Localization of neutral excitations and of surface impurities on electric charges in superfluid helium

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(Submitted 28 June 1983)
Zh. Eksp. Teor. Fiz. **86**, 165–168 (January 1984)

The conditions of localization of excitations on charges in He II are calculated. The strong attraction produces around the charge a macroscopic region of radius 50–70 Å with a high concentration of excited atoms. The two-dimensional bound states that result from the interaction between charges localized near the surface of He II and ^{3}He surface impurities are considered.

1. At macroscopic distances, the energy of a bulk impurity excitation in He II in the field of a charge at rest is, in the main approximation, equal to

$$\varepsilon=\frac{\partial\Delta}{\partial N_4}\delta N_4-\frac{\varkappa_i e^2}{2r^4}, \tag{1}$$

where e is the charge of the ion, $\varkappa_i$ is the polarizability of the impurity particle, N_4 is the atomic density of ^{4}He, Δ is the binding energy of the quasiparticle in He II. The change δN_4 of the density in the electrostatic field is determined from the condition of thermodynamic equilibrium

$$\mu_4-\varkappa_4 e^2/2r^4=\text{const}, \tag{2}$$

where μ_4 is the chemical potential of ^{4}He. Obtaining δN_4 from (2) and substituting the result in (1), we get

$$\varepsilon=-\frac{\varkappa_i e^2}{2r^4}\left(1-\alpha\frac{\varkappa_4}{\varkappa_i}\right)\equiv-\frac{\gamma}{r^4},\quad \alpha=\frac{N_4}{m_4 s_0{}^2}\frac{\partial\Delta}{\partial N_4}=\frac{v_i}{v_4}. \tag{3}$$

Here m_4 is the mass of the ^{4}He atom, s_0 is the velocity of first sound in pure ^{4}He, v_i and v_4 are the volumes per impurity particle and ^{4}He atom, respectively. If ^{3}He atoms are the impurity particles, then $\varkappa_3\approx\varkappa_4$ and $v_3=1.28v_4$ so that $\gamma<0$ and the potential (3) corresponds to repulsion everywhere, which excludes the possibility of formation of bound states.[1] The relation $v_3>v_4$ is due to the fact that the amplitude of the zero-point oscillations for the lighter ^{3}He impurities is greater than for the ^{4}He atoms.

The situation changes if we are dealing with neutral ^{4}He excitations in superfluid helium. Actually, for not too highly excited states (when the electron is localized near the nucleus at distances of atomic dimensions and cannot by itself form bubbles) $v_i\sim v_4$ while $\varkappa_i\gg\varkappa_4$, since the mean distance from the electron to the nucleus in the excited state of the atom is greater than for the ground state. Thus, for example, for an atom of ^{4}He in various states, we have[2]:

$$\varkappa_4(^1S)=0.205\ \text{Å}^3;\quad \varkappa_i(2^3S)=46.8\ \text{Å}^3;\quad \varkappa_i(2^1S)=119.0\ \text{Å}^3.$$

Thus, the interaction between the charge and the neutral excitation corresponds in this case to attraction between them, $\gamma>0$. For definiteness, we consider the interaction of an excited ^{4}He atom with a positive charge.

The spectrum of localized states of the neutral quasiparticle ^{4}He is determined by the solution of the Schrödinger equation with the potential

$$U(\mathbf{r})=\begin{cases}-\gamma/r^4, & r>R\\ \infty, & r=R'\end{cases} \tag{4}$$

where R is the radius of the solid sphere formed by the positive charge in helium. One can determine the energy E of the shallow S level in the potential field (4) by analytic means. At large distances, where $|E|\gg|U(r)|$, the wave function falls off rapidly in exponential fashion:

$$\psi(r)=(a/r)e^{-\nu r},\qquad \nu^2=2ME/\hbar^2, \tag{5}$$

where M is the effective mass of the excitation. It is not difficult to establish the fact that at small distances, where $|E|\ll|U(r)|$, the Schrödinger equation has (at $E=0$) a solution that satisfies the boundary condition $\psi(R)=0$ and has the form

$$\psi(r)=A\left[e^{-i\beta/r}-e^{-2i\beta/R}e^{i\beta/r}\right],\qquad \beta^2=2M\gamma/\hbar^2. \tag{6}$$

Matching the values of ψ and ψ' at some r_1, so that $\beta\ll r_1\ll\nu^{-1}$, $r_1>R$, we find the energy of the ground state

$$\nu=\beta^{-1}\operatorname{tg}(\beta/R). \tag{7}$$

Equation (7) agrees with the assumptions that have been made if

$$\beta/R=\pi n+\delta,\quad \delta\ll\pi/4,\quad n=1,\ 2,\ 3,\dots .$$

Here $\nu=\beta^{-1}\delta^2$, while the minimal values of γ at which the first discrete level appears in the potential (4) is equal to $\gamma_{\min}=(\pi\hbar R)^2/2M$.

The existence of an attraction field $U(r)$ leads to a significant increase in the concentration of neutral excitations near the charge. If the lifetime of the excited state is large in comparison with the momentum relaxation time of the quasiparticles then, from the conditions (2) and the relation

$$\mu_i-\varkappa_i e^2/2r^4=\text{const}$$

we obtain, in the Boltzmann region,

$$N_i(r)=N_i(\infty)\exp(\gamma/r^4T)\approx N_i(\infty)(1+\gamma/r^4T). \tag{8}$$

The value $r=r_c$ at which the argument of the exponential in (8) becomes equal to unity turns out to be very large. Thus, at $T=0.5$ K for ^{4}He(2^1S), the value is $r_c=67$ Å. *Therefore*, even at macroscopic distances, $r\lesssim r_c$, the density of the ^{4}He* particles grows rapidly in exponential fashion, the ideal-gas approximation of the quasiparticles becomes inapplicable and the macroscopic structure of the charge is determined

0038-5646/84/010094-02$04.00

by the phase diagram of the concentrated solution ^{4}He*–^{4}He (in principle, such a solution, can, for example, be stratified at distances $r \lesssim r_c$). Since $v_i \approx v_4$, it follows that for any form of the phase diagram of the ^{4}He*–^{4}He mixture a crystalline region is formed around the charge with dimension of the order of 10 Å and with a large concentration of excited atoms. It is obvious that all that has been said about the structure of the charge is also valid for an electron at distances $r_b < r \lesssim r_c$, where r_b is the dimension of the bubble. We note that because of the high concentration of ^{4}He* at $r \lesssim r_c$, recombination and de-excitation of the excited atoms will take place precisely in these macroscopic regions around the charges. If a phase transition takes place within the region $r \lesssim r_c$, then the radius of the droplet of the new phase that is formed can turn out to be much smaller than r_c because of effects connected with the surface tension of the interface.

2. Very interesting bound states arise upon interaction of charges located near the surface of liquid He II[1,3] with ^{3}He surface impurities.[4]

The energy of the localization of the ion near the surface (in the Coulomb field of the image) and the binding energy of the surface impurities considerably exceed the potential of the polarization charge-impurity interaction. In this case we can show that the relative motion of particles parallel to the surface is described by a Schrödinger equation with the interaction Hamiltonian $U_{\text{eff}}(\boldsymbol{\rho})$, which constitutes the energy of interaction of the quasiparticles with the charge, averaged over the motion of the perpendicular surface (along the z axis). The change in the energy of the surface quasiparticle is composed of the polarization term $-\varkappa_i e^2/2r^4$ and terms due to the deformation of the surface in the electric field of the ion; the latter terms we shall neglect in what follows. This neglect is valid if the radius of the two-dimensional bound state (for motion along the surface), calculated in this approximation, turns out to be much smaller than the radius characterizing the deformation of the plane surface (the radius of the dimple). Because of the large value of the capillary constant in liquid He, this condition is practically always satisfied if the charge is separated from the surface by a macroscopic distance. If the ion is in the interior of the liquid and is localized near the surface of the clamping electric field,[1] then the potential $U_{\text{eff}}(\boldsymbol{\rho})$ for motion of the lighter surface impurity has the form

$$U_{eff}(\rho) = -\frac{\varkappa_3 e^2}{2} \int_{-\infty}^{\infty} \frac{\psi_0^*(z)\,\psi_0(z)}{(\rho^2+z^2)^2}\,dz, \tag{9}$$

where $\psi_0(z)$ is the unperturbed wave function of the quasiparticle for motion along the z axis. The potential (9) always has discrete levels. At $\hbar^2/Mz_0^2 \ll \varkappa_3 e^2/z_0^4$, where z_0 is the mean distance from the ion to the surface, there are deep levels which are classified the same as the spectrum of a two-dimensional oscillator

$$E = -\frac{\varkappa_3 e^2}{2}\langle z^{-4}\rangle + \hbar\omega\left(n_\rho + \frac{|l|+1}{2}\right), \qquad \omega^2 = 8\varkappa_3 e^2 M^{-1}\langle z^{-6}\rangle. \tag{10}$$

The region of applicability of (10) turns out to be rather narrow:

$$R^2 \ll z_0{}^2 \ll Me^2\varkappa_3/\hbar^2,$$

where R is the macroscopic dimension of the ion. At $z_0 \approx 30$ Å we have $\hbar\omega \approx 0.06$ K. Thus, the ion and the ^{3}He surface impurity form a two-dimensional bound state and carry out finite relative motion parallel to the surface, even if they move here in different "planes" separated by a macroscopic distance z_0. For motion of the electron over the surface of liquid He II in the attracting field of the surface quasiparticle ^{3}He, the opposite case is realized: $\hbar^2/mz_0^2 \gg |U_{\text{eff}}|$, where m is the mass of the electron, $m/M \ll 1$. The energy of the bound S state turns out then to be exponentially small in $\int_0^\infty U_{\text{eff}}(\boldsymbol{\rho})\rho d\rho$ (Ref. 5) and is equal to

$$|E| \approx \frac{\hbar^2}{m}\langle z^{-2}\rangle \exp\left[-\frac{\hbar^2}{m}\frac{4}{\varkappa_3 e^2\langle z^{-2}\rangle}\right]. \tag{11}$$

The author thanks A. F. Andreev, L. P. Pitaveskiĭ and I. A. Fomin for attentive discussion and valued remarks, and K. O. Keshishev for making him acquainted with the experimental possibilities.

[1] V. V. Shikin, Usp. Fiz. Nauk **121**, 457 (1977) [Sov. Phys. Uspekhi **20**, 226 (1977)].
[2] A. A. Radtsig and B. M. Smirnov, Spravochnik po atomnoĭ i molekulyarnoĭ fizike (Handbook of Atomic and Molecular Physics) Moscow, Atomizdat, 1980.
[3] V. S. Édel'man, Usp. Fiz. Nauk, **130**, 675 (1980) [Sov. Phys. Uspekhi **23**, 227 (1980)].
[4] A. F. Andreev, Zh. Eksp. Teor. Fiz. **50**, 1415 (1966) [Sov. Phys. JETP **23**, 938 (1967)].
[5] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics Moscow, Nauka 1974. (English translation, Pergamon, Oxford, 1977).

Translated by R. T. Beyer