z}} G_e(n',k_{1z};\omega_l-\omega_{LO})G_e^*(m',k_{1z}+p_z;\omega_l-\omega_{LO})\Big].$$

The summation over k_y, p_y, q_x, and q_y involves only the vertices and leads to the same integral as in (2.15). It can be shown that the main contribution to the sum over p_z is made by $p_z \sim \gamma l_H m_e/\hbar k(n)$ at $n=m$ and $n'=m'$. The main contribution to the sum over k_{0z} and k_{1z} is made by the vicinities of the poles of the corresponding Green's functions. We write down the equations for the poles of the Green's functions in cuts 2 and 2′ of the diagram of Fig. 3a:

$$\hbar k_{1z}^2/2m_e+(n'+{}^1/_2)\omega_{eH}+i\gamma_1(n')/2=\omega_l-\omega_g-\omega_{LO},$$
$$\hbar(k_{1z}+p_z)^2/2m_e+(m'+{}^1/_2)\omega_{eH}-i\gamma_1(m')/2=\omega_l-\omega_g-\omega_{LO}.$$

Under the conditions $n'=m'$ and $p_z \sim \gamma l_H m_e/\hbar k(n)$ these poles are equal to within a quantity of the order of the reciprocal relaxation time γ. Similarly, if $n=m$ and $p_z \sim \gamma l_H m_e/\hbar k(n)$, upon integration with respect to k_{0z}, the poles of the Green's functions in the cuts 1 and 1′ for the process with direct creation of EHP and in cuts 3 and 3′ for the process with direct annihilation of EHP are equal to within a quantity of the order of γ. If the cuts 1 and 1′ are resonant (this corresponds to direct creation), the cuts 3 and 3′ are nonresonant (indirect annihilation), and vice versa. The described equality of the poles corresponds to the fact that the main contribution to the second-order scattering cross section is made by processes in which two out of three states that are intermediate in the scattering process are real. P takes then the form ($n=m$, $n'=m'$; $p_z \sim \gamma l_H m_e/\hbar k(n)$):

$$P=\frac{m_e^2L_z^3}{2\pi\hbar^2\omega_{LO}^2}\int dp_z\Big[\frac{\gamma_0(n)}{(\hbar k_0(n)p_z/m_el_H)^2+\gamma_0^2(n)}\frac{l_H}{k_0(n)}$$
$$+\frac{\gamma_2(n)}{(\hbar k_2(n)p_z/m_el_H)^2+\gamma_2^2(n)}\frac{l_H}{k_2(n)}\Big]$$
$$\times\frac{\gamma_1(n')}{(\hbar k_1(n')p_z/m_el_H)^2+\gamma_1^2(n')}\frac{l_H}{k_1(n')},\qquad(2.16)$$

where $L_xL_yL_z=V_0$. It can be seen that $P\propto\gamma^{-1}\propto\alpha^{-1}$, and when α^2 is taken into account we obtain from the vertices of the electron-phonon interaction that $\sigma_2\propto\alpha$.

In processes in which both phonons are emitted by a hole, all the intermediate states in the scattering process are virtual (we recall that the hole can emit an LO phonon only as a result of a virtual transition, since $m_n \to \infty$), therefore the contribution of such processes to the cross section is of second order in the coupling constant α.

Calculating the contribution of the interference graph (Fig. 3b), we can see that if the LO-phonon damping is neglected this contribution is exactly equal to the contribution of graph 3a [this is a direct consequence of the first equality of (2.9)]. A doubling of this type when allowance is made for graphs with crossing phonon lines was obtained in Ref. 14 for RRLS processes of second order in the case of scattering via excitonic states in a zero magnetic field. It can be shown that the contribution of any of the six remaining graphs for two-phonon scattering differs from the contribution of the graph of Fig. 3a only by an integer factor, and for four out of the six graphs it differs in sign; when added up, these six graphs make a zero contribution [this cancellation of the contributions follows from the second equality of (2.9)]. The final result coincides with (2.15).

We can now show, without resorting to quantitative result, how the scattering cross section for an N-th order process with $N \geqslant 2$ depends on the coupling constant α. We consider a graph of the ladder type (Fig. 3a for $N=2$). In analogy with the two-phonon process it can be shown that the main contribution to the sum over p_z corresponds to $p_z \sim \gamma l_H m_e/\hbar k(n)$, $n_s = m_s$, and that increasing the order of the scattering by unity adds each time to P an additional factor in the form

$$\sum_{k_{sz}} G_e(n_s, k_{sz}; \omega_l - s\omega_{LO}) G_e^*(m_s, k_{sz}+p_z, \omega_l - s\omega_{LO}) \approx \frac{2m_e L_z}{\hbar}$$

$$\times \frac{\gamma_s(n_s)}{(\hbar k_s(n_s) p_z/m_e l_H)^2 + \gamma_s^2(n_s)} \cdot \frac{l_H}{k_s(n_s)}.$$

This factor $\propto \gamma^{-1} \propto \gamma^{-1}$. If account is taken of the additional factor $\propto \alpha$ from two vertices, it is clear that an increase in the order of the scattering does not change the dependence of the scattering cross section on α. We note that at $p_z \sim \gamma l_H m_e/\hbar k(n)$ and $n_s = m_s$ the poles of both Green's functions in the additional factor of P agree accurate to $\sim \gamma$. Therefore in N-th order MRRSL ($N \geqslant 2$) in a strong magnetic field, N out of the $N+1$ intermediate states are real and one intermediate state (in indirect creation or in indirect annihilation) is virtual, and $\sigma_N \propto \alpha$, $N \geqslant 2$. We note for comparison that in the case of a zero magnetic field N out of $N+1$ intermediate states are real and $\sigma_N \propto \alpha^3$ at $M \geqslant 4$ (see Ref. 11).

We now shed light on the physical reason why the degree of the Fröhlich coupling constant α is lowered[12] in the cross section for the MRRSL process when a strong magnetic field is turned on. It was shown in Ref. 11 that at $H=0$ real wandering of the electrons over the crystal takes place in MRRLS of N-th order ($N \geqslant 4$) when N out of the $N+1$ intermediate states are real. The MRRLS cross section is proportional to the probability of return of the electron to the EHP creation point after emission of $N-1$ LO phonons (the heavy hole remains at the point of EHP creation). It is shown in Ref. 11 that the probability of electron return to the EHP creation point is directly proportional to the cube of the coupling constant α, therefore the cross section is $\sigma_N \propto \alpha^3$ for $N \geqslant 4$. In other words, at $N \geqslant 4$ the characteristic volume[11] V_{EHP} occupied by the EHP created by the light is proportional after emission of several LO phonons to the cube of the mean free path, and since the mean free path in polar semiconductors is inversely proportional to the first power of α, we have $V_{\text{EHP}} \propto \alpha^{-3}$ and the EHP annihilation probability is $\propto V_{\text{EHP}}^{-1} \propto \alpha^3$. For $N=2$ or 3 the main contribution to the cross section is made by processes in which all $N+1$ intermediate states are virtual, so that $\sigma_2 \propto \alpha^2$ for $N=2$ and $\sigma^3 \propto \alpha^3 \ln \alpha - 1$ for $N=3$.

In the case of a strong magnetic field, as already noted, N out of the $N+1$ intermediate states are real in an MRRSL of N-th order ($N \geqslant 2$), and we can use the cited qualitative arguments connected with random walk of the electron over the crystal. In the language of helical quasiclassical electron trajectories in a strong magnetic field, it can be stated that when a strong magnetic field is turned on the electron motion becomes quasi-one-dimensional: the motion remains free along the field but is bounded in a plane perpendicular to the **H** direction. The character of the distribution in the relative distance between the electron and the hole is different along the $z \| \mathbf{H}$ axis from that in the plane perpendicular to **H**. The z-projection of the vector relative to the distance between the electron and the hole after emission of several LO phonons is on the average proportional to the mean free path, i.e., $\propto \alpha^{-1}$, and with increasing number N of emitted phonons it increases only numerically. In a plane perpendicular to the **H** direction, the relative positions of the electron and hole after emission of a finite number N of phonons depends only on the number of phonons emitted, but not on the probability of emitting an LO phonon (and hence also on α), inasmuch as in each collision with a phonon the position of the electron-orbit center changes jumpwise (the hole is immobile), while in the interval between the collisions with the phonons the electron revolves around the center of the orbit and its average distance to the hole does not change. The area occupied by the EHP in the plane perpendicular to **H** is bounded and does not depend on the strength of the interaction of the electrons and holes with the LO phonons. With increasing number of emitted phonons this area increases only numerically. Consequently $V_{\text{EHP}} \propto \alpha^{-1}$ and $\sigma_N \propto \alpha$ for $N \geqslant 2$ in a strong magnetic field. In other words, the probability that the electron will return to the EHP creation point after emission of $(N-1)$ LO phonons is directly proportional to the coupling constant α, hence $\sigma_N \propto \alpha$.

Comparison of the expression for second-order MRRSL in a strong magnetic field (2.15) with the corresponding expression in a zero magnetic field shows that if $\omega_{eH} \sim \omega_{LO}$ turning on a strong magnetic field can increase the scattering intensity by α^{-1} times. For a *polar semiconductor* such as InSb($\alpha \approx 0.013$) in a field ~ 30 kG one can expect the scattering to be increased by approximately 100 times. For N-th order MRRSL ($N \geqslant 4$) the scattering can be expected to increase by α^{-2} times, i.e., by 10^4 times for InSb.

3. INTERFERENCE EFFECTS IN TWO-LEVEL RRSL IN A STRONG MAGNETIC FIELD

It was shown in Fig. 2 that in an arbitrary strong magnetic field the main contribution to the cross section for two-phonon RRSL is made by processes in which two out of the three intermediate states in the scattering process are real, and that this contribution is of first order in the coupling constant α. The main contribution of the processes in which two intermediate states are real corresponds to the main contribution made to (2.12) by the term with $n = m$ and $n' = m'$. Contributions to (2.12) from terms with $n \neq m$ and (or) $n \neq m'$ are of second order in α and are negligibly small in the considered case $\alpha \ll 1$. An analysis of the graphs for the scattering tensor $S_{\alpha\gamma\beta\lambda}$ has shown that the main contribution to the scattering cross sections is made by graphs of two types: of the "ladder" type shown in Fig. 3a and of the "fan" type shown in Fig. 3b and corresponding to interference of processes with differing order of LO-phonon emission. Neglecting the LO-phonon damping, the contributions from both graphs are equal in size. We turn anew to expression (2.12) for the scattering cross section. It turns out that the interference effects in two-phonon RRSL are not restricted to the fact that when the damping of the LO phonons is neglected the fan graph for the light-scattering tensor leads to a doubling of the cross section. Assume that the magnetic field is subject to the following condition:

$$\frac{\omega_{LO}}{\omega_{eH}} = L, \quad H = \frac{m_e c \omega_{LO}}{|e|} \frac{1}{L} = \frac{H_0}{L}, \tag{3.1}$$

where $L = 1, 2, 3, \ldots$.

An analysis of expression (2.12) shows that under the condition (3.1) there appear a number of additional contributions to the cross section σ_2 which is of first order in the coupling constant α. If $m' = n - L$, $m = n' + L$ we have $\chi_1^{*2} - \beta_1^2 \sim \gamma$ [cf. (2.14)], and this corresponds to a contribution of first order to the cross section of first order in α from processes with indirect creation and indirect annihilation of EHP. If $m' = n + L$, $m = n' + L$ we have $\chi_2^{*2} - \beta_2^2 \sim \gamma$, which corresponds to an additional contribution, of first order in α, from processes with direct creation and direct annihilation of EHP. In an arbitrary strong magnetic field the third and fourth terms in the curly brackets of (2.12) made a contribution of second order in α, which was neglected. If the condition (3.1) is satisfied, the contribution of these terms also becomes of first order in the coupling constant α. We consider those terms of the sum for which $m = n + 2L$, $m' = n'$ or $m = n' + L$, $m' = n + L$; for any pair of conditions the third term in the curly bracket of (2.12) makes a contribution of first order in α to the cross section. This contribution is negative and there is no interference of processes with direct creation and direct annihilation of EHP. A similar contribution is made by the fourth term in the curly bracket of (2.12) at $m = n - 2L$, $m' = n'$ or $m = n' - L$, $m' = n - L$. It can be seen from the conditions $m = n + 2L$, $m' = n'$ or $m = n - 2L$, $m' = n'$ that negative cross-section contributions of first order in α occur not only for fields determined by integer L in (3.1), but also for fields determined by half-integer L. To describe more lucidly the cause of the additional contributions to the cross section of first order in α under condition (3.1), we turn to the graphs for the scattering tensor $S_{\alpha\gamma\beta\lambda}$. In an arbitrary strong magnetic field the main contribution to the sum over p_z was made by the region $p_z \sim \gamma l_H m_e / \hbar k(n)$. In formal language this means that the integral with respect to p_z was taken over the poles and only pure imaginary poles $p_z \sim i\gamma$ led to cross-section contributions of first order in α. It can be shown that under condition (3.1) an important role is assumed by a host of poles that differ from the preceding by a real part $\sim k(n)$. Their contribution to the scattering cross section under condition (3.1) is found to be of first order in α. These poles can be easily taken into account by making the change of variable $p_z' = p_z + k_{0z} + k_{1z}$ in the expressions that correspond to the graphs for the tensor $S_{\alpha\gamma\beta\lambda}$ (e.g., to the graphs of Fig. 3). The main contribution to the integration with respect to p_z' is then made by the poles $p_z' \sim i\gamma$.

We have already shown that the additional contributions to the cross section (corresponding to the contribution of the poles with large real part) under conditions (3.1) become contributions of first order in the coupling constant α. One can expect two out of the three intermediate states in the scattering process to be real. Let us verify this. We write down the condition for the vanishing of the denominators of the Green's function for the graph of Fig. 3a (after making the change of variable $p_z \to p_z'$) for a process with direct creation:

$$\begin{gathered} \hbar k_{0z}^2/2m_e + (n+{}^1/_2)\,\omega_{eH} + i\gamma_0(n)/2 = \omega_l, \\ \hbar(-k_{0z}+p_z')^2/2m_e + (m'+{}^1/_2)\,\omega_{eH} - i\gamma_1(m')/2 = \omega_l - \omega_{LO}, \\ \hbar k_{1z}^2/2m_e + (n'+{}^1/_2)\,\omega_{eH} + i\gamma_1(n')/2 = \omega_l - \omega_{LO}, \\ \hbar(-k_{1z}+p_z')^2/2m_e + (m+{}^1/_2)\,\omega_{eH} - i\gamma_0(m)/2 = \omega_l. \end{gathered} \tag{3.2}$$

All these equations are compatible at $p_z' \sim \gamma l_H m_e / \hbar k(n)$ and $n = m' + L$, $n' = m - L$. We can similarly obtain all the conditions on n, n', m, and m' if (3.1) holds. These conditions separate the main contribution of first order in α and correspond to interference of processes in which two out of the three intermediate states in the scattering process are real. The interfering processes can differ only in the order of the emission of the LO phonons, but also in the partition of the energy among the longitudinal and transverse motions in real intermediate states. This can be easily seen by analyzing the expressions of type (3.2) (the interference contributions considered in Sec. 2—fan-type graph on Fig. 3b— correspond to interference of processes that differ only in the sequence of the emission of the LO phonons). In addition, contributions of first order in α come also from interference of processes in which EHP are directly created and annihilated.

Let us explain why contributions corresponding to interference between the direct creation and direct annihilation processes are negative. We note for this purpose that, for example for a process with direct production, each of the two electron Green functions corresponding to a virtual state of the EHP will yield a factor $(-1/2\omega_{LO})$, and $(1/2\omega_{LO})^2$ in the result. For the contributions corresponding to the interference of processes with direct production and direct annihilation of EHP, the virtual state of one of the interfering processes yields a factor $(1/2\omega_{LO})$ and that of the other $(-1/2\omega_{LO})$. The resultant factor $-(1/(2\omega_{LO})^2)$ differs in sign from the corresponding factor for processes with direct creation (or with direct annihilation) of EHP.

We have considered the interference contributions to the scattering cross section of first order in α under condition (3.1) for the graphs on Figs. 2a and 3b. In analogy with the case of an arbitrary strong magnetic field it can be shown that if the LO-phonon damping is neglected the additional interference contributions are the same for both graphs, and the contributions of the same character from the remaining six graphs differ from the contribution of the graph of Fig. 3a only by an integer factor and are of different sign, thus adding up to a zero contribution.

Denoting by $\sigma_2^{(b)}$ the interference contributions to the cross section σ_2 (2.12), which were obtained in this section under condition (3.1), we can write

$$\sigma_2^{(b)}=2\sigma_2^{(0)}\omega_{eH}\int_0^\infty dx\left\{\sum_{\substack{L\leqslant n\leqslant s_1\\0\leqslant n'\leqslant s_2}}\frac{T(n,n',x)T(n'+L,n-L,x)f_0(n,n',x)}{{}^1/_2[\gamma_0(n'+L)+\gamma_1(n')]k_0(n)+{}^1/_2[\gamma_0(n)+\gamma_1(n-L)]k_1(n')}\right.$$
$$+\sum_{\substack{0\leqslant n\leqslant s_3\\L\leqslant n'\leqslant s_2}}\frac{T(n,n',x)T(n'-L,n+L,x)f_2(n,n',x)}{{}^1/_2[\gamma_1(n')+\gamma_2(n'-L)]k_2(n)+{}^1/_2[\gamma_1(n+L)+\gamma_2(n)]k_1(n')}$$
$$-2\sum_{\substack{0\leqslant n\leqslant s_3\\0\leqslant n'\leqslant s_2}}\frac{T(n,n',x)T(n'+L,n+L,x)f_2(n,n',x)}{{}^1/_2[\gamma_0(n'+L)+\gamma_1(n')]k_2(n)+{}^1/_2[\gamma_1(n+L)+\gamma_2(n)]k_1(n')}$$
$$\left.-2\sum_{\substack{0\leqslant n\leqslant s_3\\0\leqslant n'\leqslant s_2}}\frac{T(n,n',x)T(n+2L,n',x)f_2(n,n',x)}{\gamma_1(n')k_2(n)+{}^1/_2[\gamma_0(n+2L)+\gamma_2(n)]k_1(n')}\right\}. \quad (3.3)$$

The first term in (3.3) corresponds to interference of processes with direct EHP production, the second to interference of processes with direct EHP annihilation, and the remaining terms correspond to interference processes with direct creation and direct annihilation of EHP.

Equation (3.3) determines the interference contribution to the scattering cross section only in a discrete set of magnetic-field values determined by the condition (3.1). A more detailed analysis shows that the dependence of the contribution $\sigma_2^{(b)}$ on the value of H at a fixed frequency ω_l has the character of resonances of width $\sim\alpha H_0/L$.

We have so far neglected throughout the dispersion of the LO phonons on the interference contributions $\sigma_2^{(b)}$, we turn, for example, to expressions (3.2) and replace the limiting frequency ω_{LO} in them by $\omega_{LO}+\Delta(\mathbf{q})$, where $\Delta(\mathbf{q})$ is the dispersion. For Eqs. (3.2) to be compatible to within values of the order of γ at arbitrary $\mathbf{q}$ we must have then $\Delta(\mathbf{q})\lesssim\gamma\sim\alpha\omega_{LO}$. It can be shown that in the processes considered the significant phonons are those with wave vectors $|\mathbf{q}|\sim(2m_eE/\hbar^2)^{1/2}$, where E is the electron energy reckoned from the bottom of the conduction band. At $E\sim1$ eV we have $|\mathbf{q}|\sim10^6$–10^7 cm^{-1} ($m_e/m_0\sim0.1$–0.01), or ~0.01–0.1 of the characteristic dimension of the first Brillouin zone. One can expect $\Delta(\mathbf{q})$ to be small and the effect of the dispersion to be negligible in this case.

Figure 4 shows the results of a numerical calculation for the dimensionless quantity $(\sigma_2^{(a)}+\sigma_2^{(b)})\alpha\omega_{LO}/2\omega_{eH}\sigma_2^{(0)}$, carried out for the model Hamiltonian of the interaction of electrons and holes with LO phonons.[7] This Hamiltonian does not depend on the phonon wave vector (to transform to the model of Ref. 7 it suffices to put $f_0=f_2$ in (2.15) and (3.3)). It was assumed in the numerical calculations that $\omega_l-\omega_g=3.8\omega_{LO}$. It was found that the total interference contribution $\sigma_2^{(b)}$ (3.3) to the scattering cross section at $L=1$ and $L=1.5$ is about 40% of the contribution of $\sigma_2^{(a)}$ (2.15) and is negative, i.e., a resonant decrease of the cross section takes place in the vicinity of magnetic field values defined by

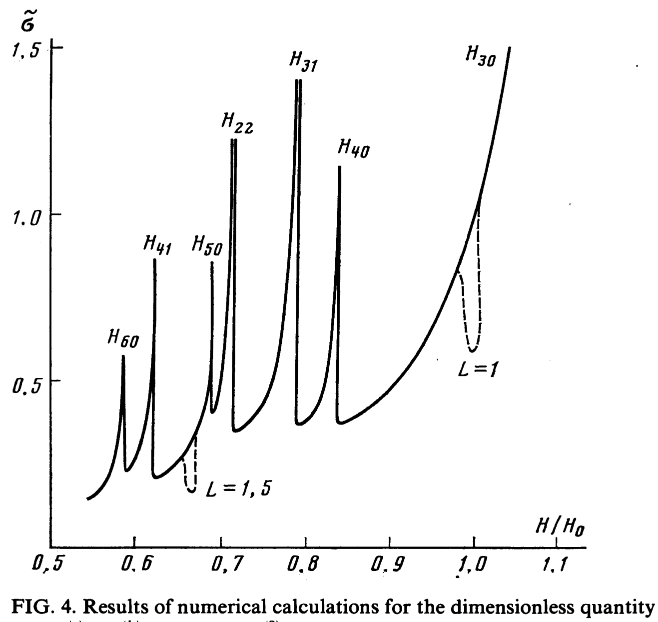

FIG. 4. Results of numerical calculations for the dimensionless quantity $\tilde{\sigma}=(\sigma_2^{(a)}+\sigma_2^{(b)})\alpha\omega_{LO}/2\omega e_H\sigma_2^{(0)}$ vs H/H_0, $H_0=m_ec\omega_{LO}/|e|$ at $\omega_l-\omega_g=3.8\omega_{LO}$. The positions of the maxima of the oscillations connected with the singularities of the density of states near the bottom of the Landau bands are determined by the condition $H/H_0=H_{ns}=((\omega_l-\omega_g)/\omega_{LO}-s)/(n+\frac{1}{2})$, $s=0$, 1 or 2, depending on the number of emitted phonons, and n is the number of the Landau bands. The dashed lines show the resonances connected with the additional interference-type contributions to the scattering cross section in a small vicinity $\sim\alpha H_0$ of the discrete set of magnetic-field values determined by the condition $H=H_0/M$, where $M=0.5$, 1, 1.5, 2, 2.5, ...

condition (3.1). At $L = 1.5$ all the interference contributions are negative. For $L = 1$ there are both negative and positive contributions, the latter amounting to approximately 100% and the former to about 60% of the $\sigma_2^{(a)}$ (2.15) contributions. In the case of the Fröhlich interaction (2.2) at $\omega_l - \omega_g = 3\omega_{LO}$ and $L = 1$ the numerical estimates show that the interference contributions lead to a resonant decrease of the scattering intensity, by $\approx 60\%$ of the contribution (2.15). To obtain more complete information on the relative values of the positive and negative interference contributions to (3.3) and concerning their sum as a function of the values of ω_l and L, detailed numerical calculations are necessary, with the true Fröhlich Hamiltonian of the interaction of the electrons and holes with the LO phonons. But it can be seen even from the cited numerical estimates for the simplest case that the interference effects can lead to a substantial decrease of the scattering intensity in a small vicinity $\sim \alpha H_0/L$ of the discrete set of the magnetic-field values determined by condition (3.1). The interference contributions (3.3) are shown in the figure by dashed lines. It is important to emphasize that these additional interference-type contributions to the scattering cross sections are in no way connected with the infinite density of states at the bottom of the Landau band. The condition for the resolution of resonances with different values of L coincides with the strong-field condition. We note also that with changing frequency of the primary radiation ω_l a change takes place also in the position of the oscillation maxima on the plot against the magnetic field, whereas the positions of the negative interference-type resonances (3.3) do not depend on ω_l and are determined by condition (3.1).

[1)]It is assumed for simplicity that in the last real state that precedes the annihilation the exciton has a kinetic energy sufficient for the emission of one LO phonon.

[2)]In the described process, the intensity of the scattered light is linear in the intensity of the exciting radiation. Only processes of this type will be considered hereafter.

[3)]By arbitrary strong magnetic field will be meant here a field subject to conditions no other than (2.1).

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