An anisotropic mechanism for the generation of a magnetic field in a collisional plasma in a high-frequency field

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In the framework of magnetostatic instability theories we analyze the problem of the generation of a magnetic field in a collisional plasma in a high-frequency field. Using the solution of the collisional kinetic equation we evaluate the anisotropic part of the electron equilibrium function. We find a kinetic equation for the (perturbed) magnetostatic part of the distribution function which takes into account the equilibrium anisotropy of the particles and the anisotropy of the energy spectrum of the high-frequency waves. We solve this equation in the limiting cases of small and large ratios of the wavelength of the perturbation to the mean free path. In the corresponding limiting cases we observe collisionless and collisional anisotropic instabilities. The collisionless instability is similar to a Weibel-type instability. The collisional instability is caused by the electron viscosity. We study the conditions for the evolution of these instabilities and find their characteristic wave numbers and growth rates.

1. INTRODUCTION

The present paper is devoted to the problem of the generation of a quasi-stationary field in a plasma in a high-frequency (HF) field. The theoretical analysis of this problem was stimulated to begin with by the observation of strong magnetic fields in experiments on the interaction of laser radiation and matter.[1–3] It was assumed in the first theoretical papers on this problem that the physical reason for the field generation in this kind of experiment is the spatial inhomogeneity of the macroscopic parameters of the plasma and the HF field (such as the plasma density or temperature and the HF field pressure).[4–7] In later papers the generation of a magnetic field in an inhomogeneous plasma was considered using a Rayleigh-Taylor type instability caused by the plasma acceleration.[8–10]

According to the general ideas of the theory of plasma instabilities[11] the cause of the instabilities can be either the spatial inhomogeneity of the plasma or its non-equilibrium in velocity space (a non-Maxwellian character of the particle momentum distribution), i.e., a local thermodynamic non-equilibrium. A special case of such a non-equilibrium is anisotropy in momenta leading to different kinds of anisotropic instabilities, such as the Weibel-type magnetostatic instabilities[12] (for a detailed bibliography see Refs. 11, 13) the evolution of which corresponds to magnetic field generation.

In the traditional theory of plasma instabilities[11,13] one considers situations when the plasma is clear of any fields altogether or is in a constant magnetic field. When there is a HF field present the analysis of anistropy effects is important; these arise when this field acts upon the plasma. Such an analysis was carried out in Ref. 14 for the case of a collisionless plasma. We introduced there the concept of the anisotropy of the HF waves (the simplest example of an anisotropic HF field is a single monochromatic wave) and the anisotropy of the particle momentum distribution caused by it. According to Ref. 14 the effects of the anisotropy of the waves and the anisotropy of the particles turn out to compete in the problem of the magnetic field generation and completely cancel one another when we neglect relativistic effects in the plasma. The field generation is therefore observed only when we take relativistic effects into account. A consequence of this is the presence of additional small parameters of the order of v_T/c and $(v_T/c)^3$ in the expressions for the wave number and growth rate of the magnetostatic perturbations which are responsible for the magnetic field generation.

In contrast to Ref. 14, in the present paper we consider a collisional plasma. In that case the anisotropic part of the equilibrium distribution function turns out to be different from the part corresponding to the case of a collisionless plasma. Thanks to this the effects of the anisotropy of the waves and of the particles do not cancel one another even when relativistic effects are neglected. Correspondingly, the wave numbers and growth rates of the magnetostatic perturbations in a collisional plasma turn out to be appreciably larger than when there are no collisions.

Taking interparticle collisions into account we assume that the characteristic time of the existence of a quasi-stationary state of the plasma is larger than the characteristic time of the electron collisions. Owing to electron-ion collisions, the energy of the HF waves is dissipated. Therefore, if we assume the HF field to be quasi-stationary we must assume the presence of a continuously acting source for the HF field. The problem of a quasi-stationary state of a collisional plasma with an anisotropic source of HF waves has been considered before in the theory of beam-plasma type plasmochemical reactors.[15] In that case the beam instability plays the role of the HF wave source. The main attention in Ref. 15 was paid to the calculation of the isotropic part of the equilibrium electron distribution function. For our purposes, however, it is necessary to know the anisotropic part of this function. We evaluate it in Sec. 2.

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We carry out the analysis of instabilities leading to magnetic field generation on the basis of a kinetic equation for the magnetostatic part of the electron distribution function. We give a derivation of that equation in Sec. 3.

The magnetic field generation process in a collisional plasma looks different depending on the ratio of the particle mean free path l and the characteristic wavelength λ of the magnetostatic perturbations. When $l > \lambda$ the effect of the collisions on the perturbation is unimportant so that the corresponding instability looks like the collisionless one. Such an instability belongs to the number of so-called "collisionless instabilities of a collisional equilibrium." (Among this type we have the loss-cone instability; see §12.3 of Ref. 11.) It can be studied using the kinetic approach given in Ref. 14. We consider this instability in Sec. 4. To study perturbations with $\lambda > l$ which correspond to a collisional anisotropic instability we must change from a kinetic to a hydrodynamic description. The analysis of such perturbations is carried out in Section 5. We discuss the results of this paper in Sec. 6.

We note also some important aspects of the analysis given in Secs. 4 and 5. We obtain the basic results of the theory of the collisionless anisotropic instability in the framework of a kinetic-electrodynamic approach (Sec. 4.1). In such an approach the plasma is described by means of a perturbed distribution function and the only macroscopic quantity which characterizes the perturbation of the plasma is the electric current density expressed in terms of the perturbed electric field. In the majority of theoretical papers on magnetic field generation in a laser plasma, amongst them the ones mentioned above (see, e.g., Ref. 5) and also in Sec. 5 of the present paper, one uses a different approach—the magnetohydrodynamic (MHD) one which does not operate with the distribution function but with its moments which are expressed in terms of the perturbed magnetic field. In this connection it is interesting to give a magnetohydrodynamic treatment of the collisionless anisotropic instability. In subsection 4.2 we derive the simplest set of MHD equations for a plasma in a HF field and show that the basic hydrodynamic equation in the case of a weakly collisional plasma is not the electron equation of motion (see Sec. 5) but the stress tensor transfer equation. Such an equation is well known in the theory of instabilities of a high-pressure plasma in a static magnetic field.[16] In subsection 4.2 we elucidate how it is modified when a HF field is present and show how one can use this equation to obtain the results of subsection 4.1.

We neglect in the present paper the spatial inhomogeneity of the plasma and the HF field. The problem of the instabilities then reduces to an analysis of the dispersion relation for the spatial Fourier components of the perturbations. When there is a spatial inhomogeneity such an approach is inapplicable. In this connection it becomes necessary to develop alternative methods to study the collisionless anisotropic instability. We show in subsection 4.3 that one can construct a quadratic form which has the meaning of the potential energy of the perturbations. From this it follows that the analysis of the collisionless anisotropic instability can, when the plasma and the HF field are inhomogeneous, be performed using the energy method (by determining the sign of the potential energy) similar to how this is done in the theory of the MHD instabilities of a plasma contained in a curved magnetic field[17] and in the tearing-mode instability theory.[18]

The transfer equation for the stress tensor is important also in the case of a strongly collisional plasma when the mean free path is small compared to the wavelength of the perturbations (Sec. 5). In that case the electron equation of motion also becomes important in which the electron viscosity must be taken into account. In the original papers on the magnetic field generation theory in a laser plasma the electron viscosity was as a rule neglected (see, for instance, Ref. 9). In that case one assumes, in fact, that the viscosity is caused by the spatial derivatives of the macroscopic electron velocity.[19] Such an approach is valid when there is no HF field in the plasma. When there is a HF field present the electron viscosity contains extra terms which are not connected with the macroscopic electron velocity. We evaluate these terms and show that a new mechanism which we call the viscosity mechanism for magnetic field generation is connected with them.[20]

We note also that the rule of the electron viscosity in the magnetic field generation problem has recently also been discussed in the papers of some other authors.[21–24] However, the generation mechanism studied by us was not recognized in those papers.

2. QUASI-STATIONARY STATE

We assume that electromagnetic waves or Langmuir oscillations play the role of the HF waves. For the sake of simplicity we assume the electric field $\tilde{\mathbf{E}}$ of the HF waves to be directed along the z axis: $\tilde{\mathbf{E}} = (0,0,\tilde{E})$.

We write the electron distribution function in the HF field f in the form

$$f = F(1+\Phi) + \tilde{f}, \tag{2.1}$$

where F is a Maxwellian function corresponding to a density n and temperature T; $\tilde{f}$ is the oscillating part of the distribution function which is linear in the HF field; Φ is a small quantity which characterizes the deviation of the (averaged over the HF waves) equilibrium distribution function from the Maxwellian. It follows from Boltzmann's kinetic equation that the functions $\tilde{f}$ and Φ are given by the relations

$$\frac{\partial \tilde{f}}{\partial t} + \frac{e}{m}\tilde{E}\frac{\partial F}{\partial v_z} = C_{ei}(\tilde{f}), \tag{2.2}$$

$$\frac{e}{m}\left\langle \tilde{E}\frac{\partial \tilde{f}}{\partial v_z}\right\rangle = \langle C_{ee}(\tilde{f},\tilde{f})\rangle + I_{ee}(\Phi) - I_{ei}(\Phi). \tag{2.3}$$

Here e, m are the electron charge and mass, $\mathbf{v}$ the particle velocity, C_{ei}, I_{ei}, C_{ee}, I_{ee} the electron-ion and electron-electron collision operators, defined following Braginskiĭ,[19] and the angle brackets indicate averaging over the HF waves.

We solve Eq. (2.2) by expanding in a series in the collision frequency. We then find

$$\tilde{f} = \tilde{f}^{(0)} + \tilde{f}^{(1)}, \tag{2.4}$$

$$\tilde{f}^{(0)} = 2\tilde{v}v_z F/v_T^2, \tag{2.5}$$

$$f^{(1)}=2C_{ei}(v_zF)\int_{-\infty}\tilde{v}(t')\frac{1}{v_T^2}dt', \tag{2.6}$$

where $v_T = (2T/m)^{1/2}$ and $\tilde{v}$ the oscillating particle velocity determined by the relation

$$\partial\tilde{v}/\partial t=e\tilde{E}/m. \tag{2.7}$$

Using (2.4)–(2.7) we get

$$\frac{e}{m}\left\langle \tilde{E}\frac{\partial\tilde{f}}{\partial v_z}\right\rangle=-2w\frac{\partial}{\partial v_z}C_{ei}(v_zF), \tag{2.8}$$

where $w\equiv\langle\tilde{v}^2\rangle/v_T^2$.

We write Φ in the form

$$\Phi=\Phi^{ee}+\Phi^{ei}, \tag{2.9}$$

$$\Phi^{ee}=2w(v_z^2-{}^1/_3v^2)/v_T^2. \tag{2.10}$$

The function Φ^{ee} satisfies the equation

$$I_{ee}(\Phi^{ee})+\langle C_{ee}(f^{(0)},\ f^{(0)})\rangle=0. \tag{2.11}$$

Using (2.8), (2.11) we write (2.3) in the form

$$I_{ee}(\Phi^{ei})+I_{ei}(\Phi^{ei})=-2w\left\{\frac{F}{v_T^2}C_{ei}(v_z^2)+\frac{\partial}{\partial v_z}[FC_{ei}(v_z)]\right\}. \tag{2.12}$$

Following Ref. 19 we solve (2.12) in the two-Sonine-Laguerre polynomials approximation. As a result we get

$$\Phi^{ei}=\alpha w(v_z^2-{}^1/_3v^2)/v_T^2, \tag{2.13}$$

where

$$\alpha=0.19+0.50L_1^{5/2}(x). \tag{2.14}$$

Here $L_1^{5/2}$ is the appropriate polynomial of the argument $x = v^2/v_T^2$.

Equations (2.9), (2.10), (2.13), (2.14) indicate the presence of anisotropy in the electron equilibrium distribution function. The function Φ^{ee} corresponds to the "collisionless" part of the anisotropy. It will become clear from the following analysis that the magnetic field generation process, an effect caused by Φ^{ee}, is cancelled by the effect of the anisotropy of the waves (see also section 1). Therefore only that part of the anisotropy described by the function Φ^{ei} is responsible for the magnetic field generation.

3. KINETIC EQUATION FOR THE MAGNETOSTATIC PART OF THE DISTRIBUTION FUNCTION

When there are magnetostatic perturbations present, Eq. (2.1) for the electron distribution function is modified as follows:

$$f=F(1+\Phi)+\tilde{f}+f^m+f^{hm}. \tag{3.1}$$

Here f^m, f^{hm} are, respectively, the low- and high-frequency parts of the distribution function which satisfy the following kinetic equations which follow from the Boltzmann equation:

$$\frac{\partial f^m}{\partial t}+(\mathbf{v}\nabla)f^m-\frac{e}{m}\mathbf{E}^m\frac{\partial F}{\partial\mathbf{v}}-\frac{eF}{mc}[\mathbf{vB}^m]\frac{\partial\Phi}{\partial\mathbf{v}}=S_{QL}{}^m+S_c{}^m, \tag{3.2}$$

$$\frac{\partial f^{hm}}{\partial t}-\frac{e}{mc}[\mathbf{vB}^m]\frac{\partial\tilde{f}}{\partial\mathbf{v}}=0. \tag{3.3}$$

Here $\mathbf{E}^m$, $\mathbf{B}^m$ are the electric and magnetic fields of the magneto-static perturbations, S_C^m the magnetostatic part of the Coulomb collision term, and S_{QL}^m the analogous part of the quasi-linear collision term defined by the equation

$$S_{QL}{}^m=\frac{e}{m}\left\langle \tilde{E}\frac{\partial f^{hm}}{\partial v_z}\right\rangle. \tag{3.4}$$

Using (2.5) and putting $\mathbf{B}^m = (0,B_y^m,0)$ we find from (3.3)

$$f^{hm}=\frac{eF}{cT}v_xB_y{}^m\int_{-\infty}^{t}\tilde{v}(t')\,dt'. \tag{3.5}$$

It then follows from (3.4) that

$$S_{QL}{}^m=\frac{emv_xv_z}{cT^2}F\langle\tilde{v}^2\rangle B_y{}^m. \tag{3.6}$$

Substituting (3.6) into (3.2) and expressing Φ in the form (2.9) we note that the contributions S_{QL}^m and Φ^{ee} cancel one another in (3.2), i.e.,

$$S_{QL}{}^m=-\frac{eF}{mc}[\mathbf{vB}^m]\frac{\partial\Phi^{ee}}{\partial\mathbf{v}}. \tag{3.7}$$

Therefore (3.2) becomes

$$\frac{\partial f^m}{\partial t}+(\mathbf{v}\nabla)f^m-\frac{e}{m}\mathbf{E}^m\frac{\partial F}{\partial\mathbf{v}}-\frac{eF}{mc}[\mathbf{vB}^m]\frac{\partial\Phi^{ei}}{\partial\mathbf{v}}=S_c{}^m. \tag{3.8}$$

Equation (3.8) is the main starting equation for our analysis of the magneto-static instabilities.

4. COLLISIONLESS ANISOTROPIC INSTABILITY

4.1. Kinetic-electrodynamic approach

We neglect in (3.8) the Coulomb collisions. We write the space-time dependence of the perturbations in the form $\exp(-i\omega t + i\mathbf{k\cdot r})$. We put $\mathbf{k} = (k,0,0)$. Then $\mathbf{E}^m = (0,0,E_z^m)$, $B_y^m = -ckE_z^m/\omega$. It then follows from (3.8) that

$$f^m=-\frac{ieFv_zE_z{}^m}{(\omega-kv_T)T}\left(1+\alpha w\frac{kv_x}{\omega}\right). \tag{4.1}$$

Using (4.1) and assuming $\omega\ll kv_T$ we find the electric current density of the perturbations

$$j_z{}^m=-e\int v_zf^m\,dv=\frac{\omega_p{}^2}{4\pi\omega}E_z{}^m\left[\pi^{1/2}\frac{\omega}{|k|v_T}-0.69iw\right], \tag{4.2}$$

where $\omega_p^2 = 4\pi e^2n/m$ is the square of the plasma frequency and n the plasma density. Substituting (4.2) into the Maxwell equation

$$\frac{\partial B_y}{\partial x}=\frac{4\pi}{c}j_z{}^m, \tag{4.3}$$

we get the dispersion relation (cf. Ref. 14)

$$\varepsilon_0+\varepsilon_1-c^2k^2/\omega^2=0, \tag{4.4}$$

where

$$\varepsilon_0=i\pi^{1/2}\omega_p{}^2/|k|\omega v_T, \tag{4.5}$$

$$\varepsilon_1=0.69w\omega_p^2/\omega^2. \tag{4.6}$$

It follows from (4.4)–(4.6) that

$$\omega=i\frac{|k|v_T}{\pi^{1/2}}\left(0.69w-\frac{c^2k^2}{\omega^2}\right). \tag{4.7}$$

Hence we find that the perturbations grow with time provided

$$k<k_*, \tag{4.8}$$

where k_* is determined by the relation

$$k_*^2=0.69w\omega_p^2/c^2. \tag{4.9}$$

The growth rate $\gamma = \mathrm{Im}\,\omega$ has a maximum of the order of

$$\gamma_{max}\approx\omega_p(v_T/c)w^{3/2}, \tag{4.10}$$

which is reached when $k = k_*/3^{1/2}$.

The condition for the applicability of the assumption of a collisionless nature of the perturbations means that

$$k>\nu_e/v_T, \tag{4.11}$$

where ν_e is the electron collision frequency. It follows from (4.8), (4.11) that the instability considered here can occur provided that

$$\tilde{v}>c\nu_e/\omega_p. \tag{4.12}$$

4.2. Hydrodynamic approach

For the sake of simplicity we assume that the system is at the limit of stability. Then $\partial f^m/\partial t = 0$, $\mathbf{E}^m = 0$ so that in the case considered of weak collisions Eq. (3.8) reduces to the form

$$v_x\frac{\partial f^m}{\partial x}=\frac{eF}{mc}[\mathbf{v}\mathbf{B}^m]\frac{\partial\Phi^{ei}}{\partial\mathbf{v}}. \tag{4.13}$$

Using (4.13) to construct the standard hydrodynamic continuity, motion, and heat balance equations we verify that the first and third equations do not give positive information while the equation of motion reduces to the condition that the viscosity tensor vanishes. However, we get nontrivial information from the equation for the z,x component of the stress tensor which we obtain by multiplying (4.13) by mv_zv_x and subsequently integrating over the velocities:

$$\frac{\partial V_z^m}{\partial x}=-0.69\frac{eB_y^m}{mc}w, \tag{4.14}$$

where V_z^m is the macroscopic electron velocity. Using the fact that

$$V_z^m=-j_z^m/en, \tag{4.15}$$

and using Eq. (4.3) we bring (4.14) to the form

$$\frac{\partial^2B_y^m}{\partial x^2}=-0.69\frac{\omega_p^2}{c^2}B_y^mw. \tag{4.16}$$

When $B_y^m \propto \exp(ikx)$ Eq. (4.16) means that $k^2 = k_*^2$, where k_*^2 is given by Eq. (4.9). It is clear from what has been said that the physical process considered here is identical with that discussed in subsection 4.1.

4.3. Energy method

To supplement subsection 4.2 we evaluate the linear dissipation of the perturbations caused by resonance particles. We then arrive at a Maxwell equation of the form

$$-\frac{\partial^2A_z^m}{\partial x^2}=\frac{4\pi}{c}(j_z^{m(1)}+\hat{\sigma}_{zz}^{(0)}E_z^m). \tag{4.17}$$

Here A_z^m is the vector potential of the perturbations, $J_z^{m(1)}$ that part of the current density which corresponds to the electron velocity (4.14), i.e.,

$$j_z^{m(1)}=0.69\frac{\omega_p^2}{4\pi c}wA_z^m, \tag{4.18}$$

and $\sigma_{zz}^{(0)}$ is the linear conductivity of the plasma in a coordinate-time representation which in the standard way is connected with the quantity ε_0 given by Eq. (4.5) (see, e.g., Ref. 11). Multiplying (4.17) by A_z^m and integrating over space we get

$$\frac{2\pi}{c^2}\frac{\partial}{\partial t}\int A_z^m\hat{\sigma}_{zz}^{(0)}A_z^m\,dx=-W, \tag{4.19}$$

where

$$W=\int\left[\left(\frac{\partial A_z^m}{\partial x}\right)^2-\frac{4\pi}{c}j_z^{m(1)}A_z^m\right]dx. \tag{4.20}$$

We can consider the quantity W as the potential energy of the perturbations of the system. It is positive when there is no HF field. The sign of the integral on the left-hand side of (4.19) is also positive. The perturbations considered are therefore damped when there is no HF field. When there is a HF field present the damping is replaced by a growth, if $W<0$, i.e., if

$$\left(\frac{\partial A_z^m}{\partial x}\right)^2-\frac{4\pi}{c}j_z^{m(1)}A_z^m<0. \tag{4.21}$$

Putting

$$A_z^m=A_{z0}\exp(ikx)+\mathrm{c.c.} \tag{4.22}$$

and substituting (4.18) into (4.21) we find that the condition for instability has the form $k < k_*$ in accordance with subsection 4.1.

5. COLLISIONAL ANISOTROPIC INSTABILITY

We now assume the Coulomb collision term in (3.8) to be large. We consider a perturbation with $\mathbf{k} = (0,0,k)$, $\mathbf{E}^m = (E_x^m,0,0)$. In that case the Maxwell equations in the coordinate-time representation have the form

$$\frac{\partial B_y^m}{\partial t}=-c\frac{\partial E_x^m}{\partial z}, \tag{5.1}$$

$$-\frac{\partial B_y^m}{\partial z}=\frac{4\pi}{c}j_x^m. \tag{5.2}$$

We add to (5.1) (5.2) the electron equation of motion[19]

$$enE_x^m+\partial\pi_{xz}^m/\partial z+0.51mn\nu_e V_x^m=0, \tag{5.3}$$

where π_{xz}^m is the magnetostatic part of the xz components of the electron viscosity tensor, V_x^m the average electron motion velocity in the magnetostatic perturbations, and $\nu_e = 1/\tau_e$, where τ_e is given in Ref. 19.

As we assume the perturbations to be purely electronic we have

$$V_x^m=-j_x^m/en. \tag{5.4}$$

Using (5.2) we then have

$$V_x^m=\frac{c}{4\pi en}\frac{\partial B_y^m}{\partial z}. \tag{5.5}$$

Substituting (5.5) into (5.3) we get

$$E_x^m=-\frac{c}{4\pi\sigma}\frac{\partial B_y^m}{\partial z}-\frac{1}{en}\frac{\partial\pi_{xz}^m}{\partial z}, \tag{5.6}$$

where $\sigma = e^2n/0.51m\nu_e$ is the conductivity. It follows from (5.1), (5.6) that

$$\frac{\partial B_y^m}{\partial t}=\frac{c^2}{4\pi\sigma}\frac{\partial^2 B_y^m}{\partial z^2}+\frac{c}{en}\frac{\partial^2\pi_{xz}^m}{\partial z^2}. \tag{5.7}$$

We use Eq. (3.8) which, neglecting unimportant terms, we write in the form

$$S_C^m=-\frac{eF}{cT}w\alpha v_x v_z B_y^m \tag{5.8}$$

to evaluate the quantity π_{xz}^m. We put the function f^m which occurs in S_C^m equal to

$$f^m=F\Phi^m. \tag{5.9}$$

By analogy with Sec. 2 we evaluate Φ^m in the two-Sonine-Laguerre polynomials approximation. The required tensor component π_{xz}^m is connected with ϕ^m through

$$\pi_{xz}^m=m\int v_x v_z F\Phi^m\,dv. \tag{5.10}$$

As a result we find

$$\pi_{xz}^m=-0{,}11neB_y^m\langle\tilde{v}^2\rangle/c\nu_e. \tag{5.11}$$

Using (5.11) we get from (5.7)

$$\frac{\partial B_y^m}{\partial t}=\left(\frac{c^2}{4\pi\sigma}-0.11\frac{\langle\tilde{v}^2\rangle}{\nu_e}\right)\frac{\partial^2 B_y^m}{\partial z^2}. \tag{5.12}$$

Hence it follows that instability occurs, provided that [cf. (4.12)]

$$\langle\tilde{v}^2\rangle>4.7c^2\nu_e^2/\omega_p^2. \tag{5.13}$$

Then

$$\gamma\approx 0.1k^2\langle\tilde{v}^2\rangle/\nu_e. \tag{5.14}$$

Hence it is clear that the growth rate increases with increasing wave number k. However, we assume that $k\lesssim\nu_e/v_T$, for otherwise the collisional approximation assumed in the present section is violated. Putting $k\approx\nu_e/v_T$ we find that the maximum growth rate given by Eq. (5.14) is, as to order of magnitude, equal to

$$\gamma\approx\nu_e\langle\tilde{v}^2\rangle/v_T^2. \tag{5.15}$$

6. DISCUSSION OF THE RESULTS

The analysis given here indicates a new type of instability for a plasma in a HF field. The cause of these instabilities is the anisotropy of the plasma caused by the HF field. A result of the instability is a growth of magnetostatic perturbations which corresponds to the generation of a quasi-stationary magnetic field.

The instabilities considered are realized provided the amplitude of the HF field is not too small so that inequalities (4.12) or (5.13) are satisfied. If these inequalities are strong the wave number dependence of the growth rate has the shape schematically shown in the figure. It is clear from the figure that the maximum growth rate occurs for "collisionless" perturbations with $k\approx k_*$ where k_* is characterized by Eq. (4.9). The maximum growth rate is given by Eq. (4.10).

One can obtain an estimate of the magnitude of the generated magnetic field when the instabilities considered develop by taking into account their similarity to Weibel type instabilities[12] and using the estimates of Ref. 25. According to Ref. 25 the magnitude of the generated magnetic field when a Weibel type instability develops is of the order of $B\approx(4\pi n\Delta T)^{1/2}$ where ΔT is the temperature anisotropy. In our case the role of the temperature anisotropy is played by a quantity of the order of $m\tilde{v}^2$. We then get an estimate $B\approx\tilde{E}$.

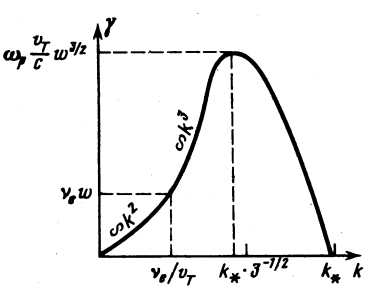

FIG. 1. Schematic wave number dependence of the growth rates of anisotropic instabilities. The region $k>\nu_e/v_T$ corresponds to the collisionless and the region $k<\nu_e/v_T$ to the collisional instability.

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